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Four-phase intersection traffic control based on mixed logical dynamical modeling and predictive control approach

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Abstract—The urban population is increasing continuously, thus the importance of urban traffic control has greatly increased. One of the major issues in urban traffic control is intersections traffic control. At the intersection traffic control systems, continuous variables such as flow, queue length, average speed and discrete variables such as intersection light switching are exist. The MLD model can consider relationship between discrete variables and continuous variables. It provides a linear model from variables that is good to optimize in MPC framework. In this paper, MLD model as a class of hybrid systems used to model intersection traffic control. Then predictive control approach based on MIQP optimization is used to verify the rationality of MLD model. The results of MATLAB simulation show that MLD model with predictive control method can improve intersection traffic control.

Keyword: MLD, urban traffic control, predictive control, intersection.

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I. INTRODUCTION

By increasing trend of urbanization, increasing use of private cars and the lack of transport infrastructure in urban areas, the time required for interurban trips increased. So people spend a lot of time in crowded streets. Furthermore the queue length of cars waiting to move in the streets increased which will increase fuel consumption. All of these factors has led to urban air pollution, noise pollution and waste time along the streets.

To solve this problem, in general, two approaches can be considered:

- Developing urban transport infrastructure.
- Improving urban transport infrastructure.

In the first approach, development is critical but it should be noted that in heavily populated urban areas such as city centers, this approach is very expensive and difficult to implement. So the best approach to solve this problem is the effective use of the existing infrastructure, by improving the use of infrastructure (improved the way of using urban roads) to improve the quality of traffic flow.

Urban traffic system is a kind of hybrid systems. At the intersection traffic control system, continuous variables such as flow, queue length, average speed and discrete variables such as intersection light switching are exist. Traditional dynamic models such as vehicle-following model, traffic wave model, lane changing model are based on analysis of traffic flow characteristics.

Methods of hybrid systems modeling divided into two groups, namely extended discrete event dynamic system modeling approach and extended continuous variable dynamic system modeling approach. Most researchers use petri nets or hybrid automata to control the intersection traffic signal [1-6]. It is based on extended discrete event dynamic system modeling method. This method needs to accurate design and analysis of the intersection phase switching. So in this method it is difficult for controller to consider relationship between discrete variables and continuous variables very well. In this paper, extended continuous variable dynamic system modeling approach is considered. The hybrid systems modeling based on this approach include MLD model. MLD model can consider relationship between discrete variables and continuous variables very well [7]. Predictive control method can stable MLD systems on reference paths [8].

This paper is organized as follows: in section II the mixed logical dynamical model is introduced and represented for intersection traffic. The simulation results in order to show the effectiveness of proposed method is presented in section III and the conclusion is presented in section IV.

II. INTERSECTION SIGNAL CONTROL MODELING

In this paper, MLD systems used for intersection signal control modeling.

A. Mixed logical dynamical model(MLD)

MLD systems are one of the hybrid system's classes that are combination of logical components, dynamic and constraints. MLD model can be expressed as the following linear relationship [8]:

$$\begin{cases} x(k+1) = \mathrm{A}x(k) + B_1 u(k) + B_2 \delta(k) + B_3 \mathrm{z(k)} \\ y(k) = Cx(k) + D_1 u(k) + D_2 \delta(k) + D_3 z(k) \\ E_1 x(k) + E_2 u(k) + E_3 \delta(k) + E_4 z(k) \le E_5 \end{cases} \tag{1}$$

Wherein, $\mathrm{A}, \mathrm{B}_1, \mathrm{B}_2, \mathrm{B}_3, \mathrm{C}, \mathrm{D}_1, \mathrm{D}_2, \mathrm{D}_3, \mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \mathrm{E}_4$ and E_5 are matrices with appropriate dimensions, x is the state variables vector, y is the output variables vector, u is the control variables vector, δ is the auxiliary logic variables vector, z is the auxiliary continuous variables vector. In this relationship, first equation is the state equation, second equation is the output equation and third equation is the inequality constraints which contains both the system state and the input output constraints. x, y and u has continuous parts and logic parts that shown in equation (2) [8]:

$$\begin{aligned} x &= \begin{bmatrix} x_c \\ x_l \end{bmatrix}, x_c \in \mathbb{R}^{n_c}, x_l \in \{0,1\}^{n_l}, n = n_c + n_l \\ y &= \begin{bmatrix} y_c \\ y_l \end{bmatrix}, y_c \in \mathbb{R}^{p_c}, y_l \in \{0,1\}^{p_l}, p = p_c + p_l \\ u &= \begin{bmatrix} u_c \\ u_l \end{bmatrix}, u_c \in \mathbb{R}^{m_c}, u_l \in \{0,1\}^{m_l}, m = m_c + m_l \end{aligned} \tag{2}$$

In MLD systems, state variables, inputs and outputs can be quite continuous or quite logical [8].

B. Mixed logical dynamical modeling of intersection traffic

Queue length model for one-lane road shown in equation (3) [9].

$$\mathrm{L_d}(\mathrm{t}) = \frac{\mathrm{N_0} + \mathrm{N_u}(\mathrm{t}) - \mathrm{N_d}(\mathrm{t}) - \mathrm{K_m L}}{\mathrm{K_j} - \mathrm{K_m}} \tag{3}$$

Wherein, $\mathrm{N_0}$ is the number of vehicles in the road at initial moment of time t=0, L is the length of road, $\mathrm{N_u(t)}$ is the number of vehicles accumulated in input of the road at time t, $\mathrm{N_d(t)}$ is the number of vehicles accumulated in output of the road at time t, $\mathrm{K_j}$ is the average of road blocking density, $\mathrm{K_m}$ is the average of road optimum density, $\mathrm{L_d(t)}$ is the average queue length in the road at time t. Queue length model for M-lane road shown in equation (4) [9].

$$\mathrm{L_d}(\mathrm{t}) = \frac{\mathrm{N_0} + \sum_{\mathrm{i=1}}^{\mathrm{M}} \mathrm{N_u}(\mathrm{i,t}) - \sum_{\mathrm{i=1}}^{\mathrm{M}} \mathrm{N_d}(\mathrm{i,t}) - \mathrm{MK_m L}}{\mathrm{M}(\mathrm{K_j} - \mathrm{K_m})} \tag{4}$$

Wherein, $\mathrm{N_0}$ is the number of vehicles in the road at initial moment of time t=0, L is the length of road, $\mathrm{N_u(i,t)}$ is the number of vehicles accumulated in input of the i-th road lane at time t, $\mathrm{N_d(i,t)}$ is the number of vehicles accumulated in output of the i-th road lane at time t, $\mathrm{K_j}$ is the average of a lane blocking density, $\mathrm{K_m}$ is the average of a lane Optimum density, M is the number of road lanes, $\mathrm{L_d(t)}$ is the average queue length in each lane of the road at time t. mixed logical dynamical model of the average queue length in each lane of road shown in equation (5) [10].

$$\mathrm{L_d}(\mathrm{k+1}) = \mathrm{L_d}(\mathrm{k}) + \frac{\mathrm{T_s}}{\mathrm{M}(\mathrm{K_j} - \mathrm{K_m})}(\mathrm{q_{in}} - \mathrm{M\,m\,v}) \tag{5}$$

Wherein, m is the export discharge saturation flow rate at each lane of road, $\mathrm{T_s}$ is the sample rate, $\mathrm{q_{in}}$ is a road entrance flow rate in terms of the number of cars at hour and v is a logic variable (v=0 stands for red light, v=1 stands for green light) [10].

Fig. 1 shows the signal control structure model of intersection with four phases. This intersection has four lights, namely $\mathrm{v_1}$,$\mathrm{v_2}$, $\mathrm{v_3}$ and $\mathrm{v_4}$ that each of them installed on roads leading to the intersection.

In four phases mode, each of these lights are both for direct movement and the left-turn movement. It means light $\mathrm{v_2}$ is both for move directly from the intersection B to D and the same for the left-turn movement from intersection B to F. This intersection has the following four phases:

- light $\mathrm{v_1}$ is green and other lights are red
- light $\mathrm{v_2}$ is green and other lights are red
- light $\mathrm{v_3}$ is green and other lights are red
- light $\mathrm{v_4}$ is green and other lights are red

The average queue length model for the roads leading to intersection G shown in equation (6).

$$\begin{cases} \mathrm{L_1}(\mathrm{k+1}) = \mathrm{L_1}(\mathrm{k}) + \frac{\mathrm{T_s}}{\mathrm{M}_{f,\mathrm{g}}(\mathrm{K_{jf,g}} - \mathrm{K_{mf,g}})}(\mathrm{q}_{\mathrm{in},f}(\mathrm{k}) \\ \qquad - \mathrm{M}_{f,\mathrm{g}} \times \mathrm{m}_{f,\mathrm{g}} \times \mathrm{v_1(k)}) \\ \mathrm{L_2}(\mathrm{k+1}) = \mathrm{L_2}(\mathrm{k}) + \frac{\mathrm{T_s}}{\mathrm{M_{b,g}}(\mathrm{K_{jb,g}} - \mathrm{K_{mb,g}})}(\mathrm{q_{in,b}(k)} \\ \qquad - \mathrm{M_{b,g}} \times \mathrm{m_{b,g}} \times \mathrm{v_2(k)}) \\ \mathrm{L_3}(\mathrm{k+1}) = \mathrm{L_3}(\mathrm{k}) + \frac{\mathrm{T_s}}{\mathrm{M_{d,g}}(\mathrm{K_{jd,g}} - \mathrm{K_{md,g}})}(\mathrm{q_{in,d}(k)} \\ \qquad - \mathrm{M_{d,g}} \times \mathrm{m_{d,g}} \times \mathrm{v_3(k)}) \\ \mathrm{L_4}(\mathrm{k+1}) = \mathrm{L_4}(\mathrm{k}) + \frac{\mathrm{T_s}}{\mathrm{M_{k,g}}(\mathrm{K_{jk,g}} - \mathrm{K_{mk,g}})}(\mathrm{q_{in,k}(k)} \\ \qquad - \mathrm{M_{k,g}} \times \mathrm{m_{k,g}} \times \mathrm{v_4(k)}) \end{cases} \tag{6}$$

Wherein, $\mathrm{q_{in,f}}$, $\mathrm{q_{in,b}}$, $\mathrm{q_{in,d}}$ and $\mathrm{q_{in,k}}$ respectively are input flow rates from intersections F, B, D and K toward intersection G. $\mathrm{m_{f,g}}$, $\mathrm{m_{b,g}}$, $\mathrm{m_{d,g}}$ and $\mathrm{m_{k,g}}$ respectively are the export discharge saturation flow rate at each lane of roads FG, BG, DG and KG. $\mathrm{K_{jf,g}}$, $\mathrm{K_{jb,g}}$, $\mathrm{K_{jd,g}}$ and $\mathrm{K_{jk,g}}$ respectively are the average of a lane blocking density in roads FG, BG, DG and KG and also $\mathrm{K_m}$ is the average of a lane optimum density. $\mathrm{M_{f,g}}$, $\mathrm{M_{b,g}}$, $\mathrm{M_{d,g}}$ and $\mathrm{M_{k,g}}$ respectively are the number of lanes for roads FG, BG, DG and KG.

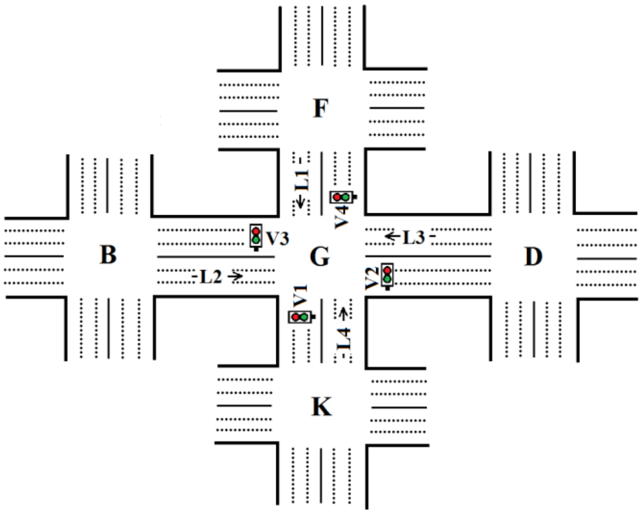

Figure 1. Intersection with four phases.

Equations (6) can be expressed as the following matrix form:

$$x(k+1) = Ax(k) + Bu(k) + E \tag{7}$$

Wherein, x is the state vector and u is the control signal.

$$x = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix}, u = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{8}$$

Matrices A, B and E are obtained as

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$B = \begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & b_4 \end{bmatrix} \tag{10}$$

$$E = \begin{bmatrix} \dfrac{T_s \times q_{in,f}(k)}{M_{f,g}\left(K_{jf,g} - K_{mf,g}\right)} \\ \dfrac{T_s \times q_{in,b}(k)}{M_{b,g}\left(K_{jb,g} - K_{mb,g}\right)} \\ \dfrac{T_s \times q_{in,d}(k)}{M_{d,g}\left(K_{jd,g} - K_{md,g}\right)} \\ \dfrac{T_s \times q_{in,k}(k)}{M_{k,g}\left(K_{jk,g} - K_{mk,g}\right)} \end{bmatrix} \tag{11}$$

Wherein, b_1, b_2, b_3 and b_4 are as following:

$$b_1 = \frac{-T_s \times M_{f,g} \times m_{f,g}}{M_{f,g}\left(K_{jf,g} - K_{mf,g}\right)} \tag{12}$$

$$b_2 = \frac{-T_s \times M_{b,g} \times m_{b,g}}{M_{b,g}\left(K_{jb,g} - K_{mb,g}\right)} \tag{13}$$

$$b_3 = \frac{-T_s \times M_{d,g} \times m_{d,g}}{M_{d,g}\left(K_{jd,g} - K_{md,g}\right)} \tag{14}$$

$$b_4 = \frac{-T_s \times M_{k,g} \times m_{k,g}}{M_{k,g}\left(K_{jk,g} - K_{mk,g}\right)} \tag{15}$$

For intersection with four phases based on MLD modeling two modes for MPC controller can be considered. First, optimization problem in predictive control isn't constrained to observe a specific phase sequence and second, the controller is constrained to observe a specific phase sequence to green the lights for example in each cycle, the lights should be green with phase sequence v_1,v_2, v_3 and v_4. In the first mode, the controller determine phase sequence and light's green time but in second mode just determine the green time of lights. In both mentioned mode at any time just one light most be green, so following constraints must be considered:

$$v_1(k) + v_2(k) + v_3(k) + v_4(k) = 1 \tag{16}$$

In MLD modeling the constraint should be expressed as linear inequalities. So equation (16) reformulated to linear inequalities (17).

$$\begin{cases} v_1(k) + v_2(k) + v_3(k) + v_4(k) \le 1 \\ -v_1(k) - v_2(k) - v_3(k) - v_4(k) \le -1 \end{cases} \tag{17}$$

Linear inequalities (17) can rewrite to matrix form (18).

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \\ v_4(k) \end{bmatrix} \le \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{18}$$

In second mode, if at last moment light v_1 was green, now light v_1 or v_2 should be green and lights v_3 and v_4 shouldn't be green, i.e.

$$\text{if } \; v_1(k-1) = 1 \;\wedge\; v_1(k) = 0 \;\Rightarrow v_2(k) = 1 \tag{19}$$

Logic relation (19) is proportional with logic relation (20).

$$v_1(k-1)(1-v_1(k)) \Rightarrow v_2(k) \tag{20}$$

Logic relation (20) can be expressed as inequality (21).

$$v_1(k-1)\left(1-v_1(k)\right)-v_2(k) \le 0 \tag{21}$$

In order to observe phase sequence v_1,v_2, v_3 and v_4, the following constraints should exert to controller.

$$\begin{cases} v_1(k-1)\left(1-v_1(k)\right)-v_2(k) \le 0 \\ v_2(k-1)\left(1-v_2(k)\right)-v_3(k) \le 0 \\ v_3(k-1)\left(1-v_3(k)\right)-v_4(k) \le 0 \\ v_4(k-1)\left(1-v_4(k)\right)-v_1(k) \le 0 \end{cases} \tag{22}$$

Linear inequalities (22) can rewrite to matrix form (23).

$$\begin{bmatrix} -v_1(k-1) & -1 & 0 & 0 \\ 0 & -v_2(k-1) & -1 & 0 \\ 0 & 0 & -v_3(k-1) & -1 \\ -1 & 0 & 0 & -v_4(k-1) \end{bmatrix} \times \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \\ v_4(k) \end{bmatrix} \le \begin{bmatrix} -v_1(k-1) \\ -v_2(k-1) \\ -v_3(k-1) \\ -v_4(k-1) \end{bmatrix} \tag{23}$$

So linear inequalities (18) provided conditions that at any time only a light being green, if inequalities (23) are also consider the controller is obliged by the phase sequence v_1,v_2, v_3 and v_4 lights up green.

C. *Predictive control of intersection traffic*

MPC controller can stabilize MLD systems on a desired reference paths while operating constraints are met. For MPC controller consider following cost function:

$$J\left(u_0^{N_p-1}, x_0\right) = \sum_{t=0}^{N_p-1} \left\|u(t)\right\|_{Q_1}^2 + \left\|x(t)-x_f\right\|_{Q_2}^2 \tag{24}$$

Q_1 And Q_2 respectively are suitable weighting matrices for control signal and state variables. From equation (7), for time-invariant systems we have the solution formula

$$x(t) = A^t x(0) + \sum_{i=0}^{t-1} A^i (Bu(t-i-1)+E) \tag{25}$$

By plugging equation (25) into equation (24), the cost function is obtained in terms of control signal u. With simplification, cost function of the form (26) can be obtained.

$$J = U^T HU + F^T U \tag{26}$$

The matrices H and F are obtained after simplification. By minimizing the cost function (24) in the presence of constraints that are posed, optimal control sequence U will be obtained.

$$U = \begin{bmatrix} u(0) \\ \vdots \\ u(N_p-1) \end{bmatrix} \tag{27}$$

According to the receding horizon philosophy, set

$$u(t) = u(0) \tag{28}$$

Disregard the subsequent optimal inputs $u(1),\ldots,u(N_p-1)$, and repeat the whole optimization procedure at time t+1.

III. Simulation

Results for intersection with four phase that has shown in figure 1, with features such as all roads leading to the intersection have three lanes, the export discharge saturation flow rate at each lane of road is 2400 vehicles per hour, the average of a lane blocking density in all roads is 200 vehicles per kilometer and the average of a lane optimum density in all roads is 50 vehicles per kilometer, are investigated.

For intersection G in figure 1, results of two modes, i.e. without constraint on phase sequence of lights up green and with constraint on phase sequence of lights up green, with input flow rates 2000 and 3000 vehicle per hour shown in Fig. 2 and Fig. 3.

Fig. 2 and Fig. 3 show that the average queue length for both modes i.e. without constraint on phase sequence and with constraint on phase sequence is similar.

Lights signal for input flow rates 2000 and 3000 vehicle per hour shown in Fig. 4 and Fig. 5 respectively.

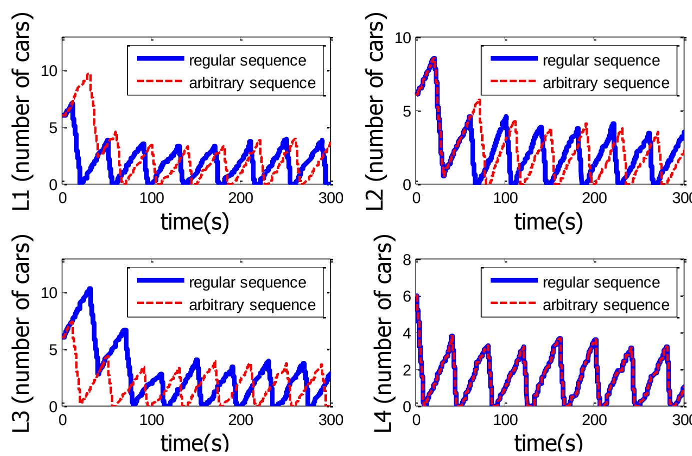

Figure 2. The average queue length in each lane of the roads leading to intersection G while input flow rates are 2000 vehicle per hour for all roads.

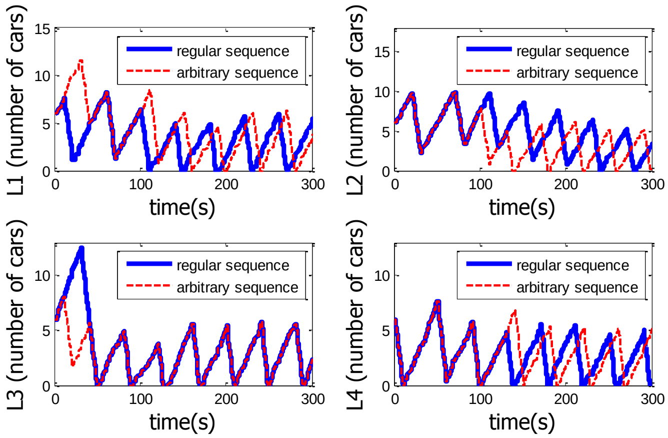

Figure 3. The average queue length in each lane of the roads leading to intersection G while input flow rates are 3000 vehicle per hour for all roads.

Fig. 6 shows the results for the case that input flow rate are different, input flow rate of road FG is 4500 vehicle per hour, road BG is 1800 vehicle per hour, road DG is 2000 vehicle per hour and road KG is 1800 vehicle per hour. Intersection lights signal for different input flow rates shown in Fig. 7.

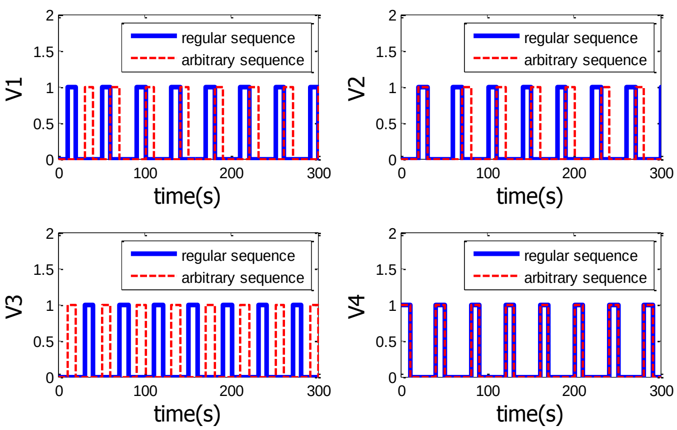

Figure 4. Light signals for intersection with four phase while input flow rates are 2000 vehicle per hour for all roads.

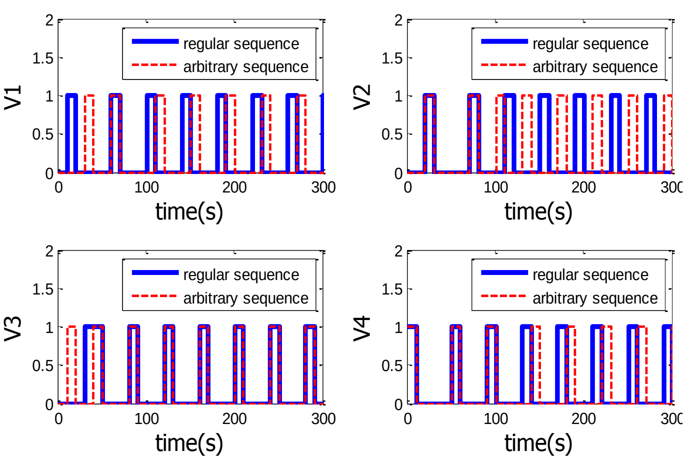

Figure 5. Light signals for intersection with four phase while input flow rates are 3000 vehicle per hour for all roads.

For different input flow rates, result of the mode without constraint on phase sequence of lights up green has better performance than a mode with constraint on phase sequence of lights up green. The reason of better results is the freedom of MPC controller in the no constraint mode.

Sometimes queue length in one of road that leading to intersection has more importance. This means that in this road queue length should not exceed a certain amount. An advantage of the method presented in this paper is that it can consider constraints for the maximum queue length, i.e. MPC controller with taking into account this constraint obtain the sequence of lights up green that queue length in a certain road should not exceed a certain amount. Now results for case that input flow rates are 3000 vehicle per hour to all roads and number of four vehicle considered as maximum amount of queue length in road FG shown in Fig. 8.

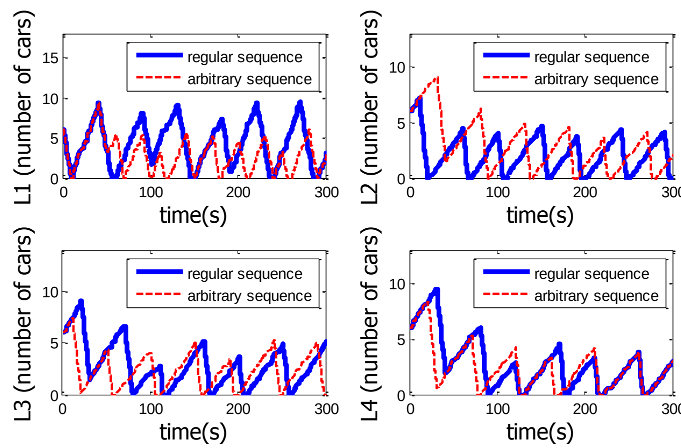

Figure 6. The average queue length in each lane of the roads leading to intersection G while input flow rates are 4500, 1800, 2000 and 1800 vehicle per hour for roads FG, BG, DG and KG respectively.

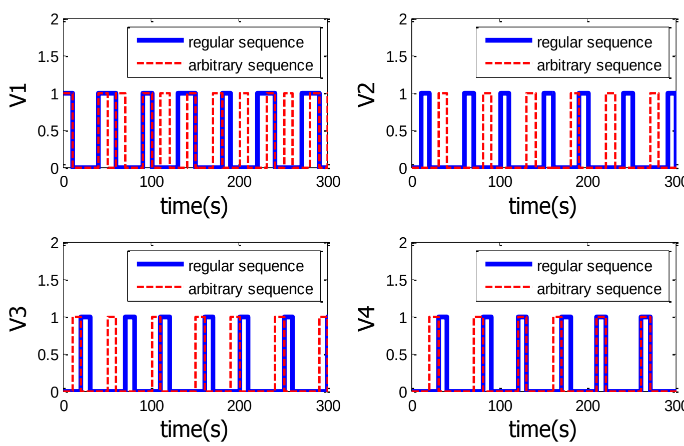

Figure 7. Lights signal for intersection G while input flow rates are 4500, 1800, 2000 and 1800 vehicle per hour for roads FG, BG, DG and KG respectively.

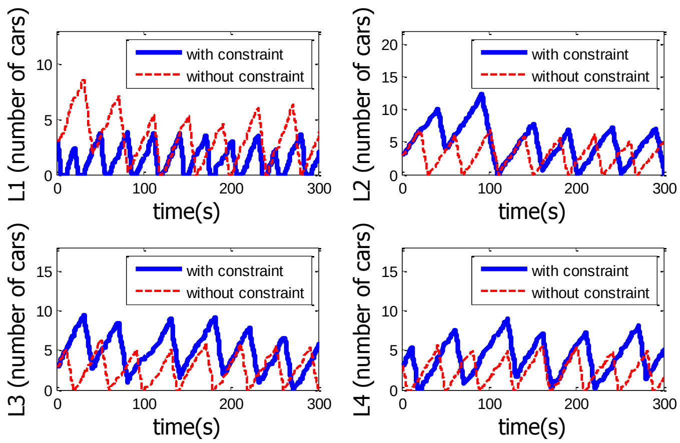

Figure 8. The length of the queue in the roads of a four-phase intersection for constraint and unconstraint condition.

Continuous lines show that in the case of with constraint and discontinuous lines show without constraint results on the maximum amount of queue length case. In which case that has constraint on maximum amount of queue length observed that the length of the queue in the road FG is less than the number of four vehicles in each lane. But as expected, the average queue length in other roads has increased. Lights signal for this case shown in Fig. 9.

IV. Conclusion

Urban traffic system is a kind of hybrid systems. In this paper mixed logical dynamical modeling method is presented to the intersection based on the lights. Then the model predictive control methods based on MIQP optimization by MATLAB simulation researched. Simulation results show that MLD method can effectively establish the intersection based on the lights traffic control model, and the predictive control method based on MIQP optimization can effectively control MLD systems.

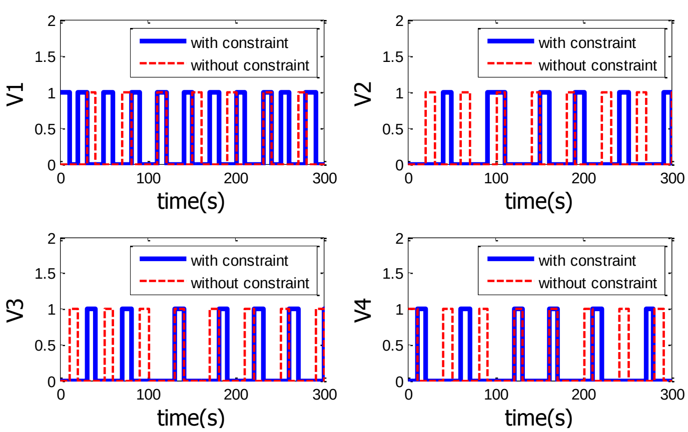

Figure 9. lights signal of a four-phase intersection for constraint and unconstraint condition.

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