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New Application of Random Decrement Technique to Structural Health Monitoring using Control Chart

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Abstract
This study aims to introduce a new application of random decrement (RD) technique to structural health monitoring for improving the results of statistical process control (SPC). The RD technique transforms random time series data into free vibration forms by averaging them at a specific time. Autoregressive (AR) model is utilized to fit a mathematical model to the averaged time series data in order to extract residual errors of AR model as a damage-sensitive feature. After that, Shewhart X-bar control chart is applied to identify the damage by subgrouping the residual errors. Acceleration time series data from a laboratory experiment conducted on a three-story frame are exploited to validate the proposed algorithms for the process of damage identification. Based on these data and mentioned procedures, it is possible to demonstrate the superior performance of random decrement technique in the statistical process control as an applicable and useful method for structural health monitoring. Results show that the RD technique improves the process of time series modelling. This technique one can assist to attain better uncorrelated residual errors that should be used in the statistical process control. Furthermore, RD technique reduces the influences of operational and environmental conditions for damage identification due to having the averaging and normalization processes. In addition, the quality control with aid of the X-bar control chart is performed precisely in association with the random decrement technique that clearly discriminates the damaged from undamaged states.

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This study aims to introduce a new application of random decrement (RD) technique to structural health monitoring for improving the results of statistical process control (SPC). The RD technique transforms random time series data into free vibration forms by averaging them at a specific time. Autoregressive (AR) model is utilized to fit a mathematical model to the averaged time series data in order to extract residual errors of AR model as a damage-sensitive feature. After that, Shewhart X-bar control chart is applied to identify the damage by subgrouping the residual errors. Acceleration time series data from a laboratory experiment conducted on a three-story frame are exploited to validate the proposed algorithms for the process of damage identification. Based on these data and mentioned procedures, it is possible to demonstrate the superior performance of random decrement technique in the statistical process control as an applicable and useful method for structural health monitoring. Results show that the RD technique improves the process of time series modelling. This technique one can assist to attain better uncorrelated residual errors that should be used in the statistical process control. Furthermore, RD technique reduces the influences of operational and environmental conditions for damage identification due to having the averaging and normalization processes. In addition, the quality control with aid of the X-bar control chart is performed precisely in association with the random decrement technique that clearly discriminates the damaged from undamaged states.

Keywords: Structural health monitoring; Random decrement; Residual errors; Statistical process control.

1. Introduction

The problem of damage identification takes into account a necessity for many civil infrastructures due to safety, economy, and importance of these structures. Structural health monitoring (SHM) is an experimental-theoretical process that intends to assess the structural conditions for identifying the damage on the basis of vibration data. The fundamental premise of SHM relies on this fact that undesirable changes in the structure including materials, geometric, boundary conditions, system connectivity, and physical characteristics lead to adverse changes in the measured dynamic response of the structure. Recently, data-based methods have received more attention in comparison with model-based approaches in the procedure of structural health monitoring. The data-based methods exploit the raw vibration measurements from dynamic tests or simulation approaches. Thus, it is not necessary to build a finite element model for implementing the damage detection procedure based on changes in the physical properties of structure. The data-based method, on the other hand, focuses on using statistical pattern recognition paradigm in the four-step process: (1) operational evaluation, (2) data acquisition, (3) feature extraction, and (4) statistical modelling for feature classification [1-3].

For reason of using time series data in the most application of SHM, time series analysis should be applied to extract damage-sensitive features. These features are statistical properties of time series models that directly depend on the damage and accurately distinguish a damaged structure from an undamaged one. The damage-sensitive features are significant characteristics, because if they do not have any relation with damage, even the most influential pattern recognition and mathematical algorithms will not improve the damage detection process. Random decrement (RD) technique is one of the methods that can be used to choose appropriate and accurate damage feature. The RD is a dynamic-based technique that transforms a large number of time series data with random property into a free decay response [4].

There are numerous studies and researches about statistical pattern recognition paradigm with aid of time series data. Sohn et al. [5] used the coefficients of an autoregressive (AR) model as the damage-sensitive feature to exploit in the statistical process control for only detecting the presence of damages in the structure. Fugate et al. [1] applied statistical process control such as X-bar and S control charts to detect the damage at a concrete bridge column by residual errors of AR model fitted to the acceleration time series data. Nair et al. [6] utilized an autoregressive moving average (ARMA) model and applied the first three AR parameters as the damage-sensitive feature, which were able to identify and locate the damage. Kullaa [7] detected the damaged occurred in the Z24 bridge using some control charts as on-line and off-line damage detection procedures. Gul and Catbas [8] proposed the random decrement technique for averaging the time series data to extract the damage-sensitive features and also used the Mahalanobis distance method to identify difference types of structural changes on different test structures. Yao and Pakzad [9] proposed new feature extraction techniques using AR model spectra along with its residual autocorrelation function and applied several statistical pattern recognition algorithms to validate their proposed methods.

This study intends to introduce a new application of random decrement technique to structural health monitoring for improving the results of statistical process control (SPC). Prior to feature extraction, the time series data are averaged by RD technique and then AR model is fitted the averaged responses to extract the residual errors as the damage-sensitive feature. By dividing the residuals into subgroups, statistical process control by Shewhart X-bar control chart is applied to identify the structural damage. To validate the proposed algorithms in this study, the acceleration responses of a three-story laboratory frame which simulates the nonlinear damages due to cracks. Results show that the RD technique improves the process of time series modelling by decreasing the model order. This technique one can assist to attain better uncorrelated residual errors that should be used in the statistical process control. Furthermore, RD technique reduces the influences of operational and environmental conditions for damage identification due to having the averaging and normalization process.

2. Random decrement technique

The random decrement (RD) technique is a signal processing procedure, which is applied to transform the random time series data into a free decay (free vibration) response. This technique consists in a rather simple process of averaging time segments of the measured structural responses in the time domain, which in essence contains only information about structural dynamic properties. The RD technique was initially proposed by Cole [10] concerning the analysis of dynamic response of space structures subjected to ambient loads. The RD technique enables to remove the random part of vibration measurements and only utilize the deterministic part of vibration data.

To obtain the random decrement from the stationary random response, the response is divided into a number of segments, N, each of length τ. All of these segments should have a same initial condition, which is known as triggering value or triggering condition. Considering the response time series $x(t)$ and the triggering condition $Tx(t_i)$, the RD function can be mathematically expressed by the following expression:

$$x(\tau)=\frac{1}{N}\sum_{i=1}^{N}x_i(t_i+\tau)\left|Tx(t_i)\right. \tag{1}$$

One important aspect of the RD technique is the definition of triggering condition. There are different equations that one can apply to determine the triggering conditions [11]. However, in this study, level crossing method is applied that uses the standard deviation of the time series data under analysis. Thus:

$$Tx(t_i)=\{x(t_i)=a\} \tag{2}$$

In this case, the optimum level crossing triggering condition is $a = \sqrt{2}\sigma$ [12]. One of the important advantage of RD technique is to reduce the effects of operational and environmental variability. Since the measured dynamic responses in time domain are usually contaminated by noise, RD method removes the noisy part of vibration data by averaging (normalization) procedure. Another advantage of random decrement technique is that it can produce a normalized time series data, which assist to estimate a more efficient time series model with optimal orders. Furthermore, this technique is a useful tool for using in the most statistical methods such as quality control and time series analysis, which leads to achieving more reliable and applicable results in the context of structural health monitoring.

3. Time series analysis by autoregressive model

Autoregressive model is a linear stationary model that is known as the simplest model among all type of time series models [13]. An AR model estimates the value of a mathematical function at time t based on linear combination of its previous values [14]. The basic equation of AR model with p order is defined in the following form:

$$x(t)=\sum_{i=1}^{p}\varphi_{xi}x(t-i)+e(t) \tag{3}$$

where $x(t)$ is the measured time signal at time t; φ_{xi} denotes the AR coefficients; $e(t)$ is the random error or residual of AR model. The residual error, which is the difference between the measured and the predicted signal, is calculated at time t as follows:

$$e(t)=x(t)-\bar{x}(t) \tag{4}$$

In this equation, $\bar{x}(t)$ is the predicted time series data that is computed by the estimated AR model. Typically, the coefficients and residuals of the AR model are introduced as the damage-sensitive features in the SHM [3]. Here, the residual errors of healthy and damaged structures are chosen as the damage features that are computed after the signal processing with aid of the random decrement technique. The idea beyond the residual errors as the damage-sensitive feature is that an AR model is fitted the healthy condition of structure (baseline condition). When this model is desirably modelled, the residuals between measured and predicted data are insubstantial quantities. In the following, this model is applied in the damaged structure to predict the time series responses. As a result, it can be observed that the residual errors associated with the damaged structure will be increased that is indicative of damage occurrence.

In this study, an improved feature extraction method is proposed in order to extract the residuals of AR model. On the basis of this method, all channels of baseline condition are selected as reference points. An AR model for all channels is then fitted so that the residual errors of this model become uncorrelated. For this purpose, Ljung-Box Q-test is used to evaluate the independency of residual errors. Based on this statistical test, an uncorrelated residual has got a P-value less than 0.05. This process can be done by default MATLAB code named as *lbqtest*.

4. Statistical process control

The statistical process control (SPC) is a method of quality control and is applied in order to monitor and control a process [15]. In the structural health monitoring, SPC makes an attempt to discriminate the damaged from the undamaged conditions. This method usually uses control chart limits (threshold values) to monitor the observation of both healthy and damaged structures. If the process is in control (healthy state), the process data vary randomly within the control limits, otherwise, the process data put in out of control limits that indicate an abnormally (damaged) state in the structure.

The Shewhart X-bar control chart is one of the applicable and useful control methods that is utilized to identify when data points fall outside the control limits by using sample means of residuals errors. Assume that a random variable (e.g. residual errors) X is characterized by a normal probability distribution, mean μ, and standard deviation σ. By dividing m samples of X with n observations, the control charts limits such as upper and lower control limits (UCL and LCL) and centreline (CL) are defined as follows:

$$UCL = CL + 3\frac{\bar{\sigma}}{\sqrt{n}} \tag{5}$$

$$CL = \bar{\mu} = \frac{1}{m}\sum_{i=1}^{m}\alpha_i \tag{6}$$

$$LCL = CL - 3\frac{\bar{\sigma}}{\sqrt{n}} \tag{7}$$

In these equations, α_i is the sample average value for the i^{th} sample; $\bar{\sigma}$ is the standard deviation (s) of the distribution of the α_i values, which one can obtain as follows:

$$\bar{\sigma} = \frac{1}{m}\sum_{i=1}^{m}s_i \tag{8}$$

The Shewhart X-bar control charts are constructed by the subgroups of residuals from the AR model. Subgroup of 4 or 5 data points each are recommended as discussed in [15]. To display the X-bar control chart, the sample mean of each subgroup is inserted in the chart with respect to the number of subgroups.

5. Experimental verification

In order to validate the proposed procedures, the acceleration time series data of a three-story laboratory frame are applied. The experimental model and testing process belong to the Engineering

Institute (EI) at Los Alamos National Laboratory (LANL) and test data have been downloaded from http://institute.lanl.gov/ei/software-and-data/data. The structure is an unscaled laboratory frame whose schematic and sensor locations are shown in Fig 1. A comprehensive details of descriptions of the structure are given in [16, 17]. A random vibration load was applied by means of an electro-dynamics shaker to the base floor along the centre line of the frame. The structure was instrumented with four accelerometers mounted at the centre line of each floor on the opposite side from the excitation source to measure the acceleration time history response. The shaker and frame were mounted together on an aluminium baseplate and the entire system rests on rigid foam. The sensor signals were sampled at 320 Hz for 25.6 seconds in duration, which were discretized into 8192 data sampled at 3.125 microsecond intervals.

To induce the structural damage, a centre column was suspended from the third floor. This column was contacted a bumper mounted on the second floor so that the position of the bumper can be adjusted to define diverse structural damages. The source of the damage is a simulation of fatigue cracks to induce nonlinear behaviour that subsequently open and close under excitation forces. The baseline and some damaged conditions of the laboratory frame are shown in Table 1.

Table 1. Damage cases for damage detection in the laboratory frame

State No.	Condition	Description
1	Baseline	Baseline or healthy condition
10	Damaged	Distance between bumper and column tip 0.20 mm
14	Damaged	Distance between bumper and column tip 0.05 mm
17	Damaged	Bumper 0.10 mm from column tip, 1.2 kg added at the base

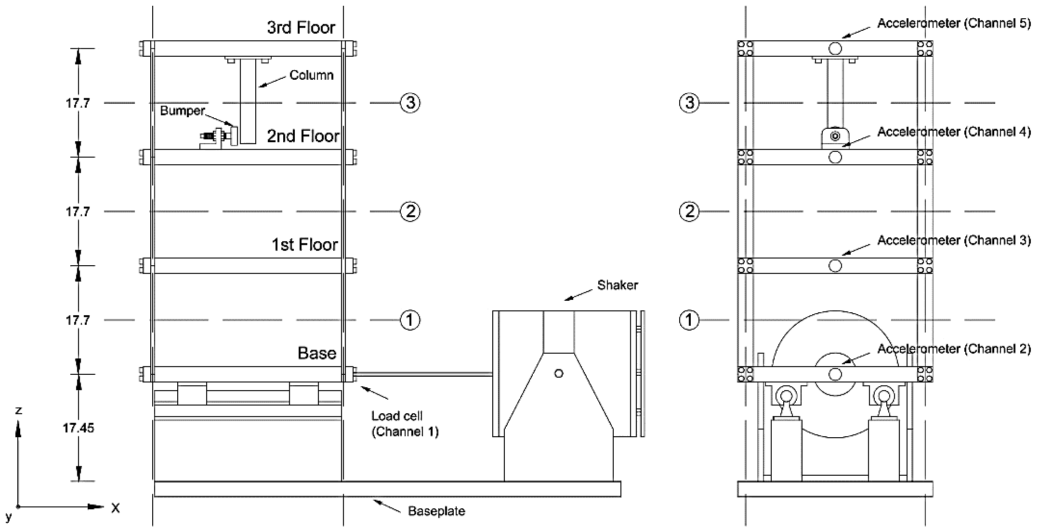


Figure 1. The three-story laboratory frame [16]

With respect to the obtained data, Figs. 2(a) and 2(b) illustrate the acceleration time series responses at the channel 5 in the state #1 (baseline condition) and the state #14 (the highest level of damaged condition), respectively. As can be seen, it is difficult and roughly impossible to visually discriminate the damaged from the undamaged conditions based on the acceleration time series data. The time series analysis assists to extract some useful statistical features such as the coefficients and the residual errors of time series models that are sensitive to the structural damages.

Before fitting an AR model to the acquired acceleration time series data, the random decrement technique is implemented to average the random time series data. On the basis of this technique, it is possible to create free decay time responses that one can normalize the original time series data. Hence, Figs. 2(a) and 2(b) show the averaged acceleration time series data at one sec for states #1 and #14, respectively.

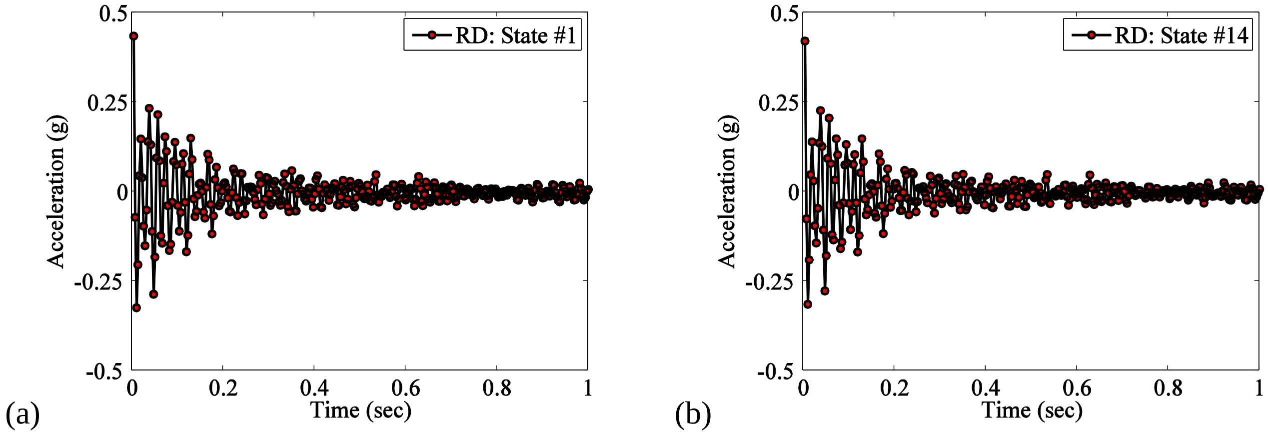
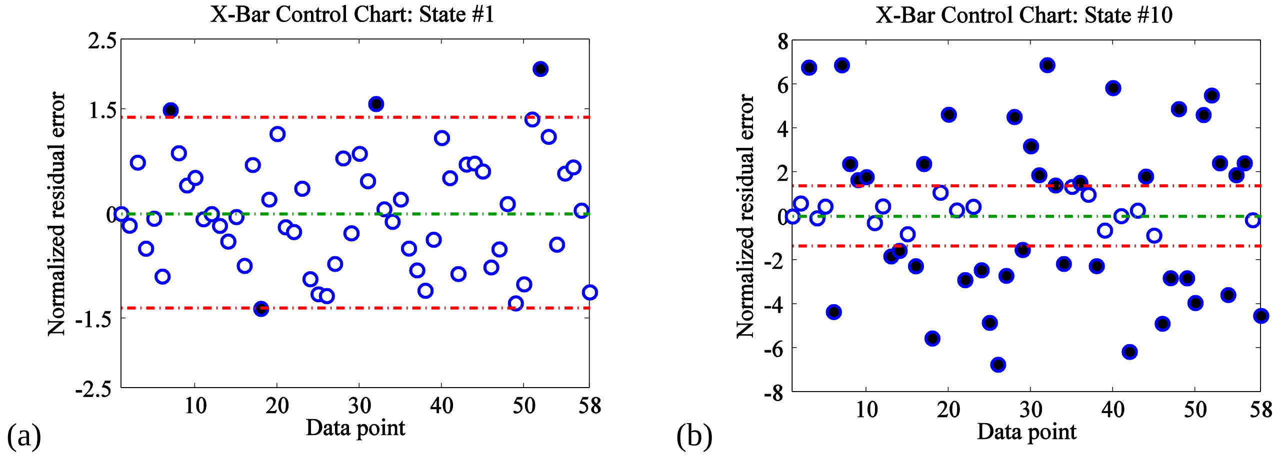
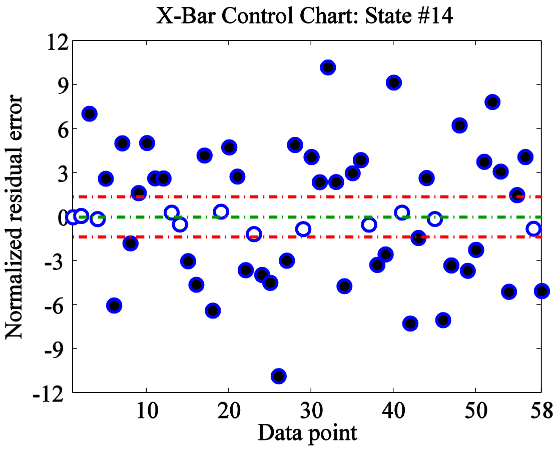


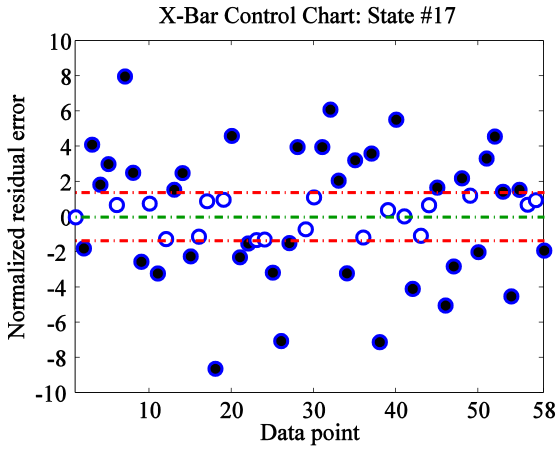
Figure 2. Random decrement technique of vibration data at the channel #5: (a) state #1, (b) state #14

Appearance of the free vibration form is a reasonable reason for choosing the averaging time in the random decrement technique. As can be observed from Figs. 2(a) and 2(b), both the acceleration responses have the free vibration style and this averaging time (1 sec) is valid for other acceleration time histories. After constructing the free vibration responses for all states, an AR model is fitted the normalized data in such a way that the uncorrelated residual errors can be extracted from this model. It is important to note that the residuals of AR model should be uncorrelated (independent) for using in the statistical process control [1, 15]. In such circumstance, the order of AR model plays a significant role in producing the uncorrelated residual errors. Basically, it is a desirable property that one can fit a time series model with little coefficients to the vibration data. In other words, a high-order model may better fit the data, but may not conveniently generalize other time series data. By contrast, a low-order model may not capture the underlying dynamic properties of structure [17]. On the basis of improved feature extraction algorithm described in the Section 3, AR(30) is selected for all acceleration responses of healthy state, which have been normalized by random decrement. In the following, the normalized acceleration responses of all damaged conditions are predicted with AR(30). Eventually, the residual errors of both the undamaged and damaged structures are obtained so that an increasing form can be indicated in the residuals of damaged states. For using the residuals in the X-bar control chart, each residual is reorganized in subgroups of 5. Based on the normalized time series data by RD technique, there are 320 acceleration measurements. Residuals can be computed for acceleration measurements 30-320. This process produces 58 subgroups of the uncorrelated residuals each of size 5. Prior to constructing subgroups, the residuals are normalized by subtracting the mean and dividing by the standard deviation of the residuals from baseline condition. This data normalization procedure based on the baseline condition is applied to all residuals data of the damaged states.





(c)



(d)

Figure 3. Statistical process control by X-bar chart: (a) state #1, (b) state #10, (c) state #14, (d) state #17

Figs. 3(a)-(d) indicate the X-bar control charts for all structural conditions shown in Table 1, respectively. The centreline of the charts is zero because the sample mean of the associated normalized errors is zero. Furthermore, the upper and lower control chart limits are given by 1.3642 and -1.3642. Fig. 3(a) shows that all sample means of the residuals of baseline condition fall into the control limits and therefore, there is no damage in this state. On the other hand, Figs. 3(b), 3(c) and 3(d) illustrate the damaged conditions in the laboratory frame, because the majority of sample means of the subgroups fall outside of the UCL and LCL that obviously demonstrate the damage occurrence in the structure.

It should be noted that the state #17 represents the damaged situation along with the operational and environmental variability. Hence, it can be inferred from Fig. 3(d) that many outliers (damaged points) have put outside of the control limits. Such result one can approve the capability of the random decrement technique for decreasing the influences of operational and environmental conditions, because the result of quality control in the state #17 is similar to the states #10 and #14, which the operational and environmental variability have not been defined in these states.

6. Conclusions

This paper introduces a new application of random decrement technique to structural health monitoring for improvement of the statistical process control results. The Shewhart X-bar control chart is used to identify the damage based on residual errors of autoregressive (AR) model that are determined after transforming the original acceleration time series data into free vibration responses through the random decrement technique. The main reason for using AR model is that the content of time series data is consistent with this model. For validation of the proposed method, the acceleration responses of a three-story laboratory frame are utilized. Random decrement technique improves the time series modelling due to its inherent property about averaging the time series data. Number of model's order reduces when the original time series data are normalized with RD. It helps to attain better and more convenience uncorrelated data that should be used in the quality control. Moreover, the random decrement technique leads to achieving more little residual errors so that one can remove some unusual data from the time series data, which have not any relation with the damage-sensitive features. One limitation in the control chart is the presence of some observations outside the control limits that may be the effects of operational and environmental conditions. On the basis of control chart results, the RD technique decreases the influences of operational and environmental variations on the damage identification process. Eventually, the procedure of damage identification by X-bar control chart has accurate and precise results, which prove the new application of random decrement technique for improving the results of statistical process control.

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