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Gyrostabilized Two Wheeled Inverted Pendulum Robot

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***Abstract*— The increase in air pollution has motivated the design of many electrical transportation systems. In this paper, a "two wheeled inverted pendulum" personal transporter is considered. This system is inherently unstable and requires precise torque control of the two motor wheels to maintain its balance. To assist the control system, a mechanical controller or a "gyrostabilizer" is added to this robot. It is shown that the overall system stability is improved.**

Keywords—two wheeled inverted pendulum; equations of motion; gyrostabilizer; Euler's equations; rigid body dynamics

I. INTRODUCTION

A "two wheeled inverted pendulum" is a type of robot in which the two wheels are connected laterally to a chassis and a pendulum is pinned to that. Fig.1 shows a "two wheeled inverted pendulum". This is an unstable robot. As it will be seen in this paper the robot will lose its balance unless the motors of the robot exert a specific amount of torque to the robot. This torque depends on the inclination angle of the robot pendulum.

Fig. 1. Two wheeled inverted pendulum.

For this robot various controllers are designed and implemented. Some of these controllers are PID control [1], pole placement control [2], LQR control [3], fuzzy control [4], robust control [5], sliding mode control [6], nonlinear control [7], etc.

As it will be seen in this paper, the calculations show that sometimes the required torque to maintain robot balance is significantly large. This large amount of torque may need strong batteries and motors that increase the costs. Also, as this robot carries human, some mechanical control methods are needed to enhance the reliability of the robot balance. One of these methods could be using a gyrostabilizer.

In this paper, at first, in section II a simplified model of the robot is studied. The equations of motion are derived and stability condition is specified. After that a detailed model of the two wheeled inverted pendulum robot is analyzed and the equations of the motion are derived for the detailed one. Then in section III an introduction to "Euler's equations" from advanced dynamics concepts is presented. In section IV Euler's equations are used to prove that the gyrostabilizer can exert enough torque to the robot to prevent its falling. Finally, in section V declaration that gyrostabilizer can prevent robot falling is verified by a simulated model of the robot using MATLAB, Simulink.

II. INVERTED PENDULUM ROBOT EQUATIONS OF MOTION

To discuss the usage of gyrostabilizer possibility, the robot equations of motion should be derived. Then necessary condition for the robot balance must be specified and after that usage of gyrostabilizer could be studied.

To get familiar with the robot mechanism, firstly the equations of motion of a simplified model of the robot are derived. Then the equations of motion of the detailed model of the robot will be derived. In the simplified model of the robot, a cart inverted pendulum robot is studied instead of two wheeled inverted pendulum.

A. Simplified Model of the Robot Equations of Motion

To introduce two inverted pendulum robot balancing mechanism, firstly a simple model of the robot is studied. Fig. 2. [8] shows a simple model of the robot free body diagram.

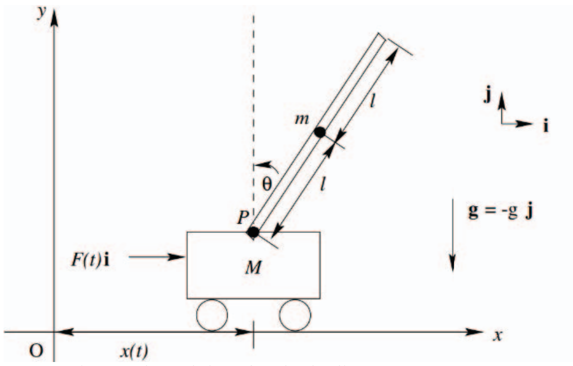

Fig. 2. Cart inverted pendulum free body diagram [8].

Table I describes the parameters used in the free body diagram. In this case, the equations of motion are [8]:

$$(M + m)\ddot{x} + \varepsilon\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F(t) \tag{1}$$

$$ml\ddot{x}\cos\theta + 4/3\,\mathrm{m}\,l^2\ddot{\theta} - mgl\sin\theta = 0 \tag{2}$$

Equations of motion show that the force "F(t)" is required to maintain cart inverted pendulum balance. The magnitude of the force is shown in (1).

After substituting the value of the parameters represented in table I into (1) and (2) and neglecting the terms "$\dot{\theta}^2$" and "$\ddot{\theta}$", we have:

$$Eq.2 \rightarrow 100\times0.9\times\ddot{x}\times0.996 - 100\times9.81\times0.9\times0.087 = 0$$
$$\Rightarrow \ddot{x} = 0.857$$
$$Eq.1 \rightarrow (40+100)\ddot{x} = F(t) \Rightarrow F(t) = 119.98$$

B. Detailed Model of Inverted Pendulum Robot

Fig. 3 [9] is a more detailed model of the two wheeled inverted pendulum robot. To derive equations of motion, in this model the robot is divided in two parts. "Wheels" and "chassis and pendulum" [9]. Fig. 3 also shows detailed model free body diagram.

Table I Equations of motion parameters for cart inverted pendulum.

Variable	Definition (unit)	Reasonable or assumed value
M	Mass of cart (kg)	40
m	Mass of pendulum (kg)	100
l	Distance from pivot "P" to pendulum center of mass (m)	0.9
θ	Pendulum angle (deg.)	5
ϵ	Air drag and other frictional coefficient	0
x	Center of mass displacement	-
F	Force exerted to the cart	-
g	Gravity constant	9.81

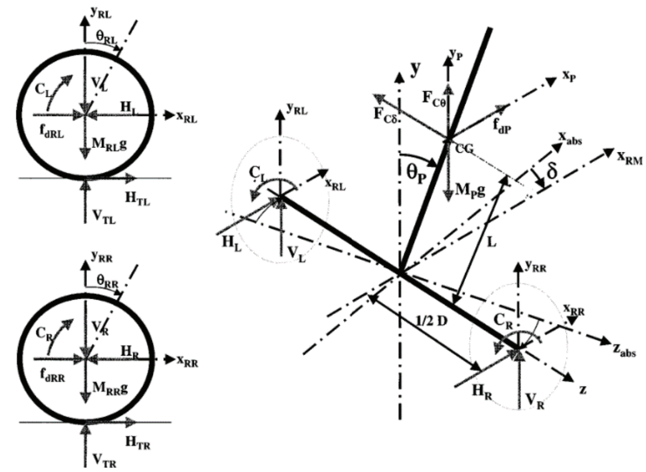

Fig. 3. Detailed model of the two wheeled inverted pendulum [9].

In this case the equations of motion for both wheels and chassis could be derived. As the equations of the left and right wheels are the same, only the equations of the left wheel are written below. By assuming rolling without sleeping, the equations of motion for the left wheel are [9]:

$$\ddot{x}_{RL}M_{RL} = f_{dRL} - H_L + H_{TL} \tag{3}$$
$$\ddot{y}_{RL}M_{RL} = V_{TL} - M_{RL}g - V_L \tag{4}$$
$$\ddot{\theta}_{RL}J_{RL} = C_L - H_{TL}R \tag{5}$$

For the chassis the equations of motion could be derived in the following form:

$$\ddot{x}_P M_P = f_{dP} + H_R + H_L \tag{6}$$
$$\ddot{y}_P M_P = V_R + V_L - M_P g + F_{C\theta} \tag{7}$$
$$\ddot{\theta}_P J_{P\theta} = (V_R + V_L)L\sin\theta_P$$
$$-(H_R + H_L)L\cos(\theta_P) - (C_R + C_L) \tag{8}$$
$$\ddot{\delta}J_{P\delta} = (H_L - H_R)D/2 \tag{9}$$

Table II describes the variables used in the above equations. By using (3) to (9) and according to table II, the required torque to maintain robot balance during its motion could be calculated.

Table II Equations of motion parameters for detailed model of two wheeled inverted pendulum

Variable	Definition
M_{RR}, M_{RL}	Right and left wheel masses
V_R, V_L	Reaction forces between the chassis and wheels in Y direction (vertical reaction)
H_R, H_L	Reaction forces between the chassis and wheels in X direction (horizontal reaction)
V_{TR}, V_{TL}	Ground reaction force to the wheels
H_{TR}, H_{TL}	Friction force between the wheels and ground
f_{dRR}, f_{dRL}	Disturbance forces exerted to the wheels
C_R, C_L	Right and left motor torque
D	Lateral distance between the wheels
M_P	Chassis and pendulum mass
f_{dP}	Disturbance force exerted to the center of mass
R	Wheel radius

III. Rigid Body Dynamics of Gyrostabilized Inverted Pendulum

Previous section specified the necessary condition for the two wheeled inverted pendulum robot balance. In this section we will discuss if the gyrostabilizer can maintain the robot balance or not.

A. Rigid Body Dynamics

As advanced dynamics concepts affirms, “the central moment of momentum” for a rigid body is:

$$\vec{H}^{C} = \vec{I}^{C}.\vec{\omega} \tag{10}$$

Where “ω” is the body angular velocity with respect to the coordinate system and “I” is the body inertia tensor. Also time derivative of a general vector like “A” that rotates with the angular velocity “ψ” is:

$$\dot{\vec{A}} = \dot{A}\,\overrightarrow{e_A} + \overrightarrow{\psi_A} \times \vec{A} \tag{11}$$

Moment of momentum principle of a rigid body states that:

$$\vec{M}_A = \dot{\vec{H}}_A + \vec{V}_A \times \vec{P} \tag{12}$$

Where “P” is the rigid body linear momentum and defines:

$$\vec{P} = m\vec{V}_C \tag{13}$$

If point “A” is the center of mass or a “fixed point of a body in pure rotation [10]” or if the velocity vector of point “A” is parallel to “P” then we have:

$$\vec{M}_A = \dot{\vec{H}}_A \tag{14}$$

Also, if we assume that the coordinate system is fixed to the body, then the term “$\dot{A}$” in (11) becomes zero. From (12) and (14), we have:

$$\vec{M}_A = \vec{\Omega} \times (\vec{I} \times \vec{\omega}) \tag{15}$$

Where “Ω” is the coordinate system angular velocity.

B. Generalized Form of Euler’s Equations

If the coordinate system is fixed to the body and by using (15), generalized form of Euler’s equation states:

$$\vec{M}_A = I_{ij}^{A}\,\alpha_j\,\overrightarrow{e_i} + \vec{\Omega} \times (\vec{I}_A.\vec{\omega}) \tag{16}$$

Or:

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix} \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{17}$$

Expanding (17) gives us:

$$\begin{aligned} M_1^A &= I_{11}^A\alpha_1 + I_{12}^A\alpha_2 + I_{13}^A\alpha_3 + \\ &\Omega_2(I_{31}^A\omega_1 + I_{32}^A\omega_2 + I_{33}^A\omega_3) \\ &-\Omega_3(I_{21}^A\omega_1 + I_{22}^A\omega_2 + I_{23}^A\omega_3) \end{aligned} \tag{18}$$

$$\begin{aligned} M_2^A &= I_{21}^A\alpha_1 + I_{22}^A\alpha_2 + I_{23}^A\alpha_3 + \\ &\Omega_3(I_{11}^A\omega_1 + I_{12}^A\omega_2 + I_{13}^A\omega_3) \\ &-\Omega_1(I_{31}^A\omega_1 + I_{32}^A\omega_2 + I_{33}^A\omega_3) \end{aligned} \tag{19}$$

$$\begin{aligned} M_3^A &= I_{31}^A\alpha_1 + I_{32}^A\alpha_2 + I_{33}^A\alpha_3 + \\ &\Omega_1(I_{21}^A\omega_1 + I_{22}^A\omega_2 + I_{23}^A\omega_3) \\ &-\Omega_2(I_{11}^A\omega_1 + I_{12}^A\omega_2 + I_{13}^A\omega_3) \end{aligned} \tag{20}$$

C. Modified Euler’s Equations

If a rigid body has at least two equal principal moments of inertia at the mass center, modified Euler’s equations could be replaced by Euler’s equations. Fig. 4. Shows the case that modified Euler’s equations can be used. By assuming “$I_2=I_3$” and axis 1 “x” as the axis of symmetry, from (16) we have [10, 11]:

$$I_1 = I_\alpha \tag{21}$$

$$I_2 = I_3 = I_t \tag{22}$$

$$M_1 = I_\alpha(\dot{\Omega}_1 + \dot{\omega}_{B/f}) \tag{23}$$

$$M_2 = I_t\dot{\Omega}_2 + (I_\alpha - I_t)\Omega_1\Omega_3 + I_\alpha\omega_{B/f}\,\Omega \tag{24}$$

$$M_3 = I_t\dot{\Omega}_3 + (I_t - I_\alpha)\Omega_1\Omega_3 - I_\alpha\omega_{B/f}\,\Omega_2 \tag{25}$$

Where “Ω” is the angular velocity of the coordinate system and “$\omega_{B/f}$” is the angular velocity of the body with respect to the coordinate system.

The above equations show that a gyrostabilizer can exert torque to our case study inverted pendulum. If this torque will be equal to the required torque to maintain the robot balance that studied in section II then we declare that gyrostabilizer can maintain robot balance.

IV. Gyrostabilizer in Two Wheeled Inverted Pendulum

In order to test the possibility of using a gyrostabilizer in two wheeled inverted pendulum, a mechanism like Fig. 4 is designed [12]. This gyrostabilizer could easily be mounted on a two wheeled inverted pendulum. The mechanism consists of the following components:

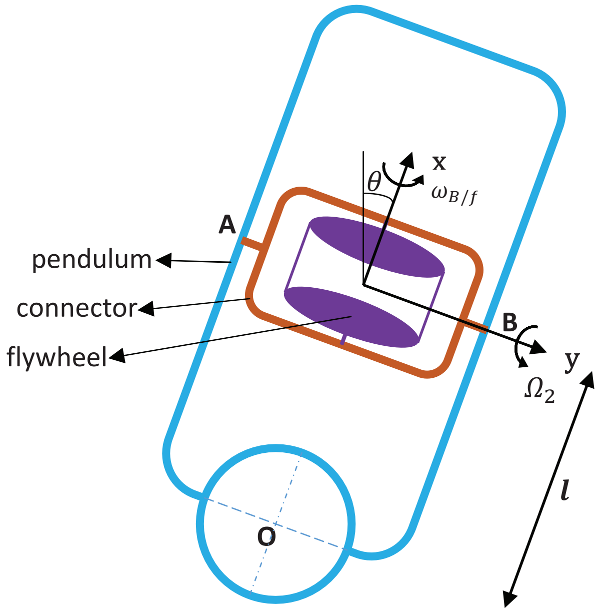

Fig. 4. Gyrostabilizer in inverted pendulum robot.

- A thin-walled cylinder that is assumed to be pendulum. It is connected to the wheels by pin in point O and rotates freely in the "Z" direction.
- A connector that is pinned to the pendulum in points A and B and rotates freely in the "Y" direction.
- A disk that is assumed to be flywheel. It is connected to the connector via a pin and rotates freely in "X" direction.

A. *Euler's Equations*

To use the Euler's equations, "KRF" which stands for "kinematics reference frame" should be defined. For this case study the "KRF" is fixed to the connector mass center. It means that flywheel angular velocity in "X" direction represents "$\omega_{B/f}$" and the connector angular velocity in "Y" direction represents "Ω_2". These angular velocities are the inputs of the case study and are set by the designer. By assuming "$\omega_{B/f} = 20\, rad/s$" and "$\Omega_2 = 28.5\, rad/s$" and substituting the gyrostabilizer data from table III and IV into (20), we have:

$$M_3^A = I_{31}^A \alpha_1 + I_{32}^A \alpha_2 + I_{33}^A \alpha_3$$
$$+\Omega_1 (I_{21}^A \omega_1 + I_{22}^A \omega_2 + I_{23}^A \omega_3)$$
$$-\Omega_2 (I_{11}^A \omega_1 + I_{12}^A \omega_2 + I_{13}^A \omega_3)$$

$$M_3^A = -28.5 \times 0.1500 \times 20 = -85.5 N.m$$

B. *Modified Euler's Equations*

By neglecting the connector mass with respect to the flywheel mass, we can approximate the flywheel as a gyrostabilizer that verifies the "modified Euler's equations" conditions.

Table III Two wheeled inverted pendulum mass and inertia.

Body	**Mass (kg)**	**Moments of inertia (kg.m^2)** $\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$
Pendulum	70.00	$\begin{bmatrix} 1.3790 & 0 & 0 \\ 0 & 24.0230 & 0 \\ 0 & 0 & 24.0230 \end{bmatrix}$
Flywheel and connector	30.00	$\begin{bmatrix} 0.1500 & 0 & 0 \\ 0 & 0.0813 & 0 \\ 0 & 0 & 0.0813 \end{bmatrix}$

Table IV Gyrostabilizer data

Parameter	**Description**
I_t	Two equal principal moments of inertia (0.0813)
I_α	Principal moments of inertia in the axis of symmetry direction (0.1500)
$\Omega_{B/f}$	Flywheel angular velocity in X direction with respect to the connector (assumed value=20)
Ω_1	Flywheel angular velocity in X direction (value=0)
Ω_2	Flywheel angular velocity in Y direction (assumed value=28.5)
$\dot{\Omega}_3$	Flywheel angular acceleration in Z direction (value=0)

To use the modified Euler's equations, "KRF" should be defined too. In this model we will attach it to the connector center of mass. Like Euler's equation the flywheel angular velocity in "X" direction represents "$\omega_{B/f}$" and similarly the connector angular velocity in "Y" direction represents "Ω_2". The angular velocities are the model input and are set by the designer. Like Euler's equation it is assumed that "$\omega_{B/f} = 20\, rad/s$" and "$\Omega_2 = 28.5\, rad/s$" and substituting the gyrostabilizer data from table IV into (25) we have:

$$M_3 = 0.0813 \times 0 + (0.0813 - 0.1500)\Omega_1 \times 0$$
$$-0.1500 \omega_{B/f} \Omega_2 = -85.5 N.m$$

$$M_g = mg \sin(\theta) \times l = 100 \times 9.81 \times \sin(5) \times 1 = 85.5$$

The above calculations show that the gyrostabilizer exerts 85.5 Newton meter torque to the robot. Also, if we incline the pendulum by 5°, another 85.5 Newton meter torque will be exerted to the mechanism in the reverse direction because of gravity. This means that the robot should remain in a steady position in this situation.

V. Gyrostabilizer in Inverted Pendulum Simulation

To verify the equations of motion results and model verification, the gyrostabilizer is modeled in MATLAB, Simulink. The structure of the block diagram model is shown in Fig. 5. All the parameters of this block diagram model are taken out of the model discussed in section IV. Four different simulations are studied here:

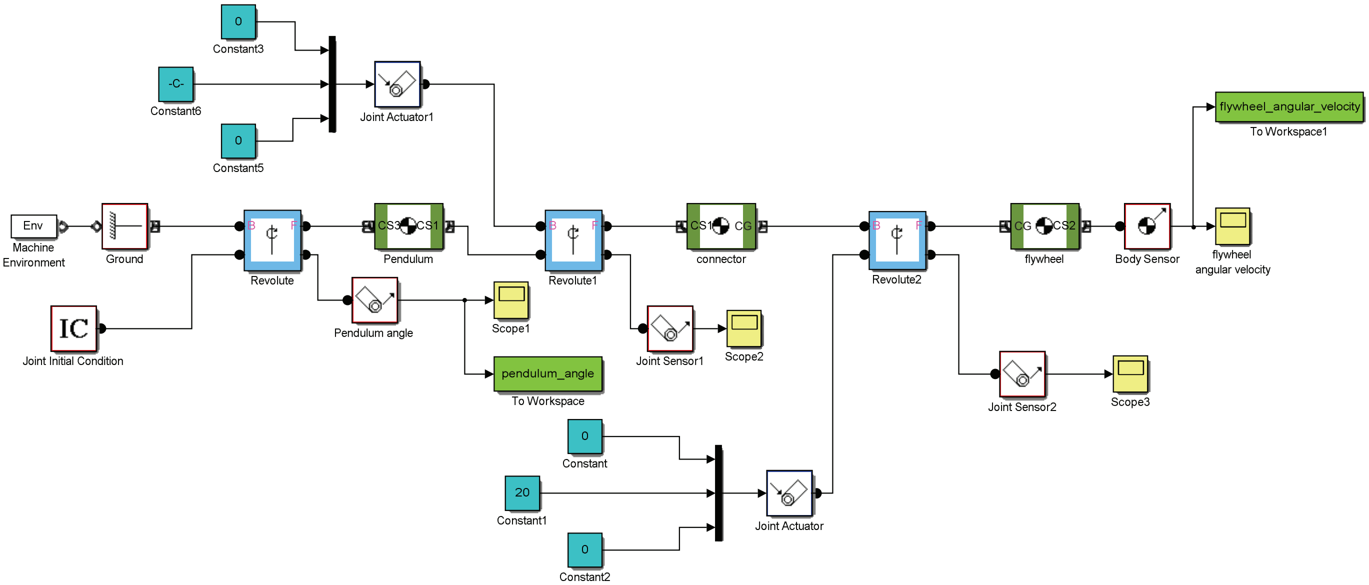

Fig. 5. Gyrostabilizer mechanism in MATLAB Simulink.

A. *Angle =5°, Angular Velocities =0rad/s, 0rad/s*

For the first simulation the inclination angle is set to "5°" with no flywheel angular velocity. Logically, we expect that pendulum inclination angle increases rapidly because of gravitational force. Fig. 6. shows the pendulum angle results in this case.

$$\sum M_Z = M_{gravity} \neq 0$$

B. *Angle =0°, Angular Velocities =20rad/s, 28.5rad/s*

In the second simulation the inclination angle is set to "0°" and the angular velocities to "20 rad/s" and "28.5 rad/s". It is expected that flywheel exerts a moment to the pendulum in Z direction. It causes the pendulum to lose its balance. Fig. 7. shows the pendulum angle result in this case.

$$\sum M_Z = M_{gyrostabilizer} \neq 0$$

C. *Angle=5°, Angular Velocities =20rad/s ,28.5rad/s*

For the 3rd simulation the inclination angle is set to "5°" and the angular velocities to "20 rad/s" and "28.5 rad/s". It is expected that the gyrostabilizer moment neutralize the pendulum moment caused by gravity force in Z direction. It causes the pendulum angle to remain in its initial angle. Fig. 8. shows the pendulum angle result in this case.

$$\sum M_Z = M_{gyrostabilizer} + M_{gravity} = 0$$

D. *Angular Velocities =20rad/s, 28.5rad/s, External Torque*

For the 4th simulation the inclination angle is set to "0°" and the angular velocities are set to "20 rad/s" and "28.5 rad/s". Also an 85.5 Newton meter torque is exerted to the pendulum joint in the reverse direction. The 85.5 Newton meter torque is extracted from (25). It is expected that the gyrostabilizer moment neutralize the external moment and the pendulum angle remains in its initial angle. Fig. 9. shows the pendulum angle result in this case.

$$\sum M_Z = M_{gyrostabilizer} + M_{external} = 0$$

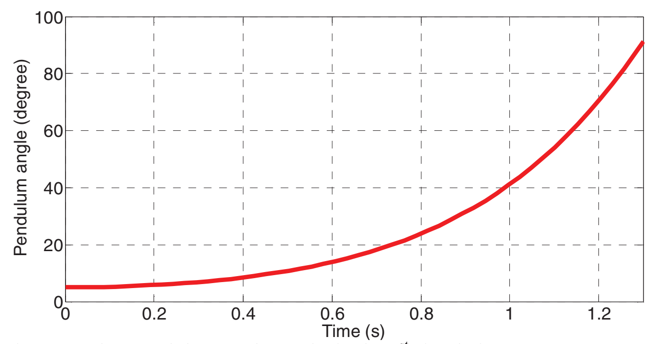

Fig. 6. Robot pendulum angle results in the 1st simulation.

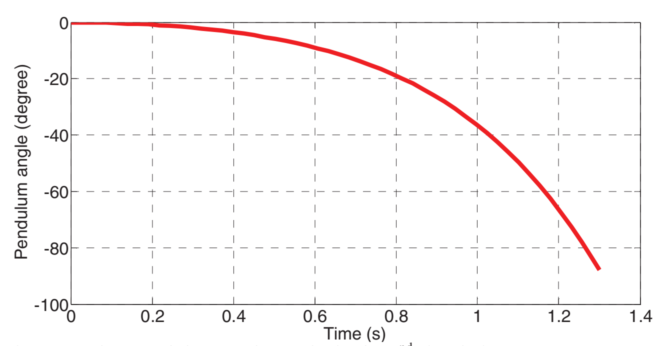

Fig. 7. Robot pendulum angle results in the 2nd simulation.

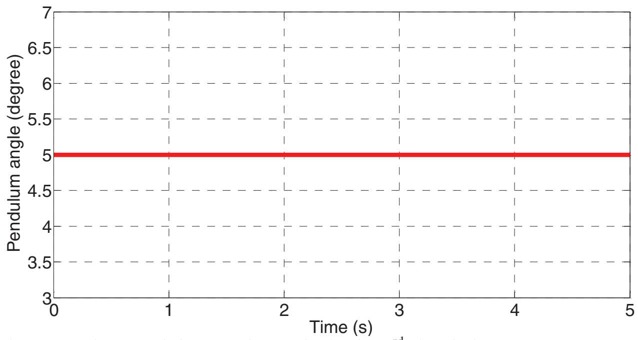

Fig. 8. Robot pendulum angle results in the 3rd simulation.

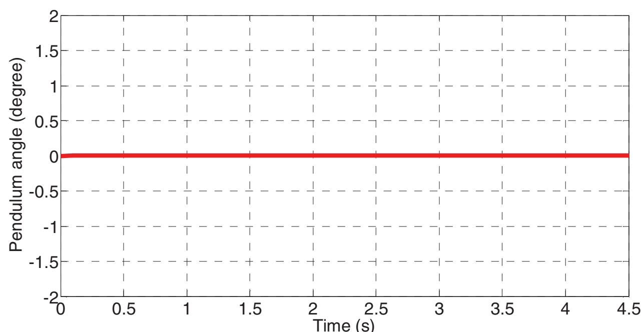

Fig. 9. Robot pendulum angle results in the 4th simulation.

VI. Conclusion

A two wheeled inverted pendulum robot is inherently an unstable system. The ordinary control theory says to maintain robot balance during its motion, a specified torque should be exerted to the robot wheels. This torque depends on the robot pendulum inclination angle. Our novel mechanical control says this robot can maintain its balance if a gyrostabilizer is mounted on the robot. It is shown that the required angular velocity of the gyrostabilizer depends on the pendulum inclination angle and the robot inertia tensor. This gyrostabilizer can be mount on robot and become activated in case of electronic controller failure.

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