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Numerical Simulation of Ambient Vibration Test based on Dynamic Analysis in Time Domain

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Abstract

Structural health monitoring (SHM) is a process based on damage identification algorithms to provide an economical and possibly more reliable solution to infrastructure condition assessment. The main principle of SHM is based on using vibration data that are acquired by dynamic tests. Ambient loads are one of the useful and applicable vibration forces that are widely used in the civil infrastructure. The main objective of this study is to compare some numerical methods including state space method (SSM), linear Newmark method (LNM), and central difference method (CDM) for the simulation of acceleration time history caused by applying the ambient vibration. In order to demonstrate the results of numerical simulation, a numerical model of the reinforced concrete beam is constructed based of finite element modelling. Caughey damping model is applied rather than Rayleigh model due to considering two more modes in the dynamic behaviour of structure. To excite the structure, different Gaussian white noises are subjected to all the transitional degree of freedoms that virtually equipped with sensors. Numerical results show that both SSM and LNM one can reliably simulate the acceleration responses; however, the state space method obtains more accurate responses. In contrast, the CDM cannot provide meaningful and trustworthy results.

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Abstract

Structural health monitoring (SHM) is a process based on damage identification algorithms to provide an economical and possibly more reliable solution to infrastructure condition assessment. The main principle of SHM is based on using vibration data that are acquired by dynamic tests. Ambient loads are one of the useful and applicable vibration forces that are widely used in the civil infrastructure. The main objective of this study is to compare some numerical methods including state space method (SSM), linear Newmark method (LNM), and central difference method (CDM) for the simulation of acceleration time history caused by applying the ambient vibration. In order to demonstrate the results of numerical simulation, a numerical model of the reinforced concrete beam is constructed based of finite element modelling. Caughey damping model is applied rather than Rayleigh model due to considering two more modes in the dynamic behaviour of structure. To excite the structure, different Gaussian white noises are subjected to all the transitional degree of freedoms that virtually equipped with sensors. Numerical results show that both SSM and LNM one can reliably simulate the acceleration responses; however, the state space method obtains more accurate responses. In contrast, the CDM cannot provide meaningful and trustworthy results.

Keywords: Dynamic analysis; time domain; ambient vibration; numerical simulation.

1. Introduction

The ambient vibration tests describe the linear behaviour of structures due to inducing low-amplitude vibration on the structures. Furthermore, these tests can be applied to describe the linear behaviour of damaged structures and use in structural health monitoring. The process of implementing a damage detection strategy by vibration data is referred to as the structural health monitoring [1]. As a matter of fact, SHM evaluates the integrity of the structures by observing their measured vibration data by an array of sensors and assesses the current health state or damage identification. Implementation of SHM process is usually carried out by vibration data. In this regard, the dynamic tests including forced vibration tests and ambient vibration tests are utilized to measure the vibration data in the time or frequency domains.

In the resent years, many researchers have paid more attention to use raw vibration measurements in time domain in the context of SHM with aid of statistical pattern recognition paradigm [2-5]. Although, the forced and ambient vibration tests can be subjected to the structure for acquiring the vibration responses, the forced vibration tests may require large forces to extract essential and useful vibration information from tested structure whereas the ambient vibration tests require low-amplitude vibration sources with employing light equipment along with smaller number of operators. On the other hand, implementation of dynamic tests, particularly ambient vibration test, may be a complex, expensive and time-consuming process. Sometimes, it is not possible to carry out the dynamic test and acquire the measured vibration data. In such circumstances, simulation of dynamic tests is a useful and applicable tool that is investigated by many researchers.

On the basis of using ambient vibration in the SHM, Caicedo et al. [6] simulated the acceleration response data from an analytical model of an existing physical structure applying a benchmark study belonging to IASC–ASCE Task Group on Structural Health Monitoring. Zheng and Mita [7] presented two statistical distance method for structural damage detection based on numerical simulation of a five-story shear building through simulation of Gaussian white noise as a source of ambient vibration. Yao and Pakzad [5] proposed new damage-sensitive features and verified their research by modelling a four degree of freedom mass-spring-damper system subjected to white noise excitation. Makki Alamdari et al. [8] utilized two popular civil engineering systems including a beam and a planner truss for demonstrating the accuracy of proposed symbolic time series analysis for structural health monitoring. The simulation procedure was based on using the random white noise for simulating the ambient vibration tests in such a way that the time series responses were extracted at each nodal point by introducing state-space vector.

Although the numerous simulation techniques are available, there are some weaknesses and ambiguities that should be obviated. Therefore, this study intends to introduce a useful way for simulation of the ambient vibration test by Gaussian white noise in order to extract the time series responses of the tested structure. The main objective of this article is to deal with the presence drawbacks in the numerical simulation of ambient tests. Accurate modelling of damping matrix, excitation of structure by white noise subjected to the whole of structure (all nodes), considering linear model of damaged structure used in numerical simulation are the most problems that will be investigated. In this regard, the classical damping model using Caughey damping method is utilized that one can assist to provide more realistic vibration measurements. Dynamic analysis of structure in the time domain is performed by Newmark-beta approach with respect to the linear behaviour of healthy and damaged structures. In the following, FE models of a reinforced concrete beam containing the healthy and damaged conditions are numerically modelled to extract the acceleration time series responses. The difference Gaussian white noise is subjected to all nodes of beam to simulate the ambient vibration. Numerical results indicate that the obviated problems such as Caughey damping model, Newmark-beta for linear systems, and Gaussian white noise can be applicable tools for simulation of ambient vibration test in the research areas.

2. Modelling of structural systems

Finite element model of a structure is the main part of numerical simulation of dynamic tests. It is important because the dynamic responses depend directly on the physical properties of structure. For instance, the eigenvalue problem expresses a direct relationship between mass and stiffness matrices with modal data including modal frequency and mode shape. In similar, the dynamic responses in time domain are also dependent to the physical feature of structures. In dynamic problems, these feature are usually known as spatial model of structure. With this model, it is possible to construct an analytical model that treats as a real structure. For this purpose, finite element method (FEM) is used to determine the physical properties of structure such as mass, stiffness and damping matrices. There are many technical literatures and publications associated with dynamics of structures and FEM that can be referred to model the popular dynamic systems in the civil engineering field such as beam, truss, and frame. For more details about FEM modelling, refer to [9].

By considering the classical damping matrix, Rayleigh or Caughey damping methods can be used in the modelling of the structure. The Rayleigh damping method only utilizes two modes, while the Caughey approach is an extensive method of Rayleigh approach that uses more than two modes for computing the damping matrix [10]. Many authors employ Rayleigh method due to its simplification, however, for large structures this method may not provide appropriate simulation of damping matrix. Therefore, in this study, Caughey method is applied to specify values for damping matrix at the first j^{th} modes. It is worthwhile remaking that the eigenvalue problem with aid of the mass and stiffness matrices are used to identify the modal data. On the other hand, since the proportional (classical) damping matrix is utilized in the numerical modelling of structure, the modal data are extracted as real data.

3. Numerical methods for simulation of dynamic analysis

Most of the numerical methods for simulation of time response in dynamic systems attempt to calculate the responses from the second order differential equation with N degree of freedoms (DOFs). This equation one can be expressed as follows:

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=F(t) \tag{1}$$

where M, C and K are the mass, damping and stiffness matrices of linear multi degree of freedom system, respectively. These are known as inherent properties of the dynamic systems that are mostly obtained from finite element method (FEM). Moreover, F is the input force vector. The responses of system are the displacement $x(t)$, the velocity $\dot{x}(t)$ and the acceleration $\ddot{x}(t)$. Among these responses, the time series acceleration response is one of the most applicable quantities that are widely used in the new branch of vibration-based techniques, particularly in the structural health monitoring. In the following, three well-known numerical approaches named as difference central, Newmark-β, and state space methods are compared to choose an appropriate method that is more consistent with the simulation of ambient vibration. These methods

3.1 Difference central method

The difference central method is based on a finite difference approximation of the time derivative of displacement. Taking constant time step into consideration, the central difference expressions for velocity and acceleration are given by:

$$\dot{x}_i=\frac{x_{i+1}-x_{i-1}}{2\Delta t} \tag{2}$$

$$\ddot{x}_i=\frac{x_{i+1}-2x_i+x_{i-1}}{\Delta t^2} \tag{3}$$

Substituting these equations into Eq. (1), the dynamic equilibrium formulation is only expressed on the basis of displacement at the difference sample time. On the basis of the central difference method, it is necessary to establish the numerical model of dynamic system such as mass, damping, and stiffness matrix along with initial displacement and velocity responses. In should be noted that for multivariate response data, the initial displacement and initial velocity will no longer scalar quantities and should be expressed as vectors with m data point, where m is the number of data point in these matrices. Table 1 summarizes the calculation of dynamic responses by the central difference method.

Table 1. Central difference method for dynamic analysis in the time domain

Step 1:	Consider mass, damping, stiffness, initial displacement and velocity
Step 2:	Solve $M\ddot{x}_0 = F_0 - C\dot{x}_0 - Kx_0$ to determine the initial acceleration response $\ddot{x}_0$ at t = 0
Step 3:	Initial calculations:
	$x_{-1} = x_0 - \Delta t\,\dot{x}_0 + \frac{\Delta t^2}{2}\ddot{x}_0$; $G = \frac{M}{\Delta t^2} + \frac{C}{2\Delta t}$; $a = \frac{M}{\Delta t^2} - \frac{C}{2\Delta t}$ and $b = K - \frac{2M}{\Delta t^2}$
Step 4:	Computations for time step *i*:
	Solve: $h_i = F_i - a\,x_{i-1} - b\,x_i$ and $G\,x_{i+1} = h_i$
	Use Eqs. (2) and (3) for the calculation of velocity and acceleration responses.
Step 5:	Repetition for the next time step.

3.2 Linear Newmark method

The linear Newmark method is a numerical technique used to calculate responses of a structure for both linear and nonlinear conditions [11]. This method has is categorized into two group including acceleration-based and displacement-based approaches. Due to importance of the acceleration time histories in most application of vibration-based techniques, here this study only focuses on the first group to calculate the acceleration time histories. The Newmark's method one can use based on two special cases such as constant average acceleration and linear acceleration methods whose formulations depend on using two coefficients γ and β. These define the variation of acceleration over a time step and determine the stability and accuracy characterises of the method [10]. Basically, the Newmark method is based on a solution of an incremental form of the dynamic equilibrium equation as:

$$M\ddot{x}_{i+1} + C\dot{x}_{i+1} + Kx_{i+1} = F_{i+1} \tag{4}$$

in which

$$\ddot{x}_{i+1} = \frac{1}{\beta\Delta t^2}\left(x_{i+1} - x_i\right) - \frac{1}{\beta\Delta t}\dot{x}_i - \left(\frac{1}{2\beta} - 1\right)\ddot{x}_i \tag{5}$$

$$\dot{x}_{i+1} = \frac{\gamma}{\beta\Delta t}\left(x_{i+1} - x_i\right) + \left(1 - \frac{\gamma}{\beta}\right)\dot{x}_i + \Delta t\left(1 - \frac{\gamma}{2\beta}\right)\ddot{x}_i \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (4) one can determine the displacement at time *i+1* on the basis of several different constant parameters. Table 2 summarizes the calculation of dynamic responses by the Newmark's method.

Table 2. Linear Newmark method for dynamic analysis in the time domain

Step 1:	Consider mass, damping, stiffness, initial displacement and velocity
Step 2:	Select the coefficients γ and β:
	Constant average acceleration method : $\gamma = 1/2$, $\beta = 1/4$
	Linear acceleration method: $\gamma = 1/2$, $\beta = 1/6$
Step 3:	Solve $M \cdot \ddot{x}_0 = F_0 - C\dot{x}_0 - Kx_0$ to calculate the initial acceleration response $\ddot{x}_0$ at $t = 0$.
Step 4:	Initial calculations:
	$a_1 = \frac{1}{\beta\Delta t^2}M + \frac{\gamma}{\beta\Delta t}C;$ $a_2 = \frac{1}{\beta\Delta t}M + \left(\frac{\gamma}{\beta} - 1\right)C;$ $a_3 = \left(\frac{1}{2\beta} - 1\right)M + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)C$,
	$G = K + a_1$
Step 5:	Calculation for each time step, $i = 0,1,2,\ldots$
	$h_{i+1} = F_{i+1} + a_1 x_i + a_2\dot{x}_i + a_3\ddot{x}_i$
	and solve $G.x_{i+1} = h_{i+1}$ to determine the displacement x_{i+1}
	Use Eqs. (5) and (6) for the calculation of acceleration and velocity responses.
Step 6:	Repetition for the next time step.

3.3 State space method

As the dynamic system becomes more complex, using simple and approximate method with many assumptions impose cumbersome efforts. This is even more true for a system with a number of inputs and outputs. To overcome these issues, state space method one can be introduce to simulate dynamic responses in the time domain.

In this method, the second differential dynamic equation transform to the first differential equation in the state space domain. Thus, the decreased form of Eq. (1) one can write as follows:

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu \\ y &= H\,z(t) \end{aligned} \tag{7}$$

where $z(t) = \{x(t) \quad \dot{x}(t)\}^T$ is the state vector; u is a vector representing the input and y is a vector representing the observed output. The matrices A, B and H determine the relationships between the input, output and the state variables. The state space formulation in Eq. (6) can be rewritten as:

$$\begin{aligned} \begin{Bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{Bmatrix} &= \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} F \\ y &= \begin{bmatrix} I & 0 \end{bmatrix} \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} \end{aligned} \tag{8}$$

where *I* is the identity matrix. For calculation of the dynamic responses, it is necessary to solve the first expression in Eq. (8) to determine the space vector *z*(*t*), which consists of the displacement and velocity responses. Considering the initial condition for state vector, z(0), and input vector u, the solution the first-order differential equation of state space formulation is given by:

$$z(t) = e^{At} z(0) + e^{At} \int_0^t e^{-A\tau} Bu(\tau) d\tau \tag{9}$$

The full state response consists of two part: the first is the homogenous solution of the first-order differential equation, and the second term is the particular solution that is dependent on the input vector. In most cases, the initial condition of state vector is zero; therefore, only the second term of Eq. (9) is used to determine the state vector. The use of this equation in the numerical simulation of dynamic responses is not common. For sake of simplicity, a default code named as *lsim* available in MATLAB commercial software is applied to obtain the state vector. To compute the acceleration responses, the state vector are inserted into the first expression of the state space equation.

4. Simulation of ambient vibration by white noise

The dynamic tests by ambient vibration (such as traffic, wind, microtremors and etc.) are based on acquiring the dynamic responses of the structure under tests with non-controllable and immeasurable vibration loadings. Since the ambient vibration is immeasurable and unknown quantities, it is only possible to measure the response (output) of structure. Under this theory, the structure can be excited numerically with some reasonable assumptions such as stationary distribution and zero mean for vibration loadings. Based on these assumptions, white noise takes into account a good source for excitation of structure.

A white noise vector is a random vector, which its components have a probability distribution with constant mean and finite variance. If the white noise vector has got a normal distribution with zero mean and the same variance, the vector is called as a Gaussian white noise vector. If the structure is excited by white noise, that is, all modes are equally excited and the dynamic responses contain full information about the structure [12]. Thus, the white noise data are appropriate vibration sources for using in the numerical applications in order to simulate the ambient vibration.

5. Numerical simulation of a reinforced concrete beam

To simulate acceleration time histories based on the ambient vibration, a numerical model of the reinforced concrete beam is constructed with dimensions 3000 mm × 250 mm × 250 mm, as shown in Fig. 1. The material properties consist of the module of elasticity E = 22.5 GPa, the concrete density ρ = 2400 Kg/m^3 and Poisson ratio υ = 0.2. The beam is discretized by 15 elements and 16 nodes with two lateral and rotational degrees of freedom (DOFs) at each element. The simple boundary condition including two hinged support conditions are assigned at the ends of the beam. The Euler-Bernoulli beam theory is applied to establish the finite element (FE) model with aid of an FE code implemented in MATLAB. The Caughey damping is considered to determine the damping matrix with assumption of identifying the first five modes. Having obtained the mass and stiffness matrices of beam, the eigenvalue problem is utilized to determine the modal data such as natural frequencies and mode shapes. Assume that the damping ratio of each mode is 5%.

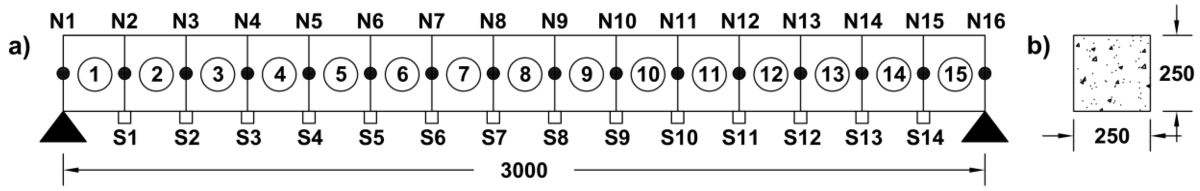

Figure 1. The simulated beam: (a) the FE model along with the sensors location, (b) the cross section

Different continuous-time Gaussian white noise are adopted to all transitional degree of freedoms by a default MATLAB code named as *wgn* with 12000 data points and power of vibration loads 0.1. The time sampling for each white noise vectors is 0.01 sec, and 120 sec is chosen to acquire time series data. Fig. 2 indicates two sample Gaussian white noise subjected to the location of sensor 6 and 10, respectively.

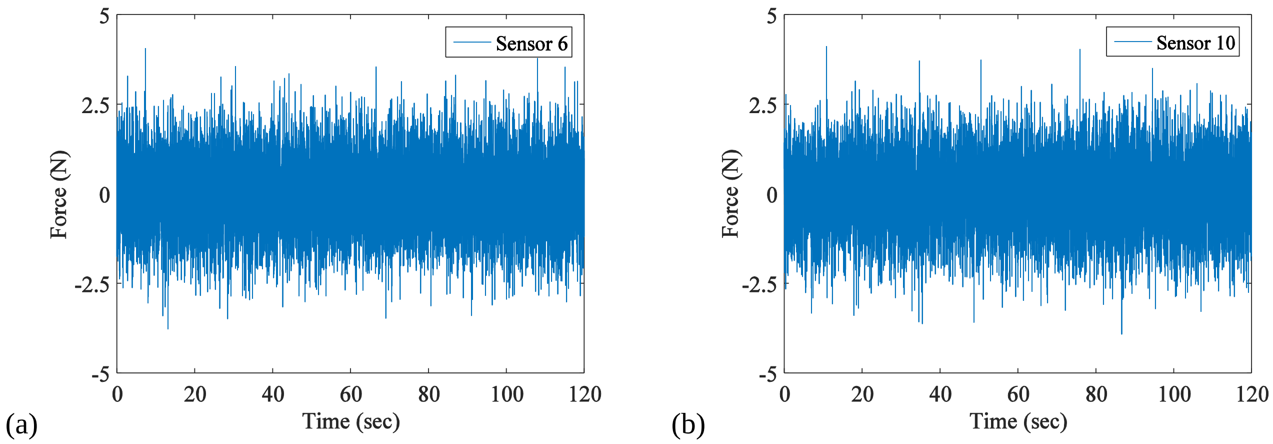

Figure 2. The simulation of ambient vibration by Gaussian white noise: (a) sensor #6, (b) sensor #10

It is assumed that the numerical model of beam is a time-invariant dynamic system. This assumption is valid for both undamaged and damaged conditions because the physical properties of structure in both the conditions are constant during applying the ambient vibration to structure. As stated, the ambient vibrations are low-amplitude vibration sources and make a linear behaviour in the structure. Therefore, it is an accurate assumption that the structure remains invariant when the ambient vibration applied to the damaged structure. After modelling the beam and inducing the simulated ambient excitations, the acceleration time histories are calculated by the methods mentioned in the previous section. Fig. 4 shows the result of acceleration response simulation by the central difference method at the location of sensors 6 and 10, respectively.

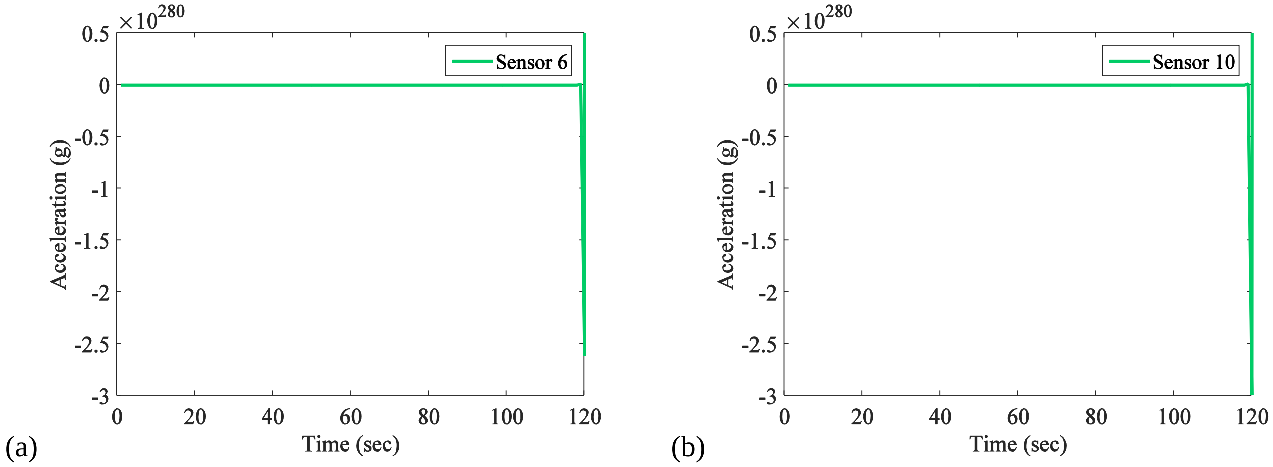

Figure 3. The acceleration time histories computed by CDM: (a) sensor 6, (b) sensor 10

As can been observed from Fig. 3, it is clear that the central difference method is not able to determine the acceleration response. For linear and nonlinear conditions, the acceleration responses should be had stationary properties, which consist of zero mean and constant variance. Due to the lack of any stationary features in the acceleration time histories, it can be seen that the CDM is not robust method for dynamic analysis of complex and continuous structures. Another main reason for this issue is a large difference between magnitude of the mass and stiffness matrices. By comparing the values of elements of these matrices one can understand that the elements of stiffness matrix are much more than the corresponding elements in the mass matrix. Thus, this situation leads to achieving inaccurate results. This process is also valid for other structures with such circumstances.

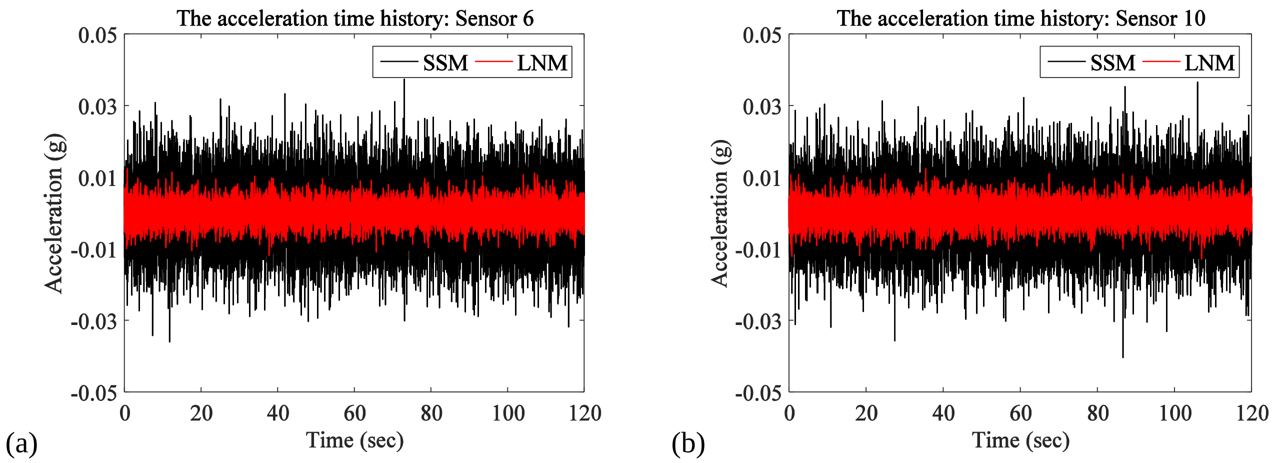

Figure 4. The acceleration time histories computed by LNM and SSM: (a) sensor 6, (b) sensor 10

Fig. 4 presents the results of dynamic analysis by the state space and the linear Newmark methods at the sensors 6 and 10, respectively. In this figure, the red plot belongs to the linear Newmark method (LNM) and the black plot shows the acceleration time histories gained by the state space method (SSM). Both plots have stationary random properties since the variations of acceleration responses are constant. The type of dynamic system and the content of structural properties do not have any influence on the results of dynamic simulation. The underlying difference between SSM and LNM is concerned with the magnitude of acceleration responses. The SSM provide the acceleration quantities more than the corresponding ones in the LNM. It may be caused by the mathematical structure of both methods. The state space method is a robust and efficient approach so that there is no assumption and limitation in its formulation, while the linear Newmark method uses several numerical assumptions that may be affected the results. In addition, the principle of

LNM relies on two numerical coefficients in such a way that other computations depend directly on these coefficients. In conclusion, it is better to state that among these methods, the SSM provide more accurate and reliable results in comparison with LNM.

6. Conclusions

The main objective of this article is to compare several numerical methods in order that simulating the dynamic responses in the time domain along with a simulation of ambient vibration. Another aim of this study is to overcome some weaknesses and ambiguities in the previous techniques such as choosing the appropriate damping model and the accurate way of vibration of a structure with Gaussian white noise as a good simulation of the ambient vibration. To achieve these purposes, three numerical methods named as state space method (SSM), linear Newmark method (LNM) and central difference method (CDM) are applied.

In order to demonstrate the capability and efficiency of the simulation approaches, a numerical model of the reinforced concrete beam was constructed by Euler-Bernoulli beam theory with aid of FE method. For modelling the damping matrix, the Caughey technique was applied rather than using the Rayleigh method. To excite the beam, different Gaussian white noises are only subjected to the transitional degree of freedoms, which make the more realistic simulation process regarding the ambient vibration. The research presented in this article leads to the following conclusions:

- Among all the mentioned methods, CDM could not provide the accurate and reliable acceleration time histories since there were no meaningful results in the responses. The presence of small quantities in the elements of mass matrix in comparison with the corresponding ones in the stiffness matrix was the main reason to obtain the wrong results. This consequence is also valid for another type of structure such as shear buildings, dynamic discrete or continuous systems all of them with small mass matrix compared with the stiffness matrix. Therefore, the central difference method is not a reliable approach for the light and flexible structures.
- Both the state space and linear Newmark methods provided the meaningful and reliable acceleration responses due to the presence of stationary property in the responses. Therefore, the type of structure and its inherent properties did not any influences on the dynamic responses.
- The SSM obtained more accurate results in comparison with the LNM since it did not use any assumptions and numerical coefficients, which mainly applied to the linear Newmark method.
- The Caughey damping model was more suitable for dynamic analysis of continuous systems since there were two more of the mode shape in the dynamic response of structure.
- To make a realistic simulation, it is better to apply the diverse white noises to whole of the structure due to the effect of random content available in the ambient vibration.

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