

## The Study of Bubble Coalescence in Coaxial and Side-by-Side Motions

Amin Deyranlou<sup>1</sup>, Mohammad Passandideh-Fard<sup>2</sup>

<sup>1</sup>M.Sc. Student, Ferdowsi University of Mashhad, Mashhad, Iran; [deyranlou.amin@gmail.com](mailto:deyranlou.amin@gmail.com)

<sup>2</sup>Associate Professor, Ferdowsi University of Mashhad, Mashhad, Iran; [mpfard@um.ac.ir](mailto:mpfard@um.ac.ir)

### Abstract

The bubble coalescence is discussed in many chemical and metallurgical phenomena. In this paper, the coalescence process of two bubbles moving in a cylindrical tube is studied using both 2D and 3D numerical simulations. The Navier-Stokes equations along with an equation for the interface advection by the Volume-of-Fluid (VOF) method are solved. The results are compared with experimental data reported in the literatures. Simulations are performed for various cases with different configurations of the two bubbles. First, the coaxial bubble motion is studied. The Reynolds number, the density ratio and the viscosity ratio are held constant. The results show that by increasing the Bond number the coalescence time decreases and the leading bubble reveals a more concave interface. Both of these effects make the two bubbles to coalesce faster. Next, the motion of the two bubbles rising side-by-side is studied. On the basis of different Weber numbers based on the approach velocity of the bubbles and the rise velocity, either coalescence or separation will occur.

**Keywords:** Bubble Coalescence, Bond Number, Weber number, VOF method, numerical simulation

### Introduction

Coalescence of bubbles is a phenomenon in which two bubbles combine and form one single bubble. The importance of this phenomenon is in its wide applications in the industry. Some examples include: polymerization processes; dispersion; extraction; enhanced oil recovery; production of detergent and cosmetic products and boiling and condensation. In petrochemical industries, different processes in which fluids interact with each other (such as the process in bubble column reactors), the formation of bubbles is inevitable. Therefore, understanding the characteristics of the bubble coalescence is a useful mean to control the process and assure a better implementation.

In studying bubble dynamics, because of the limitations of the experimental equipment many researchers used computational methods to study different aspects of the phenomenon. The factors that affect the possibility of merging bubbles and the quality of their coalescence are summarized here. Generally three criteria have been proposed for two

bubble coalescence [1]. The most popular theory is cited by Shinnar and Church (1960) [2]. They proposed the film drainage model. In this model, after bubble collision a thin liquid film appears with a thickness of about 0.01-0.001 mm. The coalescence will occur if the attraction forces between two bubbles increases until the film drains out and bubbles coalesce. For simplicity, three successive steps are outlined as: bubble approach; trapping and film drainage; and finally film rupture. These processes occur just in a small fraction of a second. Indeed collision of the bubbles is limited to a short time, because of prevailing fluctuating forces that exist in the domain of control volume. Thus, coalescence occurs only if the film thickness reaches to its critical value where the film rupture happens. On the other hand, Howarth (1964) [3] believes that attraction forces between two bubbles in the interface, originates from the molecular nature of fluids not on the turbulences. In the more recent works done by Lehr et al. (2002) [4], Lehr and Mewes (1999) [5], they introduced a critical approach velocity. They stated that a small approach velocity leads to a more effective coalescence.

The objective of this paper is first simulating the dynamics of two bubbles coalescence in various bubbles positions with respect to each other in the gravity conditions. Next, a comparison is performed between the numerical results with those of the empirical and computational data reported in the literatures. The effects of dimensionless numbers of Bond, Morton and Reynolds on the bubble coalescence will also be investigated in this paper.

### Computational Model

The main assumptions used in simulating the bubble coalescence are that the flow is laminar with low Reynolds numbers, and the fluid is incompressible and isothermal. In addition, the mass transfer (Marangoni) effect is neglected. The two bubbles are assumed to have the same initial radius in the cases considered in this paper. The tube radius where the bubbles are assumed to rise is considered large enough such that the tube wall effects are neglected.

The governing equations are the Navier-Stokes equations along with an equation for the bubbles free-surface deformation. The Volume-of-Fluid (VOF) method first introduced by Hirt & Nicholes (1981) is

used to for this purpose. The governing equations are summarized below:

$$\nabla \cdot \vec{v} = 0 \quad (1)$$

$$\frac{\partial \vec{v}}{\partial t} + \nabla(\vec{v}\vec{v}) = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \tau + \frac{1}{\rho} \vec{F}_b \quad (2)$$

Where,  $\vec{v}$  is the velocity vector,  $p$  is the pressure and  $\vec{F}_b$  represents body forces acting on the fluids. The bubble interface is advected using the VOF method by means of a scalar field  $f$  whose value is unity in the liquid phase and zero in the gas. When a cell is partially filled with liquid  $f$  will have a value between zero and one.

$$f = \begin{cases} f_l = 1 \\ 0 < f_{l-g} < 1 \\ f_g = 0 \end{cases} \quad (3)$$

The discontinuity in  $f$  is propagating through the computational domain according to:

$$\nabla \cdot \vec{v} f + \frac{\partial f}{\partial t} = 0 \quad (4)$$

$$\rho = \sum f_i \rho_i \quad (5)$$

And effective non-dimensional numbers are:

$$M = \frac{g \mu_l^2 (\rho_L - \rho_G)}{\rho_L^2 \sigma^3}, Bo = \frac{g (\rho_L - \rho_G) d_b^2}{\sigma}, Re = \frac{V_t d_b}{\nu_L} \quad (6)$$

$$We = \frac{\rho V_t^2 R}{\sigma}$$

Where  $\rho_L$  and  $\rho_G$  are the density of liquid and gas, respectively and  $g$  is gravitational acceleration. Also  $\mu_l$  and  $\sigma$  are the viscosity and surface tension of liquid, respectively.  $V_t$  is the terminal velocity of the bubble and  $R$  is the equivalent radius. The geometries as a control volume (domain of solution) is created by the Gambit software and the motion of bubbles are simulated by FLUENT commercial code version 6.3. When two bubbles located horizontally, the initial distance between the bubbles is considered  $2.36R_b$  [6]. The computational mesh used in the simulations was such that there were eight cells per bubble radius [7]. Other computational parameters used in the simulation are mentioned in Table 1.

Table 1: computational parameters in Fluent
mesh type: quadrilateral (for 2D) hexahedral (for 3D)
solver: unsteady, time steps: 0.0001 (s)
equation discretization: pressure → body force weighted momentum → first order upwind
pressure – velocity coupling: PISO
VOF: geometry reconstruction

## Results and Discussion

### Coaxial bubble coalescence

The two coaxial bubbles motion is studied under gravity using both two- and three-dimensional simulations with different Bond numbers. Both the density ratio and the viscosity ratio are held constant and the Bond number effect is studied by varying surface tension. Tables 2 and 3 show the conditions of simulations.

Table 2: non-dimensional parameters					
Test	Re	Bo	$\frac{\mu_l}{\mu_g}$	$\frac{\rho_l}{\rho_g}$	M
1	8	5	100	847	1.25e-02
2	8	50	100	847	1.25e+01

Table 3: properties of the materials in the current simulation					
Test	$\mu_l \left( \frac{Kg}{m/s} \right)$	$\mu_g \left( \frac{Kg}{m/s} \right)$	$\rho_l \left( \frac{Kg}{m^3} \right)$	$\rho_g \left( \frac{Kg}{m^3} \right)$	$\sigma \left( \frac{N}{m} \right)$
1	1.79e-03	1.79e-05	1220	1.44	1.88e-04
2	1.79e-03	1.79e-05	1220	1.44	1.88e-05

The results of simulations are presented in Figs. 1-4. The figures show that by increasing the Bond number, the coalescence time will decrease and the leading bubble reveals more concave interface. The basis of a concave interface shape for the leading bubble roots in the value of surface tension. Large surface tension precludes formation of strong jet behind the bubbles and causes the bubbles to rise slower and the two bubbles coalesce at a longer period of time (Figs. 1 and 2). In addition, higher surface tension energy delays the stretch of the top of the trailer bubble; this will lead to a later coalescence. For lower surface tension values, the coalescence occurs sooner due to a stronger jet formed behind the trailer bubble (Figs. 3 and 4).

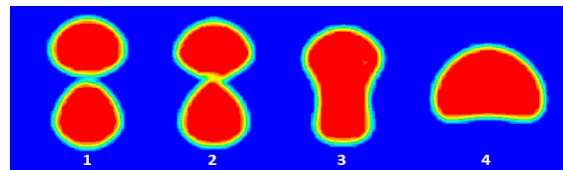


Figure 1 - 2D simulation, Bo = 5

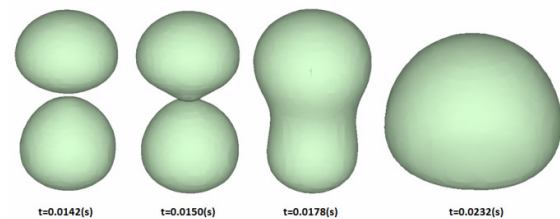


Figure 2 – 3D simulation, Bo = 5

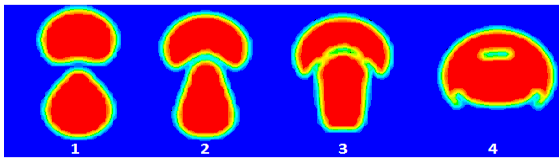


Figure 3 – 2D simulation, Bo = 50

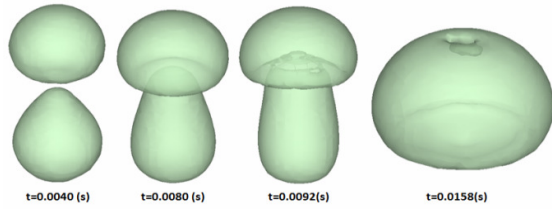


Figure 4 – 3D simulation, Bo =50

Results obtained from numerical simulation denote that at  $Bo=5$ , it takes 0.0232 sec for the bubbles to coalesce. However, at  $Bo=50$ , time for the bubbles coalescence is 0.0158 sec. Figure 5 shows the velocity magnitude of the coalesced bubble for the two cases.

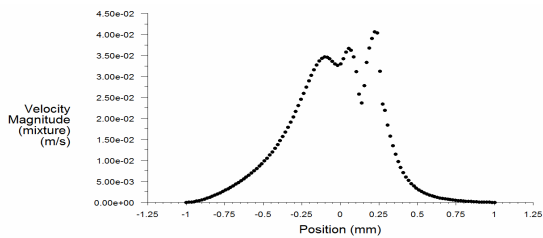


Figure 5 – Velocity of the coalesced bubble, Bo = 5

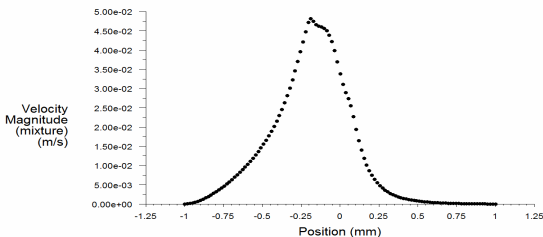


Figure 6 – Velocity of the coalesced bubble, Bo = 50

In Figs. 6 and 7, the velocity magnitude is depicted for  $t=0.0232$  sec and  $t=0.0158$  sec, respectively. In these times, the bubbles have coalesced and reached their terminal velocity. The values of the terminal velocities are in a good agreement with the experimental results reported by Li Chen et al. The apex on the diagrams in the figures shows instances where the single bubble is formed. It is seen that at higher bond number, bubbles coalesce faster.

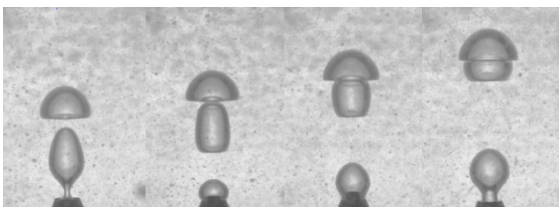


Figure 7 – Experimental results of bubble coalescence Bo = 5, Mo = 0.0041, Li Chen et al. (1998) [8]

Figure 7 shows the experimental results for the bubble coalescence. The flat interface of the leading bubble appears for  $Bo=5$  is observed in the figure.

### Side-by-side bubbles motion

The bubbles interaction as bouncing or coalescence is an important sub-process in the bubbly flows. Duinveld (1988) experimentally investigated the behavior of two bubbles rise side by side in pure water. The experiments were performed for different Weber numbers and different initial distances between two bubbles. The experiments revealed that the two bubbles rising side-by-side either can coalesce or separate. For the first case, two 1.8 mm air bubbles are selected. The bubbles start in a spherical shape, and then approach each other in a short time and coalesce. In this case, the two bubbles coalesce without an initial bounce.

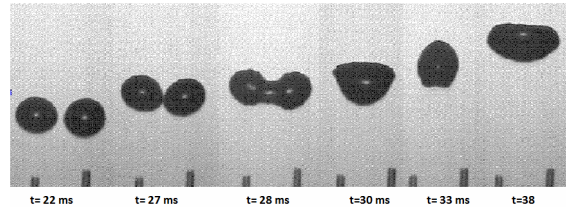


Figure 8 –Bubble coalescence,  $R=0.9$  mm – experimental results, Duinveld (1988) [9]

Figure 9, shows numerical simulation of the bubble coalescence without initial bouncing. A good agreement is seen between the experimental data (Fig. 8) and the numerical simulations (Fig. 9).

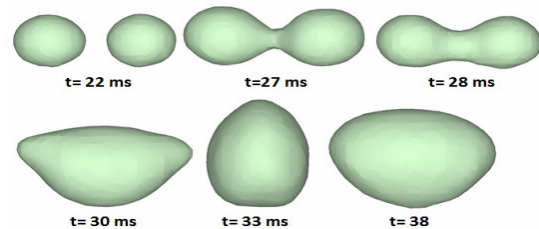


Figure 9 – 3D numerical simulation,  $R=0.9$  mm

Figures 8 and 9 show that when the bubbles approaching each other, the increasing pressure in the film between two bubbles causes film drainage and a repelling force on the bubbles. Because of the inertia, the bubbles continue to approach, which continues until a film thickness of order 10 nm is reached. At this distance, the Van Der Waals forces become important resulting in coalescence.

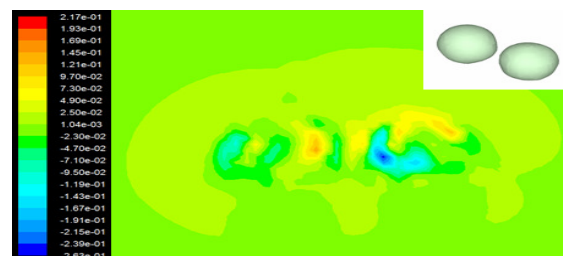


Figure 10 – Contour of approach velocity (V)

$$We = \rho V^2 R / \sigma = \frac{1000 \times 0.08^2 \times 9 \times 10^{-4}}{0.073} = 0.078$$

Therefore, the Weber number on the basis of approach velocity is below the critical value as mentioned in [8] and so the bubbles coalesced. The above numerical result is in the complete agreement with the experimental result was achieved by Li Chen et al. Through experimental investigations, they concluded that if the bubbles radius equal  $R=0.9$  mm, the approach velocity is  $V = 0.08 \pm 0.01$  and the critical weber number for bouncing coalescence would be  $We = 0.08 \pm 0.01$ . Therefore as calculated above the weber number based on the velocity of approach is below the critical value, so the coalescence without bouncing will occurs.

Next, the bouncing-separation case of the bubbles is simulated. In this case, bubbles are initially close to each other but due to the higher Weber number (on the basis of approach velocity) they bounce. After bouncing, since the Weber number on the basis of the bubble rise velocity is higher than the critical value, they separate.

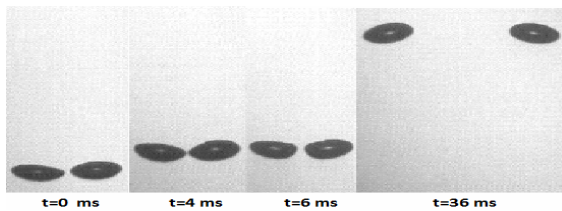


Figure 11 – Bouncing-separation,  $R=1$ mm – experimental results, Duinveld (1988)

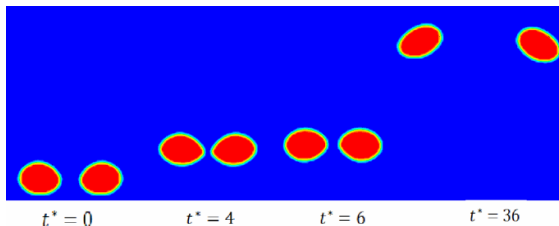


Figure 12 –Bouncing-separations,  $R=1$  mm – 3D simulation results – intersected by (x-z) plane

The coalescence and separation of side-by-side rising bubbles depend on the Weber number value. In pure water if the weber number,  $We = \rho V^2 R / \sigma$ , based on the approach velocity,  $V$ , is below the critical value of  $W_{Cr} = 0.18$ , the bubbles will coalesce.  $R$  is the equivalent radius defined as  $R^{-1} = (1/2) (1/R_1 + 1/R_2)$ . After coalescence, the bubble shape oscillates during the rising. When  $W_{Cr}$  is exceeded, bubbles bounce each other. Bubbles can either coalesce or separate. This was found to depend on the weber number,  $We = \rho U^2 R / \sigma$  based on the rise velocity,  $U$ . If this number is below the critical value, the coalescence will occur after bouncing.

## Conclusion

In this paper, first the two coaxial bubbles coalescence was studied for two Bond numbers, 5 & 50 where the

Reynolds number, the density ratio and the viscosity ratio were held constant. Results from both the numerical simulations and the experimental data show that by increasing the Bond number the coalescence will occur faster and the leading bubble reveals more concave interface. Next, the motion of two side-by-side bubbles that were rising in a cylindrical tube was studied. Results show that by controlling the Weber number, different states will occur for bubbles interaction. If the critical Weber number on the basis of the approach velocity for the  $4R$  initial separation was less than 0.08, the bubbles coalesce during rising, however, for higher Weber numbers the bubbles bounce each other. After bouncing, either coalescence or separation will occur. If the Weber number on the basis of rise velocity, was less than the critical value, the coalescence will occur, otherwise the bubbles will separate.

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