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**Buckling analysis of shear loaded finite plates with circular cutout based on complex analysis**

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**Abstract**

In this paper, the complex variable method is used to study the buckling of finite isotropic shear loaded plates with circular cutout. The prebuckling stress distribution is calculated by introducing the generalized complex-potential functions and the in-plane boundary conditions are applied by using the principle of virtual work. The energy of the plate is calculated based on the first order shear deformation theory (FSDT) and the Ritz method is employed to calculate the buckling load. The effects of the cutout size and the boundary conditions on the buckling load are investigated.

**Keywords:** potential functions, cutout, buckling,Ritz method)

**Introduction**

Thin plates are commonly used in architectural structures, airplanes, ships, etc. Cutouts with different shapes are usually made in the plates for access to different subsystems or to decrease the weight. The stress distribution in the plate is affected by the existence of cutout and as a result, the buckling analysis of a perforated plate is complicated compared with a plate without cutout. Lekhnitskii [[1](#_ENREF_1)] and Savin [[2](#_ENREF_2)] used the complex potential method to calculate the stress concentration factor around a circular hole in an infinite orthotropic plate. Ukadgaonker and Rao [[3](#_ENREF_3)] presented a solution for stresses around an arbitrarily shaped hole in symmetric laminates based on Savin’s formulation. The stress distribution in finite composite laminates with multiple cutouts was studied by Xu and Yue [[4](#_ENREF_4)]. Pan et al. [[5](#_ENREF_5)] used modified stress functions and boundary collocation method to calculate the stress distribution in a finite plate with a rectangular hole. Batista [[6](#_ENREF_6)] presented stress concentration around cutouts with different shapes by using the Schwartz–Christoffel mapping function. Sharma [[7](#_ENREF_7)] used the complex variable method to study stress distribution around polygonal holes in an infinite plate under arbitrary biaxial loading. An approximate method was used by Nemeth et al. [[8](#_ENREF_8)] to study the buckling of a rectangular composite plate with circular cutout. El-sawy and Nazmy [[9](#_ENREF_9)] investigated the buckling of rectangular plates with arbitrarily located circular and rectangular cutouts by Finite Elements (FE) method. Also Anil et al. [[10](#_ENREF_10)] used the FE formulation to calculate the buckling load of composite laminates with rectangular cutout subjected to biaxial loading. Buckling of the plates containing single and multiple holes was demonstrated by Moen and Schafer [[11](#_ENREF_11)]. Komur and Sonmez [[12](#_ENREF_12)] presented numerical solution for buckling of rectangular perforated plates under linearly varying in-plane load. Maiorana et al. [[13](#_ENREF_13)] used the FE method to study the elastic stability of perforated plates under axial compression and bending moment. Barut and Madenci [[14](#_ENREF_14)] developed a complex potential-variational formulation to study the pre-buckling and buckling of flat laminates with an elliptical cutout. Ovesy and Fazilati [[15](#_ENREF_15)] used the finite strip method to calculate the buckling load and natural frequencies of composite plates with circular and square cutouts. Guo [[16](#_ENREF_16)] performed the numerical and experimental studies on the effect of reinforcements around cutouts on buckling of shear loaded plates.

In this paper, the prebuckling stress distribution for a finite isotropic plate with circular cutout under shear load is obtained based on the complex potentials and the Ritz method is used to calculate the buckling load from total potential energy of plate. The effect of cutout size and simply supported and clamped boundary conditions on buckling load is investigated.

**Formulation**

prebuckling

The equilibrium equations for a plate under plane stress conditions are expressed as [[17](#_ENREF_17)]

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These equations are solved by introducing the stress function  as follows:

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the only compatibility equation for plane stress state is written as the following:

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Where  are normal strains and  is shear strain. The stress-strain relations are expressed as

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Now by using the stress-strain relation in Eq. and definition of stress components in Eq. , the compatibility equation is written as a function of  as follows:

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Which is called the biharmonic equation. By introducing the complex variable z=x+iy in Eq., this equation is written as follows

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The solution of this equation is represented as

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Where  is the complex variable and  are arbitrary functions of .

Now by defining , the stress components are calculated from Eq. as the following:

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And the boundary condition is expressed as:

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Where  are resultant force components acting on a segment of the boundary from A to B (Figure 1). The problem solution now reduces to finding two complex potential functions  and  that satisfy the boundary conditions at cutout edge and also at plate edges. In order to calculate the stress distribution for a finite plate with cutout, the generalized complex potentials are expressed as

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Figure. 1: Traction force vectors applied at plate boundaries [[17](#_ENREF_17)]

For an infinite plate under shear loading, the complex potentials reduce to

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Where  is the shear stress applied at infinity and R is the radius of cutout. For a finite plate, the constants  and  in Eq. are calculated from the in-plane boundary conditions. At first, to apply the boundary condition at cutout edge, the complex potentials are substituted in the expression of boundary condition in Eq. , where for a traction free cutout. A system of equations is obtained in which the number of equations is less than the number of unknowns and by solving this system, some of the constants are calculated. Then, to apply the boundary condition at plate edges, a boundary integral is obtained based on the principle of virtual work that is used to calculate the remaining constants. According to the principle of virtual work, in an equilibrium state the relation  holds, where and are the virtual work of internal and external forces, respectively and are expressed as [[18](#_ENREF_18)]

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Where  and  are the stress resultants applied at the plate edges and  are in-plane displacements. By substituting the strain components in the virtual work relation and using the Green’s theorem, we have

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Where  are stress resultants which are expressed as

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 Where h is the plate’s thickness. The coefficients of  and  in each integral of Eq. must be equated to zero. this gives the equilibrium equations and boundary conditions of the problem. The equilibrium equations are satisfied by the obtained solution in previous section. The second integral in Eq. is used to apply the boundary conditions at the plate outer edges. To do this, the displacements  and  must be known. Based on the stress-strain relations, the strain components are calculated and the displacements are calculated by integrating the strains. To apply the boundary conditions at the plate edges, the values of resultant forces  and  and also the variations  and  are substituted in the boundary integral (the second integral in Eq. ). By equating the coefficients of  and  to zero in the obtained expression, a system of algebraic equations is acquired and the unknown constants are calculated.

Energy of plate based on FSDT

According to FSDT, the displacement field of a deformed plate with length *a*, width *b* and thickness *h*, is expressed as [[18](#_ENREF_18)]:

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Where  are the displacement components of a point on the mid-plane (z=0) and  are the rotation of a line initially perpendicular to the mid-plane relative to y and x axes, respectively. The total potential energy of the plate during lateral deflection is  where  is the strain energy due to bending deformations and  is the potential energy due to the work of in-plane loads during lateral deflection. These energies are expressed as [[19](#_ENREF_19)]

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Where S is the mid-plane area. The moment and transverse shear resultants are defined as:

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Where ks =5/6 is the shear correction factor [[18](#_ENREF_18)]. By substituting Eqs. and into Eq. , the total potential energy can be written as a function of prebuckling stress components and displacement field. In the Ritz method, the potential energy of the plate is approximated by assuming appropriate functions for displacements. Here the displacements are expressed as

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Where  are orthogonal functions that satisfy the essential boundary conditions of the problem. In this paper, the vibrational mode shapes of a beam with boundary conditions similar to that of the plate are used to express  and. These functions for simply supported and clamped beams are as follows

a) simply supported at both ends:

 Where *a* is the length of the beam.

b) clamped at both ends:

 

Where  is the solution of .

The functions  are constructed by replacing *x, a* and *m* by *y, b* and *n,* respectively, in the corresponding  function. At an equilibrium state, the total potential energy of the system is stationary , i.e

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By substituting the expression for total potential energy into Eqs. , an eigenvalue problem is obtained and by solving this problem, the buckling loads and mode shapes are calculated.

**Results and discussion**

Prebuckling

The method of complex potentials was used to obtain the prebuckling stress distribution as was explained in previous section. The value of  and  in Eq. are selected as 20. The maximum error in satisfying the boundary conditions at plate edges for this value is less than 2%. This error is calculated as:
E(%)= (actual value-calculated value)/actual value\*100 the variation of normalized tangential stress () around circular cutout for different *d/a* ratios is represented in Figure 3, where  is the shear stress applied at plate edges and *d,a* are the cutout diameter and plate’s length, respectively (Figure 2). The value of maximum tangential stress for an infinite plate is equal to 4. For small cutouts (*d/a*<0.1) the plate can be assumed as infinite, but as the value of *d/a* increases, the maximum stress increases too and this assumption cease to be correct. Figure 3 shows that the maximum stress happens at 45o and repeats on 900 intervals.



Figure 2: Geometry of the plate with cutout under in-plane shear loading



Figure 3: variation of tangential stress around circular cutout.

The contour plot of normalized prebuckling stress components for a plate with *d/a*=0.6 is shown in Figure 4. The stress distribution is symmetric about *x* and *y* axes.

Buckling

The potential energy of the plate is calculated by substituting the prebuckling stress components in Eq. and the gauss quadrature method is used for integration. After that, by applying the Ritz minimization method, an eigenvalue problem is obtained and the buckling load is calculated by solving this system. In Table 1, the buckling load of square simply supported (SSSS) and clamped plates (CCCC) on all edges and with different d/a ratios is presented (a/h=40). The results are also compared with Finite Element (FE) solutions obtained by modeling the plate in the ABAQUS package. The relative error is computed as: error(%)=(FE-present)/FE\*100. The buckling load is normalized as  where  is the flexural rigidity of the plate. Table 1 shows that the buckling load decreases as the cutout size increases for both simply supported and clamped plates. Also plates with clamped boundary conditions are more stable and buckle in higher loads compared with simply supported plates.

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Figure 4: contour plot of prebuckling stress components

The sensitivity of the buckling load to prebuckling stress distribution is studied in Figure 5. In this Figure, the buckling load is calculated by assuming constant prebuckling stress field, i.e  and this value is compared with the buckling load from previous results. When the  ratio increases, the non-uniformity in the prebuckling stress field increases and as a result, the sensitivity of the buckling load to prebuckling stress field increases, which is seen in Figure 5.

Table 1: Normalized buckling load () of a square plate with circular cutout under shear loading

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| --- | --- |
| d/a | Boundary condition |
| SSSS | CCCC |
| Present | FE | Error(%) |  | Present | FE | Error(%) |  |
| **0** | **9.235** | **9.240** |  **0.06** |  | **14.371** | **14.358** | **-0.09** |  |
| **0.1** | **8.683** | **8.479** | **-2.40** |  | **13.317** | **13.015** | **-2.32** |  |
| **0.2** | **7.054** | **6.948** | **-1.53** |  | **10.785** | **10.693** | **-0.87** |  |
| **0.3** | **5.465** | **5.403** | **-1.13** |  | **8.662** | **8.535** | **-1.49** |  |
| **0.4** | **4.089** | **4.051** | **-0.95** |  | **6.853** | **6.701** | **-2.27** |  |
| **0.5** | **2.998** | **2.969** | **-0.98** |  | **5.158** | **5.292** |  **2.52** |  |
| **0.6** | **2.185** | **2.162** | **-1.10** |  | **4.059** | **4.264** |  **4.81** |  |

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| $$\overbar{N}\_{cr}$$a) SSSS |
| $$\overbar{N}\_{cr}$$b) CCCC |

Figure 5: Sensitivity of buckling load to prebuckling stress field

**Conclusion**

In this paper, buckling of shear loaded plates with circular cutout was investigated. the complex potential method was used to calculate the prebuckling stress distribution for a finite plate. The potential energy of plate was calculated based on the first order shear deformation theory and the Ritz method was used to calculate the buckling load for simply supported and clamped plates. The results show that the presented solution can predict the stress distribution for a finite plate and also the buckling load with good accuracy. The presence of cutout decreases the buckling load of the plate and as the cutout size increases, the plate buckles in lower loads.

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