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On the Stress-Force-Fabric relationship in anisotropic granular materials

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Abstract. In review of micromechanical laws for granular materials, there is a relationship that describes the stress state, fabric, and contact forces among particles, which is called "Stress-Force-Fabric" (SFF) relationship. SFF relationship can properly describe the media including the anisotropic particles such as polygonal particles, but the particles should be arranged randomly which constitutes isotopic granular media. The question is if this relationship is still true for anisotropic media, in which the elongated particles are arranged directionally. In this paper, reliability of SFF relationship is examined for both isotropic and anisotropic samples. To this aim, Discrete Element Method (DEM) is used to simulate the behavior of isotropic and anisotropic samples. The geometry of the particles are considered to be polygonal as well as elongated. The behavior of such samples are studied in terms of stress ratio, which is calculated from macroscopic and microscopic points of view. The results show that the so called SFF can well predict the behavior of isotropic sample but the predictions for anisotropic samples are not acceptable.

Keywords: Inherent Anisotropy, Stress-Force-Fabric relationship, Discrete Element Method (DEM).

1 Introduction

Anisotropy is a common phenomenon in a granular material since it consists of individual discrete bodies. One type of anisotropy especially in naturally deposited sands is inherent anisotropy. It pertains to the initial spatial arrangement of particles, voids, and associated contacts. This is generally initiated during the deposition of soil particles under gravity so that the long axis of particles tends to align in a

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specific direction. Another type of anisotropy called as induced anisotropy, occurs during loading process.

Understanding of mechanical behavior of granular media is highly dependent to the understanding of micromechanical response of the system. The main key is to study the fabric, i.e., the spatial arrangement of soil particles, contact points, and associated voids. Many attempts have been made in order to quantitatively describe the fabric in a granular material. For instance, different forms of the so-called 'fabric tensor', which describes either the distribution of contacts among particles or the orientation of particles, were introduced [e.g., 1-4]. Hill [5] defined the average stress tensor in terms of applied forces over a homogeneous granular system. Weber [6]introduced a macroscopic stress tensor based on geometrical arrangement of contacting particles. Based on weber's work, Rothenburg [7] showed that the average stress tensor for an assembly of circular or sphere particles has the properties of the stress tensor as used in the continuum mechanics, but is derived from consideration of discrete contact forces, contact geometry and principles of static equilibrium. He developed useful relationships for the assemblies with planar particles (circular disks). By assuming that the distributions of average contact force components and contact normals have the same directions of anisotropy, a simple form of stress–force–fabric relationship (SFF) was introduced [8] and its applicability was examined for the assemblies with circular [9], elliptical [10] as well as rigid and breakable polygonal particles [11,12]. All studies mentioned above with non-circular particles imply that the SFF relationship is applicable if the fabric of assemblies (particle orientation) has an isotropic condition rather than being directionally anisotropic.

In order to verify quantitatively the SFF equation, a set of detailed information is needed about the distribution of contact properties among a large number of particles. Since experimental tests could hardly give the required information, numerical simulations of granular media by Discrete Element Method (DEM) was inevitably used for verification.

In this paper, the accuracy of SFF relationship is investigated for anisotropic assemblies of granular materials in which, elongated polygonal particles are inclined along a specific bedding angle. To this aim, DEM was used in order to simulate several anisotropic assembles. In the end, the variation of stress ratio along the loading stages are compared from micro and macro viewpoints.

2 Brief Review

Inter-particle load transfer between particles can be described by a contact force vector $\vec{f}$ applied to contact point. In addition, it is required to introduce a contact normal $\vec{n}$, denoting the unit vector orthogonal to the contact tangent plane and a contact vector $\vec{l}$ describing the line pointing from the mass center of the contacting particle to the contact point [7]. Fig. 1 shows these vectors for two contacting particles.

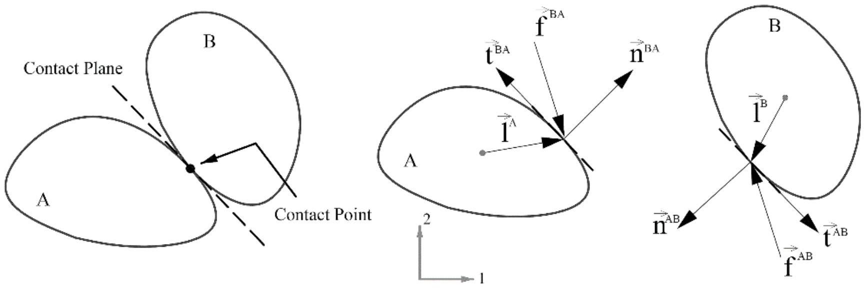

Fig. 1 Schematics contact, force, contact normal, and tangential contact vectors for two contacting particles

Conditions of static equilibrium in a granular assembly lead to the expression of the Cauchy stress tensor related to microscopic averages, which describes the geometry and force distributions in a granular assembly as follows [7]:

$$\sigma_{ij} = m_v \int_0^{2\pi} \bar{f}_i(\theta)\bar{l}_j(\theta)E(\theta)d\theta \qquad i,j = 1,2 \tag{1}$$

The term m_v is the density of contacts (the number of contacts per unit area). $E(\theta)$ is the normalized contact orientation distribution defining relative frequency of contacts with orientation θ. The contact orientation is defined by the contact normal components as $\vec{n} = (\cos\theta, \sin\theta)$. $\bar{f}_i(\theta)$ and $\bar{l}_j(\theta)$ represent the polar distributions of average components of force vector and contact vector, respectively.

The average contact force acting on contacts with orientation θ can be decomposed into an average normal force component $\bar{f}_n(\theta)$ and an average tangential force component $\bar{f}_t(\theta)$. Therefore:

$$\bar{f}_i(\theta) = \bar{f}_n(\theta)n_i + \bar{f}_t(\theta)t_i \tag{2}$$

$\vec{t} = (-\sin\theta, \cos\theta)$ represents the direction orthogonal to $\vec{n}$, according to Fig. 1. Hence, by substituting Eq. 3 to Eq. 2, it can be simplified:

$$\sigma_{ij} = m_v\bar{l}_0 \int_0^{2\pi} \left\{\bar{f}_n(\theta)n_i(\theta) + \bar{f}_t(\theta)t_i(\theta)\right\}E(\theta)d\theta \qquad i,j = 1,2 \tag{3}$$

In order to study the degree of anisotropy, it is common to draw a polar distribution of fabric quantities .i.e., frequency of contacts (E), average normal contact force $(\bar{f}_n)$, and average tangential contact force $(\bar{f}_t)$, such as shown in Fig. 2. Based on the measured distribution, it can be expressed in terms of coefficient of anisotropy a_i and major principal direction of anisotropy θ_i. The parameter θ_i indicates the angle between the long axis of the histogram with respect to the horizontal direction. The parameter a_i is defined as $a_i = (A_1 - A_2)/(A_1 + A_2)$, in which A_1 and A_2 are the minimum and maximum widths of the histogram, respectively. The procedure how to obtain the anisotropy parameters based on the histogram data is explained by Seyedi Hosseininia [13].

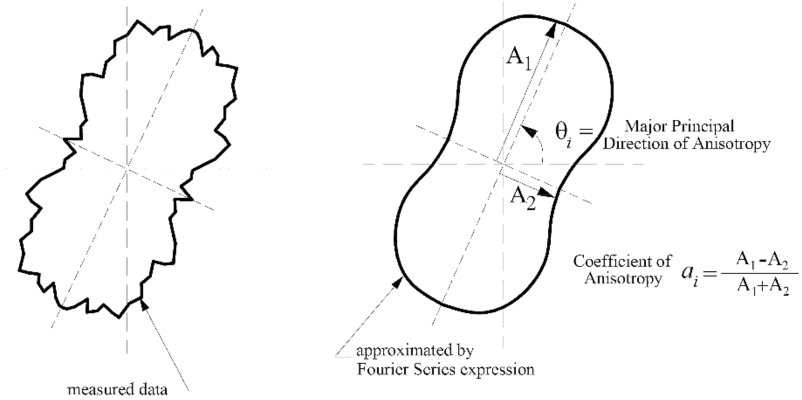

Fig.2 Definition of histogram parameters for polar functions

Polar distributions of fabric quantities can be approximated by second-order Fourier series expressions as follows [7]:

$$E(\theta) = \frac{1}{2\pi}[1 + a_c \cos 2(\theta - \theta_c)] \tag{4a}$$

$$\bar{f}_n(\theta) = \bar{f}_0[1 + a_n \cos 2(\theta - \theta_n)] \tag{4b}$$

$$\bar{f}_t(\theta) = \bar{f}_0[a_w - a_t \sin 2(\theta - \theta_t)] \tag{4c}$$

a_c describes the anisotropy in contact orientations and θ_c is the major principal direction of anisotropy. $\bar{f}_0$ represents the average normal force over all contacts. Terms a_n, a_t and a_w are non-dimensional coefficient of contact force anisotropy. Similar to θ_c, terms θ_n and θ_t represent preferred directions of contact force distributions for normal and tangential components, respectively. The term a_w is not independent and can be defined in terms of a_c and a_t from moment equilibrium condition of all contacts. Generally, the value of a_w is small and close to zero.

According to the Mohr stress circle, invariants of the average stress tensor have the following forms:

$$\sigma_n = \frac{\sigma_{11} + \sigma_{22}}{2} \quad , \quad \sigma_t = \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}^2} \tag{5}$$

The ratio of the above two stresses is generally known as the mobilized friction angle for cohesionless granular materials. By putting the Fourier series expressions in Eq. 4 into the integral of Eq. 3, the stress tensor components can be obtained. Finally, after having some mathematical manipulations and ignoring the product of anisotropy coefficients for the third and higher orders, the stress ratio can be calculated as follows:

$$\frac{\sigma_t}{\sigma_n} = \frac{\sqrt{a_c^2 + a_n^2 + a_t^2 + 2a_c a_n \cos 2(\theta_c - \theta_n)}}{2 + a_c a_n \cos 2(\theta_c - \theta_n)} \quad \frac{\overline{+2a_c a_t \cos 2(\theta_c - \theta_t) + 2a_n a_t \cos 2(\theta_n - \theta_t)}}{2 + a_c a_n \cos 2(\theta_c - \theta_n)} \tag{6}$$

The equation above is the origin of the so-called stress-force–fabric relationship which links the macroscopic mobilized stress to the microscopic anisotropy. It is emphasized that in Eq. 6, it was assumed that the contact vector and contact normal are coaxial. If directions of anisotropy for contact normals and contact forces are coaxial, i.e., $\theta_c = \theta_n = \theta_t$, like what happens for circular particles, then the simplified expression is simplified to:

$$\frac{\sigma_t}{\sigma_n} = \frac{a_c + a_n + a_t}{2 + a_c a_n} \tag{7}$$

3 Simulations

A series of biaxial compression tests were simulated by using DEM. The key point was that the particles are not circle or ellipse in shape, but they have geometry of convex irregular polygon. Schematics of particles geometry and dimensions are depicted in Fig. 3. Three degrees of sizes have been used with the scale ratio of 0.75, 1.0, and 1.25 with respect to those presented in Fig. 3. The gradation is characterized by the uniformity coefficient (D_{60} / D_{10}) of 1.35 and the curvature coefficient $(D_{30}^2 / D_{10} D_{60})$ of 1.2. D_x indicates the long axis length (diameter of an equivalent circumscribed circle) of soil particles for which x% of the particles are finer. Totally, five series of samples were generated. All the samples had the form of circle with diameter of 160 mm. About 2000 particles exist in each sample. Having the same frequency distribution of particles, these samples were distinguished by initial inclination of particles before shearing. In one assembly, the particles were inclined randomly, which constitutes an isotropic-like fabric. However, the other four assemblies contain the particles whose elongation is inclined along a predefined direction, i.e., α = 0, 30, 60, and 90°. α stands for the bedding angle which is defined by the angle between the long axis of the particle and the horizontal direction (1-1 axis). As a consequence, the latter four samples are inherently anisotropic.

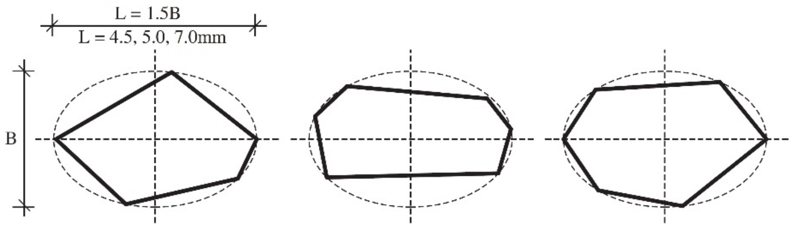

Fig. 3 Schematics of particles used in the assemblies

After the generation of samples, each assembly was compressed under the confining pressure of 400 kPa ($=\sigma_{11}=\sigma_{22}$). The compression process is continued until there is almost no volume change in the assembly. Fig. 4a represents a quarter of the compacted assemblies for all inherently-anisotropic samples under the confining pressure of 400kPa. Afterwards, the compacted assembly was sheared biaxially, by keeping the lateral stress (σ_{11}) equal to 400kPa and simultaneously, the boundary of the sample was forced to move along 2-2 axis by a constant vertical displacement rate (Fig. 4b). A detailed description of simulations is explained elsewhere [14].

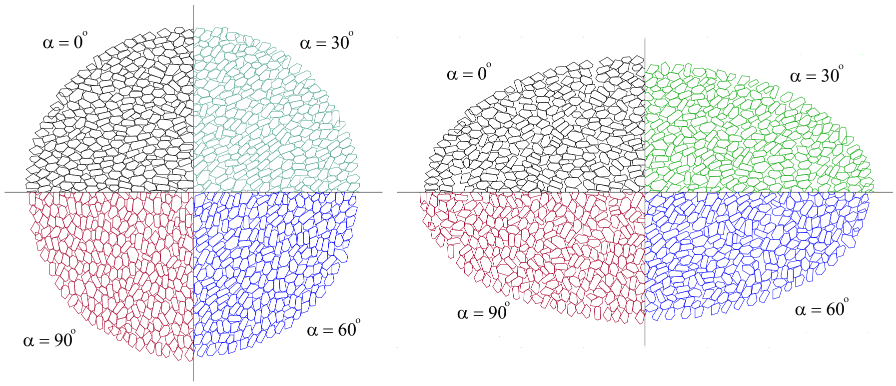

Fig.4 Presentation of inherent anisotropic assemblies: (a) isotopically compacted; (b) sheared

4 Results

Based on the loading path, it is possible to directly calculate the variation of the stress ratio σ_t/σ_n from Eq. 6. Moreover, such stress ratio can be calculated from a micromechanical point of view using Eqs. (6) and (7). From both viewpoints, the results are sketched in Fig. 5 in terms of stress ratio versus the axial strain for isotropic as well as anisotropic samples (α = 0, 30, 60, and 90°).
According to Fig. 5, it can be seen that none of the SFF forms can predict appropriately the stress ratio variation in all the anisotropic samples. The prediction of Eqs.

6 and 7 almost coincide for the samples $\alpha = 0, 30^o$, but they get far from each other for $\alpha = 60, 90^o$. It is reminded that the difference between Eq. 6 and Eq. 7 exists in the consideration of non-coaxiality among anisotropy quantities. However, it is important to note that in both of the forms of the SFF expressions, it is assumed that the contact vector is coaxial with the contact normal. Such a comparison between the results clearly expresses the importance of consideration of non-coaxially between contact vector and the contact normal. In contrary to anisotropic samples, both forms of the SFF (Eqs. 6 and 7) can well predict the stress ratio with the axial strain for the sample including randomly-distributed particles. It means that although the particles are elongated (anisotropic), the overall distribution of contact and contact normals are coincident (coaxial) and thus, both forms of the SFF expressions ae valid for such samples.

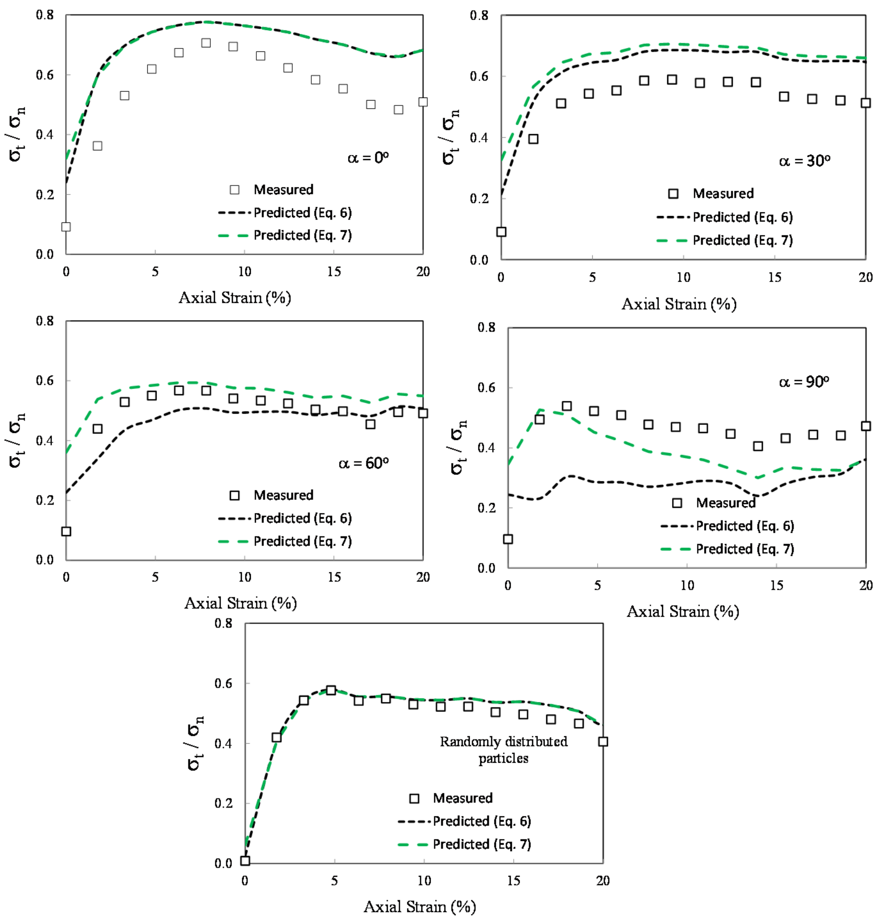

Fig.5 Comparison of stress ratio variation with axial strain for all fore anisotropic and isotropic samples

5 Conclusion

In this paper, the stress-force-fabric (SFF) relationship was examined for anisotropic assemblies including elongated polygonal particles. The accuracy of two forms of SFF including isotropic and anisotropic versions were investigated. The results show that as expected, these two expressions can well predict the relationship between micro and macro behavior of assemblies with elongated particles when the particles and randomly distributed. In such condition, the directions of anisotropy parameters coincide and thus, the sample behaves isotopically. However, in the case of inherent anisotropy, i.e., initial anisotropy in the fabric SFF expressions can no more predict acceptable results.

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