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Key Points:
- Soil similarity is not sufficient for applicability of Miller-Miller scaling
- The interrelation between $h(S)$ and $K(S)$ functions also needs to be comparable
- Besides pore geometry, the interrelation depends on solid-liquid interactions

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A critical evaluation of the Miller and Miller similar media theory for application to natural soils

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Abstract The Miller-Miller similar media theory is widely applied to characterize the spatial variability of soil hydraulic properties. For a group of soils, a distinct scaling factor is commonly assigned to each individual soil to coalesce the soil water characteristic and hydraulic conductivity functions to single curves. It is generally assumed that the Miller-Miller theory is valid as long as soils are "similar" either with regard to their microscopic pore space geometry or the closely related macroscopic soil hydraulic functions. In this paper, it is illustrated that similarity is not the sole required condition for validity of the Miller-Miller theory. In addition, the interrelation between the soil water characteristic and the hydraulic conductivity functions considered for scaling need to be comparable. The interrelation is dependent not only on the pore space geometry, but also on solid-liquid interactions. Hence similar interrelation cannot be concluded from similarity of microscopic pore space geometry. A dimensionless parameter termed the "joint scaling factor" was defined and applied to evaluate the soundness of the interrelation condition for 26 soils from the UNSODA database that were grouped into six classes of similar soils. Obtained results clearly reveal the crucial importance of the interrelation condition for the Miller-Miller scaling theory, which has been hidden behind the "similarity" requirement, and contradict the general belief that Miller-Miller scaling is valid as long as soils are "similar."

1. Introduction

1.1. Evolution of the Miller-Miller Similar Media Theory

More than half a century ago, *Miller and Miller* [1956] created a new avenue for soil hydrology research when they presented their pioneering scaling theory that assumes that two porous media are scalable through a physical characteristic length (scaling factor) when they are similar in their detailed microscopic geometry. Figure 1 illustrates two such media that can be scaled interchangeably with any characteristic length (e.g., λ_1 and λ_2), which connects corresponding points within the two media. Similar media have identical porosities and are assumed to be in a "similar state" when the volumetric water contents, θ, are identical.

Assumptions made by *Miller and Miller* include, "the physical laws of surface tension and viscous flow" apply; "no special pore-shape assumptions are required, but one topological approximation is needed; i.e., that neither isolated drops nor isolated bubbles occur." Based on these assumptions, *Miller and Miller* showed that a consequence of media similarity is that for a given water content, the soil matrix pressure head, h, and soil hydraulic conductivity, K, for N similar soils in a similar state can be scaled as:

$$\lambda_1 h_1 = \lambda_2 h_2 = \ldots = \lambda_N h_N = \hat{\lambda} h^* \tag{1}$$

$$\frac{K_1}{\lambda_1^2} = \frac{K_2}{\lambda_2^2} = \ldots = \frac{K_N}{\lambda_N^2} = \frac{K^*}{\hat{\lambda}^2} \tag{2}$$

where λ_i is the scaling factor for the ith medium, $\hat{\lambda}$ is the mean scaling factor of all considered similar media and h^* and K^* are the scaled forms of h and K. Note that for convenience, h is defined as the absolute value of the pressure head.

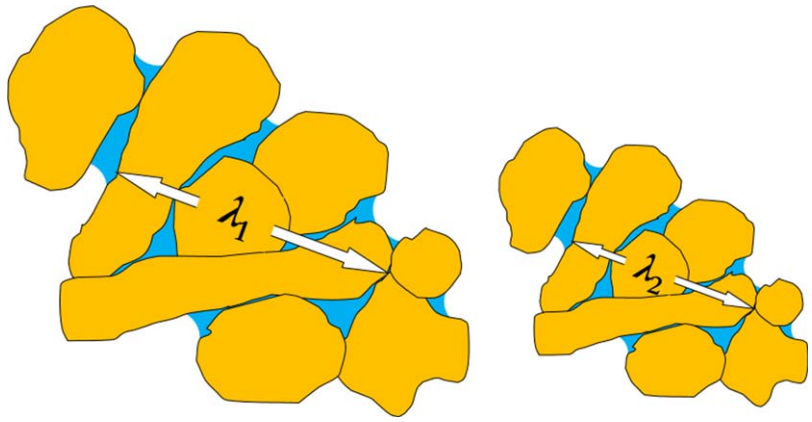

Figure 1. Sketch, illustrating two "similar media" in a "similar state". Note that the two characteristic lengths, λ_1 and λ_2, connect corresponding points within the two media [*Miller and Miller*, 1956].

Equations (1) and (2) suggest that the water characteristic, $h(\theta)$, and hydraulic conductivity function, $K(\theta)$, of various similar soils can be coalesced into unified scaled curves. Thus, all soils can be described by means of an average or reference soil tied to a single scaling factor assigned to each soil.

Miller and Miller [1956] stated: "in practice, the occurrence of detailed similarity throughout the microscopic geometries of two media has zero probability." In addition, there has been no exact methodology to explore such a geometric similarity within the microscopic domain. Hence, to evaluate the Miller-Miller theory, most researchers considered similarity of macroscopic/measurable soil properties, which are commonly tied to microscopic properties (e.g., pores size and geometry) based on fundamental physical laws.

Klute and Wilkinson [1958] were among the first to test the Miller-Miller theory for graded sands. They defined the geometrical similarity in a practical way: "similar media have size-distribution curves whose particle size axes can be made identical by multiplying one of them with a constant." They considered the average grain size as the scaling factor and showed that $h(\theta)$ and $K(\theta)$ of all sands coalesced after scaling. *Wilkinson and Klute* [1959] and *Elrick et al*. [1959] further tested the similar media theory for water flow in similar sands. They also arrived at the conclusion that the Miller-Miller scaling theory is valid yielding invariant scaled flow rates in similar sands.

Defining similarity based on *Klute and Wilkinson* [1958] is inconsistent because there is no guarantee for maintaining pore space similarity in media with similar particle-size distributions. "Because the results from ensuing tests were not particularly encouraging, except for soils composed of graded sands, the Miller and Miller scaling theory laid idle for several years" [*Nielsen et al*., 1998]. However, in the 1970s when soil physicists faced the dilemma of how to deal with naturally occurring field soil variability, Miller-Miller scaling was revived. This was because it seemed a promising approach to describe the variability of soil hydraulic functions by means of a distribution function of a single stochastic parameter, the scaling factor [*Nielsen et al*., 1998; *Wosten*, 1990].

Warrick et al. [1977] established a straightforward method for describing field spatial variability of soil hydraulic properties following the Miller-Miller theory considering two relaxations of the original theory: (*i*) owing to the fact that soils do not have identical porosities, they used degree of saturation S rather than volumetric water content θ to avoid the need for similar media to exhibit identical porosities; and (*ii*) they avoided a search for microscopic physical lengths by merely deriving scaling factors that minimized the sum of squared errors between scaled hydraulic properties of various sampling locations and field-averaged functions. The approach recognizes that the exact scaling illustrated in Figure 1 is an idealized case, which does not hold for real soils, nevertheless variability in certain cases can be approximated with scaling factors as indicated in equations (1) and (2). This regression-based scaling method was later called "functional normalization" [*Tillotson and Nielsen*, 1984].

The scaling concept rapidly gained popularity in subsequent years when various practical definitions of similarity were introduced, mostly based on the shape of soil hydraulic functions [*Simmons et al*., 1979; *Vogel et al*., 1991; *Kosugi and Hopmans*, 1998; *Tuli et al*., 2001]. A basic assumption was that the similarity of the

shape of soil hydraulic functions, $h(\theta)$ and $K(\theta)$, is a direct consequence of the similarity of the soil pore-size probability distributions. The relationship between hydraulic functions and pore-size distribution was based on the Young-Laplace equation (i.e., h is proportional to reciprocal pore size), and therefore implies the similarity defined by *Miller and Miller* (i.e., Figure 1).

Subsequently, scaling was examined and applied as a standard method for capturing spatial variability of soil hydraulic properties by assigning a single scaling factor for each soil or location [*Sharma and Luxmoore*, 1979; *Ahuja and Williams*, 1991; *Shouse and Mohanty*, 1998; *Zavattaro et al.*, 1999; *Ursino et al.*, 2000; *Tuli et al.*, 2001; *Das et al.*, 2005; *Oliveira et al.*, 2006; *Nasta et al.*, 2009; *Vogel et al.*, 2010; *Sadeghi et al.*, 2010; *Schlüter et al.*, 2012; *Nasta et al.*, 2013; *Meskini-Vishkaee et al.*, 2013; *Ahmadi et al.*, 2014; *Sadeghi et al.*, 2014]. Scaling was implemented in numerical models such as HYDRUS [*Simunek et al.*, 2008], a widely applied simulation code for unsaturated flow and transport, in order to simplify the description of the spatial variability of unsaturated soil hydraulic properties within the flow domain.

In addition to describing soil variability, scaling has facilitated a parallel avenue helping to formulate unsaturated flow and transport in similar soils in a universal, scale-invariant or soil-independent manner [*Reichardt et al.*, 1972; *Warrick and Amoozegar-Fard*, 1979; *Sharma et al.*, 1980; *Youngs and Price*, 1981; *Warrick et al.*, 1985; *Sposito and Jury*, 1985; *Kutilek et al.*, 1991; *Warrick and Hussen*, 1993; *Nachabe*, 1996; *Wu and Pan*, 1997; *Shukla et al.*, 2002; *Rasoulzadeh and Sepaskhah*, 2003; *Kozak and Ahuja*, 2005; *Roth*, 2008; *Sadeghi et al.*, 2011; *Sadeghi and Jones*, 2012]. Referring to macro Miller, Nielsen, and Warrick-type similitude, *Sposito* [1998] examined invariance of the Richards' equation with Lie group analysis leading to power-law or exponential soil hydraulic functions [*Warrick*, 2003, p. 46]. *Sadeghi et al.* [2012a,b] extended this approach beyond similar soils, indicating that under certain conditions "dissimilar soils" (e.g., sands and clays) can be scaled concurrently, leading to invariant forms of the Richards' equation. Developed scaling methods based on the Miller-Miller similar media theory were reviewed by several authors [e.g., *Miller*, 1980; *Warrick and Nielsen*, 1980; *Warrick*, 2003; *Hillel and Elrick*, 1990; *Nielsen et al.*, 1998; *Vereecken et al.*, 2007].

1.2. On the Validity of the Miller-Miller Theory

Beyond the similarity assumption, a basic postulation of the Miller-Miller theory is validity of the Young-Laplace (YL) equation:

$$h=\frac{2\tau}{\rho g R} \tag{3}$$

where τ is the liquid surface tension, ρ is the liquid density, g is the acceleration due to gravity, and R is the radius of the liquid-vapor interface curvature. Equation (3) is commonly written in terms of the effective pore radius as [*Warrick*, 2003, p. 14]:

$$h=\frac{2\tau\cos\beta}{\rho g r} \tag{4}$$

where β is the solid-liquid contact angle and r is the effective radius of the largest pore filled with water at a distinct pressure head h. The relation between h and S is that all pores smaller than the effective radius are assumed to be liquid-filled.

As discussed above, it is not possible to realistically evaluate microscopic pore space similarity. Nevertheless, considering equation (4) it may be assumed that similar soils exhibit similar shapes in their soil hydraulic functions (hereinafter termed "macro-similarity"). Therefore, the Miller-Miller scaling theory is commonly assumed to be valid for macrosimilar soils [e.g., *Kosugi and Hopmans*, 1998; *Tuli et al.*, 2001; *Das et al.*, 2005].

In contrast to previous studies cited above, we point out that similarity is not always adequate for validity of the Miller-Miller theory due to several limitations listed below:

1. Based on the YL equation, the similarity of h may imply similarity of r which, in real soils, means similarity of the sample-scale "effective" (average) pore size at a given degree of saturation, not similarity of all individual pores. Hence, detailed pore space geometry at the microscopic scale may be different in soils where macrosimilarity is satisfied.
2. Similarity of h implies similarity of r through equation (4) only when τ and β are constant for all degrees of saturation and for all considered soils. This assumption may be easily violated in natural soils where τ

varies due to temperature change and the presence of impurities (e.g., chemical coatings and biofilms) and β is sensitive to the presence of impurities, surface charge density, and microscopic heterogeneity of the solid-liquid interface [*Bachmann et al*., 2006]. The latter is well documented in *Bachmann et al*. [2013], who presented measured data exhibiting a high degree of spatial contact angle variability (from 47° to 140°) within a small sandy soil sample (2-cm radius and 5-cm tall). The effects of variable contact angles on soil water dynamics were highlighted in *Diamantopoulos and Durner* [2013]. They showed that "small changes in contact angle can strongly influence the water retention and hydraulic conductivity functions."

3. Assuming all pores smaller than *r* are liquid-filled and the remaining pores are completely empty, the YL equation does not account for contributions due to adsorbed liquid films. Several studies conducted within the past decade have provided evidence that in addition to capillary forces, adsorptive surface forces significantly impact the matric potential, especially in fine-textured soils with high specific surface areas [*Tuller et al*., 1999; *Or and Tuller*, 1999; *Tuller and Or*, 2001; *Tuller and Or*, 2005; *Peters and Durner*, 2008; *Lebeau and Konrad*, 2010]. They pointed to the coexistence of liquid water films and capillary menisci that can be defined with a combination of the conventional YL equation for the capillary pressure and the disjoining pressure concept for thin water films covering solid surfaces [*Tuller et al*., 1999; *Bachmann and van der Ploeg*, 2002].
4. Another underlying assumption introduced by *Miller and Miller* [1956] for derivation of equations (1) and (2) is phase-connectivity, which may not be valid due to issues arising from water repellency. Recent studies pointed out that most soils are neither completely hydrophilic nor completely hydrophobic, but exhibit a subcritical level of water repellency (i.e., contact angle $>0°$ and $<90°$) [*Bachmann et al*., 2013]. *Muehl et al*. [2012] reported that even a subcritical level of water repellency may lead to a decrease in the water-wetted area and disconnection of the water phase. For such cases, equations (1) and (2) may not perfectly hold even if a hypothetical microscopic similarity exists.

These shortcomings in conjunction with other issues such as measurement artifacts because of pore accessibility issues and effects of sample height (e.g., discussed in *Hunt et al*. [2013]) suggest that the Miller-Miller theory is not necessarily valid when similarity (whether microscopic or macroscopic) holds. In summary, it should be highlighted that similarity is not the sole required condition for validity of equations (1) and (2), but several other assumptions are required for successful scaling of soil hydraulic functions.

The validity of equations (1) and (2) is commonly referred to as "simultaneous scaling" [*Clausnitzer et al*., 1992], which assumes equality of scaling factors calculated from $h(S)$ or $K(S)$. This term may be used in contrast to "independent scaling" [*Clausnitzer et al*., 1992], where different but correlated scaling factors for $h(S)$ and $K(S)$ are considered. It is apparent that the ability to characterize spatial variability of soil hydraulic functions with a single set of scaling factors (simultaneous scaling) is more desirable than having two different sets of scaling factors (independent scaling) [*Clausnitzer et al*., 1992; *Tuli et al*., 2001].

Warrick et al. [1977], *Russo* and Bresler [1980], *Clausnitzer et al*. [1992], *Tuli et al*. [2001], and *Stoffregen and Wessolek* [2014] evaluated the applicability of simultaneous scaling and showed that it is not always as successful as independent scaling. However, none of these authors and other researchers have evaluated the validity of other discussed assumptions beyond similarity. Motivated by this knowledge gap, the goal of this study was to evaluate to what extent equations (1) and (2) are valid for several groups of macrosimilar soils. Within this context, the specific objectives were to examine if: (*i*) a set of scaling factors reduce $h(S)$ to a single curve; (*ii*) a set of scaling factors reduce $K(S)$ to a single curve; and (*iii*) whether the two sets of scaling factors are identical.

In addition, a new soil-specific parameter termed the "joint scaling factor" is introduced here to diagnose the validity of the Miller-Miller scaling theory, equations (1) and (2), for a series of soils merely by knowing their hydraulic parameters [e.g., *van Genuchten* parameters].

2. Theory

For the application of scaling theories, it is convenient to describe the soil hydraulic properties by continuous mathematical functions such as proposed by *Brooks and Corey* [1964], *van Genuchten* (VG) [1980], *Gardner* [1958], *Russo* [1988], or *Kosugi* [1994, 1996] (Table 1), only to name a few. These and various other functions [*Warrick*, 2003, p. 64–65], contain scale parameters (e.g. *van Genuchten* α or the saturated

hydraulic conductivity, K_s) and shape parameters (e.g., *van Genuchten n*). The scale parameters are generally multiplied with the state variables (h and K) and thus their variation will cause a linear shift (i.e., scaling) of the hydraulic functions. In contrast, the variation of shape parameters leads to a nonlinear change (i.e., shape change) of the hydraulic functions. Figure 2 illustrates how variations of the scale (h_m) and shape (σ) parameters of the *Kosugi* [1994] soil water characteristic model will change the scale and shape of the curve, respectively.

When adopting such mathematical models for scaling applications, equality of the shape parameters implies macrosimilarity, which, in conjunction with equation (4) and the invariant τ and β assumption, implies similarity of the effective pore-size distribution. This has been shown in *Kosugi and Hopmans* [1998] and *Tuli et al.* [2001] through a physically based approach where the median pore size was assumed as the characteristic length scale (λ) depicted in Figure 1. It was further assumed that pore sizes are lognormally distributed.

The soil hydraulic functions, $h(S)$ and $K(S)$, of macrosimilar soils can be scaled independently using their individual scale parameters as the scaling factor. For example, the *Kosugi* $h(S)$ scaled with h_m (i.e., $h^* = h/h_m$) and $K(S)$ scaled with K_s (i.e., $K^* = K/K_s$) are invariant functions for all macrosimilar soils (i.e., soils having the same σ; see Table 1). Hence, the macrosimilarity condition would be adequate for "independent scaling" of $h(S)$ and $K(S)$. However, as discussed in the following, it is not adequate for "simultaneous scaling" of $h(S)$ and $K(S)$ based on equations (1) and (2).

In the functions listed in Table 1 and other commonly applied models, the $h(S)$ scale parameter (η) is different from the $K(S)$ scale parameter (κ), which is equal to K_s for most models. Substitution of the scale

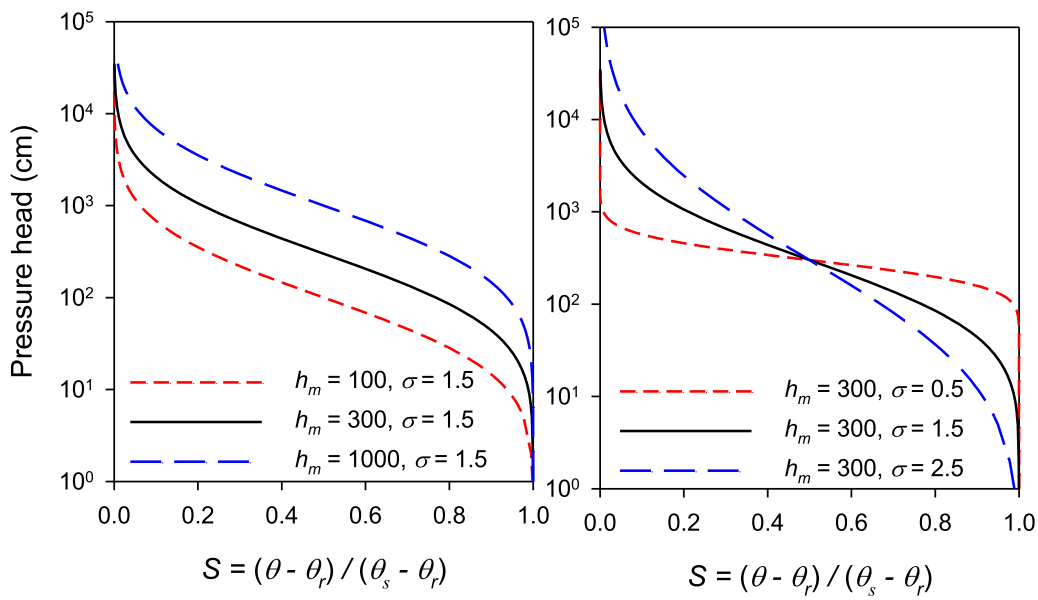

Figure 2. Effects of scale (h_m) and shape parameter (σ) variations on the *Kosugi* [1994] soil water characteristic model.

Table 1. Scaling Factor (λ), Shape Parameter (SP), and Joint Scaling Factor (γ) Specified for Four Common Hydraulic Models[a]

Model	$S = \frac{\theta-\theta_r}{\theta_s-\theta_r} =$	$K_r = \frac{K}{K_s} =$	λ	SP	γ
Brooks and Corey [1964]	$\begin{cases} 1 & h \le h_b \\ (h/h_b)^{-p} & h > h_b \end{cases}$	$S^{3+2/p}$	h_b^{-1}	p	$ch_bK_s^{0.5}$
van Genuchten [1980]	$[1+(\alpha h)^n]^{1/n-1}$	$S^{0.5}\left\{1-\left[1-S^{n/(n-1)}\right]^{1-1/n}\right\}^2$	α	n	$c\alpha^{-1}K_s^{0.5}$
Gardner [1958]; Russo [1988]	$\left(\frac{1+0.5Ah}{e^{0.5Ah}}\right)^{2/(l+2)}$	e^{-Ah}	A	l	$cA^{-1}K_s^{0.5}$
Kosugi [1994, 1996]	$0.5erfc\left[\frac{\ln(h/h_m)}{\sigma\sqrt{2}}\right]$	$S^{0.5}\left\{0.5erfc\left[erfc^{-1}(2S)+\frac{\sigma}{\sqrt{2}}\right]\right\}^2$	h_m^{-1}	σ	$ch_mK_s^{0.5}$

[a]In this table, S is the effective water saturation, θ is volumetric water content, θ_s and θ_r are the saturated and residual volumetric water contents, K_r, K, and K_s are relative, unsaturated, and saturated hydraulic conductivities, h is pressure head, h_b and h_m are the air-entry and median pressure heads, σ is the standard deviation of the log-transformed pore size, the remaining parameters p, α, n, A, and l are free model parameters obtained via nonlinear regression to measured data, and c is the constant in equation (11).

parameters η and κ (which are essentially h^{-1} and K for distinct degrees of saturation) into equations (1) and (2) yields:

$$\frac{\lambda_i}{\hat{\lambda}} = \frac{\eta_i}{\hat{\eta}} \tag{5}$$

$$\frac{\lambda_i}{\hat{\lambda}} = \sqrt{\frac{\kappa_i}{\hat{\kappa}}} \tag{6}$$

Combining (5) and (6) then yields:

$$\frac{\sqrt{\kappa_i}}{\eta_i} = \frac{\sqrt{\hat{\kappa}}}{\hat{\eta}} = \text{constant} \tag{7}$$

Equation (7) derived from equations (1) and (2) indicates that, in addition to similarity of pore space or hydraulic functions, the interrelation between $h(S)$ and $K(S)$ of all similar soils must be comparable when validity of equations (1) and (2) is expected. This condition is hereinafter termed the "interrelation" condition. From equation (7) it follows that similarity is a necessary condition for simultaneous scaling, but it is not sufficient. In addition, the interrelation condition needs to hold for similar soils. Therefore, the interrelation condition indicates validity of all required assumptions beyond similarity for derivation of equations (1) and (2) (see section 1.2).

Several analytical expressions for the relationship between η and κ exist. *Guarracino* [2007] derived the following relationship for the *van Genuchten* model:

$$K_s = c_1 \theta_s \frac{2-D}{4-D} \alpha^2 \tag{8}$$

Another relationship was proposed by *Nasta et al*. [2013] for the Brooks-Corey model:

$$K_s = c_2 \varepsilon \theta_s \frac{p}{p+2} h_b^{-2} \tag{9}$$

where θ_s is the soil porosity (saturated water content), D (ranging between 1 and 2) is the fractal dimension corresponding to the pore-size distribution, ε is a macroscopic tortuosity-connectivity fitting parameter and p is the power in the Brooks-Corey water retention model. The constants c_1 and c_2 ($m^3\ s^{-1}$) account for density, surface tension, and dynamic viscosity of water, water-solid contact angle, and gravitational acceleration.

Equations (8) and (9) were derived from idealization of the soil pore space as a bundle of cylindrical capillaries (BCC) in order to simplify the $h(r)$ and $K(r)$ relationships through equation (4) and through Poiseuille's law for an individual pore:

$$K = \frac{\rho g r^2}{8\mu} \tag{10}$$

where μ is the dynamic viscosity of water.

Equations (8) and (9) are essentially the same as equation (7) when macrosimilar soils (i.e., identical shape parameters, D and p) with identical porosities, θ_s, are assumed (ε is assumed to be constant). This means that the interrelation condition may also depend on the BCC assumption. In any case, as mentioned above, the interrelation condition is a consequence of the validity of all other assumptions beyond similarity required for derivation of equations (1) and (2).

Based on equation (7) and considering the hydraulic constants related to h and K in equation (4) and (10), a dimensionless parameter termed the "joint scaling factor" can be defined:

$$\gamma_i = \frac{\sqrt{\mu \rho g \kappa_i}}{\tau \eta_i} = c \frac{\sqrt{\kappa_i}}{\eta_i} \tag{11}$$

where $c = 1360$ ($s^{0.5}\ m^{-1.5}$) for water at 20°C.

As indicated above, equality of γ values for a set of macrosimilar soils is a direct consequence of equations (1) and (2). Hence, validity of the Miller-Miller theory for a set of macrosimilar soils can be explored by evaluating the degree of variability of the soil γ values. In other words, the more disparate the γ values for a set of soils, the less likely the validity of the Miller-Miller theory.

Table 2. Soil Textural Class, Number of h Versus θ (M) and K Versus θ (O) Observations and Kosugi Parameters for the Six Classes of Macrosimilar Soils Considered in This Study

Class	UNSODA Soil Code	Soil Texture	M	O	θ_s	θ_r	h_m (cm)	K_s (cm/d)	σ	CV_σ (%)
1	4443	Sand	21	20	0.30	0.01	24.51	518.00	0.26	
	4444	Sand	32	45	0.31	0.09	41.57	368.00	0.38	
	4445	Sand	37	23	0.28	0.00	70.58	260.00	0.34	
	3050	Sandy loam	18	25	0.46	0.29	44.75	18.68	0.39	
	Reference						**42.35**	**174.43**	**0.34**	**17.6**
2	2210	Sand	7	9	0.39	0.08	40.29	254.00	0.85	
	3130	Loamy sand	10	55	0.51	0.15	42.41	275.35	0.94	
	4441	Sand	27	37	0.30	0.00	48.59	960.00	0.84	
	4610	Loam	10	9	0.42	0.23	1493	0.03	0.79	
	4611	Sandy clay loam	14	14	0.42	0.21	2480	0.04	0.9	
	Reference						**198.4**	**9.60**	**0.86**	**7.0**
3	3120	Silty clay	10	11	0.57	0.16	606.76	4.21	1.41	
	4030	Silt loam	9	28	0.40	0.04	98.55	0.36	1.39	
	4512	Silt loam	13	16	0.41	0.19	414.72	18.15	1.53	
	4870	Silt loam	16	16	0.49	0.23	256.77	14.26	1.26	
	Reference						**282.48**	**4.45**	**1.4**	**7.9**
4	1490	Silt loam	10	38	0.44	0.18	354	2.58	1.65	
	3204	Loamy sand	13	16	0.38	0.13	77.61	4.81	1.68	
	4411	NA	7	11	0.31	0.00	110.63	144.45	1.58	
	4542	Loam	14	16	0.43	0.12	356.42	0.85	1.66	
	Reference						**181.42**	**6.25**	**1.65**	**2.4**
5	2491	Silt loam	8	9	0.43	0.20	331	11.79	1.9	
	1400	Clay	10	27	0.45	0.28	2163	0.08	2.02	
	4010	Loamy sand	9	28	0.43	0.06	139.21	50.90	1.97	
	4340	Sand	6	15	0.38	0.00	185.74	39.09	1.87	
	4410	NA	7	25	0.42	0.00	109.95	145.11	1.87	
	Reference						**289.56**	**12.26**	**1.92**	**3.6**
6	4543	Loam	10	11	0.43	0.12	909	59.92	2.79	
	3360	Silt loam	10	16	0.38	0.04	1244.28	69.00	2.76	
	3380	Silt loam	18	16	0.43	0.06	569.55	720.53	2.81	
	3395	Silt loam	15	15	0.38	0.00	5688	61.71	3.01	
	Reference						**1383.54**	**116.44**	**2.84**	**3.9**

Bold values denote the Kosugi parameters for the reference soils.

In summary, equality of the shape parameters satisfies the macrosimilarity condition, equality of γ values ensures validity of the interrelation condition, and both conditions should hold for validity of the Miller-Miller theory. The shape parameters and γ values specified for common unimodal hydraulic models are presented in Table 1.

3. Materials and Methods

To test the validity of the interrelation condition based on scaling efficiency, 26 soils covering a wide range of textures were selected from the UNSODA database [*Nemes et al.*, 2001]. The theoretical concept discussed above was primarily examined based on the well-established physically based scaling approach developed by *Kosugi and Hopmans* [1998] and *Tuli et al.* [2001], which implements the *Kosugi* [1994, 1996] lognormal hydraulic models presented in Table 1. The *van Genuchten* [1980] hydraulic functions were also examined and will be briefly discussed below in the "Results and Discussion" section.

The *Kosugi* functions were parameterized via nonlinear regression with the measured hydraulic properties of the 26 considered soils. The soils were grouped into six classes of macrosimilar soils based on their shape parameter σ such that soils within each class have practically identical σ values. Soils included in each class, denoted by their UNSODA database code, are listed in Table 2. It is evident that the coefficient of variation of σ (CV_σ) for each soil class is reasonably small, meeting the *Das et al.* [2005] criterion that states that "$CV_\sigma < 10\%$ may be used as a working definition for soil similarity." Note that reference parameters describe parameters of the so-called "reference soil" for each class calculated as [*Kosugi* and Hopmans, 1998; *Tuli et al.*, 2001]:

$$\hat{h}_m = \exp\left[(1/N)\sum_{i=1}^{N} \ln h_{mi}\right] \tag{12}$$

$$\hat{\sigma}=\left[(1/N)\sum_{i=1}^{N}\sigma_i^2\right]^{0.5} \tag{13}$$

$$\hat{K}_s=\exp\left[(1/N)\sum_{i=1}^{N}\ln K_{si}\right] \tag{14}$$

where i and ˆ denote individual soil and reference soil parameters, respectively.

Two different methods for scaling hydraulic properties were evaluated; an independent scaling method and a simultaneous scaling method. For the independent scaling method (ISM), the following scaling factors of equation (15) [*Kosugi* and Hopmans, 1998] for the water characteristic curve and equation (16) for the hydraulic conductivity curve were applied:

$$a_i=\frac{\hat{h}_m}{h_{mi}} \tag{15}$$

$$a_i'=\sqrt{\frac{K_{si}}{\hat{K}_s}} \tag{16}$$

For the simultaneous scaling method (SSM), the *Tuli et al*. [2001] approach was applied in which the scaling factors are calculated with equation (15) and applied for scaling both the soil water characteristic and hydraulic conductivity curves.

Scattering of the scaled data around the reference curve was quantified based on the weighted root mean square residual (*WRMSR*) given as [*Tuli et al*., 2001]:

$$\begin{aligned}WRMSR=&\frac{1}{SD_S}\left\{\frac{1}{\sum_{i=1}^{N}M(i)}\sum_{i=1}^{N}\left[\sum_{j=1}^{M(i)}\left(S_{ij}-\hat{S}\left(a_ih_j\right)\right)^2\right]\right\}^{1/2}\\&+\frac{1}{SD_{\ln K}}\left\{\frac{1}{\sum_{i=1}^{N}O(i)}\sum_{i=1}^{N}\left[\sum_{j=1}^{O(i)}\left(\ln K_{ij}\left(S_{ij}\right)-\ln\hat{K}\left(S_{ij}\right)-2\ln a_i\right)^2\right]\right\}^{1/2}\end{aligned} \tag{17}$$

where N is the number of soils in each group, $M(i)$ is the number of measured $h(S)$ data points for soil i, $O(i)$ is the number of measured $K(S)$ data points for soil i, S_{ij} is the jth effective water saturation of soil i, a_i is the scaling factor for the ith soil (replaced with a_i' for the SSM), $\hat{S}$ (a_ih_j) is the reference soil effective water saturation, ln K_{ij} is the natural logarithm of the jth unsaturated hydraulic conductivity of soil i, ln $\hat{K}$ (S_{ij}) is the natural logarithm of the reference soil unsaturated hydraulic conductivity at S_{ij}, and SD_S and $SD_{\ln K}$ are the standard deviations of all incorporated data of S and ln K, respectively. The *WRMSR* was also calculated for unscaled data considering $a_i = 1$ for all soils.

Note that the contributions of S and ln K data to *WRMSR* in equation (17) are normalized by their standard deviation values, which minimizes the bias due to the differences in data range and magnitude. For depicting individual scaling errors for $h(S)$ and $K(S)$, the first and second terms of equation (17) were also computed, which will be reported later as $RMSR_h$ and $RMSR_K$, respectively.

Reduction percentage (*RP*) of the *WRMSR* after scaling was considered as a criterion for success or failure of the scaling methods, defined as:

$$RP=\frac{WRMSR_{unscaled}-WRMSR_{scaled}}{WRMSR_{unscaled}}\times 100 \tag{18}$$

where $WRMSR_{unscaled}$ and $WRMSR_{scaled}$ denote the *WRMSR* before and after scaling. Note that a perfect scaling performance would yield $RP = 100\%$ (i.e., having all the scaled data coalesced to the reference curve). Any discrepancy due to invalidity of the underlying assumptions will result in a *RP* of less than 100%. A negative *RP* indicates complete failure of the scaling method where scaling will increase the *WRMSR* in opposition to its goal.

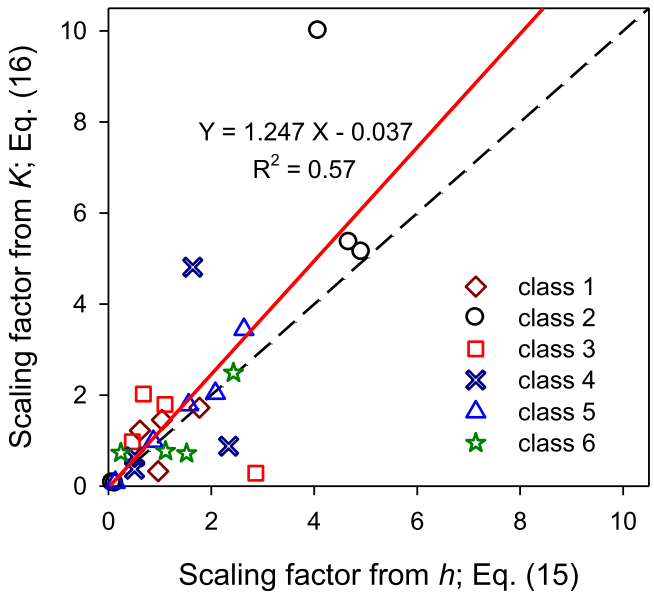

Figure 3. Scaling factors for the investigated 26 soils calculated from hydraulic conductivity data (Eq. (16)) plotted against the scaling factors derived from water characteristic data (Eq. (15)). The solid line represents the regression line and the dashed line is a 1:1 correlation.

4. Results and Discussion

4.1. Scaling Based on the *Kosugi* Soil Hydraulic Functions

As stated earlier, a consequence of the Miller-Miller similar media theory is that both the soil water characteristic and hydraulic conductivity function can be scaled with a single set of scaling factors, e.g., equality of the scaling factors from equations (15) and (16). Scaling factors for the 26 considered soils were determined from $h(S)$ data and compared with scaling factors derived from $K(S)$ data (Figure 3). The observed scatter of data points around the 1:1 line confirms that the Miller-Miller theory will not be exact even for similar soils as discussed earlier. As illustrated in the theory section above and mentioned in the following, the discrepancies between the two sets of scaling factors are mainly due to the imperfection of the interrelation condition for natural soils.

Figure 3 corresponds to Figure 10 of *Warrick et al.* [1977], who reported similarly that the scaling factors from h and K data are not necessarily identical, although they show a relatively high correlation. Similar to Figure 3, they observed a higher degree of variation for scaling factors from K data than from h data (compare the data range on the x and y axes). *Warrick et al.* [1977] suggested that "*this behavior probably stems from the fact that observations of K are much more sensitive to changes in S than those of h as well as the fact that the experimental technology required to measure K is not developed to the same degree as for h.*"

Values of γ ($= c\, h_m\, K_s^{0.5}$) for the 26 soils are shown in Figure 4. The SD values denote the standard deviation of ln γ values. As indicated earlier, equality of γ values would reflect validity of the interrelation condition. Therefore, the SD values provide a validity measure of the interrelation condition as well as validity of the Miller-Miller theory for each class. Based on the SD values presented in Figure 4, it is expected that the Miller-Miller theory (simultaneous scaling) would work well for class 5 with a small SD and less efficiently for class 3 with a large SD. Hence scaling results are presented for these two classes.

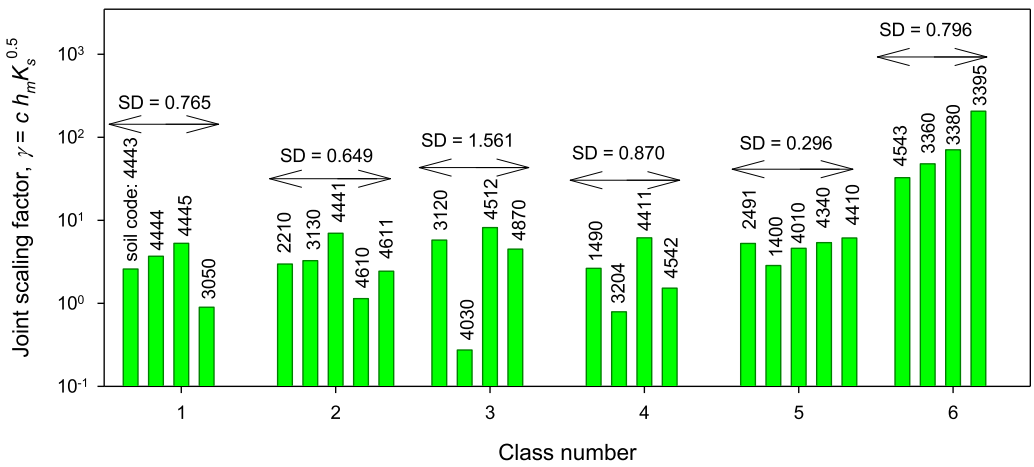

Figure 4. Joint scaling factor (γ) values for investigated soils. The SD values denote the standard deviation of ln γ for each class.

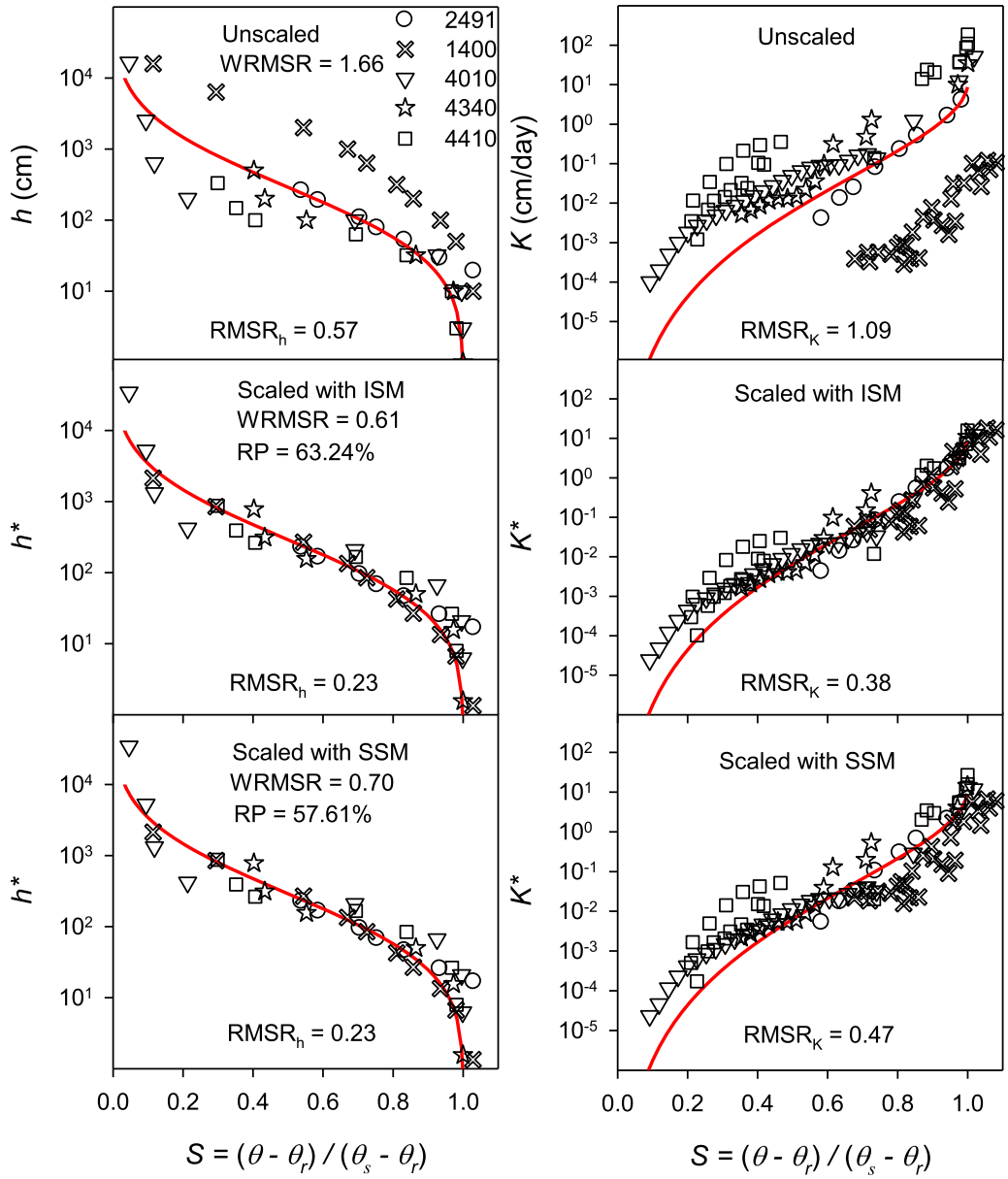

Figure 5. Unscaled soil water characteristic (left) and hydraulic conductivity (right) data for soil class 5 and their corresponding scaled data using the independent scaling method (ISM) and simultaneous scaling method (SSM) in conjunction with the Kosugi soil hydraulic functions. The solid line marks the reference curve.

Unscaled and corresponding scaled soil water characteristic and hydraulic conductivity data for soil class 5 are presented in Figure 5. The results were generated with the independent (ISM) and simultaneous (SSM) scaling methods. A comparison of *WRMSR* values before and after scaling reveals that both scaling methods significantly reduced data scattering around the reference curve. Scaling factors from both methods (obtained from equations (15) and (16)) were nearly the same, resulting in similar scaling performance and reduction percentage (*RP*) values. As indicated before, the success of the scaling theory applied to this soil class was foreseeable; σ values were nearly identical (i.e., meeting the similarity requirement) and γ values were almost the same (i.e., satisfying the interrelation condition).

Unscaled and scaled data for soil class 3 are shown in Figure 6. Since the four soils contained in this class were practically similar, as discussed earlier, the independent scaling method (ISM) worked well, positioning all data close to the reference curve with the *WRMSR* reduced by 51.51% after scaling. However, the

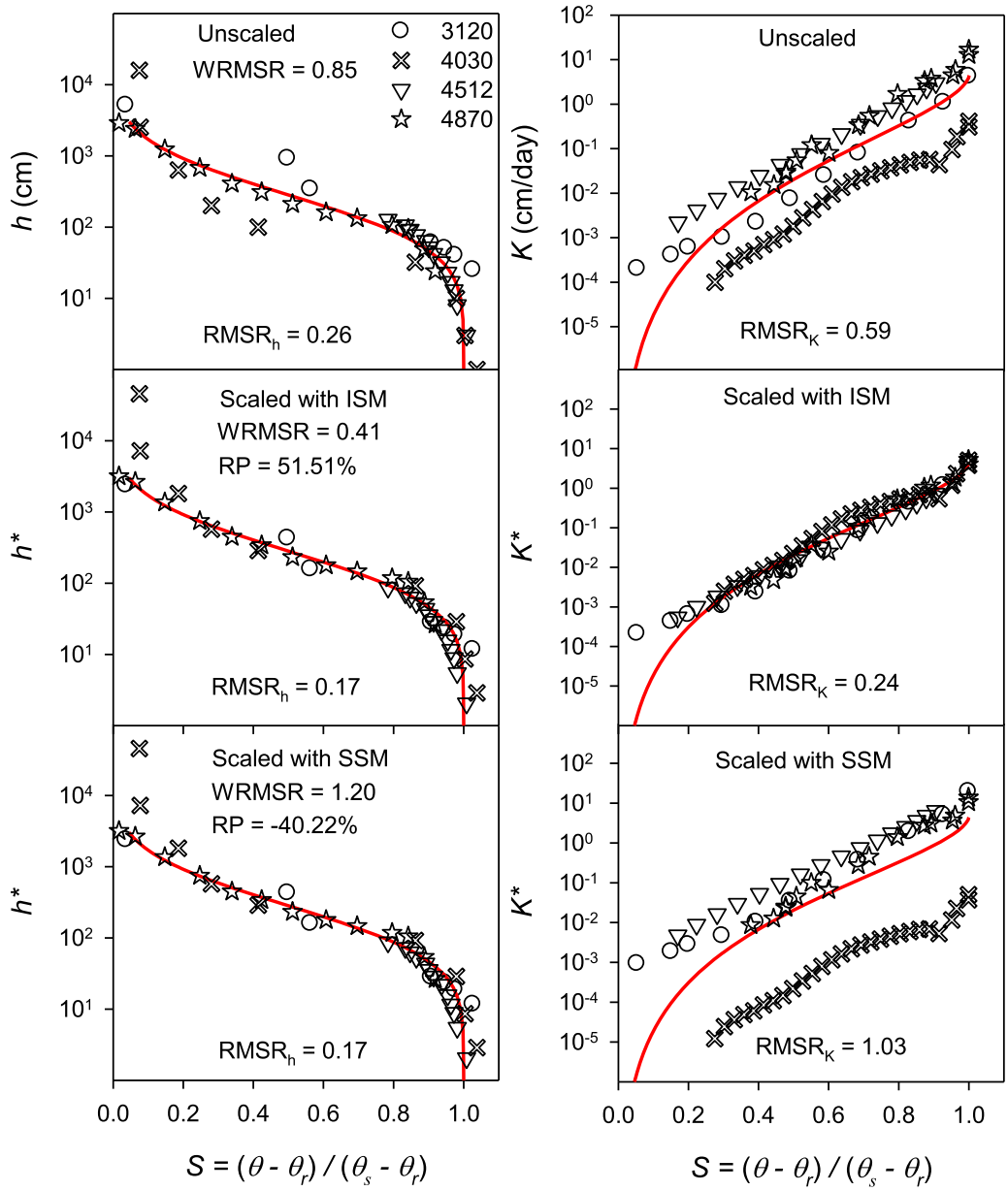

Figure 6. Unscaled soil water characteristic (left) and hydraulic conductivity (right) data for soil class 3 and their corresponding scaled data using the independent scaling method (ISM) and simultaneous scaling method (SSM) in conjunction with the Kosugi soil hydraulic functions. The solid line marks the reference curve.

negative *RP* obtained for SSM shows a complete failure of this method. Based on the theory presented in this paper, this is particularly due to the violation of the assumptions discussed in section 1.2 leading to the invalid interrelation condition for the soils in class 3 as indicated by the disparity of the γ values. As shown in Figure 4, soil 4030 has a significantly lower γ when compared to the other soils within this class. The effect of this discrepancy is clearly seen in the SSM scaled data where the hydraulic conductivity of soil 4030 deviated further from the reference curve after scaling than prior to scaling.

As discussed above, the performance of the scaling methods is affected by both the lack of a perfect similarity (error source 1) and the invalid interrelation condition (error source 2). In addition, the imperfect fit of the *Kosugi* soil hydraulic functions to measured data (error source 3) partly contributes to the total scaling error. In order to quantify contributions of each of the three error sources to the total scaling error, the ISM and SSM were also examined based on six classes containing synthetic data that were generated based on

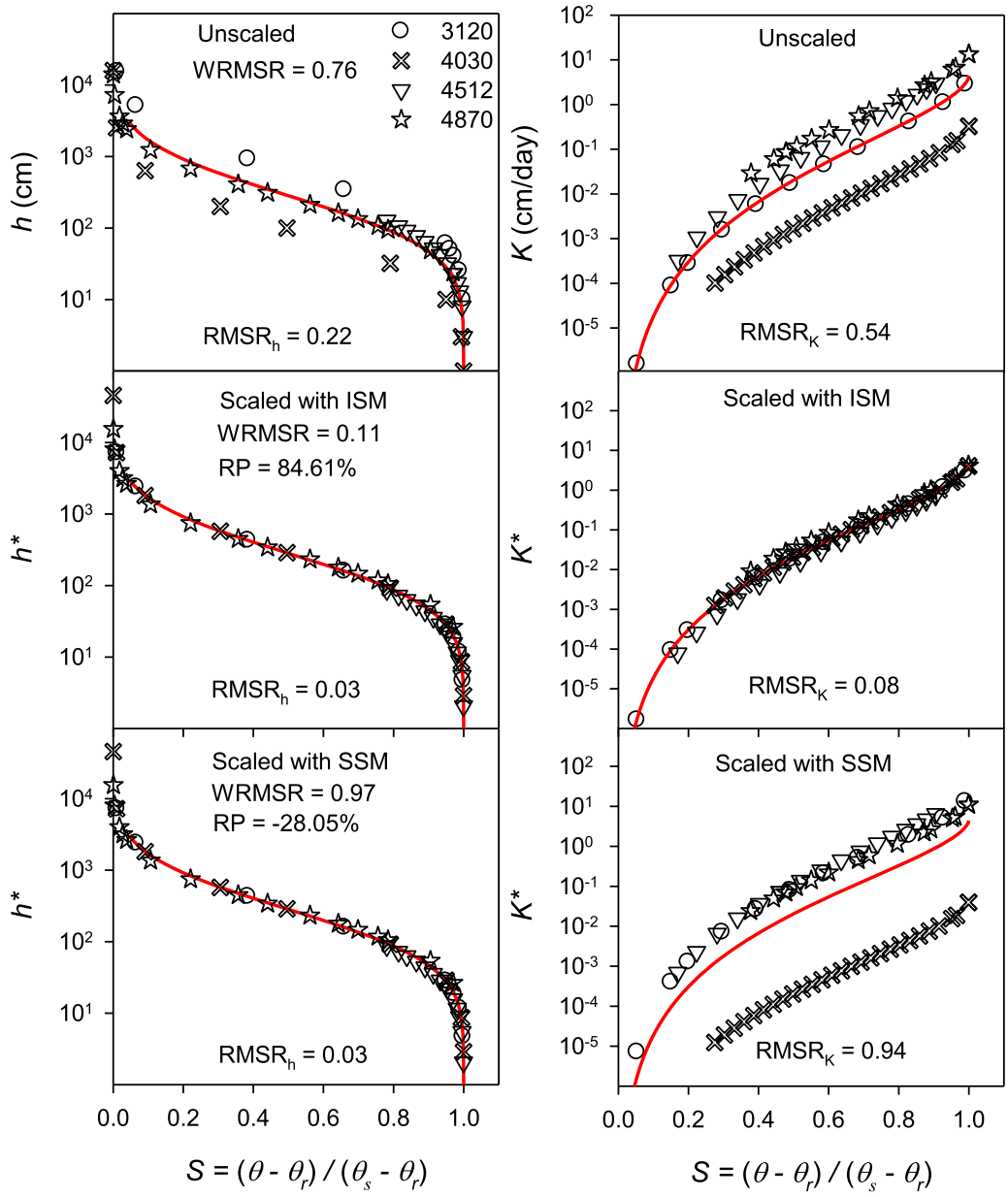

Figure 7. Unscaled synthetic data of soil water characteristic (left) and hydraulic conductivity (right) for soil class 3 and their corresponding scaled data using the independent scaling method (ISM) and simultaneous scaling method (SSM) in conjunction with the Kosugi soil hydraulic functions. The solid line marks the reference curve.

the assumption that S and K for each measured h exactly follow the *Kosugi* soil hydraulic functions (i.e., there is no fitting error).

Scaled synthetic data for class 3, for example, are depicted in Figure 7. A comparison with Figure 6 shows to what extent the inability of the *Kosugi* model to match measured data affects the performance of the ISM and SSM. Figure 7 clearly shows that while the *RP* for this class can be improved by considering hydraulic functions that are more flexible than the *Kosugi* functions, Miller-Miller scaling (i.e., SSM) would still fail (i.e., lead to a negative *RP*) because the interrelation condition is not met for this soil class.

Results of this analysis for all classes are summarized in Figure 8 that depicts the correlation between *RP* as a measure of scaling efficiency and SD of ln γ values as a measure of the validity of the interrelation condition. The contributions of the three error sources discussed above were determined by comparing the scaling efficiency for: (i) synthetic data scaled with the ISM where the lack of the similarity condition is the sole

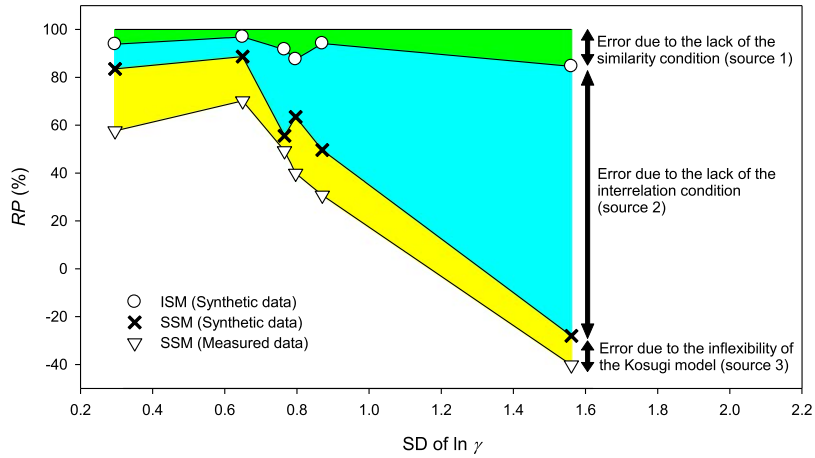

Figure 8. Correlation between *RP* (from Eq. (18)) and the standard deviation (SD) of ln γ for the independent scaling method (ISM) and simultaneous scaling method (SSM) applied to the synthetic and measured data.

error source; (ii) synthetic data scaled with the SSM where errors are due to the lack of the similarity condition and invalidity of the interrelation condition; and (iii) measured data scaled with the SSM with all three error sources contributing.

As observed in Figure 8, the scaling error is mainly governed by the invalid interrelation condition (cyan area) which is significantly larger than error sources 1 (green area) and 3 (yellow area). This observation indicates that while soils within each class are practically similar, the validity of the interrelation condition is of crucial importance for successful application of the Miller-Miller scaling theory. This finding contradicts the general belief that Miller-Miller scaling is valid as long as similarity is given.

The fact that errors due to the lack of similarity (green) are significantly smaller than errors due to invalidity of the interrelation condition (cyan) indicates that soils within each class are practically macrosimilar and thus a proper soil classification has been established. Figure 8 also highlights that the scaling results are not very sensitive to the choice of hydraulic models because the contribution of fitting errors (yellow) to *RP* is relatively small in the low-*RP* classes. To further evaluate effects associated with the choice of hydraulic models, an analysis in conjunction with the most popular unimodal *van Genuchten* hydraulic functions (see Table 1) follows below.

4.2. Scaling Based on the *van Genuchten* Soil Hydraulic Functions

As discussed in *Kosugi* [1994], the *van Genuchten* (VG) [1980] and *Kosugi* water characteristic models combined with *Mualem's* [1976] hydraulic conductivity model, closely match each other due to equivalence of corresponding parameters. Assuming that both retention functions (Table 1) yield the same *S* at pressure heads of $h = h_m$ and $h = 2h_m$, we analytically derived a relationship between scale and shape parameters of the models as follows:

$$h_m = \frac{1}{\alpha}\left[0.5^{\frac{n}{1-n}} - 1\right]^{1/n} \tag{19}$$

$$\sigma = \frac{\ln 2}{\sqrt{2}\,erfc^{-1}\left\{2\left[1+2^n\left(0.5^{\frac{n}{1-n}}-1\right)\right]^{\frac{1-n}{n}}\right\}} \tag{20}$$

Equations (19) and (20), examined for soils of group 3 in Figure 9, lead to a close relationship between the *Kosugi* and VG hydraulic functions. Hence, there is a universal relationship between shape parameters of the *Kosugi* and VG hydraulic functions, i.e., equation (20). Therefore, soils that are considered macrosimilar based on the *Kosugi* functions (i.e., soils with "practically equal" σ values), would also be considered macrosimilar based on the VG functions. In addition, according to equation (19), there is a unique relationship between the scale parameters of the *Kosugi* and VG hydraulic functions for macrosimilar soils (i.e., invariant *n*). Hence, γ values calculated based on the *Kosugi* functions are highly correlated with γ values calculated based on the VG functions. Consequently, similar scaling results are expected in theory for both hydraulic models. For example, hydraulic properties of the soils contained in class 3 were scaled based on the

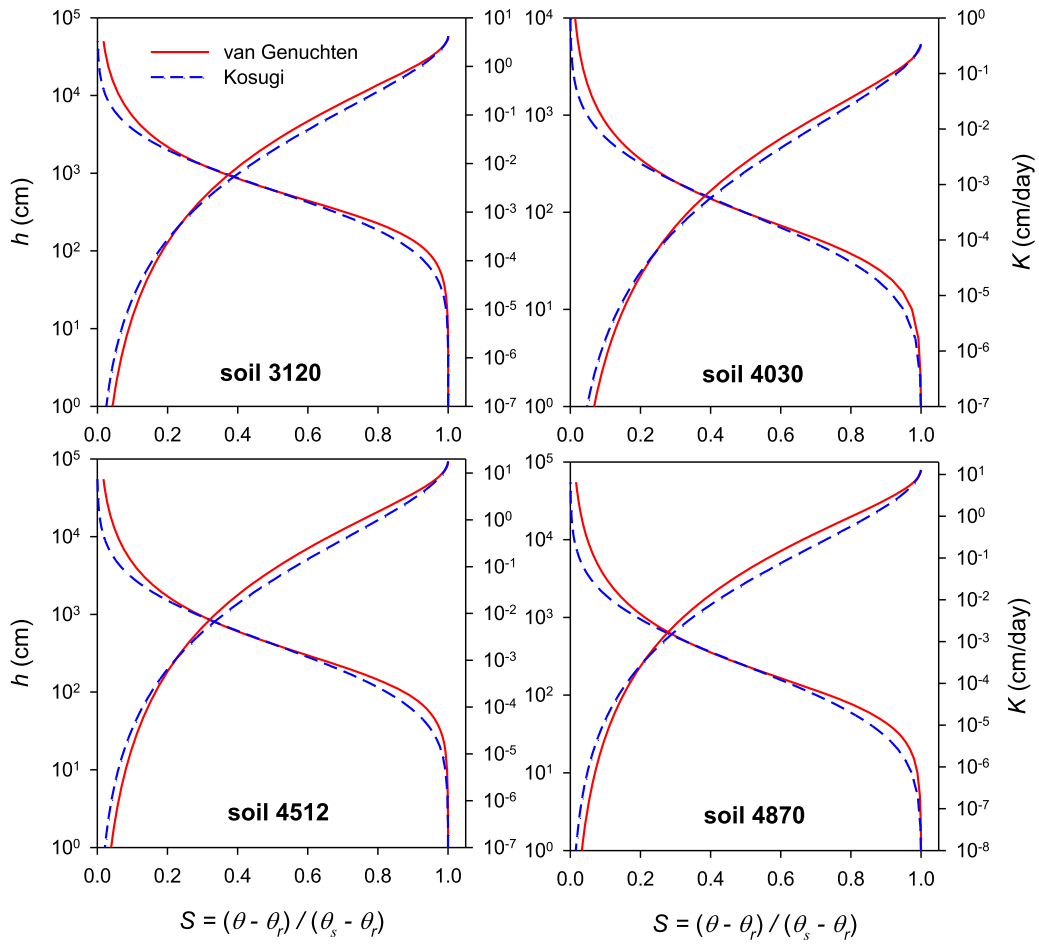

Figure 9. Relationship between the van Genuchten and Kosugi hydraulic functions based on Eqs. (19) and (20) for soils in class 3. The hydraulic functions are listed in Table 1. The Kosugi parameters reported in Table 2 were used to implicitly solve for n and α. The same K_s was applied in both models.

equivalent VG functions (Figure 9) and are shown in Figure 10. As observed, Figure 10 is very similar to Figure 6, verifying the similar performance of the *Kosugi* and VG hydraulic functions with regard to calculation of the scaling factors. Note that for VG-based scaling, equations (15) and (16) were applied for calculation of the scaling factors, while h_m was replaced with VG α in equation (15) (see Table 1). Also the scale parameters (α and K_s) and shape parameter (n) of the reference soil were considered as the arithmetic and geometric means, respectively, analogous, to equations (12) and (14).

5. Summary and Conclusions

Assuming validity of the Miller-Miller similar media theory, scaling is widely applied for characterization of the spatial variability of soil hydraulic properties by assigning single scaling factors for each soil or location. Results of this study contradict the general belief that Miller-Miller scaling is valid as long as soils are "similar." While similarity is required for validity of the Miller-Miller scaling theory, it is not sufficient. In addition, the interrelation between the soil water characteristic and the hydraulic conductivity functions of soils considered for scaling need to be comparable. The interrelation depends on several factors besides pore space geometry (e.g., hydrophobicity of soil particles, adsorptive surface forces, etc.). Hence similar interrelation cannot be concluded from similarity of microscopic pore space geometry (i.e., the interrelation condition does not depend on the similarity condition).

In practice, the validity of the pore space similarity assumption for a set of soils can be evaluated based on the equality of the shape parameters of their respective hydraulic functions (e.g., σ in the *Kosugi* hydraulic

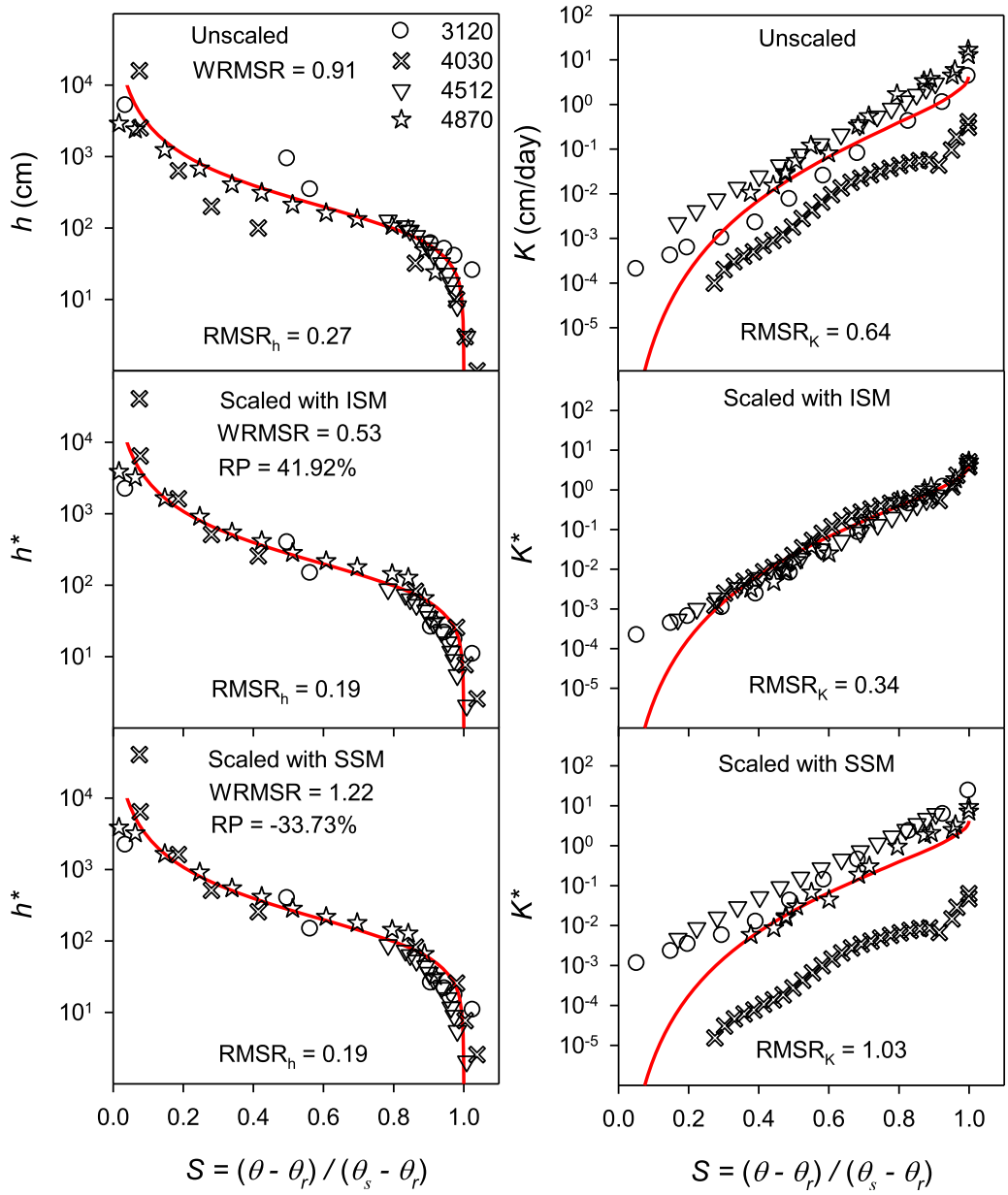

Figure 10. Unscaled soil water characteristic (left) and hydraulic conductivity (right) data for soil class 3 and their corresponding scaled data using the independent scaling method (ISM) and simultaneous scaling method (SSM) in conjunction with the van Genuchten soil hydraulic functions. The solid line marks the reference curve.

models). The interrelation condition can be explored based on the equality of the joint scaling factor, γ, introduced in this paper in equation (11). This means that Miller-Miller scaling is not applicable, unless the considered soils exhibit similar shape parameters and similar γ values.

The objective of this study was not to propose an alternative to the *Miller and Miller* [1956] scaling theory, but rather to illustrate problems arising from neglecting key assumptions beyond similarity, which are concurrently required for derivation of the Miller-Miller theory (i.e., equations (1) and (2)). However, improvement of the original theory may be feasible in the future by modifying its underlying assumptions. For example, equations (1) and (2) were derived based on the Young-Laplace equation which does not account for contributions due to adsorbed liquid films and film flow, which may significantly contribute to liquid retention and flow behavior especially in fine textured and high surface area soils.

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