Impacts of Ramp Rate Limits on Oligopolistic Opportunities in Electricity Markets

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***Abstract*—In this paper, a supply function equilibrium (SFE) model is used to study the impacts of ramp rate limits on generating firms' opportunities in oligopolistic electricity markets. Ramp rate limits are added to the SFE model. Ramp rate limits couple the SFE models of different hours. A heuristic algorithm is presented to solve the coupled SFE models. Existence and uniqueness of solutions are discussed. By applying the presented algorithm to a test system, the impacts of ramp rate limits on the strategic behavior of generating firms are studied.**

***Index Terms*—Electricity market, oligopolistic opportunities, ramp rate limits, supply function equilibrium (SFE).**

I. Introduction

In the past few decades, electric power systems have been restructured with the hope of providing a nondiscriminatory environment for competition. However, due to lack of conditions of perfect competition in the electricity markets, such as limited numbers of suppliers and transmission constraints, competition is weak in electricity markets [1].

A market is at its Nash equilibrium if no firm is better off by changing its strategy unilaterally [2].

In [1], supply function equilibrium (SFE) is modeled by a set of coupled bilevel optimizations. Each bilevel optimization consists of ISO's social welfare maximization as inner problem and a generating firm's profit maximization as outer problem. Bidding strategies of generating units at market equilibrium have been used for several studies, such as the impact of large-scale wind generation on the electricity market [3], coordinating generation and transmission planning [4], and influence of emission allowance trading on electricity markets [5], [6]. In [7], the presented SFE model in [1] is simplified by ignoring transmission constraints, and an analytical closed solution is presented for pay-as-bid electricity markets. Existence and uniqueness of equilibrium in both constrained and unconstrained electricity markets are discussed. In [7], uncertainty in demand is taken into account in SFE models, and a chance constrained technique is used to convert the stochastic problem to a deterministic problem. Similarly in [3], transmission constraints are ignored, and an analytical closed solution is presented for uniform electricity markets. The existence and uniqueness of equilibrium in both constrained and unconstrained electricity markets are discussed. A scenario-based technique is employed to consider wind generation uncertainty in the SFE model. The impacts of large-scale wind generation on electricity markets are studied using the presented model in [3].

Ramp rate limits may considerably affect economics of power generators. Hence, ramp rate limits have been widely considered in unit commitment studies. In [8], an algorithm for solving unit commitment problems in electricity markets considering the ramp rate limits is proposed. A price-based unit commitment considering ramp rate limits is presented in [9]. The influence of the generator's ramp rate constraints on their Cournot equilibrium strategy formulation is investigated in [10]. In [10], the impacts of ramp rate constraints on the existence and uniqueness of equilibrium are not studied. The generation allocation problem in competitive electricity markets considering ramp rate limits is studied using game theory and dynamic programming methods in [11]. The impacts of ramp rate limits on bidding strategies of generating firms are ignored in [11].

In this paper, the impacts of ramp rate limits on oligopolistic opportunities in electricity markets are studied. To do so, it is required to model the strategic behavior of generating firms. SFE models are the tool to model firms' behaviors. SFE models have been repeatedly used in electricity market modeling literature as an alternative to Cournot or Bertrand models because it is proved to be a more realistic model for these particular applications (e.g., see [12] for further discussions).

The main contributions of this paper are the following: 1) presenting a multiperiod SFE model considering ramp rate limits and 2) proposing a heuristic algorithm to compute the SFE. The significance of the contributions is that the presented model can answer the following questions for an electricity market: 1) How and to what extent do ramp rate limits affect the oligopolistic opportunities of generating firms?; 2) are low ramp generators, such as nuclear power plants, able to compete with other generating firms in an electricity market?; and 3) is it reasonable to ignore ramp rate limits in computing market equilibrium?

This paper is organized as follows. In Section II, the required background is reviewed. In Section III, the SFE is modeled by

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considering ramp rate limits, and an algorithm for computing market equilibrium is presented. The presented approach is applied to a test system in Section IV, where the impacts of ramp rate limits on oligopolistic opportunities are studied. Concluding remarks are provided in Section V.

II. SFE Model: Background Review

In this paper, market equilibria under different ramp rate limits are compared in order to determine how they impact market outcomes. In particular, we use the SFE for this study. This research is based on the SFE model presented in [1] and [3]. In order to take ramp-up/ramp-down rates into account, a multiperiod 24-h day-ahead market model is used in this paper. Assume that the cost of generating $Q_{Si}^{(t)}$ by unit i at hour t is $g(Q_{Si}^{(t)}) = a_i Q_{Si}^{(t)} + 0.5 b_i {Q_{Si}^{(t)}}^2$ and the utility of consuming $Q_{Dj}^{(t)}$ by consumer j at hour t is $f(Q_{Dj}^{(t)}) = c_j^{(t)} Q_{Dj}^{(t)} - 0.5 d_j^{(t)} {Q_{Dj}^{(t)}}^2$. Note that, since fuel cost is constant in short term, the coefficients of cost functions of generators are time invariant. However, since loads change considerably during a day, the coefficients of utility functions of consumers are time variant. The marginal cost function of unit i at hour t or its true bid function is $MC_i^{(t)} = \text{Bid}_i^{\text{true}(t)} = a_i + b_i Q_{Si}^{(t)}$. Each unit bids a supply function for each hour t into the day-ahead market. It is assumed that firms only manipulate the intercept of their true bid functions, i.e., they present the bid function $\text{Bid}_i^{(t)} = \alpha_i^{(t)} + b_i Q_{Si}^{(t)}$ for each hour t. There are several reasons for this assumption that has been argued in [1]. Therefore, the bid of unit i at hour t is specified with $\alpha_i^{(t)}$. The equilibrium point of hour t is specified with $[\alpha_1^{*(t)} \alpha_2^{*(t)} \ldots \alpha_{ng}^{*(t)}]^T$, where ng is the number of generating units. To compute the market equilibrium using the SFE model, a set of coupled bilevel optimization problems should be solved [1], [3]. The fth upper (outer) level problem is the profit maximization problem of the fth Gencos. The lower (inner) level problem of each upper level problem is the ISO's optimization problem. The objective of ISO is to maximize the social welfare subject to meet the demand, generation, and transmission constraints. If the transmission network is strong enough, transmission constraints can be ignored. Hence, the optimization of ISO is modeled as follows:

$$\text{Max } J_{\text{ISO}} = \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{D}} \left(c_i^{(t)} Q_{Di}^{(t)} - \frac{1}{2} d_i^{(t)} {Q_{Di}^{(t)}}^2 \right) - \sum_{i\in\mathcal{S}} \left(\alpha_i^{(t)} Q_{Si}^{(t)} + \frac{1}{2} b_i {Q_{Si}^{(t)}}^2 \right) \right) \quad (1)$$

s.t. :

$$\sum_{i\in\mathcal{S}} Q_{Si}^{(t)} - \sum_{i\in\mathcal{D}} Q_{Di}^{(t)} = 0 \qquad \forall t \in \mathcal{T} \quad (2)$$

$$Q_{Si}^{\min} \le Q_{Si}^{(t)} \le Q_{Si}^{\max} \qquad \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \quad (3)$$

$$-Drr_i . Q_{Si}^{\max} \le Q_{Si}^{(t)} - Q_{Si}^{(t-1)} \le Urr_i . Q_{Si}^{\max} \qquad \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \quad (4)$$

where J_{ISO} is the social welfare, $Q_{Si}^{\min}$ and $Q_{Si}^{\max}$ are the capacity limits of unit i, $\mathcal{S}$ is the set of generation units, $\mathcal{D}$ is the set of consumers, $\mathcal{T}$ is the set of hours in the understudy period, and Urr_i and Drr_i are the ramp-up and ramp-down rate limits of unit i in per unit per hour, respectively. The optimization problem of firm f, $\forall f \in \mathcal{F}$, where $\mathcal{F}$ is the set of generating firms, can be modeled as follows:

$$\text{Max} \pi_f = \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{S}_f} \lambda^{(t)} Q_{Si}^{(t)} - a_i Q_{Si}^{(t)} - \frac{1}{2} b_i {Q_{Si}^{(t)}}^2 \right) \quad (5)$$

s.t. :

$$\text{ISO's optimization problem } (1)-(4) \quad (6)$$

where π_f is the profit of Genco f in the study horizon, S_f is the set of generating units of Genco f, and $\lambda^{(t)}$ is the market clearing price at hour t. The first term of (5) is the revenue of firm f, and the second and third terms stand for its generation cost. In order to determine the optimal strategy of Genco f, Karush–Kuhn–Tucker (KKT) optimality conditions of optimization (1)–(4) are added to the optimization of firm f [(5) and (6)] as constraints. This problem is a mathematical programming with equilibrium constraints (MPEC). There is an MPEC problem for each Genco. To compute the SFE, the MPEC problems of all firms should be solved together. This problem is an equilibrium problem with equilibrium constraints (EPEC). The existence and uniqueness of market equilibrium can be discussed by analyzing EPEC.

In this paper, *unconstrained electricity market* means an electricity market with only meeting demand constraint, and *constrained electricity market* means an electricity market with meeting demand constraint, generation constraints, and ramp rate constraints, i.e., constraints (2)–(4). Transmission constraints are ignored in this paper. In a specified operating point, the unit with none of its generation and ramp rate limits active is referred to as *unbound unit*, and the unit with one of its generation or ramp rate limits active is referred to as *bound unit*. The ramp-down (up) rate limit of unit m is said to be active at hour t if it reaches its limit when its generation power decreases (increases) from hour $t-1$ to t.

In the following sections, the objective is to find the multiperiod SFE considering ramp rate limits. A number of lemmas and theorems are presented in the following section. Although the presented lemmas and theorems are the contributions of this paper, the associated proofs are presented in the Appendix.

III. Constrained Market Model

In this section, the optimal strategies of generating firms and SFE considering ramp rate limits are discussed. First, an equilibrium model is presented by assuming that the bound generating units at SFE are unknown. Then, the model is modified by assuming that some of the bound generating units at SFE are known. Finally, an algorithm for computing multiperiod SFE considering ramp rate limits is presented.

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A. Equilibrium Model

Lemma 1: The optimal strategy of firm f in the uniform-price electricity market model of (1)–(5) is obtained from the following optimization:

$$\text{Max } \pi_f = \sum_{t\in\mathcal{T}} \Bigg(\Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)} \Big)^{Tr} \times \mathbf{Q}_f^{(t)} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)} \Big) + \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)} \Big)^{Tr} \mathbf{R}_f^{(t)} + \Big(\Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)} \Big)^{Tr} \times \mathbf{R}_f^{\prime(t)} + \mathbf{S}_f^{\prime(t)} \Big) \mathbf{Q}_D^{(t)} + \mathbf{S}_f^{\prime\prime(t)} \mathbf{Q}_D^{(t)^2} \Bigg) \tag{7}$$

s.t. :

$$\mathbf{V}^{(t)}\mathbf{Q}_D^{(t)} + \mathbf{U}^{(t)}(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}) \leq \mathbf{Q}_S^{\max} \perp \boldsymbol{\mu}^{(t)\max} \geq 0, \qquad \forall t \in \mathcal{T} \tag{8}$$

$$\mathbf{V}^{(t)}\mathbf{Q}_D^{(t)} + \mathbf{U}^{(t)} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) \geq \mathbf{Q}_S^{\min} \perp \boldsymbol{\mu}^{(t)\min} \geq 0, \qquad \forall t \in \mathcal{T} \tag{9}$$

$$\mathbf{V}^{(t)}\mathbf{Q}_D^{(t)} - \mathbf{V}^{(t-1)}\mathbf{Q}_D^{(t-1)} + \mathbf{U}^{(t)} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) - \mathbf{U}^{(t-1)} \Big(\boldsymbol{\alpha}^{(t-1)} + \boldsymbol{\mu}^{(t-1)} + \boldsymbol{\epsilon}^{(t-1)} - \boldsymbol{\epsilon}^{(t)}\Big) \leq \boldsymbol{Urr}\mathbf{Q}_S^{\max} \perp \boldsymbol{\epsilon}^{(t)\max} \geq 0, \forall t \in \mathcal{T} - \{\text{initial time}\} \tag{10}$$

$$-\mathbf{V}^{(t)}\mathbf{Q}_D^{(t)} + \mathbf{V}^{(t-1)}\mathbf{Q}_D^{(t-1)} - \mathbf{U}^{(\mathbf{t})} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) + \mathbf{U}^{(t-1)} \Big(\boldsymbol{\alpha}^{(t-1)} + \boldsymbol{\mu}^{(t-1)} + \boldsymbol{\epsilon}^{(t-1)} - \boldsymbol{\epsilon}^{(t)}\Big) \leq \boldsymbol{Drr}\mathbf{Q}_S^{\max} \perp \boldsymbol{\epsilon}^{(t)\min} \geq 0, \forall t \in \mathcal{T} - \{\text{initial time}\} \tag{11}$$

$$\mathbf{V}^{(t)}\mathbf{Q}_D^{(t)} + \mathbf{U}^{(t)} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) - \mathbf{Q}_S^{(0)} \leq \boldsymbol{Urr}\mathbf{Q}_S^{\max}, \perp \boldsymbol{\epsilon}^{(t)\max} \geq 0, \forall t \in \{\text{initial time}\} \tag{12}$$

$$-\mathbf{V}^{(t)}\mathbf{Q}_D^{(t)} - \mathbf{U}^{(t)} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) + \mathbf{Q}_S^{(0)} \leq \boldsymbol{Drr}\mathbf{Q}_S^{\max} \perp \boldsymbol{\epsilon}^{(t)\min} \geq 0, \forall t \in \{\text{initial time}\} \tag{13}$$

where subscript f indicates firm f, $\boldsymbol{\mu}^{(t)\max}$ and $\boldsymbol{\mu}^{(t)\min}$ are dual variables of max and min generation limits at hour t, $\boldsymbol{\mu}^{(t)} = \boldsymbol{\mu}^{(t)\max} - \boldsymbol{\mu}^{(t)\min}$, $\boldsymbol{\epsilon}^{(t)\max}$ and $\boldsymbol{\epsilon}^{(t)\min}$ are dual variables of ramp-up and ramp-down rate limits at hour t, $\boldsymbol{\epsilon}^{(t)} = \boldsymbol{\epsilon}^{(t)\max} - \boldsymbol{\epsilon}^{(t)\min}$, and Tr denotes transpose. The elements of matrices $\mathbf{Q}_f^{(t)}$ and $\mathbf{U}^{(t)}$; vectors $\mathbf{V}^{(t)}$, $\mathbf{R}_f^{(t)}$, and $\mathbf{R}_f^{\prime(t)}$; and scalars $\mathbf{S}_f^{\prime(t)}$ and $\mathbf{S}_f^{\prime\prime(t)}$ depend on $a_i\, \forall i \in S_f$ and $b_i\, \forall i \in S$ and are given in (7)–(13). $\mathbf{Q}_S^{(0)}$ is the vector of generation powers at hour $t = 0$. □

Equation (7) models the profit of firm f considering generation and ramp rate limits. Equations (8) and (9) model max and min generation constraints, respectively. Equations (12) and (13) model ramp-up and ramp-down limits for initial hour, and (10) and (11) model ramp-up and ramp-down limits for other hours, respectively. The problem (7)–(13) is an MPEC problem. If firm f would like to determine its optimal strategy, it should estimate the bids of other firms and solve the MPEC problem (7)–(13). To compute the SFE, the MPEC problems of all firms should be solved together. To this end, the KKT optimality conditions of all firm's optimizations must be solved together. Writing the KKT optimality conditions of (7)–(13) for all firms and arranging them yield the equilibrium model as follows:

$$\mathbf{H}^{(t)} \Big(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) + \mathbf{R}^{(t)} + \mathbf{R}^{\prime(t)}\mathbf{Q}_D^{(t)} - \mathbf{U}^{(t)} \Big(\boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\Big) = 0, \qquad \forall t \in \mathcal{T} \tag{14}$$

$$\text{constraints (8) to (13)} \tag{15}$$

where

$$H_{ij}^{(t)} = \frac{B_f}{B^{(t)^2} b_i b_j}\ \forall i \in \mathcal{S}_f,\ \forall j \in \mathcal{S},\ i \neq j,\ \forall f \in \mathcal{F}, \forall t \in \mathcal{T}$$

$$H_{ii}^{(t)} = \frac{B_f}{B^{(t)^2} b_i^2} - \frac{1}{b_i} \qquad \forall i \in \mathcal{S}_f,\ \forall f \in \mathcal{F}, \forall t \in \mathcal{T} \tag{16}$$

and vectors $\mathbf{R}^{(t)}$ and $\mathbf{R}^{\prime(t)}$ are defined as $\mathbf{R}^{(t)} = [\mathbf{R}_{a_a}^{(t)^{Tr}} \mathbf{R}_{b_b}^{(t)^{Tr}} \ldots \mathbf{R}_{z_z}^{(t)^{Tr}}]^{Tr}$ and $\mathbf{R}^{\prime(t)} = [\mathbf{R}_{a_a}^{\prime(t)^{Tr}} \mathbf{R}_{b_b}^{\prime(t)^{Tr}} \ldots \mathbf{R}_{z_z}^{\prime(t)^{Tr}}]^{Tr}$, where subscripts a, b,. . ., and z represent the first, second, . . ., and last firms, respectively.

Note that (14) and (15) for t and $t+1$ are coupled via ramp rate constraints. Hence, (14) and (15) for $t = 1$ to $t = T$ are coupled. Therefore, solving (14) and (15) for all t considers the whole load profile and tries to meet the demand in all hours considering the generation and ramp rate constraints. If the generation and ramp rate constraints (15) are ignored, then the problem of computing the SFE is converted to a linear algebra problem [3]. The profit of each unit of firm f is a second-order function of bids. If a generating unit of firm f reaches one of its limits, its generation power becomes constant, and its profit becomes a linear function of bids. Hence, the maximum profit of firm f cannot be computed using differentiation. In this case, in order to compute the SFE, generation and ramp rate limits are considered heuristically as follows.

Consider a uniform electricity market. Suppose it has equilibrium. For the sake of simplicity, assume that only the min generation (max generation, ramp-down rate, or ramp-up rate) limit of unit m of firm f is active at hour t of the SFE. This means that generator m is willing to decrease (increase, decrease, or increase) its power at hour t to maximize the profit of firm f. However, unit m cannot decrease (increase, decrease, or increase) its power at hour t due to its generation limit. The profit of unit m at hour t depends on $Q_{Sm}^{(t)}$ and $\lambda^{(t)}$. As soon as unit m reaches its generation or ramp rate limit,

$Q_{Sm}^{(t)}$ becomes constant, $Q_{Sm}^{(t)} = Q_{Sm}^{\min}$ ($Q_{Sm}^{(t)} = Q_{Sm}^{\max}$, $Q_{Sm}^{(t)} = Q_{Sm}^{(t-1)} - Drr_m Q_{Sm}^{\max}$, or $Q_{Sm}^{(t)} = Q_{Sm}^{(t-1)} + Urr_m Q_{Sm}^{\max}$)), and $\lambda^{(t)}$ depends only on the bids of other units. Hence, unit m is not able to participate in setting the electricity price at hour t. In this case, the game continues among the other generators. This is how ramp rate limits in particular impact market operation and oligopolistic opportunities. Suppose that the game converges to an equilibrium point. If at this point the min generation (max generation, ramp-down rate, or ramp-up rate) limit of unit m is still active, unit m cannot increase the profit of firm f by changing its bid. Therefore, the profit of firm f is at its max, and this point is the market equilibrium. If at this point the min generation (max generation, ramp-down rate, or ramp-up rate) limit of unit m gets inactive, the market does not have equilibrium by contradiction. Note that first it was assumed that the min generation (max generation, ramp-down rate, or ramp-up rate) limit of unit m is inactive at hour t of equilibrium, but its limit got active at hour t of equilibrium. Then, it was assumed that the min generation (max generation, ramp-down rate, or ramp-up rate) limit of unit m is active at hour t of equilibrium but its generation limit got inactive at hour t of equilibrium.

Thus, the equilibrium point of the constrained markets is defined as follows:

$$\begin{aligned}\text{Equilibrium}:&\Big\{\alpha_i^{(t)} | d\pi_f / d\alpha_i^{(t)} = 0 \forall f \in \mathcal{F} \text{ and } \forall i \in \mathcal{U}_f^{(t)} \\ &\text{and } \forall t \in \mathcal{T}\Big\} \cup \Big\{\alpha_j^{(t)} | Q_{Sj}^{(t)} = Q_{Sj}^{\min} \text{ or } Q_{Sj}^{(t)} = Q_{Sj}^{\max} \text{ or} \\ &Q_{Sj}^{(t)} = Q_{Sj}^{(t-1)} - Drr_j Q_{Sj}^{\max} \text{ or } Q_{Sj}^{(t)} = Q_{Sj}^{(t-1)} \\ &+ Urr_j Q_{Sj}^{\max} \forall j \in \mathcal{C}_f^{(t)} \text{ and } \forall t \in \mathcal{T}\Big\} \end{aligned} \quad (17)$$

where $\mathcal{C}_f^{(t)}$ is the set of bound units of firm f at hour t and $\mathcal{U}_f^{(t)}$ is the set of unbound units of firm f at hour t. The aforementioned definition tells us how to compute equilibrium if we know which constraints are active at equilibrium.

In order to compute the SFE using the heuristic algorithm discussed previously, we need to know the active generation and ramp rate limits at equilibrium. By solving (14) and (15), the bids of all units and dual variables of all generation and ramp rate constraints for every hour t in $\mathcal{T}$ are computed. The positive dual variables of hour t show the active generation and ramp rate limits at hour t of equilibrium. Here, the ramp rate limit at hour t means the ramp rate limit from hour $t-1$ to t. Since the profit of firm f at hour t is only a function of bids of its unbound generating units, the equilibrium model must be modified after identifying the active limits at equilibrium. In the next section, the modified equilibrium model is presented.

B. Modified Equilibrium Model

Lemma 2: The profit of firm f around the market equilibrium is a second-order function of $\alpha_i^{(t)} \forall i \in \mathcal{U}^{(t)}$. If the set of active generation and ramp rate limits of each hour at the market equilibrium is known, the profit of firm f around the market equilibrium can be written as follows:

$$\begin{aligned}\pi_f = \sum_{t\in\mathcal{T}} &\check{\boldsymbol{\alpha}}^{(t)^{Tr}} \check{\mathbf{Q}}_f^{(t)} \check{\boldsymbol{\alpha}}^{(t)} + \check{\boldsymbol{\alpha}}^{(t)^{Tr}} \ddot{\mathbf{R}}_f^{(t)} \\ &+ \left(\check{\boldsymbol{\alpha}}^{(t)^{Tr}} \check{\mathbf{R}}_f^{\prime(t)} + \ddot{\mathbf{S}}_f^{\prime(t)}\right) \check{\mathbf{Q}}_D^{(t)} + \check{\mathbf{S}}_f^{\prime\prime(t)} \check{\mathbf{Q}}_D^{(t)^2} + \ddot{\mathbf{C}}_f^{(t)}\end{aligned} \quad (18)$$

$$\begin{aligned}\ddot{\mathbf{R}}_f^{(t)} &= \check{\mathbf{R}}_f^{(t)} + \check{\mathbf{V}}^{(t)} \left(\sum_{j\in\mathcal{C}_f^{(t)}} \overline{Q}_{Sj}^{(t)}\right), \ddot{\mathbf{S}}_f^{\prime(t)} \\ &= \check{\mathbf{S}}_f^{\prime(t)} + \frac{1}{\check{B}^{(t)}} \left(\sum_{j\in\mathcal{C}_f^{(t)}} \overline{Q}_{Sj}^{(t)}\right) \\ \ddot{\mathbf{C}}_f^{(t)} &= -\sum_{j\in\mathcal{C}_f^{(t)}} \left(a_j \overline{Q}_{Sj}^{(t)} + \frac{1}{2} b_j \overline{Q}_{Sj}^{(t)^2}\right) \qquad \forall t \in \mathcal{T}\end{aligned} \quad (19)$$

where $\check{\boldsymbol{\alpha}}^{(t)}$ is the vector of the bid of all unbound units at hour t, $\check{\mathbf{Q}}_D^{(t)} = \mathbf{Q}_D^{(t)} - \sum_{j\in\mathcal{O}^{(t)}} \overline{Q}_{Sj}^{(t)}$, where $\mathcal{O}^{(t)}$ is the set of all omitted units at hour t, and coefficients $\check{B}^{(t)}$, $\check{\mathbf{V}}^{(t)}$, $\check{\mathbf{Q}}_f^{(t)}$, $\check{\mathbf{R}}_f^{(t)}$, $\check{\mathbf{R}}_f^{\prime(t)}$, $\check{\mathbf{S}}_f^{\prime(t)}$, and $\check{\mathbf{S}}_f^{\prime\prime(t)}$ are computed like $B^{(t)}$, $\mathbf{V}^{(t)}$, $\mathbf{Q}_f^{(t)}$, $\mathbf{R}_f^{(t)}$, $\mathbf{R}_f^{\prime(t)}$, $\mathbf{S}_f^{\prime(t)}$, and $\mathbf{S}_f^{\prime\prime(t)}$, respectively, assuming that each firm has only its unbound units. If the max (min) generation limit of unit j is activated at hour t, $\overline{Q}_{Sj}^{(t)}$ is equal to $Q_{Sj}^{\max}$ ($Q_{Sj}^{\min}$). If the ramp-up (down) rate limit of unit j is activated at hour t, $\overline{Q}_{Sj}^{(t)}$ is equal to $Q_{Sj}^{(t-1)} + Urr_j Q_{Sj}^{\max}$ ($Q_{Sj}^{(t-1)} - Drr_j Q_{Sj}^{\max}$).

To compute the SFE, the MPEC problems of all firms should be solved together. In order to compute the SFE, assuming that all active generation and ramp rate limits of every hour of the study time horizon are known, the KKT optimality conditions of all firm's profit maximization problem(18) must be solved together. Writing the KKT optimality conditions of (18) for all firms and arranging them yield

$$\check{\mathbf{H}}^{(t)} \check{\boldsymbol{\alpha}}^{(t)} = -\ddot{\mathbf{R}}^{(t)} - \check{\mathbf{R}}^{\prime(t)} \check{\mathbf{Q}}_D^{(t)}, \qquad \forall t \in \mathcal{T} \quad (20)$$

$\check{\mathbf{H}}^{(t)}$ and $\check{\mathbf{R}}^{\prime(t)}$ are computed like $\mathbf{H}^{(t)}$ and $\mathbf{R}^{\prime(t)}$, respectively, assuming that each firm has only its unbound units and $\ddot{\mathbf{R}}^{(t)} = [\ddot{\mathbf{R}}_{a_a}^{(t)^{Tr}} \ddot{\mathbf{R}}_{b_b}^{(t)^{Tr}} \ldots \ddot{\mathbf{R}}_{z_z}^{(t)^{Tr}}]^{Tr}$. By solving the linear algebra equation given in (20) for every t in $\mathcal{T}$, the bids of unbound units at equilibrium are computed. If only some of the active generation and ramp rate limits of some hours of the study time horizon are unknown, the MPEC model given in (7)–(13) can be modified by omitting the known bound units. Here, omission of bound units means fixing their output power at the power of their active limits, subtracting the power of each active limit from the load of the related hour, and writing the firms' profits at hour t versus the bids of the unbound units at hour t. In this case, the profit function of firm f changes from (7) to (18) considering that only the known bound units are omitted. Moreover, only the generation and ramp rate limits of bound units are omitted from the constraints (8)–(13) in this case. Writing the KKT conditions of the modified MPECs for all firms leads to the modified equilibrium model as follows:

$$\check{\mathbf{H}}^{(t)}\left(\check{\boldsymbol{\alpha}}^{(t)}+\check{\boldsymbol{\mu}}^{(t)}+\check{\boldsymbol{\epsilon}}^{(t)}-\check{\boldsymbol{\epsilon}}^{(t+1)}\right)+\ddot{\mathbf{R}}^{(t)}+\check{\mathbf{R}}'^{(t)}\check{\mathbf{Q}}_D^{(t)} - \check{\mathbf{U}}^{(t)}\left(\check{\boldsymbol{\mu}}^{(t)}+\check{\boldsymbol{\epsilon}}^{(t)}-\check{\boldsymbol{\epsilon}}^{(t+1)}\right)=0,\ \forall t\in\mathcal{T} \tag{21}$$

$$\text{modified constraints } (8) \text{ to } (13) \tag{22}$$

where modified constraints (8)–(13) are the same as constraints (8)–(13) with a check symbol over each variable and parameter. $\check{\boldsymbol{\alpha}}^{(t)}$, $\check{\boldsymbol{\mu}}^{(t)}=\check{\boldsymbol{\mu}}^{(t)\max}-\check{\boldsymbol{\mu}}^{(t)\min}$, and $\check{\boldsymbol{\epsilon}}^{(t)}=\check{\boldsymbol{\epsilon}}^{(t)\max}-\check{\boldsymbol{\epsilon}}^{(t)\min}$ are the vectors of bids and dual variables of the remaining units at hour t, respectively. Matrix $\check{\mathbf{U}}^{(t)}$ is computed like $\mathbf{U}^{(t)}$, assuming that, at every hour t, the system has only the unbound units of hour t.

If the active constraints at equilibrium are known, the equilibrium can be computed from (20). However, the active constraints at equilibrium are unknown before being computed. Hence, we start using the model (14) and (15) for computing SFE. This model is an inaccurate model since the profit of firm f at hour t is only a function of bids of its unbound generating units, whereas in this model, the profit of firm f at hour t is a function of bids of all units. By solving (14) and (15), the bids of all units and dual variables of all generation and ramp rate constraints for every hour t in $\mathcal{T}$ are computed. The positive dual variables of hour t show the active generation and ramp rate limits at hour t of equilibrium. However, since this model is not an accurate model, identifying all active generation and ramp rate limits using this model could be misleading. The dual variables that are well in positive territory will remain positive in the accurate model and show the active generation and ramp rate limits at equilibrium. In [3], the unit that has the biggest dual variable is identified, and the model is modified. Since this model is a multiperiod model and takes into account ramp rate limits, omitting bound units one by one is very time-consuming. In the presented algorithm, first the maximum dual variable over all generation and ramp rate limits of all hours is identified. At each hour, the unit where the dual variable of one of its limits is greater than the g percent of the maximum dual variable is omitted from the model, and the model is modified as (21) and (22). By solving the modified model, other active generation and ramp rate limits can be identified. The procedure continues until all active generation and ramp rate limits are omitted. In this case, we reach the equilibrium model (20), and the bids of unbound units at equilibrium are computed from this linear algebra model. Two results can be concluded from the equilibrium model (20). First, the bid of every unbound unit at equilibrium is unique. This will be approved in the following theorem. Second, around the equilibrium, as far as the status of active limits is unchanged, the model can be divided to n_T decoupled models for every t in $\mathcal{T}$, where n_T is the number of hours in the study time horizon. Therefore, the equilibrium can be defined for every hour in the study time horizon.

Theorem 1: If a constrained uniform electricity market has SFE at hour t, the unbound units of hour t have a unique bid at the SFE of hour t.

This theorem is proved in the Appendix. Assuming that ISO optimization has a unique solution, it can be concluded that unbound units have unique bid and power at market equilibrium, but bound units have only unique power at market equilibrium. The existence and uniqueness of concave n-person games are addressed in [12]. It is assumed that the set of player actions is a compact and coupled constrained set. The author proves that such concave n-person game has Nash equilibrium, and it is unique if the weighted sum of payoff functions is diagonally strictly concave. A similar condition for the uniqueness of equilibrium in the Cournot model is presented in [13]. The existence and uniqueness of SFE in uniform electricity markets with and without generation constrained are discussed in [3]. In [3], generation space is divided into partitions. In each partition, the status of each generation limit (in terms of being active or not) is unchanged. According to [3], every unconstrained uniform electricity market has a unique SFE. A constrained uniform electricity market has at most one SFE in each partition. In constrained uniform electricity markets that may have multiple SFEs, [3] focuses on one of the equilibria that corresponds to the unconstrained SFE and refers to it as principal SFE.

C. Heuristic Algorithm for Computing SFE

The presented heuristic algorithm for computing market equilibrium can be itemized as follows.

1) Set iteration counter at $\nu=1$, and solve (14) and (15). By solving this mixed complementarity problem, the bids of all units and dual variables of all generation and ramp rate limits are computed for every hour t.
2) For every t in $\mathcal{T}$, identify the greatest generation or ramp rate dual variable. The units that are related to those maximum dual variables whose values are greater than or equal to the g percent of the maximum dual variables are determined. These units should be omitted. Save the bids and dual variables of these units for the related hours.
3) Increase the iteration counter $(\nu=\nu+1)$.
4) Revise (14) and (15) by omitting the bound units identified in step 2. Revising (14) and (15) yields (21) and (22).
5) Solve (21) and (22). By solving (21) and (22), the bids and dual variables of generation and ramp rate limits of all remaining units at hours t, $\forall t\in\mathcal{T}$, are computed.
6) Compute the generation power of all units for every hour of scheduling period by running ISO optimization. The computed bids for the remaining units in step 5 and the latest value(s) computed for the bid(s) of omitted unit(s) are used for ISO optimization. The dual variables of the omitted units of each hour are updated in this step. If the results of the ISO optimization show that more than one omitted units are unbound, the algorithm is back one step, and the omitted units in the last step are restricted. In this case, only the bound units that remained bound and the unit that becomes unbound and has the greatest dual

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TABLE I
PARAMETERS OF GENERATING UNITS

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
a	20	30	17.5	30	10	32.5
b	0.2	0.25	0.175	0.25	0.625	0.0834
$\mathbf{Q}_s^{min}$	0	0	0	0	0	0
$\mathbf{Q}_s^{max}$	80	40	80	30	50	55
$\boldsymbol{Urr}$	23/80	8/40	12/80	5/30	12/50	13/55
$\boldsymbol{Drr}$	25/80	16/40	17/80	9/30	15/50	17/55
$\mathbf{Q}_s^{(0)}$	79	24	73	21	41	22

* a in (\$/MWh), b in (\$/MW^2h), $\mathbf{Q}_s^{min}$ and $\mathbf{Q}_s^{max}$ in (MW), $\boldsymbol{Drr}$ and $\boldsymbol{Urr}$ in $per unit/5min$

variable among the units that become unbound are omitted. If the results of the ISO optimization show that, for every t in $\mathcal{T}$, the omitted units of hour t remained bound at hour t, then there are some other active generation or ramp rate limits at some hour t. The procedure is continued until the SFE at all hours is reached or it is realized that the market does not have SFE at some hours.

7) If the result of ISO optimization shows that all omitted units remained bound at related hour and there is no any other bound unit at any hour, the considered values for bids in ISO optimization indicate SFE in bid space.

If the result of ISO optimization shows that only one omitted unit got unbound, the market does not have SFE by contradiction.

If the result of ISO optimization shows that more than one omitted units got unbound, back to step 4. Omit only the bound units that remained bound and the unit that got unbound and has the greatest dual variable among the units that got unbound in the last ISO optimization.

If the result of ISO optimization shows that all omitted units remained bound at related hour but there are some other bound units, go to step 2, and continue the procedure until reaching SFE, or realize that the market does not have equilibrium.

IV. CASE STUDIES AND NUMERICAL RESULTS

In this section, the presented algorithm is applied to the generating units of the IEEE 30-bus test system. The parameters of the generating units are given in Table I. Transmission constraints are ignored. A day-ahead uniform electricity market is considered for this test system. It is assumed that units 1 to 6 belong to firms A to F, respectively. Load is inelastic and is given in Fig. 1 for the study period. In this section, the impacts of ramp rate limits on the electricity market are assessed. Note that, in day-ahead scheduling, it is assumed that the power outputs of generating units are constant during an hour. It is also assumed that generating units change their power during the last 5 or 10 min of each hour to follow their schedule for the next hour. Therefore, restrictions in power increase or decrease during 5 or 10 min are considered as ramp-up or ramp-down rate limits in megawatts per hour [8], [14], [15]. In this paper, the ramp rate limits given in megawatts per 5 min are considered as ramp rate limits from one hour to the next hour and are expressed in terms of megawatts per hour or per

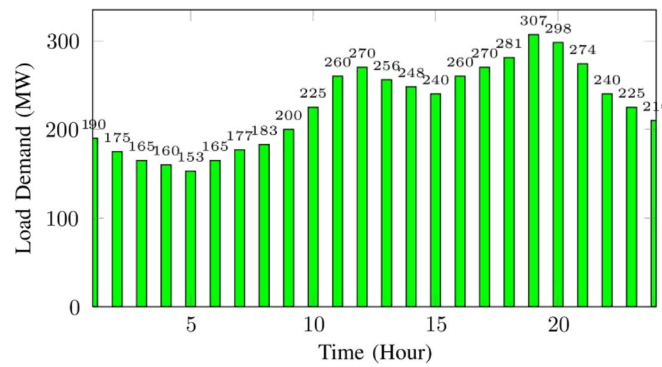

Fig. 1. IEEE 30-bus power system's hourly loads.

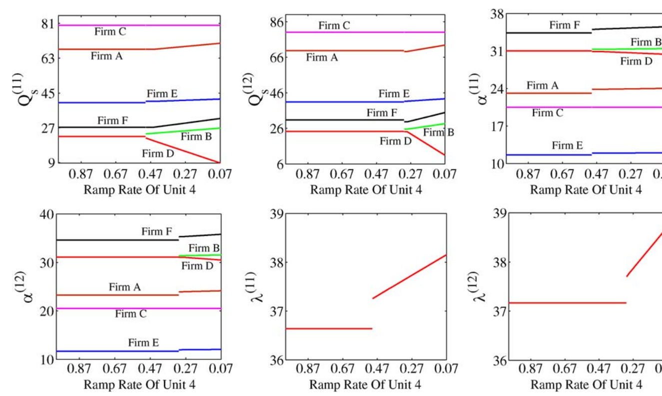

Fig. 2. Bids, generating powers, and MCP at equilibria of hours 11 and 12.

unit per hour. Solver PATH in GAMS environment is used to solve the mixed complementarity problem. The accuracies of all equilibrium points were verified using the definition of Nash equilibrium, i.e., in equilibrium, no firm can increase its profit by changing its strategy unilaterally.

A. *Impacts of Ramp Rate Limits on Oligopolistic Opportunities*

In order to study the impacts of ramp rate limits on oligopolistic opportunities of firms, the upper ramp rate limit of unit 4, firm D, is changed from 1 to 0.07 per unit/h step by step, while the ramp rates of other units are set at 1 per unit/h. In each step, the SFE is computed for the 24-h load which is given in Fig. 1. Let us focus on hour 11. The upper ramp rate limit of unit 4 will become active at the equilibrium of hour 11 if the ramp rate is equal or less than 0.47 per unit/h. Figs. 2 and 3 show the generating powers, bids, MCPs, and profits at equilibria of hours 11 and 12. As Fig. 2 shows, when the upper ramp rate limit of unit 4 gets active, MCP increases due to the decrease in competition level, which is the result of the omission of firm D. An increase in MCP increases the profits of all firms, as depicted in Fig. 3. As the upper ramp rate limit of unit 4 decreases, the generating power of firm D decreases, and consequently, the generating powers of the other units increase at equilibrium. Hence, the profit of firm D decreases, and the profits of the other firms increase. The interesting result is that, if the upper ramp rate limit of unit 4 is in the range of 0.27–0.47 per unit/h, the profit of firm D

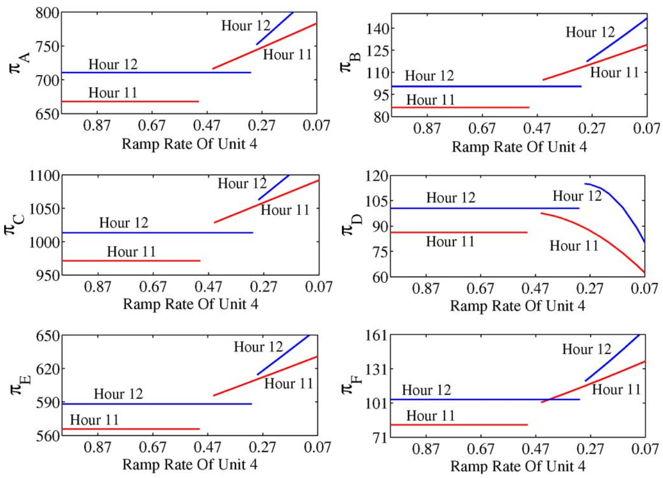

Fig. 3. Profits of generating firms at equilibria of hours 11 and 12 (red continues: hour 11; blue continues: hour 12).

increases and creates an oligopolistic opportunity for firm D. This is in contradiction with this general expectation that the operating constraints of a unit will decrease its oligopolistic opportunity. According to Fig. 3, if the upper ramp rate limit of unit 4 is less than 0.47 per unit/h, the ramp rate limit will decrease the profit of firm D and, consequently, its oligopolistic opportunity. Fig. 3 also shows that decreasing the upper ramp rate limit of unit 4 increases the oligopolistic opportunities of the other firms, which is consistent with the general expectation. Let us now turn the focus to hour 12. The upper ramp rate limit of unit 4 will become active at the equilibrium of hour 12 if the ramp rate is equal or less than 0.3 per unit/h. The same result is concluded from the analysis of hour 12. Fig. 3 shows that the profits of all firms increase from hour 11 to hour 12 due to the increase in load if no ramp rate limit is active at hours 11 and 12, i.e., if the ramp rate limit of unit 4 is greater than 0.47 per unit/h. Fig. 3 also shows that, although the load at hour 12 is greater than the load at hour 11, the profits of firms A, B, C, E, and F at hour 12 will be less than their profits at hour 11 if the ramp rate limit of unit 4 is in the range of 0.47–0.3 per unit/h. It could be concluded that the restriction on the ramp rate limit of the unit of firm D decreased the profits of the other firms. This is not true. In fact, for this case, the firms gain from the oligopolistic opportunities that resulted from the ramp rate limit of unit 4 at hour 11 but not at hour 12. This is why their profits decreased in hour 12 in spite of the increase in load.

B. Existence of Equilibrium in Constrained Electricity Markets

In this section, the existence of SFE in the test system is assessed. Two scenarios for ramp rate limits are considered. In the first scenario, the ramp rate limits are the same as given in Table I. In the second scenario, it is assumed that unit 1 is a nuclear power plant, and its upper and lower ramp rate limits are equal to 0.04 and 0.05 per unit/h, respectively. In each scenario, the SFE is computed for the 24-h load. In the first scenario, in the first iteration of computing SFE, the lower generation limit of unit 6 at hours 2 to 6 is activated. The dual variable of this constraint is very small at hours 2, 3, 4, and 6. After omitting unit 6 at hours 2, 3, 4, and 6, in the second iteration, the lower generation limit of unit 6 becomes inactive at these hours. Therefore, the system does not have SFE at hours 2, 3, 4, and 6. In other words, activation of the lower generation limit of unit 6 changes the system model and, consequently, the firms' strategies. The new strategies of firms cause the lower generation limit of unit 6 to get inactive and switch to the first system model. This process is continued and prohibits the system to approach its SFE at these hours. This happens in the second scenario at the respected hours.

Moreover, in the second scenario, the market does not have equilibrium at hours 11, 19, 21, and 22. At hour 11, the upper generation limit of unit 3 and the upper ramp rate limits of units 1, 2, 4, and 6 are active, and hence, all units, except unit 5, are omitted. At hour 19, the upper generation limits of units 3 to 5 and the upper ramp rate limits of units 1 and 2 are active, and hence, all units, except unit 6, are omitted. Since the load is inelastic, the bid of unit 5 at hour 11 and the bid of unit 6 at hour 19 tend to approach infinity, and hence, the market has no SFE at hours 11 and 19. At hours 21 and 22, the lower ramp rate limit of unit 1 becomes active, and the associated dual variable is very small. After omitting unit 1 in these hours, the lower ramp rate limit of unit 1 gets inactive at these hours, and hence, the system does not have SFE at hours 21 and 22. This example shows that the smaller ramp rate limits cause the less hours to have SFE.

Having a unit with small ramp rate limits in the second scenario causes the SFE of different hours of the second scenario to deviate from the SFE of the related hours in the first scenario. Among the hours that have SFE, the SFE of hour 11 has the biggest deviation. At the SFE of hour 11, the total absolute generation error is 8.22 MW or 3%, the total profit error is 217.86 \$/h or 8.32%, and the MCP error is 0.84 $\$ \ \mathrm{MW} \cdot \mathrm{h}$ or 2.26%. This shows that ramp rate limits can affect the SFE and, consequently, the oligopolistic opportunities of some hours considerably.

V. Conclusion

In this paper, an SFE model for electricity markets considering ramp rate limits has been presented. A heuristic algorithm for computing market equilibrium has been proposed. The existence of market equilibrium considering ramp rate limits has been examined. The results show that ramp rate limits may considerably affect the existence of market equilibrium. The smaller ramp rate limits cause more hours not to have SFE. Furthermore, the impacts of ramp rate limits on the suppliers' oligopolistic opportunities are studied in this paper. The results of the test system show that applying ramp rate limits on a particular unit could increase or decrease its profit depending on the value of the ramp rate limit. However, ramp rate limits may increase the profit of other firms. Depending on the value of the ramp rate limits, firms' profits at SFE may be considerably affected during some hours. It is worth mentioning that applying ramp rate limits on a unit could actually create an oligopolistic opportunity for its owner. This is in contradiction with this general expectation that applying operating constraints to a unit will decrease its oligopolistic opportunity.

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Appendix
Proofs of Lemmas and Theorems

Proof—Lemma 1: Consider a uniform electricity market. The first-order optimality conditions of ISO optimization (1)–(4) are as follows:

$$\lambda^{(t)} = \lambda_i^{(t)} = \left(\alpha_i^{(t)} + \mu_i^{(t)} + \epsilon_i^{(t)} - \epsilon_i^{(t+1)}\right) + b_i\, Q_{Si}^{(t)},\quad \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \tag{23}$$

$$\lambda^{(t)} = \lambda_i^{(t)} = c_i^{(t)} - d_i^{(t)} Q_{Di}^{(t)}, \forall i \in \mathcal{D}, \forall t \in \mathcal{T} \tag{24}$$

$$Q_{Si}^{(t)} - Q_{Si}^{\max} \le 0 \perp \boldsymbol{\mu}_i^{(t)\max} \ge 0, \forall t \in \mathcal{T}, \forall i \in \mathcal{S} \tag{25}$$

$$Q_{Si}^{\min} - Q_{Si}^{(t)} \le 0 \perp \boldsymbol{\mu}_i^{(t)\min} \ge 0, \forall t \in \mathcal{T}, \forall i \in \mathcal{S} \tag{26}$$

$$Q_{Si}^{(t)} - Q_{Si}^{(t-1)} - Urr_i Q_{Si}^{\max} \le 0 \perp \boldsymbol{\epsilon}_i^{(t)\max} \ge 0, \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \tag{27}$$

$$Q_{Si}^{(t-1)} - Q_{Si}^{(t)} - Drr_i Q_{Si}^{\max} \le 0 \perp \boldsymbol{\epsilon}_i^{(t)\min} \ge 0, \forall t \in \mathcal{T}, \forall i \in \mathcal{S}. \tag{28}$$

Computing $Q_{Sj}^{(t)}$ and $Q_{Dl}^{(t)}$ versus $Q_{Si}^{(t)}$ from (23) and (24) yields

$$Q_{Sj}^{(t)} = \frac{1}{b_j}\left(\left(\alpha_i^{(t)} + \mu_i^{(t)} + \epsilon_i^{(t)} - \epsilon_i^{(t+1)}\right) - \left(\alpha_j^{(t)} + \mu_j^{(t)} + \epsilon_j^{(t)} - \epsilon_j^{(t+1)}\right) + b_i Q_{Si}^{(t)}\right) \forall j \in \mathcal{S}, j \ne i, \forall t \in \mathcal{T} \tag{29}$$

$$Q_{Dl}^{(t)} = \frac{1}{d_l^{(t)}}\left(c_l^{(t)} - \left(\alpha_i^{(t)} + \mu_i^{(t)} + \epsilon_i^{(t)} - \epsilon_i^{(t+1)}\right) - b_i Q_{Si}^{(t)}\right) \forall l \in \mathcal{D}, \forall t \in \mathcal{T}. \tag{30}$$

Substituting $Q_{Sj}^{(t)}$, $Q_{Dl}^{(t)}$, and $\lambda^{(t)}$ from (29), (30), and (23) into (2) yields

$$Q_{Si}^{(t)} = v_i^{(t)} \mathbf{Q}_D^{(t)} + \mathbf{u}_i^{(t)Tr}\left(\boldsymbol{\alpha}^{(t)} + \boldsymbol{\mu}^{(t)} + \boldsymbol{\epsilon}^{(t)} - \boldsymbol{\epsilon}^{(t+1)}\right) \quad \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \tag{31}$$

where $\mathbf{Q}_D^{(t)}$, $v_i^{(t)}$, and the elements of $\mathbf{u}_i^{(t)}$ are defined as follows:

$$\mathbf{Q}_D^{(t)} = \begin{cases} \text{total demand} & \text{for inelastic loads} \\ \sum_{i\in\mathcal{D}} \frac{c_i^{(t)}}{d_i^{(t)}} & \text{for elastic loads} \end{cases} \tag{32}$$

$$v_i^{(t)} = \frac{1}{b_i B^{(t)}} \qquad \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \tag{33}$$

$$u_{i_j}^{(t)} = \frac{1}{b_i b_j B^{(t)}} \qquad \forall i \in \mathcal{S}, i \ne j, \forall t \in \mathcal{T} \tag{34}$$

$$u_{i_i}^{(t)} = \frac{-\left(b_i B^{(t)} - 1\right)}{b_i^2 B^{(t)}} \qquad \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \tag{35}$$

$$B^{(t)} = \begin{cases} \sum_{i\in\mathcal{S}} \frac{1}{b_i} & \text{for inelastic loads} \\ \sum_{i\in\mathcal{S}} \frac{1}{b_i} + \sum_{i\in\mathcal{D}} \frac{1}{d_i^{(t)}} & \text{for elastic loads} \end{cases}. \tag{36}$$

Substituting (23) and (31) into (5) and rearranging it yield (7). The elements of $\mathbf{Q}_f^{(t)}$, $\mathbf{R}_f^{(t)}$, $\mathbf{R}_f^{\prime(t)}$, $\mathbf{S}_f^{\prime(t)}$, and $\mathbf{S}_f^{\prime\prime(t)}$ are defined as follows:

$$Q_{fij}^{(t)} = \frac{1}{2B^{(t)2}} \frac{B_f}{b_i b_j} \forall i \in \mathcal{S}_f, \forall j \in \mathcal{S}_f, i \ne j, \forall t \in \mathcal{T} \tag{37}$$

$$Q_{fii}^{(t)} = \frac{1}{2}\left(\frac{B_f}{B^{(t)2} b_i^2} - \frac{1}{b_i}\right) \forall i \in \mathcal{S}_f, \forall t \in \mathcal{T} \tag{38}$$

$$Q_{fij}^{(t)} = \frac{1}{2B^{(t)2}} \frac{B_f - B^{(t)}}{b_i b_j}, \forall i \in \mathcal{S}_f, \forall j \in \mathcal{S}_{\hat{f}}, \forall t \in \mathcal{T} \tag{39}$$

$$Q_{fij}^{(t)} = \frac{1}{2B^{(t)2}} \frac{B_f + B^{(t)}}{b_i b_j} \forall i \in \mathcal{S}_{\hat{f}}, \forall j \in \mathcal{S}_f, \forall t \in \mathcal{T} \tag{40}$$

$$Q_{fij}^{(t)} = \frac{1}{2B^{(t)2}} \frac{B_f}{b_i b_j} \forall i \in \mathcal{S}_{\hat{f}}, \forall j \in \mathcal{S}_{\hat{f}}, \forall t \in \mathcal{T} \tag{41}$$

$$R_{fi}^{(t)} = \frac{1}{B^{(t)} b_i}\left(a_i B^{(t)} - C_f\right) \forall i \in \mathcal{S}_f, \forall t \in \mathcal{T} \tag{42}$$

$$R_{fi}^{(t)} = \frac{-C_f}{B^{(t)} b_i} \forall i \in \mathcal{S}_{\hat{f}}, R_{fi}^{\prime(t)} = \frac{B_f}{B^{(t)2} b_i}, \forall i \in \mathcal{S}, \forall t \in \mathcal{T} \tag{43}$$

$$S_f^{\prime(t)} = \frac{-C_f}{B^{(t)}}, \qquad S_f^{\prime\prime(t)} = \frac{B_f}{2B^{(t)2}}, \forall t \in \mathcal{T} \tag{44}$$

$$B_f = \sum_{i\in\mathcal{S}_f} \frac{1}{b_i}, \qquad C_f = \sum_{i\in\mathcal{S}_f} \frac{a_i}{b_i} \tag{45}$$

where subscript f indicates firm f and subscript $\hat{f}$ indicates all firms except firm f.

In order to consider generation and ramp rate limits, constraints (25)–(28) are moved to bid space. Substituting (31) into (25)–(28) for all $i \in S$ yields (8)–(13). □

Proof—Lemma 2: The total profit of firm f can be written as follows:

$$\pi_f = \sum_{t\in\mathcal{T}}\left(\sum_{j\in\mathcal{U}_f^{(t)}}\left(\lambda^{(t)} Q_{Sj}^{(t)} - a_j Q_{Sj}^{(t)} - \frac{1}{2} b_j Q_{Sj}^{(t)^2}\right) + \sum_{j\in\mathcal{C}_f^{(t)}}\left(\lambda^{(t)}\overline{Q}_{Sj}^{(t)} - a_j \overline{Q}_{Sj}^{(t)} - \frac{1}{2} b_j \overline{Q}_{Sj}^{(t)^2}\right)\right). \tag{46}$$

The first term of (46) can be written as (7), assuming that the system has only its unbound units at every hour t, and consequently, all related dual variables are zero, i.e., the first term of (46) can be written as follows:

$$\text{The first term of } \pi_f = \sum_{t\in\mathcal{T}}\left(\check{\boldsymbol{\alpha}}^{(t)^{Tr}} \check{\mathbf{Q}}_f^{(t)} \check{\boldsymbol{\alpha}}^{(t)} + \check{\boldsymbol{\alpha}}^{(t)^{Tr}} \check{\mathbf{R}}_f^{(t)} + \left(\check{\boldsymbol{\alpha}}^{(t)^{Tr}} \check{\mathbf{R}}_f^{\prime(t)} + \check{\mathbf{S}}_f^{\prime(t)}\right) \check{\mathbf{Q}}_D^{(t)} + \check{\mathbf{S}}_f^{\prime\prime(t)} \check{\mathbf{Q}}_D^{(t)^2}\right). \tag{47}$$

Suppose that unit i is an unbound unit of firm f; then, $\lambda^{(t)}$ can be written as (23), and $Q_{Si}^{(t)}$ can be written as (31), i.e., as follows:

$$Q_{Si}^{(t)} = \breve{v}_i^{(t)} \breve{\mathbf{Q}}_D^{(t)} + \breve{\mathbf{u}}_i^{(t)^{Tr}} \breve{\boldsymbol{\alpha}}^{(t)} \tag{48}$$

where $\breve{v}_i^{(t)}$ and $\breve{\mathbf{u}}_i^{(t)}$ are computed as $v_i^{(t)}$ and $\mathbf{u}_i^{(t)}$, assuming that the system has only its unbound units at every hour t. Substituting (48) into (23) and the result into the second sigma of (46) and rearranging it yield (18) and (19).

Proof—Theorem 1: Consider a constrained uniform electricity market, and suppose that it has SFE at hour t. Suppose that $\mathcal{C}^{(t)}$ is the set of all bound units at the SFE of hour t. Build the profit functions of firms (18) by omitting the bound generators at hour t, subtracting the sum of power of the active generation and ramp rate limits of hour t from the load of the related hour, and considering the effects of omitted generators in the profit functions of firms using (19). Differentiating from (18) versus $\alpha_i^{(t)}$ $\forall i \in \mathcal{U}_f^{(t)}$, $\forall f \in \mathcal{F}$, and $\forall t \in \mathcal{T}$ and rearranging them yield (20). Based on [3], matrix $\mathbf{H}$ is nonsingular, and hence, the unbound units of hour t have a unique bid. □

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