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Optimum parameters of nonlinear integrator using design of experiments based on Taguchi method

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Abstract
For many physical systems like vehicles, acceleration can be easily measured for the respective states. However, the outputs are usually affected by stochastic noise disturbance. The mentioned systems are often sensitive to noise and structural uncertainties. Furthermore, it is very difficult to estimate the multiple integrals of the signal, acceleration to velocity and velocity to position. In this study, emphasis was on eliminating the drifting phenomenon caused by the noise disturbance. As a result, it is essential to find a reliable integrator to evaluate the multiple integrals of the signal. The goal of this experiment was to design a continuous low-drift integrator to estimate the integrals of a proposed signal. In addition, the chattering is capable of amplifying the instability of the system and for this reason, it should be avoided. In this study, a solution method was introduced for this problem which is inspired by the designing of experiments based on the Taguchi method and therefore optimizes the parameters which are effective for minimizing the errors. The results show a reliable response in comparison to previous studies.

Keywords: *nonlinear integrator, signal drifting, Taguchi method.*

Nomenclature	
s_s	*source signal*
s_n	*noisy signal*
s_f	*filtered signal*
s_t	*tracked signal by observer*
s_{if}	*integrated signal by Simulink direct integral (filtered)*
s_{it}	*integrated signal tracked by observer*
s_{is}	*integrated source signal*

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1. Introduction

Vehicle tracking is the procedure of deciding and keeping up positional data for a vehicle moving on the street. Vehicle tracking systems (VTSs) are helpful for tracking the movements of public transport vehicles, helping drivers to have an easy parking maneuver especially when reversing, or for location-aware computing and individual rout assistance [1]. For most VTSs, GPS is the essential hotspot for acquiring transportation and speed information. Under perfect conditions, GPS provides long-term accurate and outright estimations. Outdoors, where there is a reasonable viewable pathway to four or more satellites, GPS is able to provide locations with accuracies ranging from tens of meters to tens of centimeters, depending on the type of GPS receiver [2]. In any case, GPS is regularly inaccessible in urban ranges, where the urban crowding impact may weaken the quantity of noticeable satellites transmitters. Notwithstanding, when GPS is accessible, it is frequently less exact as a result of multipath impacts.

Alternative approaches for absolute VTSs require active radiation sources such as infrared light [3], magnetic fields [4], ultra wide band (UWB) or other radio frequencies [5]. Common to these methods is that they require an initial setup of the corresponding source. Additional methods involve the use of light emitting and capturing sources such as cameras or light detection and ranging (LIDAR) systems [6]. There are few other radio frequency (RF)-based technologies, such as those using cellular telephones based on wireless networks to obtain navigation parameters. The main restriction of these cellular technologies is that they are not accurate enough for most vehicle tracking parking purposes. Wireless local area networks (WLANs) can be used for indoor navigation, where the location fix is determined by measuring signal strengths from several access points [1]. However, like GPS, this technique suffers from multipath and disappearing important properties, and pre-mapping of signal strength is often required.

An entirely different approach is the use of the relative sensor modalities, mainly IMUs. IMUs do not necessitate any external infrastruture and do not actively radiate. The problem with IMUs is that, the position and heading estimates are derived from the accelerometers and gyros of the IMU, which develop unbounded errors. Smaller and lighter IMUs, typically using the micro-electro-mechanical systems (MEMS) technology, develop large position and heading errors quickly. A common approach used in the making of MEMS-based IMUs feasible in VTSs applications is to find better ways that allow the bounding of position errors. In IMU-based VTSs, the direction of motion is most commonly obtained from the gyroscopes of the IMU. However, in the MEMS-based systems, a bias drift of the gyros is significant, and if left uncorrected can completely mask the navigation resolution [7]. Therefore, alternative equipment or additional data sources are required to achieve reasonable accuracy.

Many methods have been introduced in the literature to avoid chattering, and to eliminate drifting phenomenon for IMUs [8-10]. One of the recent and successful methods was introduced by Wang et al. [8], using a quad rotor control system. In this study, a nonlinear integrator was derived and the finite time convergence was proven. The proposed nonlinear integral has some parameters that we did not have any previous knowledge of finding their value optimally. An algorithm has been proposed in this study to find the regarded values robustly. The concept used here, was adopted from the statistical technique for designing experiments which is called the Taguchi method.

This paper is organized as follows. The nonlinear integral is introduced in the next section while the design of experiment principles is discussed in the third section of this paper. After achieving optimized values of the modeling and implementation, the nonlinear integral is supposed in the fourth and fifth sections respectively.

2. Nonlinear integral

Equation (1) shows the proposed differential equation for a single integrator [1]. This nonlinear equation gets the filtered signal $a(t)$ and two x_1 and x_2 signals which are built as the integrated and observed signals respectively. Although, they mentioned the parameters of this equation, there is no straight forward method of calculating the values given in Table 1. For this reason, a design of experiments based on Taguchi method will be address to introduce a method for achieving the optimized gains.

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\varepsilon^3 \dot{x}_2 = -K_1 \left|\varepsilon x_1\right|^{\alpha_1} sign(x_1) \\ &\quad -K_2 \left|\left(x_2 - a(t)\right)\right|^{\alpha_2} sign\left(x_2 - a(t)\right) \end{aligned} \tag{1}$$

Table 1. The Parameters value mentioned in [8]

Parameter	Wang's value
K_1	2
K_2	3
α_2	0.1
$\alpha_1 = \alpha_2 / (2 - \alpha_2)$	0.0526
ε	0.83

3. Designing of Experiments

The Taguchi method is a powerful tool for characterization, design and performance optimization [11-16]. The Taguchi experimental design method offers a wide range of applications, with advantages such as simple concept, ease of use, as well as variation reduction [17].

This technique has seven steps [17]

1. Determination a function that needs to be optimized.
2. Determination of controllable factors and their levels.
3. Selection of a suitable orthogonal array.
4. Performing the experiments and measuring outputs.
5. Calculation of S/N ratio.
6. Selecting the parameters corresponding to optimal conditions, analyzing the data and prediction of output in optimum case.
7. Conducting the confirmation experiment.

In the present study, five parameters are used as control factors. All parameters designed to have five levels are presented in Table 2. The effect of time as a simulation elapsed period is considered

By means of the Taguchi method, the L_{25} orthogonal arrays table with 25 rows is constructed for the controllable factors, as given in Table 3. Each row corresponds to the number of experiments to be performed.

In the Taguchi method, a loss function is used to assess the cost of deviation from the target. The loss function is further transformed into the S/N ratio. It provides a measure, for understanding the impact of noise factors on system performance. The larger the S/N, the more robust the product is against noise. Several S/N ratios depend on the experimental objective. For example, one may choose, Smaller is better (SB), nominal is better (NB) or Larger is better (LB). In this study, the Lower root mean square between the desired integral of a signal and the filtered integrated signal indicates a better performance. Therefore, to obtain optimum performance characteristics, "SB" for the output was selected. Using LB, the definition of the loss function (L) for RMS output, y_i, of n repeated experiments using different levels of noise factors is given in Equations (2) and (3).

$$L_{SB} = \frac{1}{n} \sum_{i=1}^{n} y_i^{\,2} \tag{2}$$

The S/N ratio η_{ij} can be expressed as (3).

$$\eta_{ij} = -10 \log\left(L_{ij}\right) \tag{3}$$

where i and j indices represent the i_{th} performance characteristic and j_{th} experiment, respectively. The optimal level of the parameters and better performance is indicated by greater values of η. The S/N ratio for each experiment of L_{25} shown in Table 3 is calculated and introduced in Table 4. As shown in Table 4, the efficient performance for the RMS is obtained at the first level of K_1, the fifth level of K_2, the fifth level of α_2, the fourth level of ε and finally, the first level time of simulation. This behavior is similar to the goal of minimizing the time elapsed in the simulation. Nevertheless, for K_2 the first level can be effective for minimizing the time.

Table 2. Variety of control parameters used in the Taguchi method

Level	K_1	K_2	α_2	ε	time
1	2	2	0.2	0.2	200
2	4	4	0.4	0.4	400
3	6	6	0.6	0.6	600
4	8	8	0.8	0.8	800
5	10	10	1	1	1000

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Besides, the category of parameters affected both the proposed and cost functions, the minimized RMS and time elapsed, are ranked in both Tables 4 and 5. In addition, Fig. 1 shows the behavior of parameters in minimizing the RMS using different levels.

Table 3. Taguchi experiments

K_1	K_2	α_2	ε	time	RMS	Time elapsed
1	1	1	1	1	4816.135	3.711016
1	2	2	2	2	443.8899	6.393078
1	3	3	3	3	237.3513	4.02252
1	4	4	4	4	161.0291	2.669304
1	5	5	5	5	104.6611	1.960742
2	1	2	3	4	28142.23	3.794936
2	2	3	4	5	5399.2	2.41274
2	3	4	5	1	304.4364	0.478725
2	4	5	1	2	2398.457	41.75755
2	5	1	2	3	79.27285	242.5222
3	1	3	5	2	7526.2	1.597425
3	2	4	1	3	36989.1	35.202
3	3	5	2	4	5989.839	8.198962
3	4	1	3	5	1317.735	6.663291
3	5	2	4	1	280.2418	1.187146
4	1	4	2	5	24834.53	4.838446
4	2	5	3	1	1780.478	0.910047
4	3	1	4	2	20322.9	2.644653
4	4	2	5	3	4714.041	1.511617
4	5	3	1	4	52117.69	142.0657
5	1	5	4	3	3367.21	1.101419
5	2	1	5	4	204220	24.07171
5	3	2	1	5	290945.7	50.3861
5	4	3	2	1	6902.216	2.722246
5	5	4	3	2	4485.583	2.530687

Table 4. SNR for minimizing the root mean square error

Level	K_1	K_2	α_2	ε	time
1	-51.73	-79.72	-73.28	-87.25	-62.81
2	-65.35	-78.03	-74.73	-66.23	-71.45
3	-71.16	-73.63	-74.16	-67.39	-64.17
4	-81.38	-64.88	-69.22	-64.89	-81.84
5	-87.17	-58.94	-63.82	-69.45	-74.92
Delta	35.44	20.78	10.91	22.36	19.03
Rank	1	3	5	2	4

Table 5. SNR for minimizing the time elapsed

Level	K_1	K_2	α_2	ε	time
1	-10.794	-8.315	-22.327	-30.366	-2.872
2	-18.589	-16.301	-13.365	-20.895	-2.872
3	-18.589	-13.292	-15.111	-9.479	-19.029
4	-13.592	-13.941	-10.964	-5.391	-21.813
5	-15.856	-21.230	-11.312	-6.948	-15.543
Delta	7.795	12.915	11.363	24.975	18.941
Rank	5	3	4	1	2

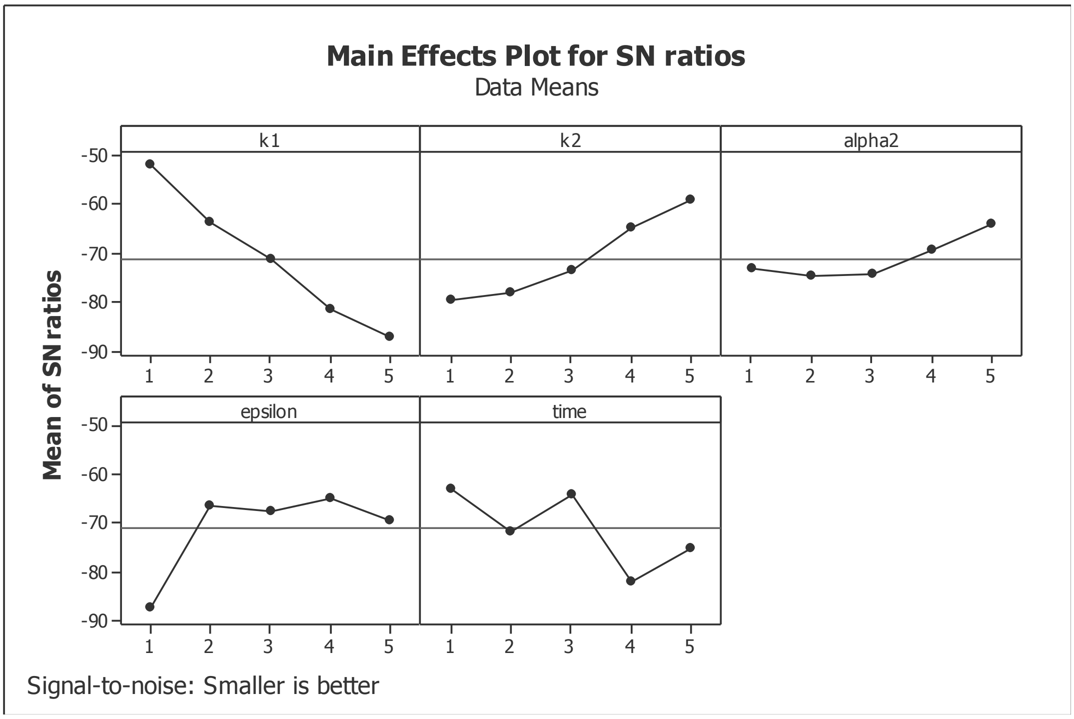

Fig. 1. Signal to Noise Ratio where the objective is to minimize RMS

Similar to Table 1. The Parameters value mentioned in [8], a new table is made by the optimized parameters. These parameters given in Table 6, are not exactly the optimized parameters, but are more informative than the previous ones.

Table 6. The optimized parameters value

Parameter	Proposed value
K_1	2
K_2	10
α_2	1
$\alpha_1 = \alpha_2 / (2 - \alpha_2)$	1
ε	0.8

4. Modeling and Simulation

Simulating a set of differential equations is the first step of analyzing the system of equations. To implement the differential equations in MATLAB, there are some methods discussed in following. One method is using ode solver in MATLAB script environment. In this method, the differential equations are considered in a MATLAB-function and user tries to solve the differential equation using an ode solvers like Runge-Kutta. In this method, the equations are written regarding to the differential order. Although this method is straight forward, the differential equations should take in the first order. Therefore for a system which has many higher differential orders, this method is annoying. Moreover, many users are interested to implement the model in SIMULINK environment to use many its facilities. In SIMULINK, using connection between the parts of equations is not easy when the system of equations is growing. To avoid this disturb, we introduce an easy solution which splits differential and algebraic parts from each other. DAS (Differential Algebraic Splitter) method can simplify the equations dramatically and let user has a few errors in implementing the system of equations. In this method, all of differential parts of equations are removed and configured by From-Goto blocks and integrate with the common Simulink integration operator. Afterwards, the rest of the set of equations are considered algebraically and modeled without any integration operator. Every equation is ended to a block where it is referred to the differential part. This method avoids connection crowding and creates a plain environment for users. Moreover, using this method user can see all of the system of equations clearly whereas by the conventional methods, many errors might be seen during the simulation.

To prove the reliability of our proposed values, we implement the nonlinear integral (formula 1) in Simulink environment by differential- algebraic separation method and use a sinusoidal signal with pi/2 phase. The pure integral of this signal is also a sinusoidal signal without any phase. To have a stochastic noise, we create a pulse generator added by random instantaneous values. To filter the signal, we used a proper butter low path filter. After filtration process, the signal is integrated by Simulink operator and on the other hand integrated by nonlinear integrator. The nonlinear integrator is examined for both of Wang's values and our new optimized values. Fig. **2** and Fig. **3** show the Simulink environment for implementing the total and nonlinear integrator system, respectively.

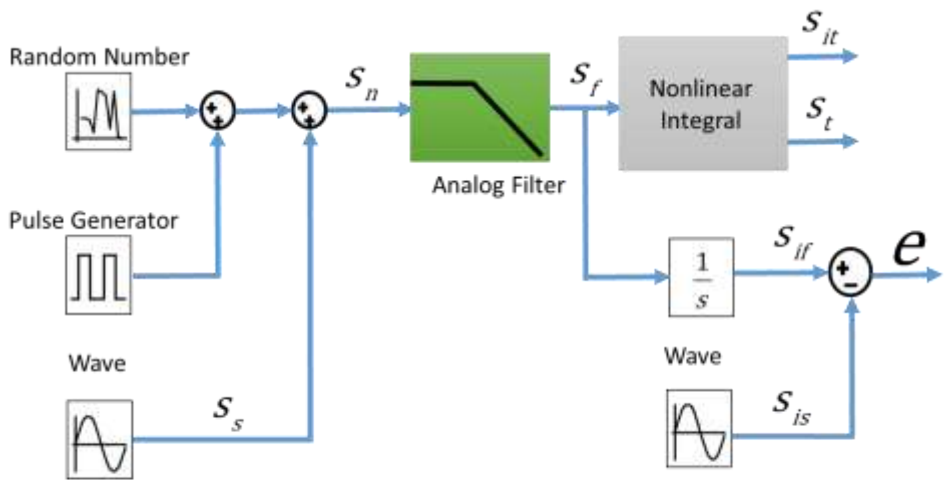

Fig. 2. Simulink environment for modeling the nonlinear integrator

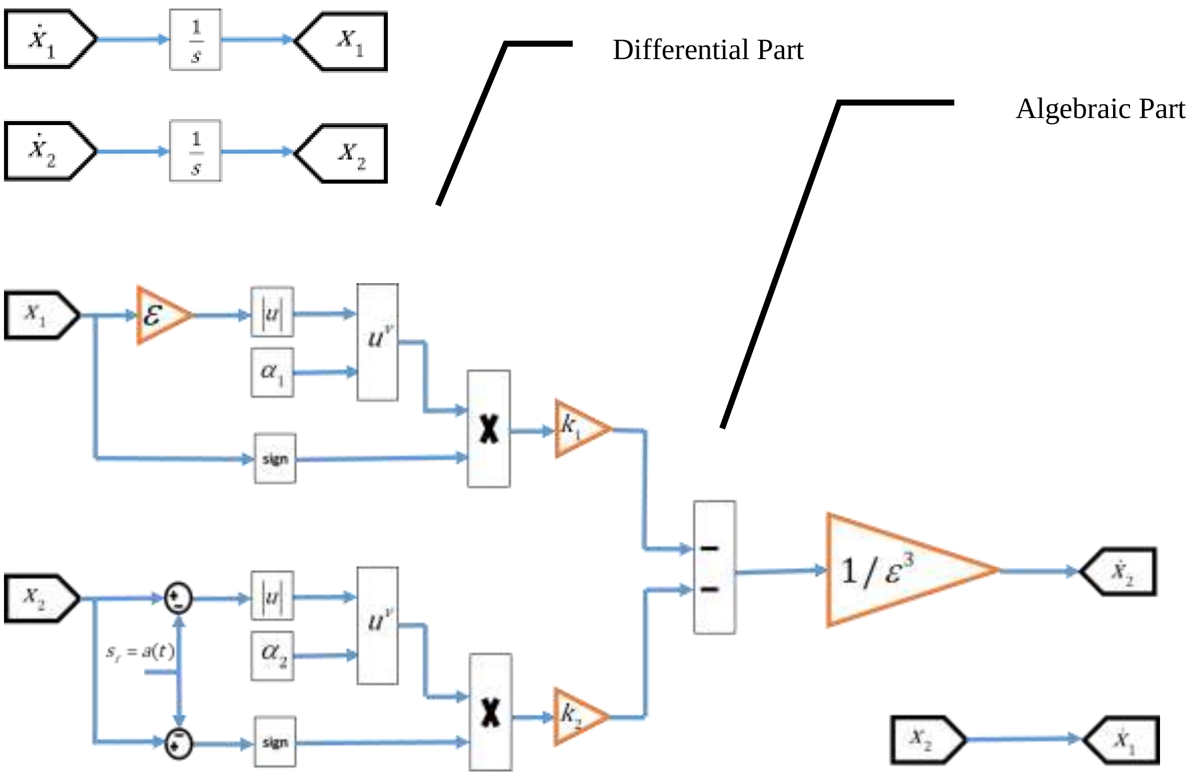

Fig. 3. Schematic of simulink environment for implementing nonlinear integrator using differential-algebraic

5. Results and Discussion

The results are shown in Fig. –8, where a sinusoidal signal is shown in Fig. 4. The noises produced by the mentioned blocks were added to this pure wave in Fig. 6. Afterwards, the noisy cosinus signal was filtered with the buffer filter in Fig. . The filtered noisy signal was integrated by the simulink integrator. As shown in Fig. , there exists a rising drift in the phenomenon. This problem has previously been discussed. As shown by this figure, the differences of the desired sinusoidal signal and integral, increased rapidly with time. The nonlinear operator used integrated the filtered noisy signal. Figure 11 illustrates the reliability of the nonlinear integrator for integrating the signal and avoiding instability like the drift phenomenon made by the simulink integrator. Although Wang's parameters could converge in a finite time, our proposed optimized values converge rapidly and precisely. This result can be more important when we want to analyze a vehicle for transient response.

6. Conclusion

In order to implement a feedback controller, the sensors must be adopted to measure the states. Afterwards, the evaluated states are used to provide feedback signals for a closed loop process. However, it is known that all states cannot be measured directly. Therefore, designing an observer for the states or estimator is a bypass procedure. In this paper, a recent method was introduced for avoiding the drift phenomenon in integrating acceleration sensors which have been used dramatically in navigation. In this nonlinear integral, some parameters should be optimized inherently. Evolutionary searching algorithms like genetic cannot be used, as a result of the low speed of the procedure. For this reason, an experimental design based on Taguchi method was introduced and using signal to noise ratio criterion, the optimized level of each parameter was determined. The nonlinear integral, simulated in Matlab-Simulink environment was adopted and equipped by these new values. The results show that previous studies recorded more success in eliminating the drifting phenomenon.

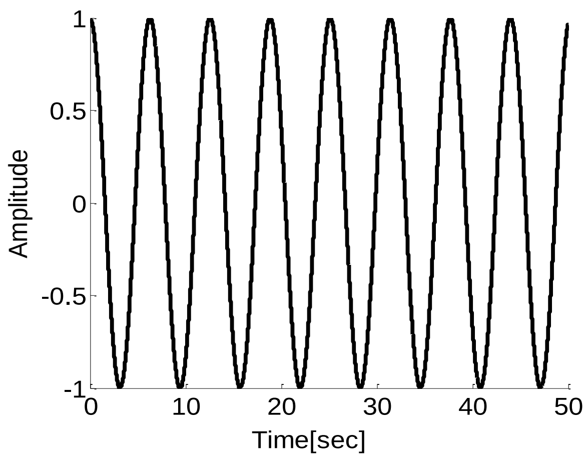

Fig. 4. Pure cosinus signal

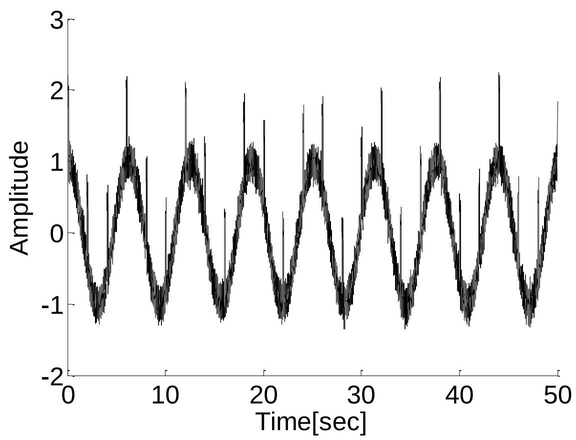

Fig. 5. Noised cosinus signal

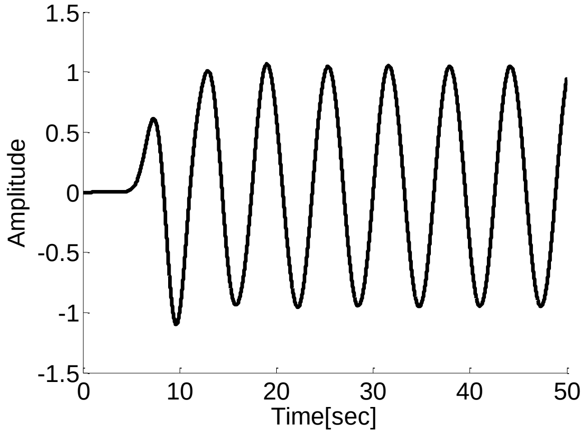

Fig. 6. Filtered cosinus signal

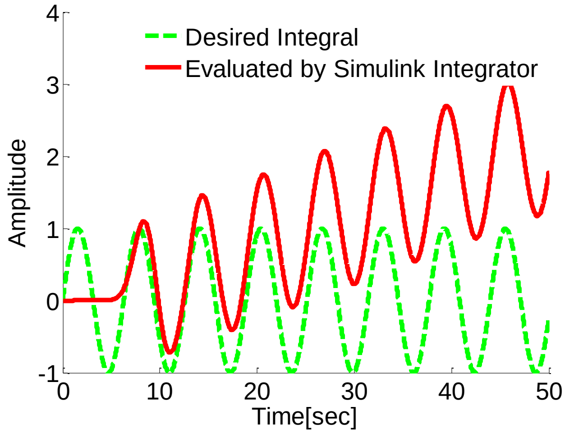

Fig. 7. Integral of filtered cosinus signal using simulink integrator versus desired sinus

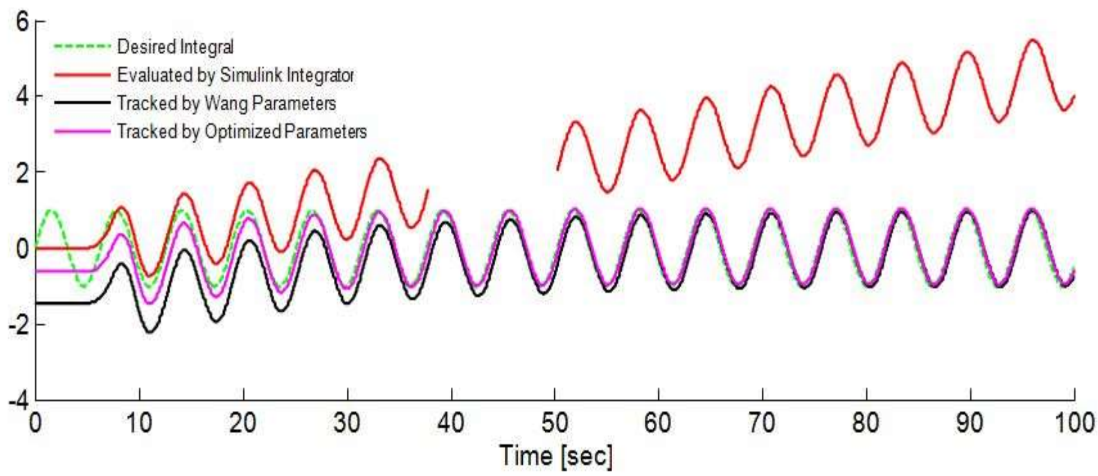

Fig. 8. Comparison of simulink integrator and nonlinear integrator by using Wang's parameters and the optimized parameters during 100 seconds

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