

# Repeatability Analysis of a SCARA Robot with Planetary Gearbox

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**Abstract**—Repeatability is one of the most important features of an industrial robot. It is a key feature stated by commercial robot manufacturers. Details of repeatability calculation method for industrial robots is presented by ISO 9283. However, the well-known robot manufactures do not state how they calculate their repeatability. In this paper, the industrial FUM SCARA robot, designed by the Ferdowsi University of Mashhad in Iran, is used and its repeatability is measured according to the ISO 9283 method. Planetary reducers are applied as a part of the power transmission system. Although planetary reducers have pre-defined backlash, experimental results show satisfactory repeatability for the FUM SCARA. An ideal and intelligible mathematical model of backlash is developed to show the low effect of backlash in the repeatability calculation. Consequently, it is shown that the result of ideal backlash model corresponds with the experimental result.

**Keywords**—repeatability; SCARA; industrial robot; backlash; ISO 9283

## I. INTRODUCTION

Nowadays robotic manipulators have an essential and undeniable role in the industry. A wide variety of robots for different applications are available in the market. Choosing the right robot is mostly based on the following parameters:

- End-effector velocity
- Pricing
- Repeatability

The parameters are listed in most industrial robots' catalogs. Higher velocity and better repeatability cause obviously higher price. Robot design considering low price with competitive velocity and repeatability is a challenging goal. FUM SCARA robot is introduced and its design process illustrated in [1]. Three joints of the FUM SCARA utilize planetary gearbox as their transmission, which has a great rigidity and minor backlash. Some theoretical models are presented for planetary gearbox backlash. Researchers try to model the backlash in order to control robots more accurately. Backlash modeling, simulation, and experimental results have been presented in [2]. This modeling is applied in motion control. Non-linear nature of the backlash is a problem for controlling systems and is studied in [3]. Meshing gears in an electromechanical system is modeled using differential equations and a nonlinear spring-damper. This model is

utilized for adaptive controller scheme. Reference [4] develops a simulation system for gear train models. This system has incorporated the nonlinear behavior caused by backlash and modeled this behavior using an impact pair. Therefore different controllers have been evaluated. Modeling the backlash of gear drive is presented in [5]. This model has been presented for a simple prismatic joint and the backlash, including a compliant shaft has been illustrated in detail. The presented paper implemented ISO 9283 and developed a backlash model for the angular motion according to [5].

The rest of the paper is organized as follows. In section II and III, ISO 9283 is investigated and implemented on FUM SCARA, respectively. Section IV presents theoretical development which corresponds experimental results. Finally concluding remarks are made in the last section of the paper.

## II. REPEATABILITY TEST OF AN INDUSTRIAL ROBOT

There are two concepts in motion control, accuracy and repeatability. Although the repeatability of most industrial manipulators is quite good, the accuracy is usually much worse and varies quite a bit from manipulator to manipulator [6].

In general, repeatability is more important than accuracy. In most cases, inaccuracies are symmetric and can be corrected or compensated because they can be predicted and measured. This is commonly done for CNC machines. On the other hand, repeatability is generally random and cannot be easily compensated [7].

Our literature search of the commercial robot catalogs, did not reveal any information on robot accuracy. This shows that for the industrial robots, accuracy is not critical to present as a robot specification.

ISO 9283 presents a systematic method to calculate accuracy and repeatability [8]. However, industrial robot manufacturers do not mention their selected repeatability calculation method. The ISO 9283 method is utilized and applied to the FUM SCARA robot as follows.

The procedure requires that first a cube to be selected in the robot's workspace. The cube must have these features:

- It has to be the biggest cube located in the workspace

- Edges of the cube have to be parallel to the base coordinate system

The cube is depicted in Fig. 1 for the FUM SCARA robot. After selecting the appropriate cube, four other subjects must be considered.

### A. Path

A path must be selected. To do this, choose a diagonal plane in the cube among all nine possible diagonal planes. Then a smaller square is selected in this plane. The exact position of the square is defined by the parameter,  $m$ , as depicted in Fig. 2. The value of  $m$  is calculated as in (1) where  $L$  is the spatial diagonal length of the cube. The desired linear path, trajectory, needs to be located within this square.

$$m = (0.1 \pm 0.02) \times L \quad (1)$$

### B. Number of cycles

Based on the ISO 9283, the desired trajectory has to be traced repeatedly. The first cycle is not considered in the repeatability calculation. The desired trajectory has to be repeated 30 times.

### C. Velocity

Rated velocity must be considered for the robot's end-effector.

### D. Load

Rated load must be considered as robot's payload.

## III. FUM SCARA REPEATABILITY TEST

Experimental results for all four DOF of the FUM SCARA repeatability test are investigated [9]. Commercial SCARA robot manufactures describe repeatability in the following formats [10]:

- XY repeatability value
- Z repeatability value
- $\theta$  repeatability value

The available measurement device for the test is a calibrated dial indicator. The device is approved with ISO 9283. Unlike the commercial robot manufactures, the repeatability is performed for all four Cartesian DOFs of the SCARA robot. Fig. 3 shows the dial indicator used for the Y axis. The repeatability test consists of a set of 30 cycles. The results are shown in table I.

The calculation process of  $X$ , the first column of the table I, is as follows. Equations (2) through (5) are applied to each set of the test results to calculate the repeatability value of each coordinate axis. The repeatability result of each axis is shown in the last row of the table I.

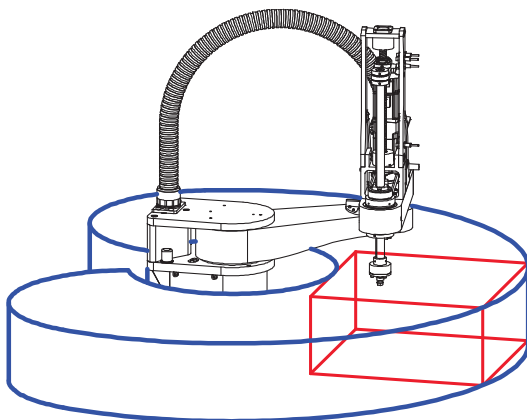


Fig. 1. Selected cube based on ISO 9283

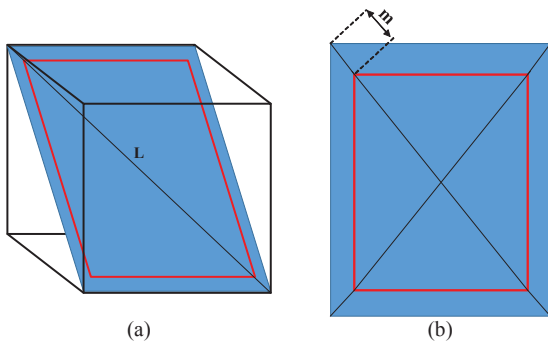
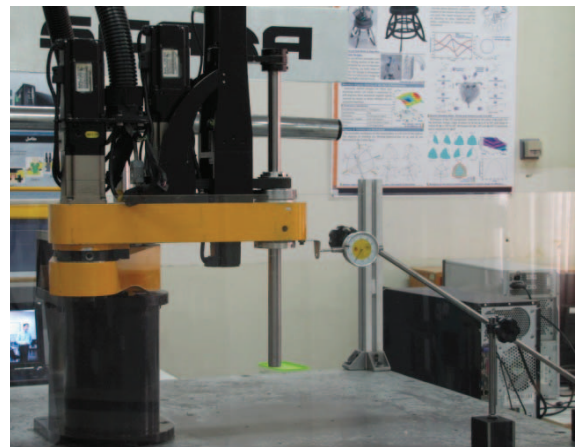
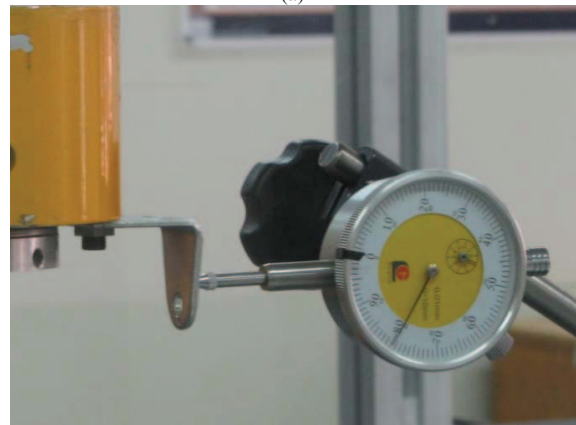


Fig. 2. Selected square on a diagonal plane of proposed cube (a) a diagonal plane of the cube (b) selected square on the diagonal plane



(a)



(b)

Fig. 3. FUM SCARA repeatability test (a) Y axis repeatability test (b)

TABLE I. FUM SCARA REPEATABILITY TEST

Number of Cycle	Axis			
	X $mm \times 10^{-2}$	Y $mm \times 10^{-2}$	Z $mm \times 10^{-2}$	$\Theta$ $mm \times 10^{-2}$
1	11	81	36	47.5
2	9	81	37	48
3	10	81	36	48
4	9	81	37	48
5	9	81	38	47.5
6	9	81	38	47.5
7	10	81	37	47.5
8	11	81	38	47.5
9	9	81	35	47.5
10	10	81	37	47
11	10	81	38	47
12	10	81	37	48
13	10	81	38	47.5
14	9	81	37	48
15	10	80	36	48
16	10	80	38	48
17	10	80	36	48
18	10	80	37	48
19	10	80	37	47.5
20	10	81	36	48
21	9	80	35	48
22	9	80	37	48
23	10	80	38	47.5
24	10	80	36	47.5
25	9	80	37	47
26	10	80	37	48
27	10	81	38	48
28	10	80	37	47.5
29	10	80	36	47.5
30	9	80	37	47
Repeatability	1.3	0.5	2.3	0.8

$$\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j = 9.733 \text{ mm} \times 10^{-2} \quad (2)$$

$$l_{x_j} = |x_j - \bar{x}| \quad (3)$$

$$\bar{l}_x = \frac{1}{n} \sum_{j=1}^n l_{x_j} = 0.544 \text{ mm} \times 10^{-2} \quad (4)$$

$$S_x = \sqrt{\frac{\sum_{j=1}^n (l_{x_j} - \bar{l}_x)^2}{n-1}} = 0.25 \text{ mm} \times 10^{-2} \quad (5)$$

$$RP_x = \bar{l}_x + 3S_x = 1.3 \text{ mm} \times 10^{-2} \quad (6)$$

#### IV. MATHEMATICAL FORMULATION

In this section repeatability test results are investigated. A development of the method proposed in [5] for the linear motion is used to perform the repeatability analysis of the rotary motion. The deviation of the robot's end-effector from

expected coordinates is studied. Through this section, motor-gearbox assembly and robot are denoted as motor and load, respectively. Two main assumptions are:

- All impacts are assumed to be plastic due to small energy impact in small machines like gears. Therefore, there is no rebound in collisions.
- Friction is not taken into account. So motor and the load are free to slide rotationally on a smooth, frictionless surface.

There are three different states in location of the motor relative to the load, called left-hand, right-hand, and no contact. These are depicted in Fig. 4.

Equations (7) and (8) are the equations of motion in no contact and right/left contact states, respectively, where  $T_c$  is the contact torque between load and motor.

$$\begin{cases} I_m \ddot{\theta}_m = T_m \\ I_L \ddot{\theta}_L = 0 \end{cases} \quad (7)$$

$$\begin{cases} I_m \ddot{\theta}_m = \pm T_c + T_m \\ I_L \ddot{\theta}_L = \mp T_c \end{cases} \quad (8)$$

$$I_m \ddot{\theta}_m + I_L \ddot{\theta}_L = T_m \quad (9)$$

Center of mass of the system can be defined as (10). New statement for the center of mass is calculated using (9) and (10).

$$\theta_{c.g.} = \frac{I_m \theta_m + I_L \theta_L}{I_m + I_L} \quad (10)$$

$$\ddot{\theta}_{c.g.} = \frac{T_m}{I_m + I_L} \quad (11)$$

Equation (11) shows that the center of mass is independent of  $T_c$ .  $\theta_{CG}$  demonstrates motion of the system, including  $I_m$  and  $I_L$ , motor and load inertia, respectively. Position of the load and motor based on the deviation from center of mass are defined as

$$\begin{cases} \theta_m \triangleq \theta_{c.g.} + \xi_m \\ \theta_L \triangleq \theta_{c.g.} + \xi_L \end{cases} \quad (12)$$

where  $\xi_m$  and  $\xi_L$  are deviation of  $I_m$  and  $I_L$ , respectively. Equation (13) is concluded using (10) and (12).

$$I_m \xi_m + I_L \xi_L = 0 \quad (13)$$

A simple motion is considered while backlash is assumed between motor and load. The motor is moved from right-contact toward left-contact state.

$$(\theta_m - \theta_L)_{right} - (\theta_m - \theta_L)_{left} = 2\Delta \quad (14)$$

Right and left subscripts express right-contact and left-contact states. Equation (15) and (16) show the motor and load deviation, respectively.

$$(\xi_m - \xi_L)_{right} - (\xi_m - \xi_L)_{left} = 2\Delta \quad (15)$$

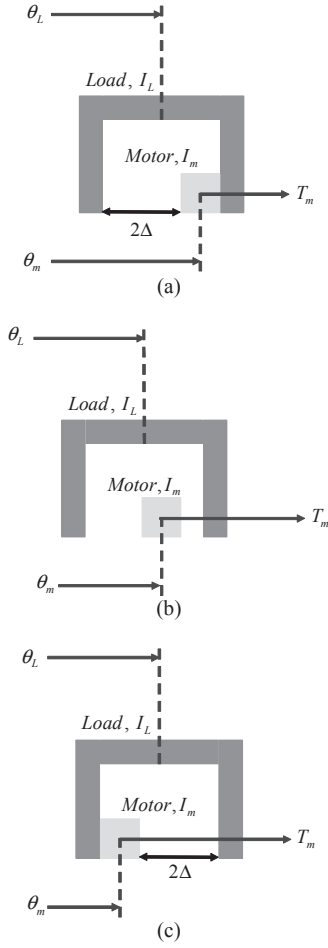


Fig. 4. Location of motor and load in three different states (a) Right-contact (b) No contact (c) Left-contact

$$(\xi_L)_{right} - (\xi_L)_{left} = \frac{-2\Delta}{1 + \frac{I_L}{I_M}} \quad (16)$$

Equation (15) shows the deviation of motor is constant through moving from the right-contact state to the left-contact state. Equation (16) shows the deviation of the load is a function of motor and load inertia. By inspection of (16) it is noted that, the higher the ratio of the load inertia to motor inertia, the lower the overall deviation of load (robot end-effector). In such a case, effect of gearbox backlash has less effect on the overall deviation of load.

The presented formulation is applied to the FUM SCARA robot and repeatability of first joint is measured. A simple rotation for the first link of the robot is assumed. The actual  $I_m$  and  $I_L$ , selected from motor catalog and actual constructed link, are

$$\begin{aligned} I_m &= 2.65 \times 10^{-4} \text{ kg.m}^2 \\ I_L &= 4.749 \text{ kg.m}^2 \end{aligned} \quad (17)$$

The backlash of the applied gearbox (APEX DYNAMICS,

part number: AE120-050) for first joint is 12 arcminutes determined by the manufacturer [11]. Therefore, it is expected that the load deviation for this robot to be

$$2\Delta = 12 \text{ arc min} = 3.491 \times 10^{-3} \text{ rad} \quad (18)$$

However, using (16), the calculated load deviation results in

$$(\xi_L)_{right} - (\xi_L)_{left} = \frac{-2\Delta}{1 + \frac{I_L}{I_M}} = -0.195 \times 10^{-6} \text{ rad} \quad (19)$$

As can be seen, the theoretical deviation is significantly lower, 18000 times less, than expected backlash as calculated in (18). For easier comparison, consider

$$\left| \frac{2\Delta}{(\xi_L)_{right} - (\xi_L)_{left}} \right| = \left| \frac{3.491 \times 10^{-3}}{-0.195 \times 10^{-6}} \right| \sim 18000 \quad (20)$$

This is explained by the high load inertia versus the motor-gearbox inertia. Considering first and second link lengths of the FUM SCARA, using the theoretical deviation and expected deviation we have

$$\begin{aligned} \delta &= (L_1 + L_2) \times |(\xi_L)_{right} - (\xi_L)_{left}| \\ &= 0.7 \text{ m} \times 0.195 \times 10^{-6} \text{ rad} = 1.37 \times 10^{-4} \text{ mm} \end{aligned} \quad (21)$$

$$\begin{aligned} \delta' &= (L_1 + L_2) \times 2\Delta \\ &= 0.7 \text{ m} \times 3.491 \times 10^{-3} \text{ rad} = 2.44 \text{ mm} \end{aligned} \quad (22)$$

Therefore, theoretically speaking, we expect the FUM SCARA robot to experience a backlash of  $1.37 \times 10^{-4}$  mm from its ideal position at its end-effector.

Repeatability in XY plane is calculated as

$$\delta'' = \sqrt{RP_x^2 + RP_y^2} = 1.39 \times 10^{-2} \text{ mm} \quad (23)$$

The actual, theoretical and expected deviation of the robot end-effector is shown in table II.

As shown in the table, the actual, measured, backlash lays within the theoretical and expected interval. The fact that the actual backlash is greater than theoretical may be explained by uncertainties due to machining tolerances and manufacturing errors, frictions, ambient temperature, and humidity. According to ISO 9283 procedure, one should not take the first cycle into account and so the measurements must be repeated thirty one cycles. This requirement eliminates the effect of the load location relative to the motor at the initial condition. Therefore, there is backlash in the system, but the cycle is repeated and

TABLE II. BACKLASH

Backlash (mm)		
Theoretical	Actual (ISO Method)	Expected
$1.37 \times 10^{-4}$	$1.39 \times 10^{-2}$	2.44

backlash does not affect repeatability.

## V. CONCLUSION

Repeatability, cost, and end-effector velocity are the main features of an industrial robot. FUM SCARA is a cost-effective industrial robot with high repeatability and velocity. Planetary reducers, with relatively lower cost, are utilized in the construction of the FUM SCARA. According to the gear box manufacture, the maximum backlash is less than 12 arcminutes. ISO 9283 method presents an organized process to calculate repeatability of industrial robots. Repeatability test is performed. Results show that even with relatively high repeatability for the gearboxes, the overall repeatability of the FUM SCARA robot is well within the repeatability of other SCARA robot manufactures. To support the experimental results, a simple theoretical method is presented in the rotary motion. It is shown that part of the high repeatability results can be justified using the theoretical formulation.

## REFERENCES

- [1] M. Shariatee, A. Akbarzadeh, A. Mousavi, S. Alimardani, "Design of an economical SCARA robot for industrial applications," ICROM, The Second International Conference on Robotics and Mechatronics, 2014.
- [2] R. Bruns, J. Diepstraten, X. Schuurbijs, and J. Wouters, "Motion control of systems with backlash," 2006.
- [3] R. Kalantari and M.S. Foomani, "Backlash nonlinearity modeling and adaptive controller design for an electromechanical power transmission system," *Scientia Iranica Transaction B: Mechanical Engineering*, 16, (6), pp. 463-469, 2009.
- [4] E.J. Burke, "Simulations of gear-based systems including backlash," 2006.
- [5] R.V. Dwivedula, "Modeling the effects of belt compliance, backlash, and slip on web tension and new methods for decentralized control of web processing lines," Oklahoma State University, 2005.
- [6] J.J. Craig, "Introduction to robotics: mechanics and control," Pearson Prentice Hall Upper Saddle River, 2005.
- [7] S. Niku, "Introduction to Robotics," John Wiley & Sons, 2010.
- [8] ISO 9283, "Manipulating industrial robots performance criteria and related test methods," 1998.
- [9] S. Alimardani, "Design, construction and control of FUM SCARA," M.S. thesis, Dept. of Mechanical Eng., Ferdowsi University of Mashhad, IRAN, 2013.
- [10] OMRON: "Industrial robots – SCARA robots for industrial applications," 2015. [online] Available: [http://industrial.omron.eu/en/products/catalogue/motion\\_and\\_drives/robots/scara\\_robots/scara\\_robots/default.html](http://industrial.omron.eu/en/products/catalogue/motion_and_drives/robots/scara_robots/scara_robots/default.html)
- [11] APEX DYNAMICS catalog, "AE / AER Series, Planetary Gearboxes, High Precision, High speed," 2011, [Online]. Available: <http://www.apexdynamicsusa.com/products.html>