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An advanced criterion based on non-AFR for anisotropic sheet metals

Farzad Moayyedian*[1] and Mehran Kadkhodayan[2a]

[1]*Department of Mechanical Engineering, Eqbal Lahoori Institute of Higher Education (ELIHE), Mashhad, Iran*
[2]*Department of Mechanical Engineering, Ferdowsi University of Mashhad (FUM), Mashhad, Iran*

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Abstract. In the current research an advanced criterion with non-associated flow rule (non-AFR) for depicting the behavior of anisotropic sheet metals is presented to consider the strength differential effects (SDEs) for these materials. Owing to the fact that Lou *et al*. (2013) yield function is dependent on structure of an anisotropic material (BCC, FCC and HCP), an advanced yield function with inspiring of Yoon *et al*. (2014) yield function is proposed which is dependent upon anisotropic structures. Furthermore, to compute Lankford coefficients, a new pressure sensitive plastic potential function which would be dependent to anisotropic structure is presented and coupled with the proposed yield function with employing a non-AFR in a novel criterion which is called here ‘advanced criterion’. Totally eighteen experimental data are required to calibrate the criterion contained of directional tensile and compressive yield stresses for the yield function and directional Lankford coefficients for the plastic potential function. To verify the criterion, three anisotropic sheet metals with different structures are taken as case studies such as Al 2008-T4 (a BCC material), Al 2090-T3 (a FCC material) and AZ31 (a HCP material).

Keywords: advanced criterion; asymmetric anisotropic sheet metals; non-AFR; tensile yield stresses; compressive yield stresses; Lankford coefficients

1. Introduction

The mechanical behaviors of asymmetric anisotropic sheet metals have been studied in recent years extensively. The issue of pressure sensitivity/insensitivity and also strength differential in tension and compression of these materials were the topic of many new researches. Some important studies on modeling of the mechanical behavior of these materials are reviewed as follows.

Spitzig and Richmond (1984) demonstrated experimentally that in both iron-based materials and aluminum the flow stress was linearly depended on the hydrostatic pressure. Liu *et al*. (1997) developed Hill’s criterion to include orthotropic plastic materials with yield stresses in tension and compression. Barlat *et al*. (2003) proposed a plane stress yield function, Yld2000-2d which was validated by experimental data of polycrystal obtained on a binary Al-2.5 wt. %Mg alloy sheet.

*Corresponding author, Assistant Professor, E-mail: farzad.moayyedian@gmail.com
[a]Professor

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Stoughton and Yoon (2004) proposed a non-AFR based on a pressure sensitive yield criterion with isotropic hardening that was consistent with Spitzig and Richmond data (1984). Hu and Wang (2005) proposed a yield function to model the strength-differential in tension and compression of materials. Hu (2005) introduced a yield criterion for anisotropic materials which described the yield condition by considering the influence of both magnitude of loading force and loading direction. Artez (2005) extended a plane stress yield function based on Hosford non-quadratic yield function called Yld2003. The applications showed that the yield function was approximately as flexible as Barlat yield function, Yld2000-2D with a simpler mathematical form. Lee *et al*. (2008) developed a yield criterion with a pressure sensitive term and considered high directional differences in the initial yield stress and also high asymmetry in tension and compression. Stoughton and Yoon (2009) extended a model for proportional loading of any biaxial stress state. The model was demonstrated to lead to an order in magnitude reduction in errors of prediction of the anisotropic stress-strain relationships in uniaxial and equal biaxial tensions. To construct a proper constitutive model, Hu and Wang (2009) defined a reasonable plastic potential to express the feature of plastic flow. Huh *et al*. (2010) computed the accuracy of an anisotropic yield criteria contain of Hill48, Yld89, Yld91, Yld96, Yld2000-2d, BBC2000 and Yld2000-18p based on the root-mean square error (RMSEs) of the yield stresses and the Lankford coefficients. Taherizadeh *et al*. (2011) compared three models for simulation of forming of anisotropic sheet metals contained of a non-AFR with both yield and potential functions in the form of Hill’s with different calibration, an AFR with a non-quadratic yield function of Yld2000 and a non-AFR non-quadratic yield function of Yld91 and plastic potential function of Yld89. Lou *et al*. (2013) proposed a method to extend symmetric yield functions to consider the Strength Differential Effect (SDE) for incompressible sheet metals with AFR. The SDE was coupled with symmetric yield functions by adding a weight pressure term. Safaei *et al*. (2013) presented a plane stress anisotropic constitutive model with mixed isotropic-kinematic hardening. The quadratic Hill 1948 and non-quadratic Yld-2000-2d yield criteria were considered in a non-AFR model to account for anisotropic behavior. Yoon *et al*. (2014) proposed an anisotropic yield function under three-dimensional loading with dependence on the first, second and third stress invariants of modified deviatoric stress tensor. Yielding was assumed to be linearly dependent on hydrostatic pressure. Safaei *et al*. (2014) proposed an approach to compute anisotropy during plastic deformation. A non-AFR based on Yld2000-2d anisotropic yield model was employed in which separate yield function and plastic potential were considered. Safaei *et al*. (2014) described two simplified methods for the relationship between the equivalent plastic strain and compliance factor in a non-AFR model. Moayyedian and Kadkhodayan (2015) introduced a Modified Yld2000-2d II with inserting modified Yld2000-2d and Yld2000-2d in place of yield and plastic potential functions respectively to model anisotropic pressure sensitive sheet metals. Moayyedian and Kadkhodayan (2015) modified the Burzynski criterion which was used for pressure sensitive isotropic materials for anisotropic pressure dependent sheet metals based on non-AFR.

In the current study a new criterion with using non-AFR for describing the behavior of anisotropic sheet metals is presented to consider their strength differential effects (SDEs). Due to the fact that Lou *et al*. (2013) yield function is dependent on structure of an anisotropic material (BCC, FCC and HCP), an advanced yield function with inspiring of Yoon *et al*. (2014) yield function is proposed which is dependent upon anisotropic structures. Furthermore, a new pressure sensitive plastic potential function which would be dependent to anisotropic structure is presented and coupled with the proposed yield function with employing a non-AFR in the novel criterion which is called here ‘advanced criterion’. Totally eighteen experimental data are required to calibrate the criterion contained of 10 data such as directional tensile and compressive yield stresses for the yield function and 8 data such as directional Lankford coefficients for the plastic potential function. To verify the criterion three anisotropic sheet metals with different structures are taken as case studies such as Al 2008-T4 (a BCC material), Al 2090-T3 (a FCC material) and AZ31 (a HCP material). Finally, it is shown that the new criterion is more successful than Yoon *et al*. (2014) and Lou *et al*. (2013) ones in prediction of experimental directional behavior of an asymmetric anisotropic sheet metals for different structures of anisotropic materials.

2. The advanced criterion

Yoon *et al*. (2014) used a modified deviatoric tensor to consider the anisotropic effects. They defined two linear transformation matrices which applied to stress tensor to obtain modified deviatoric stress tensors (s'_{ij} and s''_{ij}) in three dimensional stress space as follows

$$\begin{gathered}
\begin{Bmatrix} s'_{xx} \\ s'_{yy} \\ s'_{zz} \\ s'_{yz} \\ s'_{xz} \\ s'_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{c'_2+c'_3}{3} & -\frac{c'_3}{3} & -\frac{c'_2}{3} & 0 & 0 & 0 \\ -\frac{c'_3}{3} & \frac{c'_3+c'_1}{3} & -\frac{c'_1}{3} & 0 & 0 & 0 \\ -\frac{c'_2}{3} & -\frac{c'_1}{3} & \frac{c'_1+c'_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_6 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \\
\begin{Bmatrix} s''_{xx} \\ s''_{yy} \\ s''_{zz} \\ s''_{yz} \\ s''_{xz} \\ s''_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{c''_2+c''_3}{3} & -\frac{c''_3}{3} & -\frac{c''_2}{3} & 0 & 0 & 0 \\ -\frac{c''_3}{3} & \frac{c''_3+c''_1}{3} & -\frac{c''_1}{3} & 0 & 0 & 0 \\ -\frac{c''_2}{3} & -\frac{c''_1}{3} & \frac{c''_1+c''_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c''_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c''_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c''_6 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}
\end{gathered} \tag{1}$$

In linear transformation matrices in Eq. (1), $c'_i\left(i=1,6\right)$ and $c''_i\left(i=1,6\right)$ are anisotropy material parameters which can be determined with different experimental tests such as uniaxial directional tensile and compressive tests as well biaxial tensile and compressive yield stress tests.

Considering plane stress problem in σ_{xx}–σ_{yy} plane (i.e., $\sigma_{zz}=\tau_{xz}=\tau_{yz}=0$) for sheet metals, the modified deviatoric stress tensor components can be achieved as Eq. (2). Hence, the modified stress invariants can be defined for anisotropic sheet metals as Eq. (3). In Eq. (3) $\bar{I}_1$ is the modified first invariant of stress tensor while J'_2 and J'_3 are the modified second and third invariants of deviatoric stress tensor in an anisotropic sheet metal. Inserting Eq. (2) into Eq. (3), the modified invariants can be expressed in terms of stress components as Eq. (4), where h_x and h_y are coefficients of σ_{xx} and σ_{yy} in modified hydrostatic stress and coefficients $a_i'(i=1,4)$ and $a_i''(i=1,6)$ can be determined in terms of material parameters c_i' and c_i'' as Eq. (5).

$$\begin{cases} s'_{xx} = \dfrac{c'_2 + c'_3}{3}\sigma_{xx} - \dfrac{c'_3}{3}\sigma_{yy} \\ s'_{yy} = -\dfrac{c'_3}{3}\sigma_{xx} + \dfrac{c'_3 + c'_1}{3}\sigma_{yy} \\ s'_{zz} = -\dfrac{c'_2}{3}\sigma_{xx} - \dfrac{c'_1}{3}\sigma_{yy} \\ s'_{xy} = c'_6\tau_{xy} \end{cases}$$
$$\begin{cases} s''_{xx} = \dfrac{c''_2 + c''_3}{3}\sigma_{xx} - \dfrac{c''_3}{3}\sigma_{yy} \\ s''_{yy} = -\dfrac{c''_3}{3}\sigma_{xx} + \dfrac{c''_3 + c''_1}{3}\sigma_{yy} \\ s''_{zz} = -\dfrac{c''_2}{3}\sigma_{xx} - \dfrac{c''_1}{3}\sigma_{yy} \\ s''_{xy} = c''_6\tau_{xy} \end{cases} \tag{2}$$

$$\begin{cases} \bar{I}_1 = h_x\sigma_{xx} + h_y\sigma_{yy} \\ J'_2 = -s'_{xx}s'_{yy} - s'_{yy}s'_{zz} - s'_{xx}s'_{zz} + s'^2_{xy} \\ J''_3 = s''_{xx}s''_{yy}s''_{zz} - s''_{zz}s''^2_{xy} \end{cases} \tag{3}$$

$$\begin{cases} \bar{I}_1 = h_x\sigma_{xx} + h_y\sigma_{yy} \\ J'_2 = a'_1\sigma^2_{xx} + a'_2\sigma_{xx}\sigma_{yy} + a'_3\sigma^2_{yy} + a'_4\tau^2_{xy} \\ J''_3 = a''_1\sigma^3_{xx} + a''_2\sigma^2_{xx}\sigma_{yy} + a''_3\sigma_{xx}\sigma^2_{yy} + a''_4\sigma^3_{yy} + a''_5\sigma_{xx}\tau^2_{xy} + a''_6\sigma_{yy}\tau^2_{xy} \end{cases} \tag{4}$$

Now the yield function of advanced criterion can be presented for pressure sensitive anisotropic sheet metals as Eq. (6). The parameter *a* is newly added compared to Yoon *et al*. (2014) yield function to consider the anisotropic structures such as BCC, FCC and HCP. It is noticed that by inserting of *a*=3, the Yoon *et al*. (2014) criterion is simply achieved.

$$\begin{cases} a'_1 = \dfrac{c'^2_2 + c'^2_3 + c'_2c'_3}{9} \\ a'_2 = \dfrac{c'_1c'_2 - c'_1c'_3 - c'_2c'_3 - 2c'^2_3}{9} \\ a'_3 = \dfrac{c'^2_1 + c'^2_3 + c'_1c'_3}{9} \\ a'_4 = c'^2_6 \end{cases}$$

$$\begin{cases} a_1'' = \dfrac{c_3''c_2''^2 + c_2''c_3''^2}{27} \\ a_2'' = \dfrac{c_1''c_3''^2 - c_1''c_2''^2 - c_3''c_2''^2 - 2c_2''c_3''^2}{27} \\ a_3'' = \dfrac{c_2''c_3''^2 - c_2''c_1''^2 - c_3''c_1''^2 - 2c_1''c_3''^2}{27} \\ a_4'' = \dfrac{c_3''c_1''^2 + c_1''c_3''^2}{27} \\ a_5'' = \dfrac{c_2''c_6''^2}{3} \\ a_6'' = \dfrac{c_1''c_6''^2}{3} \end{cases} \tag{5}$$

$$F\left(\sigma_{ij}\right) = \bar{I}_1 + \left(J_2'^{\frac{a}{2}} + J_3''^{\frac{a}{3}}\right)^{\frac{1}{a}} = \sigma\left(\bar{\varepsilon}^p\right) \tag{6}$$

It is seen that in Eq. (6) the yield function linearly dependent on modified hydrostatic stress due to $\bar{I}_1$ as Spitzig and Richmond (1984). In the yield function $\bar{\varepsilon}^p$ is the effective plastic strain and $\sigma\left(\bar{\varepsilon}^p\right)$ defines the isotropic hardening. This yield function is asymmetric in σ_{xx}–σ_{yy} plane due to $\bar{I}_1$ and J_3' . Inserting the modified stress invariants from Eq. (4) into yield function of Eq. (6), the yield function of new criterion can be determined in terms of stress tensor components.

In the following a pressure insensitive plastic potential function is introduced which has not been considered by Yoon *et al*. (2014) to compute the Lankford coefficients. To define the plastic potential function, other linear matrices to be applied to stress tensor are introduced as Eq. (7). In Eq. (7), $\bar{c}_i'\left(i=1,6\right)$ and $\bar{c}_i''\left(i=1,6\right)$ are anisotropy material parameters for plastic potential function. These material parameters can be determined from different experimental tests of uniaxial and biaxial directional tensile Lankford coefficients. Considering the problem as plane stress, these modified deviatoric stress tensors can be achieved as follows

$$\begin{Bmatrix} \bar{s}_{xx}' \\ \bar{s}_{yy}' \\ \bar{s}_{zz}' \\ \bar{s}_{yz}' \\ \bar{s}_{xz}' \\ \bar{s}_{xy}' \end{Bmatrix} = \begin{bmatrix} \dfrac{\bar{c}_2' + \bar{c}_3'}{3} & -\dfrac{\bar{c}_3'}{3} & -\dfrac{\bar{c}_2'}{3} & 0 & 0 & 0 \\ -\dfrac{\bar{c}_3'}{3} & \dfrac{\bar{c}_3' + \bar{c}_1'}{3} & -\dfrac{\bar{c}_1'}{3} & 0 & 0 & 0 \\ -\dfrac{\bar{c}_2'}{3} & -\dfrac{\bar{c}_1'}{3} & \dfrac{\bar{c}_1' + \bar{c}_2'}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{c}_4' & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{c}_5' & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{c}_6' \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{s}''_{xx} \\ \bar{s}''_{yy} \\ \bar{s}''_{zz} \\ \bar{s}''_{yz} \\ \bar{s}''_{xz} \\ \bar{s}''_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\bar{c}''_2+\bar{c}''_3}{3} & -\frac{\bar{c}''_3}{3} & -\frac{\bar{c}''_2}{3} & 0 & 0 & 0 \\ -\frac{\bar{c}''_3}{3} & \frac{\bar{c}''_3+c''_1}{3} & -\frac{\bar{c}''_1}{3} & 0 & 0 & 0 \\ -\frac{\bar{c}''_2}{3} & -\frac{\bar{c}''_1}{3} & \frac{\bar{c}''_1+\bar{c}''_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{c}''_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{c}''_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{c}''_6 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \tag{7}$$

$$\begin{cases} \bar{s}'_{xx} = \frac{\bar{c}'_2+\bar{c}'_3}{3}\sigma_{xx} - \frac{\bar{c}'_3}{3}\sigma_{yy} \\ \bar{s}'_{yy} = -\frac{\bar{c}'_3}{3}\sigma_{xx} + \frac{\bar{c}'_3+\bar{c}'_1}{3}\sigma_{yy} \\ \bar{s}'_{zz} = -\frac{\bar{c}'_2}{3}\sigma_{xx} - \frac{\bar{c}'_1}{3}\sigma_{yy} \\ \bar{s}'_{xy} = \bar{c}'_6\tau_{xy} \end{cases}$$
$$\begin{cases} \bar{s}''_{xx} = \frac{\bar{c}''_2+\bar{c}''_3}{3}\sigma_{xx} - \frac{\bar{c}''_3}{3}\sigma_{yy} \\ \bar{s}''_{yy} = -\frac{\bar{c}''_3}{3}\sigma_{xx} + \frac{\bar{c}''_3+\bar{c}''_1}{3}\sigma_{yy} \\ \bar{s}''_{zz} = -\frac{\bar{c}''_2}{3}\sigma_{xx} - \frac{\bar{c}''_1}{3}\sigma_{yy} \\ \bar{s}''_{xy} = \bar{c}''_6\tau_{xy} \end{cases} \tag{8}$$

The second and third modified stress invariants of modified deviatoric tensor can be expressed as

$$\begin{cases} \bar{J}'_2 = -\bar{s}'_{xx}\bar{s}'_{yy} - \bar{s}'_{yy}\bar{s}'_{zz} - \bar{s}'_{xx}\bar{s}'_{zz} + \bar{s}'^2_{xy} \\ \bar{J}''_3 = \bar{s}''_{xx}\bar{s}''_{yy}\bar{s}''_{zz} - \bar{s}''_{zz}\bar{s}''^2_{xy} \end{cases} \tag{9}$$

Inserting Eq. (8) into Eq. (9), $\bar{J}'_2$ and $\bar{J}'_3$ can be obtained in terms of stress tensor components as follows

$$\begin{cases} \bar{J}'_2 = \bar{a}'_1\sigma^2_{xx} + \bar{a}'_2\sigma_{xx}\sigma_{yy} + \bar{a}'_3\sigma^2_{yy} + \bar{a}'_4\tau^2_{xy} \\ \bar{J}''_3 = \bar{a}''_1\sigma^3_{xx} + \bar{a}''_2\sigma^2_{xx}\sigma_{yy} + \bar{a}''_3\sigma_{xx}\sigma^2_{yy} + \bar{a}''_4\sigma^3_{yy} + \bar{a}''_5\sigma_{xx}\tau^2_{xy} + \bar{a}''_6\sigma_{yy}\tau^2_{xy} \end{cases} \tag{10}$$

In Eq. (10) the coefficients $\bar{a}'_i(i=1,4)$ and $\bar{a}''_i(i=1,6)$ are as Eq. (11).

Now, a pressure insensitive plastic potential function can be newly proposed as Eq. (12). In Eq.

(12) parameter b is defined to take care of the difference between anisotropic structures such as BCC, FCC and HCP in plastic potential function in anisotropic sheet metals. It should be noticed that the plastic potential function in advanced criterion is an asymmetric function in σ_{xx}−σ_{yy} plane due to $\bar{J}'_3$.

$$\begin{cases} \bar{a}'_1 = \dfrac{\bar{c}'^2_2 + \bar{c}'^2_3 + \bar{c}'_2\bar{c}'_3}{9} \\ \bar{a}'_2 = \dfrac{\bar{c}'_1\bar{c}'_2 - \bar{c}'_1\bar{c}'_3 - \bar{c}'_2\bar{c}'_3 - 2\bar{c}'^2_3}{9} \\ \bar{a}'_3 = \dfrac{\bar{c}'^2_1 + \bar{c}'^2_3 + \bar{c}'_1\bar{c}'_3}{9} \\ \bar{a}'_4 = \bar{c}'^2_6 \end{cases}$$

$$\begin{cases} \bar{a}''_1 = \dfrac{\bar{c}''_3\bar{c}''^2_2 + \bar{c}''_2\bar{c}''^2_3}{27} \\ \bar{a}''_2 = \dfrac{\bar{c}''_1\bar{c}''^2_3 - \bar{c}''_1\bar{c}''^2_2 - \bar{c}''_3\bar{c}''^2_2 - 2\bar{c}''_2\bar{c}''^2_3}{27} \\ \bar{a}''_3 = \dfrac{\bar{c}''_2\bar{c}''^2_3 - \bar{c}''_2\bar{c}''^2_1 - \bar{c}''_3\bar{c}''^2_1 - 2\bar{c}''_1\bar{c}''^2_3}{27} \\ \bar{a}''_4 = \dfrac{\bar{c}''_3\bar{c}''^2_1 + \bar{c}''_1\bar{c}''^2_3}{27} \\ \bar{a}''_5 = \dfrac{\bar{c}''_2\bar{c}''^2_6}{3} \\ \bar{a}''_6 = \dfrac{\bar{c}''_1\bar{c}''^2_6}{3} \end{cases} \tag{11}$$

$$G\left(\sigma_{ij}\right) = \left(\bar{J}'^{\frac{b}{2}}_2 + \bar{J}''^{\frac{b}{3}}_3\right)^{\frac{1}{b}} \tag{12}$$

It is necessary to imply that the parameters a and b may be different from each other and they can be determined via experimental results by experimental yield stresses and Lankford coefficients in yield and plastic potential functions, respectively. Finally, with substituting Eq. (10) into Eq. (12), the plastic function of advanced criterion can be expressed in stress tensor components.

To calibrate the plastic potential function, its first differentiation with respect to the stress tensor, σ_{ij} is required as derived in Eq. (13), where $\dfrac{\partial \bar{J}'_2}{\partial \sigma_{ij}}$ and $\dfrac{\partial \bar{J}''_3}{\partial \sigma_{ij}}$ for plane stress problems are as Eq. (14).

$$\begin{cases} \dfrac{\partial G}{\partial \sigma_{xx}} = G^{1-b}\left(\dfrac{1}{2}\bar{J}_2'^{\left(\frac{b}{2}-1\right)}\dfrac{\partial \bar{J}_2'}{\partial \sigma_{xx}} + \dfrac{1}{3}\bar{J}_3''^{\left(\frac{b}{3}-1\right)}\dfrac{\partial \bar{J}_3''}{\partial \sigma_{xx}}\right) \\ \dfrac{\partial G}{\partial \sigma_{yy}} = G^{1-b}\left(\dfrac{1}{2}\bar{J}_2'^{\left(\frac{b}{2}-1\right)}\dfrac{\partial \bar{J}_2'}{\partial \sigma_{yy}} + \dfrac{1}{3}\bar{J}_3''^{\left(\frac{b}{3}-1\right)}\dfrac{\partial \bar{J}_3''}{\partial \sigma_{yy}}\right) \\ \dfrac{\partial G}{\partial \tau_{xy}} = G^{1-b}\left(\dfrac{1}{2}\bar{J}_2'^{\left(\frac{b}{2}-1\right)}\dfrac{\partial \bar{J}_2'}{\partial \tau_{xy}} + \dfrac{1}{3}\bar{J}_3''^{\left(\frac{b}{3}-1\right)}\dfrac{\partial \bar{J}_3''}{\partial \tau_{xy}}\right) \end{cases} \tag{13}$$

$$\begin{cases} \dfrac{\partial \bar{J}_2'}{\partial \sigma_{xx}} = 2\bar{a}_1'\sigma_{xx} + \bar{a}_2'\sigma_{yy} \\ \dfrac{\partial \bar{J}_2'}{\partial \sigma_{yy}} = \bar{a}_2'\sigma_{xx} + 2\bar{a}_3'\sigma_{yy} \\ \dfrac{\partial \bar{J}_2'}{\partial \tau_{xy}} = 2\bar{a}_4'\tau_{xy} \end{cases} \qquad \begin{cases} \dfrac{\partial \bar{J}_3''}{\partial \sigma_{xx}} = 3\bar{a}_1''\sigma_{xx}^2 + 2\bar{a}_2''\sigma_{xx}\sigma_{yy} + \bar{a}_3''\sigma_{yy}^2 + \bar{a}_5''\tau_{xy}^2 \\ \dfrac{\partial \bar{J}_3''}{\partial \sigma_{yy}} = \bar{a}_2''\sigma_{xx}^2 + 2\bar{a}_3''\sigma_{xx}\sigma_{yy} + 3\bar{a}_4''\sigma_{yy}^2 + \bar{a}_6''\tau_{xy}^2 \\ \dfrac{\partial \bar{J}_3''}{\partial \tau_{xy}} = 2\left(\bar{a}_5''\sigma_{xx} + \bar{a}_6''\sigma_{yy}\right)\tau_{xy} \end{cases} \tag{14}$$

3. Calibration of advanced criterion

To calibrate the criterion, the tensile and compressive uniaxial and biaxial directional yield stresses for its yield function and also the tensile directional Lankford coefficients for its plastic potential function are needed. By employing the experimental yield stresses of tension and compression in different directions to calibrate the yield function, the strength differential effect of an anisotropic material can be depicted in the criterion.

In tensile test (in θ direction from rolling direction) the stress components can be found as

$$\begin{cases} \sigma_{xx} = \sigma_\theta^T cos^2\theta \\ \sigma_{yy} = \sigma_\theta^T sin^2\theta \\ \tau_{xy} = \sigma_\theta^T sin\theta cos\theta \end{cases} \tag{15}$$

and similarly in compression test it is found that

$$\begin{cases} \sigma_{xx} = -\sigma_\theta^C cos^2\theta \\ \sigma_{yy} = -\sigma_\theta^C sin^2\theta \\ \tau_{xy} = -\sigma_\theta^C sin\theta cos\theta \end{cases} \tag{16}$$

where for the tensile and compressive biaxial tests the stress components are

$$\begin{cases} \sigma_{xx} = \sigma_b^T \\ \sigma_{yy} = \sigma_b^T \\ \tau_{xy} = 0 \end{cases} \tag{17}$$

and

$$\begin{cases} \sigma_{xx} = -\sigma_c^T \\ \sigma_{yy} = -\sigma_c^T \\ \tau_{xy} = 0 \end{cases} \tag{18}$$

In the current research, the non-AFR with the proposed pressure insensitive plastic potential function is employed and the increments of the plastic strain components can be defined as

$$\begin{cases} d\varepsilon_{xx}^p = d\lambda \dfrac{\partial G}{\partial \sigma_{xx}} \\ d\varepsilon_{yy}^p = d\lambda \dfrac{\partial G}{\partial \sigma_{yy}} \\ d\varepsilon_{xy}^p = d\lambda \dfrac{\partial G}{\partial \tau_{xy}} \end{cases} \tag{19}$$

Using the pressure insensitive plastic potential function in Eq. (12), incompressibility of flow rule can be hold as follows

$$d\varepsilon_{zz}^p = -d\varepsilon_{xx}^p - d\varepsilon_{yy}^p \tag{20}$$

Moreover, from the definition of tensile uniaxial (R_θ^T) and biaxial (R_b^T) Lankford coefficients (R-values) it is found that

$$\begin{cases} R_\theta^T = \dfrac{d\varepsilon_{yy}^p}{d\varepsilon_{zz}^p} = -\dfrac{\dfrac{\partial G}{\partial \sigma_{xx}} sin^2\theta + \dfrac{\partial G}{\partial \sigma_{yy}} cos^2\theta - \dfrac{\partial G}{\partial \tau_{xy}} sin\theta cos\theta}{\dfrac{\partial G}{\partial \sigma_{xx}} + \dfrac{\partial G}{\partial \sigma_{yy}}} \\ R_b^T = \dfrac{d\varepsilon_{yy}^p}{d\varepsilon_{xx}^p} = \dfrac{\dfrac{\partial G}{\partial \sigma_{yy}}}{\dfrac{\partial G}{\partial \sigma_{xx}}} \end{cases} \tag{21}$$

Inserting Eqs. (15) to (18) into Eq. (6) and also Eq. (13) into Eq. (19) and the obtained results into Eq. (21), the uniaxial and biaxial tensile and compressive yield stresses and also the uniaxial and biaxial Lankford coefficients can be determined as follows

$$\begin{cases} \sigma_\theta^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{A + \left[B^{\frac{a}{2}} + C^{\frac{a}{3}}\right]^{\frac{1}{a}}} \\ \sigma_\theta^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{-A + \left[B^{\frac{a}{2}} + (-1)^a C^{\frac{a}{3}}\right]^{\frac{1}{a}}} \\ \sigma_b^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{h_x + h_y + \left[\left(a_1' + a_2' + a_3'\right)^{\frac{a}{2}} + \left(a_1'' + a_2'' + a_3'' + a_4''\right)^{\frac{a}{3}}\right]^{\frac{1}{a}}} \\ \sigma_b^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{-h_x - h_y + \left[+\left(a_1' + a_2' + a_3'\right)^{\frac{a}{2}} + (-1)^a\left(a_1'' + a_2'' + a_3'' + a_4''\right)^{\frac{a}{3}}\right]^{\frac{1}{a}}} \\ R_\theta^T = -\dfrac{D}{E} \\ R_b^T = \dfrac{H}{I} \end{cases} \tag{22}$$

where A, B, C, D, E, H and I are as Eq. (23).

4. Parameter evaluation and root mean square errors (RMSEs) of yield stresses and Lankford coefficients

The yield function which is an asymmetric function (pressure sensitive) is required to be calibrated with ten directional yield stress experimental tests such as uniaxial tensile $\left(\sigma_\theta^T\right)$, compressive $\left(\sigma_\theta^C\right)$ yield stresses in orientations of 0°, 15°, 45° and 90° from the rolling direction and also biaxial tensile $\left(\sigma_b^T\right)$ and compressive $\left(\sigma_b^C\right)$ yield stresses.

$$\begin{cases} A = h_x \cos^2\theta + h_y \sin^2\theta \\ B = a_1' \cos^4\theta + \left(a_2' + a_4'\right)\cos^2\theta \sin^2\theta + a_3' \sin^4\theta \\ C = a_1'' \cos^6\theta + \left(a_2'' + a_5''\right)\cos^4\theta \sin^2\theta + \left(a_3'' + a_6''\right)\cos^2\theta \sin^4\theta + a_4'' \sin^6\theta \end{cases}$$

$$\begin{cases} D=\dfrac{1}{2}\left[\bar{a}_1'\cos^4\theta+\left(\bar{a}_2'+\bar{a}_4'\right)\cos^2\theta\sin^2\theta+\bar{a}_3'\sin^4\theta\right]^{\left(\frac{b}{2}-1\right)} \\ \left(\bar{a}_2'\cos^4\theta+\left(2\bar{a}_1'+2\bar{a}_3'-2\bar{a}_4'\right)\cos^2\theta\sin^2\theta+\bar{a}_2'\sin^4\theta\right)+ \\ \dfrac{1}{3}\begin{bmatrix}\bar{a}_1''\cos^6\theta+\left(\bar{a}_2''+\bar{a}_5''\right)\cos^4\theta\sin^2\theta+ \\ \left(\bar{a}_3''+\bar{a}_6''\right)\cos^2\theta\sin^4\theta+\bar{a}_4''\sin^6\theta\end{bmatrix}^{\left(\frac{b}{3}-1\right)} \\ \begin{bmatrix}\bar{a}_2''\cos^6\theta+\left(3\bar{a}_1''+2\bar{a}_3''-2\bar{a}_5''+\bar{a}_6''\right)\cos^4\theta\sin^2\theta+ \\ \left(2\bar{a}_2''+3\bar{a}_4''+\bar{a}_5''-2\bar{a}_6''\right)\cos^2\theta\sin^4\theta+\bar{a}_3''\sin^6\theta\end{bmatrix} \\ E=\dfrac{1}{2}\left(\bar{a}_1'\cos^4\theta+\left(\bar{a}_2'+\bar{a}_4'\right)\cos^2\theta\sin^2\theta+\bar{a}_3'\sin^4\theta\right)^{\left(\frac{b}{2}-1\right)} \\ \left[\left(2\bar{a}_1'+\bar{a}_2'\right)\cos^2\theta+\left(\bar{a}_2'+2\bar{a}_3'\right)\sin^2\theta\right]+ \\ \dfrac{1}{3}\begin{pmatrix}\bar{a}_1''\cos^6\theta+\left(\bar{a}_2''+\bar{a}_5''\right)\cos^4\theta\sin^2\theta+ \\ \left(\bar{a}_3''+\bar{a}_6''\right)\cos^2\theta\sin^4\theta+\bar{a}_4''\sin^6\theta\end{pmatrix}^{\left(\frac{b}{3}-1\right)} \\ \begin{bmatrix}\left(3\bar{a}_1''+\bar{a}_2''\right)\cos^4\theta+\left(2\bar{a}_2''+2\bar{a}_3''+\bar{a}_5''+\bar{a}_6''\right)\cos^2\theta\sin^2\theta+ \\ \left(\bar{a}_3''+3\bar{a}_4''\right)\sin^4\theta\end{bmatrix} \\ H=\dfrac{1}{2}\left(\bar{a}_1'+\bar{a}_2'+\bar{a}_3'\right)^{\left(\frac{b}{2}-1\right)}\left(\bar{a}_2'+2\bar{a}_3'\right)+\dfrac{1}{3}\left(\bar{a}_1''+\bar{a}_2''+\bar{a}_3''+\bar{a}_4''\right)^{\left(\frac{b}{3}-1\right)}\left(\bar{a}_2''+2\bar{a}_3''+3\bar{a}_4''\right) \\ I=\dfrac{1}{2}\left(\bar{a}_1'+\bar{a}_2'+\bar{a}_3'\right)^{\left(\frac{b}{2}-1\right)}\left(2\bar{a}_1'+\bar{a}_2'\right)+\dfrac{1}{3}\left(\bar{a}_1''+\bar{a}_2''+\bar{a}_3''+\bar{a}_4''\right)^{\left(\frac{b}{3}-1\right)}\left(3\bar{a}_1''+2\bar{a}_2''+\bar{a}_3''\right) \end{cases} \tag{23}$$

The new proposed plastic potential function which is an asymmetric function (pressure insensitive), can be calibrated with eight experimental results such as uniaxial tensile Lankford coefficients $\left(R_\theta^T=\dfrac{d\varepsilon_{yy}^p}{d\varepsilon_{zz}^p}\right)$ in $0°$, $15°$, $30°$, $45°$, $60°$, $75°$ and $90°$ and also biaxial tensile Lankford coefficient $\left(R_b^T=\dfrac{d\varepsilon_{yy}^p}{d\varepsilon_{xx}^p}\right)$.

Having these experimental results for an anisotropic sheet metal, the 10 unknown parameters in yield function such as h_x, h_y, $c_i'\left(i=1,2,3,6\right)$ and $c_i''\left(i=1,2,3,6\right)$ and also the 8 parameters in the plastic potential function such as $\bar{c}_i'\left(i=1,2,3,6\right)$ and $\bar{c}_i''\left(i=1,2,3,6\right)$ can be determined by minimizing the following proposed error functions (E_1, E_2) with Downhill Simplex Method.

$$E_1=\left[\frac{\left(\sigma_0^T\right)_{\text{exp.}}}{\left(\sigma_0^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{15}^T\right)_{\text{exp.}}}{\left(\sigma_{15}^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{45}^T\right)_{\text{exp.}}}{\left(\sigma_{45}^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{90}^T\right)_{\text{exp.}}}{\left(\sigma_{90}^T\right)_{pred.}}-1\right]^2+$$

$$\left[\frac{\left(\sigma_b^T\right)_{\text{exp.}}}{\left(\sigma_b^T\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_0^C\right)_{\text{exp.}}}{\left(\sigma_0^C\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{15}^C\right)_{\text{exp.}}}{\left(\sigma_{15}^C\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_{45}^C\right)_{\text{exp.}}}{\left(\sigma_{45}^C\right)_{pred.}}-1\right]^2+$$
$$\left[\frac{\left(\sigma_{90}^C\right)_{\text{exp.}}}{\left(\sigma_{90}^C\right)_{pred.}}-1\right]^2+\left[\frac{\left(\sigma_b^C\right)_{\text{exp.}}}{\left(\sigma_b^C\right)_{pred.}}-1\right]^2 ? \tag{24}$$

and

$$E_2=\left[\frac{\left(R_0^T\right)_{pred.}}{\left(R_0^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{15}^T\right)_{pred.}}{\left(R_{15}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{30}^T\right)_{pred.}}{\left(R_{30}^T\right)_{exp.}}-1\right]^2+$$
$$\left[\frac{\left(R_{45}^T\right)_{pred.}}{\left(R_{45}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{60}^T\right)_{pred.}}{\left(R_{60}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_{75}^T\right)_{pred.}}{\left(R_{75}^T\right)_{exp.}}-1\right]^2+ \tag{25}$$
$$\left[\frac{\left(R_{90}^T\right)_{pred.}}{\left(R_{90}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_b^T\right)_{pred.}}{\left(R_b^T\right)_{exp.}}-1\right]^2$$

By minimizing E_1 and E_2, the unknown parameters are achieved for an anisotropic sheet metal. To understand the difference between the present calibration and the Yoon *et al*. (2014) one, it should be mentioned that Yoon *et al*. (2014) constructed an error function for obtaining yield function with eight experimental data points such as σ_0^T, σ_{45}^T, σ_{90}^T, σ_b^T, σ_0^C, σ_{45}^C, σ_{90}^C, σ_b^C. Furthermore, to obtain h_x, h_y for a pressure sensitive anisotropic material, they proposed the uniaxial tensile yield stress tests which should be carried out in a hydrostatic pressure chamber. Moreover, they did not present any plastic potential function for predicting Lankford coefficients.

After finding 18 material parameters of yield and plastic potential functions, the accuracy of present criterion in compared with experimental results can be investigated. This matter can be achieved by root-mean square errors (RMSEs) of the tensile $\left(E_\sigma^T\right)$, compressive $\left(E_\sigma^C\right)$ yield stresses, biaxial tensile yield stress $\left(E_\sigma^{Tb}\right)$, biaxial compressive yield stress $\left(E_\sigma^{Cb}\right)$, tensile Lankford coefficients $\left(E_R^T\right)$ and biaxial tensile Lankford coefficient $\left(E_\sigma^{Tb}\right)$ as Eqs. (26) to (31).

$$E_\sigma^T=\frac{1}{7}\left(\begin{array}{l}\left[\frac{\left(\sigma_0^T\right)_{exp.}-\left(\sigma_0^T\right)_{pred.}}{\left(\sigma_0^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{15}^T\right)_{exp.}-\left(\sigma_{15}^T\right)_{pred.}}{\left(\sigma_{15}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{30}^T\right)_{exp.}-\left(\sigma_{30}^T\right)_{pred.}}{\left(\sigma_{30}^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{45}^T\right)_{exp.}-\left(\sigma_{45}^T\right)_{pred.}}{\left(\sigma_{45}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{60}^T\right)_{exp.}-\left(\sigma_{60}^T\right)_{pred.}}{\left(\sigma_{60}^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{75}^T\right)_{exp.}-\left(\sigma_{75}^T\right)_{pred.}}{\left(\sigma_{75}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{90}^T\right)_{exp.}-\left(\sigma_{90}^T\right)_{pred.}}{\left(\sigma_{90}^T\right)_{exp.}}\right]^2\end{array}\right)^{\frac{1}{2}}\times 100 \tag{26}$$

and

$$E_{\sigma}^{C}=\frac{1}{7}\left(\begin{array}{l}\left[\frac{\left(\sigma_{0}^{C}\right)_{exp.}-\left(\sigma_{0}^{C}\right)_{pred.}}{\left(\sigma_{0}^{C}\right)_{exp.}}\right]^{2}+\left[\frac{\left(\sigma_{15}^{C}\right)_{exp.}-\left(\sigma_{15}^{C}\right)_{pred.}}{\left(\sigma_{15}^{C}\right)_{exp.}}\right]^{2}+\\ \left[\frac{\left(\sigma_{30}^{C}\right)_{exp.}-\left(\sigma_{30}^{C}\right)_{pred.}}{\left(\sigma_{30}^{C}\right)_{exp.}}\right]^{2}+\left[\frac{\left(\sigma_{45}^{C}\right)_{exp.}-\left(\sigma_{45}^{C}\right)_{pred.}}{\left(\sigma_{45}^{C}\right)_{exp.}}\right]^{2}+\\ \left[\frac{\left(\sigma_{60}^{C}\right)_{exp.}-\left(\sigma_{60}^{C}\right)_{pred.}}{\left(\sigma_{60}^{C}\right)_{exp.}}\right]^{2}+\left[\frac{\left(\sigma_{75}^{C}\right)_{exp.}-\left(\sigma_{75}^{C}\right)_{pred.}}{\left(\sigma_{75}^{C}\right)_{exp.}}\right]^{2}+\\ \left[\frac{\left(\sigma_{90}^{C}\right)_{exp.}-\left(\sigma_{90}^{C}\right)_{pred.}}{\left(\sigma_{90}^{C}\right)_{exp.}}\right]^{2}\end{array}\right)^{\frac{1}{2}}\times 100 \tag{27}$$

and

$$E_{\sigma}^{Tb}=\frac{\left|\left(\sigma_{b}^{T}\right)_{exp.}-\left(\sigma_{b}^{T}\right)_{pred.}\right|}{\left(\sigma_{b}^{T}\right)_{exp.}}\times 100 \tag{28}$$

and

$$E_{\sigma}^{Cb}=\frac{\left|\left(\sigma_{b}^{C}\right)_{exp.}-\left(\sigma_{b}^{C}\right)_{pred.}\right|}{\left(\sigma_{b}^{C}\right)_{exp.}}\times 100 \tag{29}$$

Using these RMSEs, the accuracy of the present criterion and other ones such as Yoon *et al*. (2014) and Lou *et al*. (2013) in compared with experimental results can be simply discussed.

5. Case studies

Using 18 material parameters mentioned in the previous section, the criterion can be calibrated. To the best knowledge of the authors these experimental values have not been determined for any pressure sensitive anisotropic sheet metal, therefore the authors study Al2008-T4, Al2090-T3 and AZ31 to validate the present criterion with experimental data. Nevertheless, it can be applied to any anisotropic sheet metals and can be calibrated with 18 mentioned experimental data properly.

$$E_{R}^{T}=\frac{1}{7}\left(\begin{array}{l}\left[\frac{\left(R_{0}^{T}\right)_{exp.}-\left(R_{0}^{T}\right)_{pred.}}{\left(R_{0}^{T}\right)_{exp.}}\right]^{2}+\left[\frac{\left(R_{15}^{T}\right)_{exp.}-\left(R_{15}^{T}\right)_{pred.}}{\left(R_{15}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\frac{\left(R_{30}^{T}\right)_{exp.}-\left(R_{30}^{T}\right)_{pred.}}{\left(R_{30}^{T}\right)_{exp.}}\right]^{2}+\left[\frac{\left(R_{45}^{T}\right)_{exp.}-\left(R_{45}^{T}\right)_{pred.}}{\left(R_{45}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\frac{\left(R_{60}^{T}\right)_{exp.}-\left(R_{60}^{T}\right)_{pred.}}{\left(R_{60}^{T}\right)_{exp.}}\right]^{2}+\left[\frac{\left(R_{75}^{T}\right)_{exp.}-\left(R_{75}^{T}\right)_{pred.}}{\left(R_{75}^{T}\right)_{exp.}}\right]^{2}+\\ \left[\frac{\left(R_{90}^{T}\right)_{exp.}-\left(R_{90}^{T}\right)_{pred.}}{\left(R_{90}^{T}\right)_{exp.}}\right]^{2}\end{array}\right)^{\frac{1}{2}}\times 100 \tag{30}$$

$$E_R^{Tb} = \frac{\left| \left(R_b^T \right)_{exp.} - \left(R_b^T \right)_{pred.} \right|}{\left(R_b^T \right)_{exp.}} \times 100 \tag{31}$$

In the following, the mechanical properties of three materials such as Al 2008-T4 (a BCC material), Al 2090-T3 (a FCC material) which are aluminum alloys and also AZ31 (a HCP material) which is a magnesium alloy are presented in Tables 1 to 3. It has to be mentioned that for anisotropic materials which their biaxial tensile and compressive yield stresses have not been computed experimentally, they can be determined from uniaxial tensile and compressive yield stresses in 0°, 45° and 90° directions $\left(\sigma_b^T = \frac{\sigma_0^T + 2\sigma_{45}^T + \sigma_{90}^T}{4}, \sigma_b^C = \frac{\sigma_0^C + 2\sigma_{45}^C + \sigma_{90}^C}{4} \right)$.

Using these mechanical properties, the unknown material parameters of the yield and plastic potential functions in Eqs. (6) and (12) for these materials can be achieved with minimizing the proposed error functions in Eqs. (24) and (25) in Tables 4 and 5, respectively.

Table 1 Experimental results for Al 2008-T4, Al 2090-T3 and AZ31 in tension presented by Lou *et al*. (2013) and Yoon *et al*. (2014)

Material	σ_0^T	σ_{15}^T	σ_{30}^T	σ_{45}^T	σ_{60}^T	σ_{75}^T	σ_{90}^T	σ_b^T
Al 2008-T4	211.67	211.33	208.50	200.03	197.30	194.30	191.56	185.00
Al 2090-T3	279.62	269.72	255.00	226.77	227.50	247.20	254.45	289.40
AZ31	170.82	-	-	177.13	-	-	191.83	179.23

Table 2 Experimental results for Al 2008-T4, Al 2090-T3 and AZ31 in compression presented by Lou *et al*. (2013) and Yoon *et al*. (2014)

Material	σ_0^T	σ_{15}^T	σ_{30}^T	σ_{45}^T	σ_{60}^T	σ_{75}^T	σ_{90}^T	σ_b^T
Al 2008-T4	213.79	219.15	227.55	229.82	222.75	220.65	214.64	222.02
Al 2090-T3	248.02	260.75	255.00	237.75	245.75	263.75	266.48	247.50
AZ31	96.58	-	-	94.45	-	-	103.38	97.47

Table 3 Experimental results for Al 2008-T4, Al 2090-T3 for Lankford coefficients in tension presented by Lou *et al*. (2013) and Yoon *et al*. (2014)

Material	R_0^T	R_{15}^T	R_{30}^T	R_{45}^T	R_{60}^T	R_{75}^T	R_{90}^T	R_b^T
Al 2008-T4	0.870	0.814	0.634	0.500	0.508	0.506	0.530	1.000
Al 2090-T3	0.210	0.330	0.690	1.580	1.050	0.550	0.690	0.670

Table 4 Material parameters in yield function of advanced criterion for Al 2008-T4, Al 2090-T3 and AZ31

Material	c_1'	c_2'	c_3'	c_6'	c_1''	c_2''	c_3''	c_6''	h_x	h_y	a
Al 2008-T4	1.9095	1.7286	1.7117	1.6715	-0.0086	-0.1139	5.1651	-0.0028	0.0426	0.0621	3
Al 2090-T3	1.8832	1.7786	1.8744	2.1654	0.0496	15.1969	0.0633	1.1501	-0.1200	0.0208	5
AZ31	2.3079	2.3781	2.4175	2.4855	-1.7834	-0.0641	-4.3644	4.0338	-0.3764	-0.0248	3

Table 5 Material parameters in plastic potential function of advanced criterion for Al 2008-T4, Al 2090-T3

Material	$\overline{c}'_1$	$\overline{c}'_2$	$\overline{c}'_3$	$\overline{c}'_6$	$\overline{c}''_1$	$\overline{c}''_2$	$\overline{c}''_3$	$\overline{c}''_6$	b
Al 2008-T4	1.2338	1.4122	0.3153	0.4779	1.4175	2.2194	-0.1905	-0.0010	3
Al2090-T3	0.5277	1.0692	-0.3533	1.0035	0.6648	0.5051	3.8000	0.7760	3

Experimental material parameters a and b which are newly introduced for the yield and plastic potential functions are obtained by using experimental data for a specific material. They make the criterion capable to predict the experimental data in compression and tension in different anisotropic structures.

5.1 Application to Al 2008-T4

By inserting the material parameters from Table 4 for Al 2008-T4 into Eq. (6), the yield function in σ_{xx}–σ_{yy} plane is obtained, Fig. 1. It is seen that, except Lou *et al*. (2013), the other criteria predict experimental data nearly accurate in σ_{xx}–σ_{yy} plane. Figs. 2 and 3 show the tensile and compressive yield stresses in different orientations and their comparison with Yoon *et al*. (2014), Lou *et al*. (2013) and experimental data. Although Lou *et al*. (2013) proposed criterion can predict tensile yield stresses more accurate than the others, but it is not successful in predicting experimental data for compressive yield stresses and it predicts them nearly independent of the orientation. As it is observed, the present criterion is the most accurate one in computing compressive yield stresses and it is more accurate than the Yoon *et al*. (2014) one in predicting tensile yield stresses. Fig. 4 shows the Lankford coefficients in different directions with material parameters of Table 5. Lou *et al*. (2013) computed the Lankford coefficients with accepting AFR and introducing the same pressure sensitive yield and plastic potential functions for anisotropic sheet metals entitled Modified Yld2000-2d. The present criterion predicts experimental data with good accuracy and better than Lou *et al*. (2013).

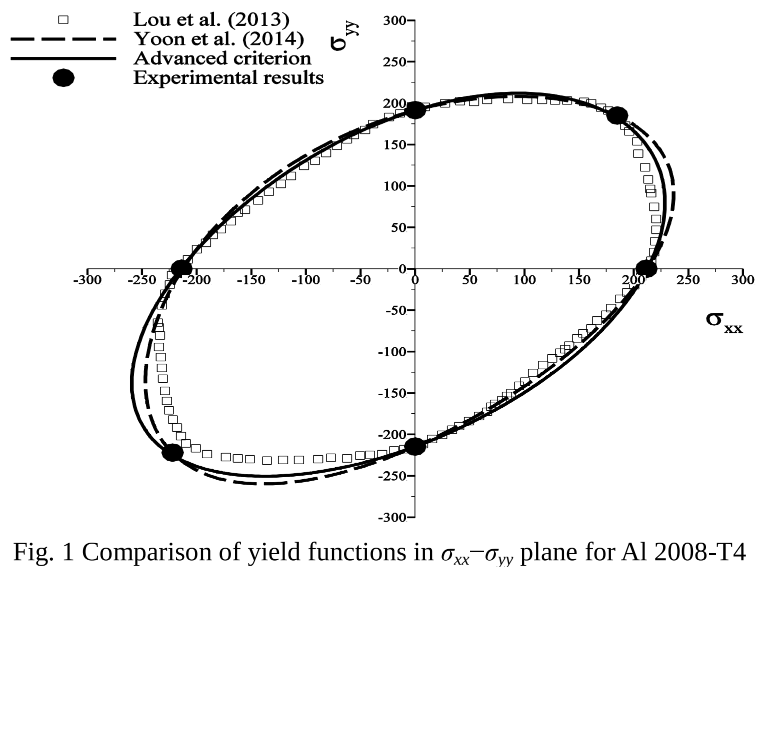

Fig. 1 Comparison of yield functions in σ_{xx}–σ_{yy} plane for Al 2008-T4

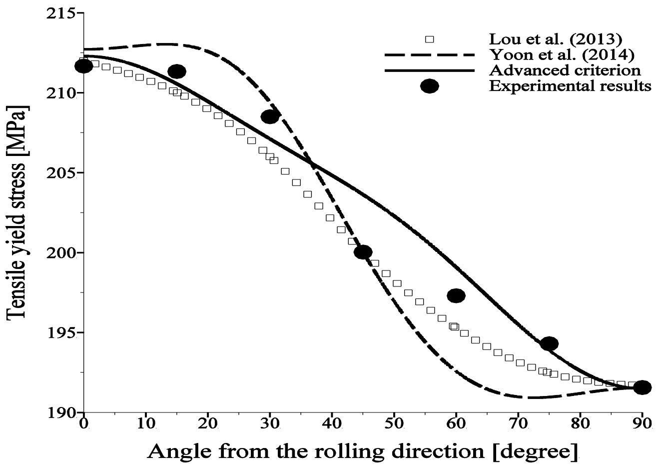

Fig. 2 Comparison of the tensile yield stress directionality for Al 2008-T4

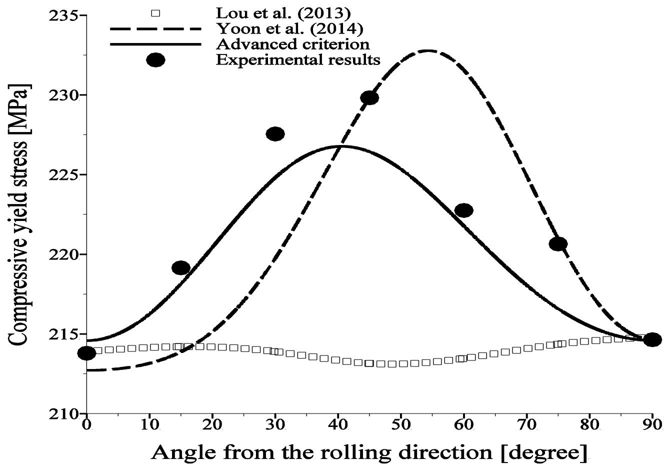

Fig. 3 Comparison of the compressive yield stress directionality for Al 2008-T4

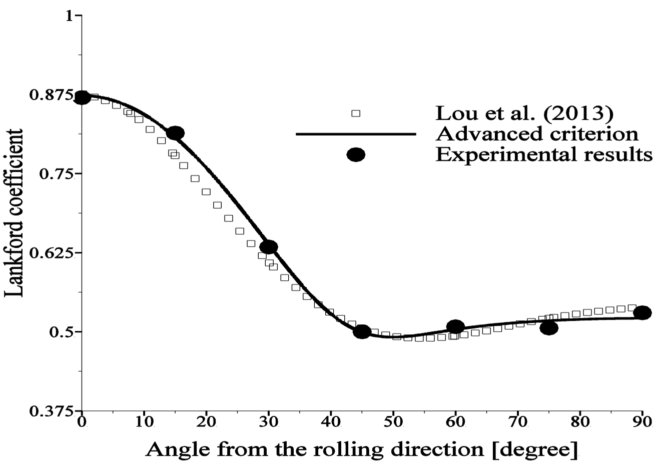

Fig. 4 Comparison of Lankford coefficients directionality for Al 2008-T4

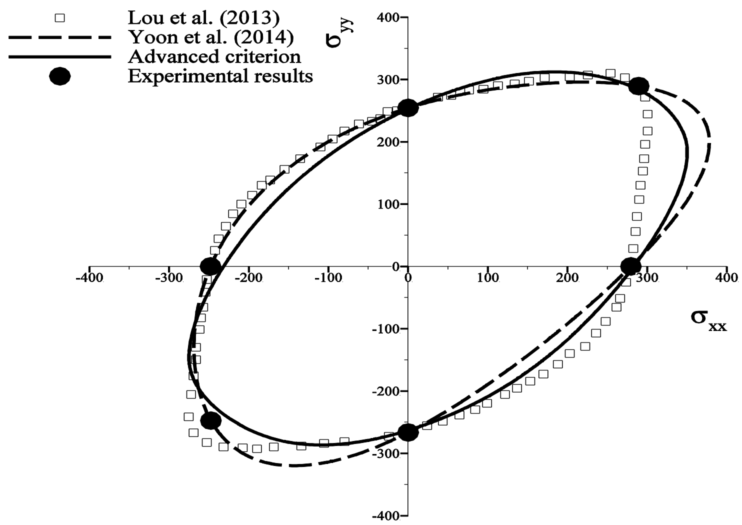

Fig. 5 Comparison of yield functions in σ_{xx}–σ_{yy} plane for Al 2090-T3

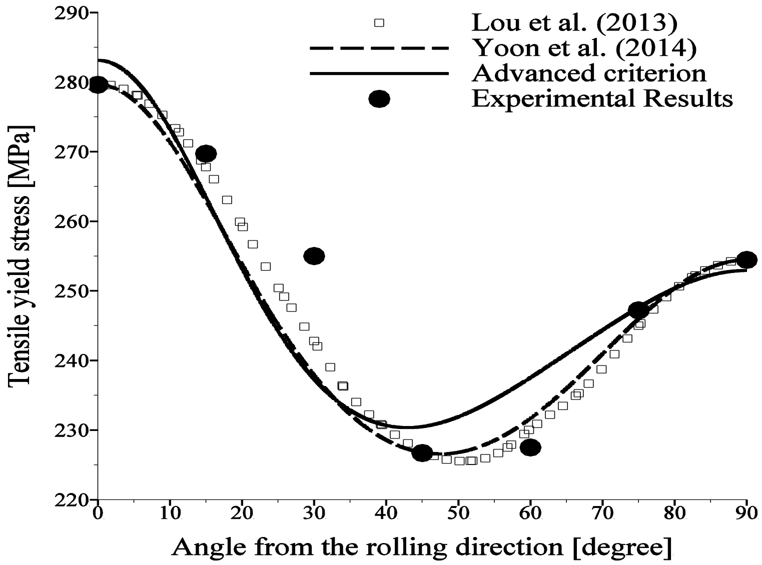

Fig. 6 Comparison of the tensile yield stress directionality for Al 2090-T3

5.2 Application to Al 2090-T3

In this part, the yield function in σ_{xx}–σ_{yy} plane and tensile and compressive yield stresses along with Lankford coefficients are investigated for Al 2090-T3 which is a FCC material. It is found that taking a=5 and b=3 for yield and plastic potential functions are appropriate to predict the experimental results. As it is observed, the experimental data can be predicted using three criteria in σ_{xx}–σ_{yy} plane with proper accuracy, Fig. 5. Furthermore, the criterion can predict the experimental compressive yield stresses for Al 2090-T3 more accurate than the others and the tensile ones more precise than Yoon *et al*. (2014), Figs. 6 and 7. Finally, Fig. 8 shows that the

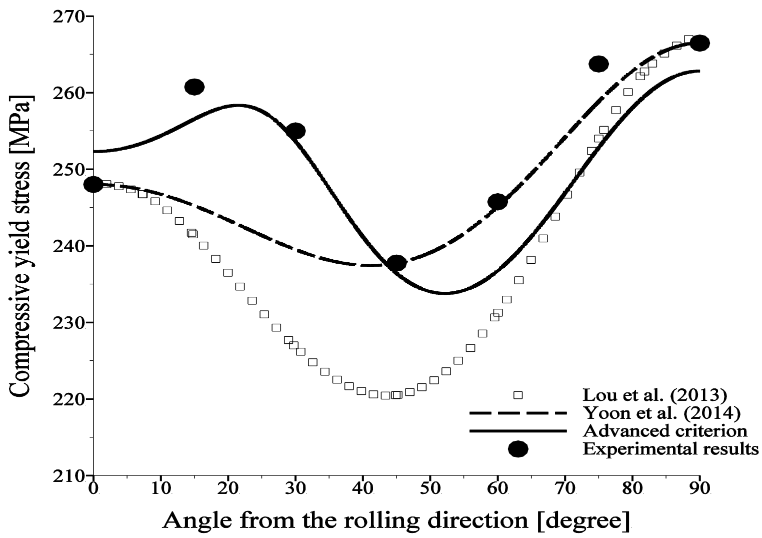

Fig. 7 Comparison of the compressive yield stress directionality for Al 2090-T3

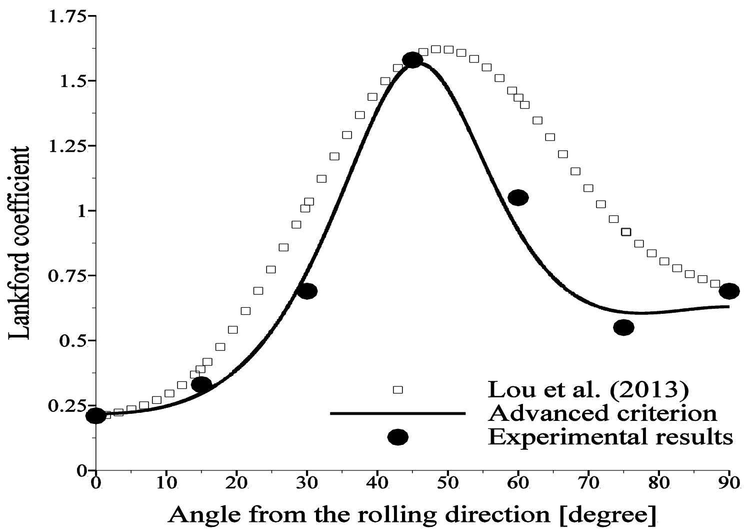

Fig. 8 Comparison of Lankford coefficients directionality for Al 2090-T3

experimental directional Lankford coefficents can be predicted more accurate than Lou *et al.* (2013).

5.3 Application to AZ31

In order to check the present criterion for a HCP material, AZ31 at 3% plastic strain is selected. Ten experimental directional yield stresses are needed to calibrate the yield function of advanced criterion as Eq. (24). To the best knowledge of the authors, however, there are not enough experimental data for AZ31 in literature therefore the following error function, instead of Eq. (24), is presented to obtain the material parameters of AZ31.

$$E_1 = \left[\frac{\left(\sigma_0^T\right)_{\text{exp.}}}{\left(\sigma_0^T\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_{45}^T\right)_{\text{exp.}}}{\left(\sigma_{45}^T\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_{90}^T\right)_{\text{exp.}}}{\left(\sigma_{90}^T\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_b^T\right)_{\text{exp.}}}{\left(\sigma_b^T\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_0^C\right)_{\text{exp.}}}{\left(\sigma_0^C\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_{45}^C\right)_{\text{exp.}}}{\left(\sigma_{45}^C\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_{90}^C\right)_{\text{exp.}}}{\left(\sigma_{90}^C\right)_{pred.}} - 1\right]^2 + \left[\frac{\left(\sigma_b^C\right)_{\text{exp.}}}{\left(\sigma_b^C\right)_{pred.}} - 1\right]^2 \tag{32}$$

The following relations are used to compute the RMSEs, instead of Eqs. (26) and (27)

$$E_\sigma^T = \frac{1}{3}\left(\left[\frac{\left(\sigma_0^T\right)_{exp.} - \left(\sigma_0^T\right)_{pred.}}{\left(\sigma_0^T\right)_{exp.}}\right]^2 + \left[\frac{\left(\sigma_{45}^T\right)_{exp.} - \left(\sigma_{45}^T\right)_{pred.}}{\left(\sigma_{45}^T\right)_{exp.}}\right]^2 + \left[\frac{\left(\sigma_{90}^T\right)_{exp.} - \left(\sigma_{90}^T\right)_{pred.}}{\left(\sigma_{90}^T\right)_{exp.}}\right]^2\right)^{\frac{1}{2}} \times 100 \tag{33}$$

and

$$E_\sigma^C = \frac{1}{3}\left(\left[\frac{\left(\sigma_0^C\right)_{exp.} - \left(\sigma_0^C\right)_{pred.}}{\left(\sigma_0^C\right)_{exp.}}\right]^2 + \left[\frac{\left(\sigma_{45}^C\right)_{exp.} - \left(\sigma_{45}^C\right)_{pred.}}{\left(\sigma_{45}^C\right)_{exp.}}\right]^2 + \left[\frac{\left(\sigma_{90}^C\right)_{exp.} - \left(\sigma_{90}^C\right)_{pred.}}{\left(\sigma_{90}^C\right)_{exp.}}\right]^2\right)^{\frac{1}{2}} \times 100 \tag{34}$$

Fig. 9 shows the comparison of the obtained yield function in σ_{xx}–σ_{yy} plane with experimental results based on the parameters of Table 4. It is observed that although the yield function for the present criterion and Yoon *et al*. (2014) are nearly the same, the directional tensile and compressive yield stress are completely different.

6. Discussions

In order to compare the mentioned criteria for different asymmetric anisotropic sheet metals with each other, RMSEs in Eq. (26)-(31) for Al 2008-T4 and Al2090-T3 and also RMSEs in Eqs. (28), (29), (33), (34) for AZ31 are employed, Tables 6-8. These relative errors express the differences between the obtained results of different criteria for these materials and experimental

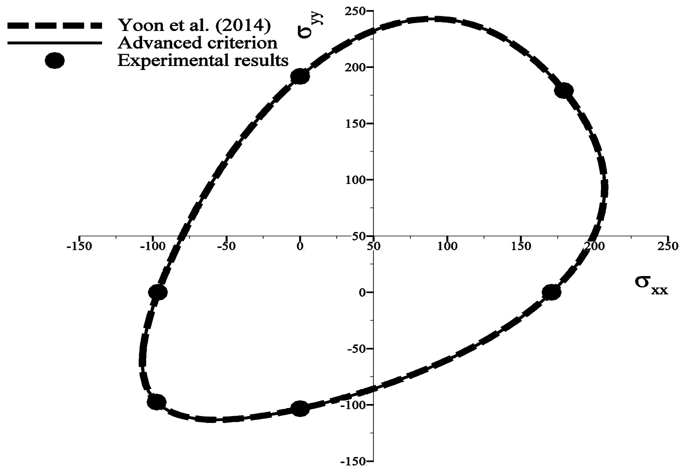

Fig. 9 Comparison of yield functions in σ_{xx}–σ_{yy} plane for AZ31

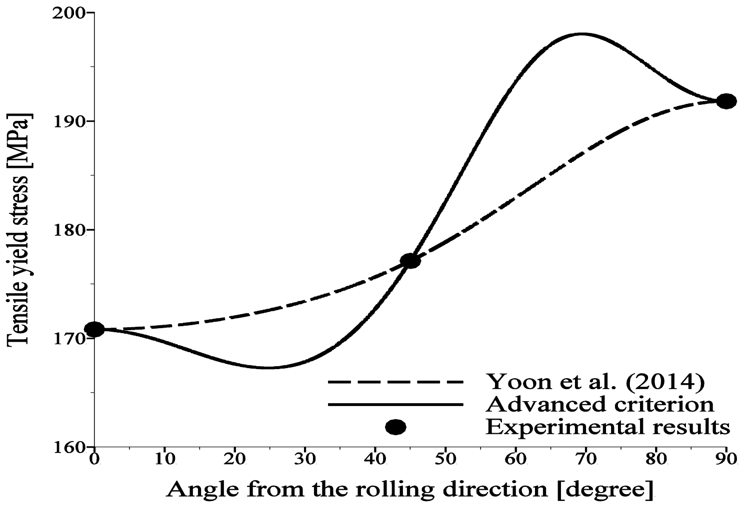

Fig. 10 Comparison of the tensile yield stress directionality for AZ31

Table 6 The obtained computation errors for Al 2008-T4 compared with experimental results (in percentage)

Criterion	E_{σ}^{T}	E_{σ}^{Tb}	E_{σ}^{C}	E_{σ}^{Cb}	E_{R}^{T}	E_{R}^{Tb}
Yoon *et al*. (2014)	0.4460	0.0694	0.8255	0.1170	-	-
Lou *et al*. (2013)	0.2704	0.0778	1.5915	6.0219	3.9852	13.6586
Advanced criterion	0.3119	0.9695	0.3926	0.7801	0.4507	0.0635

data in Tables 1-3. It can be observed that the present criterion is the most proper one for predicting directional compressive yield stresses and also directional Lankford coefficients while the Lou *et al*. (2013) criterion is appropriate in predicting tensile yield stresses. Furthermore, it is seen that the present criterion is more suitable than Yoon *et al*. (2014) in predicting directional tensile yield stresses.

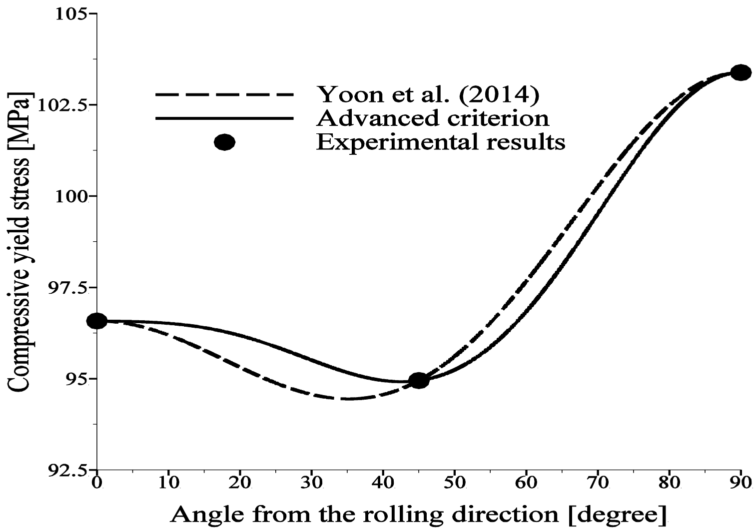

Fig. 11 Comparison of the compressive yield stress directionality for AZ31

Table 7 The obtained computation errors for Al 2090-T3 compared with experimental results (in percentage)

Criterion	E_σ^T	E_σ^{Tb}	E_σ^C	E_σ^{Cb}	E_R^T	E_R^{Tb}
Yoon *et al.* (2014)	1.0599	0.0009	1.2441	0.0005	-	-
Lou *et al.* (2013)	0.7350	0.0015	2.4651	8.2141	12.8890	4.1151
Advanced criterion	1.2580	3.4022	0.8881	3.2407	3.3954	1.5620

Table 8 The obtained computation errors for AZ31 compared with experimental results (in percentage)

Criterion	E_σ^T	E_σ^{Tb}	E_σ^C	E_σ^{Tb}
Yoon *et al.* (2014)	0.0007	0.0014	0.0005	0.0051
Advanced criterion	0.0019	0.0075	0.0006	0.0009

To better understand the difference between the present criterion and Yoon *et al.* (2014) one, the obtained results of Al 2008-T4, Al 2090-T3 and AZ31 are investigated.

The difference between the present yield function and the Yoon *et al.* (2014) one for Al 2008-T4 and AZ31 is solely due to employing the different experimental data for calibrating the yield function. For Al 2090-T3, in spite of different calibration, the experimental parameter a in yield function has taken as 5 while in Yoon *et al.* (2014) a remains 3 for all cases. Yoon *et al.* (2014) yield function considered Al 2008-T4 and Al 2090-T3 as anisotropic pressure insensitive sheet metals but in the present criterion, as it is seen in Table 4, h_x and h_y are not zero and therefore these materials are considered as pressure sensitive materials. With introducing a pressure insensitive plastic potential function based on non-AFR, Lankford coefficients can be determined while they have not been computed by Yoon *et al.* (2014). In the presented pressure sensitive and insensitive yield and plastic potential functions two experimental parameters of a and b are added which make capable the criterion to have more flexibility to predict experimental directional yield stresses and also Lankford coefficients. Finally, it is seen that the directional tensile and compressive yield stresses could be predicted more accurate and the directional Lankford coefficients could also be computed.

Using the material constants identified above, the yield function of advanced criterion can be successfully applied to describe the plastic behavior of metals under plane stress condition, but it cannot be used to model anisotropic-asymmetric plastic deformation under three-dimensional loading since the through-thickness parameters of c_4', c_5', c_4'', c_5'' have not been calibrated yet. Normally these materials constants are computed based on uniaxial tensile and compressive yield stresses in the x–z and y–z planes along 45° from the rolling direction. The uniaxial tensile and compressive yield stresses in the x–z plane along 45° are represented by σ_{xz45}^{T} and σ_{xz45}^{C}, which are calculated by the yield function of advanced criterion in following forms

$$\begin{cases} \sigma_{xz45}^{T} = \dfrac{\sigma(\bar{\varepsilon})}{\dfrac{h_x+h_y}{2}+\left[\left(\dfrac{c_1'^2+c_1'c_3'+c_3'^2+9c_5'^2}{36}\right)^{\frac{a}{2}}+\left(\dfrac{(c_1''+c_3'')(c_1''c_3''-6c_5''^2)}{216}\right)^{\frac{a}{3}}\right]^{\frac{1}{a}}} \\ \sigma_{xz45}^{C} = \dfrac{\sigma(\bar{\varepsilon})}{-\dfrac{h_x+h_y}{2}+\left[\left(\dfrac{c_1'^2+c_1'c_3'+c_3'^2+9c_5'^2}{36}\right)^{\frac{a}{2}}+(-1)^a\left(\dfrac{(c_1''+c_3'')(c_1''c_3''-6c_5''^2)}{216}\right)^{\frac{a}{3}}\right]^{\frac{1}{a}}} \end{cases} \tag{35}$$

Similarly, the uniaxial tensile and compressive yield stresses in the y–z plane in 45° denoted by σ_{yz45}^{T} and σ_{yz45}^{C} are assessed by yield function of advanced criterion as

$$\begin{cases} \sigma_{yz45}^{T} = \dfrac{\sigma(\bar{\varepsilon})}{\dfrac{h_x+h_y}{2}+\left[\left(\dfrac{c_2'^2+c_2'c_3'+c_3'^2+9c_4'^2}{36}\right)^{\frac{a}{2}}+\left(\dfrac{(c_2''+c_3'')(c_2''c_3''-6c_4''^2)}{216}\right)^{\frac{a}{3}}\right]^{\frac{1}{a}}} \\ \sigma_{yz45}^{C} = \dfrac{\sigma(\bar{\varepsilon})}{-\dfrac{h_x+h_y}{2}+\left[\left(\dfrac{c_2'^2+c_2'c_3'+c_3'^2+9c_4'^2}{36}\right)^{\frac{a}{2}}+(-1)^a\left(\dfrac{(c_2''+c_3'')(c_2''c_3''-6c_4''^2)}{216}\right)^{\frac{a}{3}}\right]^{\frac{1}{a}}} \end{cases} \tag{36}$$

Then the material constants c_5', c_5'' are identified by σ_{xz45}^{T} and σ_{xz45}^{C} from Eq. (35) while c_4', c_4'' are evaluated by σ_{yz45}^{T} and σ_{yz45}^{C} by Eq. (36). However, these tests are difficult for sheet metals due to the fact that sheet metals are normally thin to manufacture specimens for these tests. Thus, these four parameters related with through-thickness properties can be assumed to be a value such that the material properties in thickness direction are identical with those of in-plane ones, i.e. $c_4' = c_5' = c_4'' = c_5'' = c_6''$. Additionally, it has to be mentioned that setting c_4', c_5', c_4'', c_5'' as unity does not mean that the through-thickness behavior is isotropic or identical with in-plane plastic behavior, which is different with other yield functions, such as Barlat *et al*. (2003). This is because the anisotropic material constants here also affect the asymmetric behavior of metals. It is noticeable to state that similar procedure can be employed to determine the values of $\bar{c}'_4, \bar{c}'_5, \bar{c}''_4, \bar{c}''_5$ in plastic potential function of advanced criterion for non-plane problems.

7. Conclusions

A non-AFR criterion with introducing a pressure sensitive function for yield function and a pressure insensitive function for plastic potential function as well was proposed newly for considering strength differential effect for anisotropic materials and called here 'Advanced criterion'. In the present yield and plastic potential functions two experimental parameters were added which made capable the criterion to predict the experimental directional yield stresses and also Lankford coefficients more accurately. The yield and plastic potential functions can be calibrated with ten and eight experimental data respectively and also two experimental parameters for these functions can be found for any materials. To verify the criterion three anisotropic materials were selected contain of Al 2008-T4 (a BCC material), Al 2090-T3 (a FCC material) and AZ31 (a HCP material). It was shown that the present criterion could predict the tensile and compressive yield stresses better than Yoon *et al*. (2014) and the Lankford coefficients better than Lou *et al*. (2013).

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