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Investigation of market efficiency and Financial Stability between S&P 500 and London Stock Exchange: Monthly and yearly Forecasting of Time Series Stock Returns using ARMA model

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HIGHLIGHTS

- Market Efficiency and Financial Stability between S&P 500 and London Stock Exchange was studied.
- In London Stock Exchange forecasting of monthly stock returns outperforms to the yearly.
- S&P 500 capable for predicting medium or long horizons using real known values.
- Forecasting stock returns in S&P 500 outperforms the London stock exchange.
- We find both markets are efficient and have Financial Stability during periods of boom and bust.

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ABSTRACT

We investigated the presence and changes in, long memory features in the returns and volatility dynamics of S&P 500 and London Stock Exchange using ARMA model. Recently, multifractal analysis has been evolved as an important way to explain the complexity of financial markets which can hardly be described by linear methods of efficient market theory. In financial markets, the weak form of the efficient market hypothesis implies that price returns are serially uncorrelated sequences. In other words, prices should follow a random walk behavior. The random walk hypothesis is evaluated against alternatives accommodating either unifractality or multifractality. Several studies find that the return volatility of stocks tends to exhibit long-range dependence, heavy tails, and clustering. Because stochastic processes with self-similarity possess long-range dependence and heavy tails, it has been suggested that self-similar processes be employed to capture these characteristics in return volatility modeling. The present study applies monthly and yearly forecasting of Time Series Stock Returns in S&P 500 and London Stock Exchange using ARMA model. The statistical analysis of S&P 500 shows that the ARMA model for S&P 500 outperforms the London stock exchange and it is capable for predicting medium or long horizons using real known values. The statistical analysis in London Stock Exchange shows that the ARMA model for monthly stock returns outperforms the yearly. A comparison

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between S&P 500 and London Stock Exchange shows that both markets are efficient and have Financial Stability during periods of boom and bust.

1. Introduction

Nowadays, investigation of stock markets is an essential part of the nations' economy, and undoubtedly the greatest amount of capital is exchanged through stock markets all over the world. Thus, national economy is directly affected by stock market performance. Owing to the fact that stock market is accessible as a macro and microeconomic activity, people can take a distinct advantage if they have impressive business acumen and can predict the future status of stock market. Prediction has been always of considerable necessity in daily lives as a common field of science which specifically deals with financial issues and stock market exchanges. Not only are stock markets affected by macro-parameters, but also by thousands of other factors [1].

Stock market is one of the most important parts of capital market which mainly plays a role in attracting and directing the distributed liquidity and savings into optimal paths so that scarce financial resources are adequately allocated to the most profitable activities and projects [2]. People who would like to invest in common stock are first required to determine the real value of the share, and then compare it with the market price. So, one of the important issues in the stock market is predicted stock price of listed companies and their estimated real value, since prices are a signal to guide the effective allocation of capital and liquidity which could be considered as a powerful tool in the efficient allocation of resources [3].

Nowadays stock market is an important and active part of financial market. Both investors and speculators in the market would like to make better profits by analyzing market information [4]. The optimal allocation of resources is one of the most basic economic issues that have concerned natural and legal persons, economic decision-makers and officials involved in the capital market. Resource allocation is possible when resources are directed toward high returns investments with rational risk. The important role of the processed data that are combined with a valid measure is quite evident here. Since the capital market plays an important role in providing financial resources, directing capitals and promoting investments, measuring the effectiveness of stimuli with high predictive power for active decision-making process is one of the most important issues in capital markets in many countries [5].

Efficient contracting was the general premise underlying some of the early work on the economic consequences of accounting method choice [6]. Capital markets research in accounting includes several topics, including research on earnings response coefficients and properties of analysts' forecasts, fundamental analysis and valuation research, and market efficiency tests [7]. Mainstream accounting and economic thought is shaped by classical information economics the study of normative behavior under full rationality assumptions. While this powerful paradigm has proved instructive', it has also engendered an unfortunate tendency to attribute unlimited processing ability to decision makers [8].

There is a long history of predicting stock returns in finance. This is not surprising because the characterization of stock return predictability is important for making portfolio allocation decisions and for understanding the risk–return trade-off and market inefficiency as well. So far, attempts to predict excess returns have produced an enormous literature documenting that stock returns are predictable by economic variables such as dividend–price ratios, nominal interest rates, term and default spreads, and an assortment of other indicators. Though there is still some controversy on the predictability of stock returns due to concerns on spurious regressions, data mining, and instability of return predictability, the prevailing tone in the literature (e.g., Refs. [9–13]) is that stock returns have a predictable component.

By considering the returns as a time series, most of the forecasting methods utilize statistical models to reveal the hidden properties in the sequence, such as dependency and data distribution. Several kinds of statistical models, such as AR process [14], auto-regression moving average (ARMA), Markov process and the artificial neural network model [15], generalized auto-regression conditional heteroscedasticity (GARCH) model [16,17]. The empirical results indicated that nonlinear models outperformed linear models [18]. Financial analysts who can effectively process all kinds of information related to stock market are generally regarded as an important information source for stock investors [19].

A time series can be defined as a chronological sequence of the observed data from any periodical task or behavior, or activity in fields like Engineering, Biology, Economics, or Social Sciences, among many others. Therefore, the task of predicting values of the series based on past and present values in order to achieve the information of the underlying model can be understood under the concept of time series forecasting. Dealing with time series forecasting implies considering three important aspects, where the first one is the choice of the time periods (or lags) that must be used in order to forecast the values. The second aspect to take into account is the trend, i.e., whether the time series tends to grow or decrease considering a long period of time. Finally, the prediction period must be considered. Usually, there exists a tendency to forecast using short horizons due to the difficulty of utilizing longer periods and, therefore, the results of the former tend to be more reliable. A time series is a sequence of observations taken sequentially in time (daily, monthly, yearly etc.). Moreover, prediction horizon can be classified into short-term, medium-term, and long-term. Generally, forecasting tends toward short-term prediction such as one-step-ahead prediction, since longer period prediction (medium-term or long-term) is more difficult, and sometimes may not be reliable because of the error propagation. There exist a wide number of techniques that have been developed to model and forecast the time series. These techniques can be coarsely grouped into descriptive traditional technologies, linear and nonlinear modern models, and technologies coming from the soft computing area. Among all of these technologies, Autoregressive Integrated Moving Average (ARIMA), by Box and Jenkins, ARMA model is probably the most well known and widely used method. The method combines autoregressive and moving average terms into an equation in order to build a linear model to forecast new values. The autoregressive part of the equation relates the future value to the past and present ones, while the moving average component relates the future value to the errors of previous forecasting. Nevertheless, the models provided by the ARMA method are simplistic linear models, unable to find complex subtle patterns in the time series data.

2. Theoretical background and review of literature

Attempts to produce improved forecasts of stock returns have spawned a huge literature that originated from studies by Campbell [20], Campbell and Shiller [21], Fama and French [22], Fama and French (1989), Ferson and Harvey [23], and Keim and Stambaugh [24], who provide convincing economic arguments and in-sample empirical results that some of the fluctuations in returns are predictable because of persistent time variation in expected returns. In sample evidence for predictability is accumulating as various new variables have been suggested as predictors of excess returns ([25]; Lamont, 1998; [26,27], among others). Out of sample predictability evidence, however, has been much less conclusive. Paye and Timmermann [28] and Lettau and Van Nieuwerburgh [29] argue that predictability weakened or disappeared during the 1990s. Predictability of future returns using ex ante information (e.g., analyst forecasts) violates market efficiency. Kothari et al. [30] show that predictability can be due to non-random data deletion, especially in skewed distributions of long-horizon security returns. Passive deletion arises because some firms do not survive the post-event long horizon.

Bordman and Claude [31] assessed the factors affecting the stock price performance of share issued privatizations. They confirmed the effect of factors on dependent variable of the research. Egeli et al. [32] examined stock market prediction using artificial neural networks in Istanbul Stock Exchange. They studied the prediction results of various classic methods and neural networks patterns and found the superiority of neural patterns over linear patterns. Olson and Mossman [33] investigated neural network of Canadian stock returns using accounting ratios, and found that the artificial neural network could be more accurate than other methods. Wang [34] studied the stock price index in Taiwan Stock Exchange and concluded that existing volatilities among the data demonstrate a trend consistent with ARCH patterns which could enhance price prediction in case of being mixed with artificial neural network. Roh [35] conducted a study and forecasted the volatility of stock price index in Korea Stock Exchange, and asserted the superiority of the combination of artificial neural network and time series patterns in prediction over the other patterns.

In economics a problem of the asymmetric information of inefficiency of efficiency market hypothesis still exists [36]. The need to understand the dynamics of complex data coming from the biological, the financial, the environmental or the medical fields, has promoted the development of many visualization and analysis methods. Some of the linear methods—such as PCA or Classical Multi-dimensional Scaling—and non-linear methods—such as Stochastic Neighbor Embedding or Isomap used for this purpose, can have some drawbacks, like not preserving both local and global scale properties of complex data or depending on many undetermined parameters. These problems can leave a large part of analysis open to subjective interpretation [37].

Most of the economic and financial variables could not be predicted so far, since they cannot be accurately known. Some scholars believe that this unpredictability is rooted in the existence of a random procedure in the time series of these variables. Recently, structural models, which were relatively used to clarify the current conditions, could not be considerably successful in forecasting. Comparing these models, economists have shown increasing interest in univariate models of time series in the field of forecasting.

3. The efficient market hypothesis

In our opinion the EMH is simply the attempt to mathematize the idea that normal, liquid, finance markets are very hard to beat. If there is no useful information in market prices, then those prices can be understood as noise, the product of 'noise trading'. A martingale formulation of the EMH embodies the idea that the market is hard to beat, is overwhelmingly noise, but leaves open the question of hard to find correlations that might be exploited for exceptional profit [38].

Efficient Market Hypothesis (EMH) due to Samuelson [39], Fama [40] and others. Although there exist many forms of EMH, in broad terms they all assert that a market is efficient if prices immediately, for all practical purposes, reflect all relevant information about the assets on the market. The EMH thus requires that, on average, the population is always correct about the price (even if no single person is) and as new information appears, the market participants revise their expectations appropriately to maintain this state of affairs. The degree to which the EMH holds true in practice has been debated in the academic literature over the course of decades. The observation that perfectly efficient asset prices imply purely random price fluctuations, and the subsequent conflicting rejection of the random walk property of observed asset prices [41], the existence of bubbles and crashes in asset prices [42], and the unusual profitability of simple technical strategies, are among the key sources for criticism for the EMH. A conclusion of these studies is that the degree to which markets are efficient is likely not constant over time. In particular, as new information is being processed by market participants, there is a

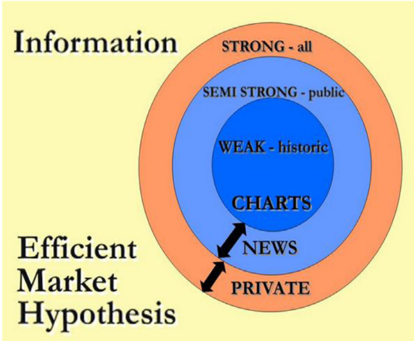

Fig. 1. Efficient market hypothesis.

transient period (of unknown and varying duration) during which price may not reflect true value. In spite of the progress made on understanding the nature of market efficiency, the actual mechanism by which prices adjust to new information—i.e. information processing by market participants during the tâtonnement process appears to be relatively unknown. In particular, to the author's knowledge there is no comprehensive mathematical model for price discovery based on the market participants' behavior [43] (see Fig. 1).

4. Financial stability and efficiency

Economic upswings and financial liberalizing policies positively influence efficiency. Cajueiro et al. [44] explored and deduced a positive impact of financial liberalization on market efficiency in Greece. Our results largely conform to this previous finding, with two distinct deviations from the pattern. First, the Malaysian market shows lower efficiency following the Asian financial crisis. During the Asian financial crisis, the Malaysian government took an unconventional method of implementing capital controls, which affected investor sentiment in the financial markets and reduced volume of trading substantially. Illiquidity in the market tends to create inefficiencies, which the Malaysian market experienced [45].

Lim et al. [46] investigates the effects of the 1997 financial crisis on the efficiency of eight Asian stock markets, applying the rolling bicorrelation test statistics for the three sub-periods of pre-crisis, crisis, and post-crisis. On a country-by-country basis, the results demonstrate that the crisis adversely affected the efficiency of most Asian stock markets, with Hong Kong being the hardest hit, followed by the Philippines, Malaysia, Singapore, Thailand and Korea. However, most of these markets recovered in the post-crisis period in terms of improved market efficiency. Given that the evidence of nonlinear serial dependences indicates equilibrium deviation resulted from external shocks, the present findings of higher inefficiency during the crisis are not surprising as in the chaotic financial environment at that time, investors would overreact not only to local news, but also to news originating in the other markets, especially when the news events were adverse.

However, on a multi-horizon analysis, an interesting observation is the increase in inefficiency for long-term investors. This may be attributed to an increasing number of retail and shorter horizon investors. A surge in the shorter horizon investor base, on the other hand, may increase efficiency in the short term but may adversely affect long-term efficiency [45].

5. Research method and research variables

The population of this research includes 350 firms listed in London Stock Exchange and S&P 500 from 2007 until the end of 2013.

The current study examines, monthly and yearly forecasting time series Stock Returns on London Stock Exchange and S&P 500 over a period from 2007 to 2013 using ARMA model.

6. ARMA model

The ARMA model is one of conventional random time-sequence models, invented by Box and Jenkins, known also as B–J method. There are three basic types of ARMA model: Auto-Regressive (AR) model, Moving Average (MA) model and Auto-Regressive Moving Average (ARMA) model.

The principle of ARMA mathematical model is as follows.

Let Y_t $(t = 0, \pm 1, \pm 2 \ldots)$ be a stationary sequence with zero average, which satisfies the following model:

$$Y_t - \varphi_1 Y_{t-1} - \cdots - \varphi_p Y_{t-p} = \alpha_t - \theta_1 \alpha_{t-1} - \cdots - \theta_q \alpha_{t-q}, \tag{1}$$

in which $\alpha_t, \alpha_{t-1}, \ldots, \alpha_{t-q}$ are stationary white noises with a mean value of zero and a variance of σ_ξ^2 then Y_t is named as the auto-regressive moving average sequence with the model's auto-regressive order of p and the model's moving average order of q, and abbreviated as ARMA (p, q) sequence. When $q = 0$, it becomes the AR (p) sequence; while $p = 0$, it is the

Table 1
Error measures criteria.

MAE	Mean Absolute Error	Mean $(\lvert e_t\rvert)$
MAPE	Mean Absolute Percentage Error	Mean $(\lvert p_t\rvert)$
MDAPE	Median Absolute Percentage Error	Median $(\lvert p_t\rvert)$
SMDAPE	Symmetric Median Absolute Percentage Error	Median $(200\,\lvert Y_t - F_t\rvert\,(Y_t + F_t))$
MASE	Mean Absolute Scaled Error	Mean $(\lvert q_t\rvert)$

MA (q) sequence. $\theta_1, \theta_2, \ldots, \theta_q$ are auto-regressive coefficients, $\varphi_1,\ \varphi_2, \ldots,\ \varphi_p$ are moving average coefficients, they are all the parameters to be estimated. Only a time sequence of stationary process can the ARMA model treat. If a non-stationary time sequence is to be analyzed, it is necessary to make it stationary, the most frequently used and simplest method is the difference operation on the original non-stationary time sequence, namely to get an ARMA (p, d, q) model, in which d is the order number of difference. In the ARMA model, the trend term is extracted by means of a difference operation on the time sequence, so as to transform it into a stationary one. Then, the ARMA model is estimated, and after the estimation it is transformed again to suit the original sequence model before the difference operation [47].

7. Quality measures for time series forecasting

In order to determine the accuracy of the forecast method applied to time series data, many measures have been proposed. Most textbooks recommended the use of the Mean Absolute Percentage Error (MAPE) and this was the primary measure in the M-competition. Other studies recommended other measures such as the Geometric Mean Relative Absolute Error (GMRAE), Median Relative Absolute Error (MDRAE), and Median Absolute Percentage Error (MDAPE). Later, the MDRAE, SMAPE (Symmetric Mean Absolute Percentage Error), and SMDAPE (Symmetric Median Absolute Percentage Error) were proposed. Nevertheless, Hyndman and Koehler [48] in their work determined that all measures mentioned before were not generally applicable since they can be infinite or undefined and can produce misleading results. For this reason, they proposed a new measure suitable for all situations: the Mean Absolute Scaled Error (MASE), which was less sensitive to outliers, less variable on small samples, and more easily interpreted. Among all of the different error measures that can be found, those used in this work are MAE, MAPE, MDAPE, SMDAPE, and MAPE. Their equations are shown in Table 1 and are calculated according to the following definitions: Y_t is the observation at time $t = 1, 2, \ldots, n$; F_t is the forecast of Y_t; e_t is the forecast error (*i.e.*, $e_t = Y_t - F_t$); $p_t = 100e_t/Y_t$ is the percentage error, and finally q_t is determined as Eq. (2):

$$q_t = \frac{e_t}{\frac{1}{n-1}\sum\limits_{i-2}^{n} \lvert Y_i - Y_{i-1}\rvert}. \tag{2}$$

Several studies find that the return volatility of stocks tends to exhibit long-range dependence, heavy tails, and clustering. Because stochastic processes with self-similarity possess long-range dependence and heavy tails, it has been suggested that self-similar processes be employed to capture these characteristics in return volatility modeling. So, the present study applies monthly and yearly Forecasting of Time Series Stock Returns in S&P 500 and London Stock Exchange Using ARMA model.

8. Results

The present study applies monthly and yearly Forecasting of Time Series Stock Returns in S&P 500 and London Stock Exchange using ARMA model. Time series occurs in a wide variety of phenomena, ranging from economics to physical science and from engineering to sociology. In this paper financial time series is studied. Many statistical models such as exponential smoothing, generalized regression and ARMA are frequently used to describe the stationary time series process.

The statistical analysis of S&P 500 shows that the ARMA model for S&P 500 outperforms the London stock exchange and it is capable for predicting medium or long horizons using real known values. The statistical analysis of London Stock Exchange shows that the ARMA model for monthly stock returns outperforms to the yearly. The comparison between S&P 500 and London Stock Exchange shows that both market is efficient and have Financial Stability during Periods of boom and bust.

8.1. Model of monthly data from London stock exchange by the ARMA model

ACF (autocorrelation function) and PACF (partial autocorrelation) graphs of incoming data include (see Fig. 2):
According to these two graphs, should the model ARMA (4, 4) be used for the data. The resulted equation for this model can be displayed as following:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \theta_4 a_{t-4} + a_t \quad a_t \sim N(\mu, \sigma).$$

The resulted coefficients for this model are given in Table 2:

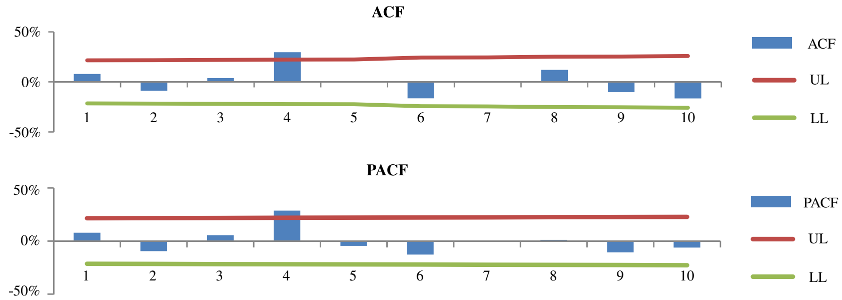

Fig. 2. ACF (autocorrelation function) and PACF (partial autocorrelation) monthly graphs-London stock exchange.

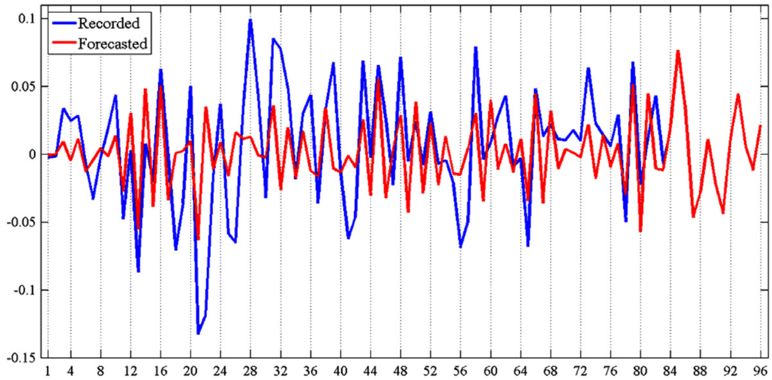

Fig. 3. Forecasted monthly data graph-London stock exchange.

Table 2
Coefficients for monthly data-London stock exchange.

φ_1	1.29
φ_2	−1.48
φ_3	1.19
φ_4	−0.76
θ_1	−1.23
θ_2	1.34
θ_3	−1.00
θ_4	0.79
μ	0.01
σ	0.04

Table 3
Expected values for 2014-London stock exchange (monthly).

2014m1	0.0766
2014m2	0.0307
2014m3	−0.0466
2014m4	−0.0283
2014m5	0.0107
2014m6	−0.023
2014m7	−0.0438
2014m8	0.0118
2014m9	0.0446
2014m10	0.0054
2014m11	−0.0117
2014m12	0.021

In this model, y values are the registered data, and a values are the errors of expectancy.

According to this model, the expected data's curve will be attained as the following, which of course has been compared with the registered data too (see Fig. 3).

The expected values for 2014 is as Table 3.

The error's measures (see Table 4):

Table 4
Error measures criteria-London stock exchange (monthly).

MAE	Mean ($\|e_t\|$)	0.0307
MAPE	Mean ($\|p_t\|$)	16.8%
MDAPE	Median ($\|p_t\|$)	8.01%
SMDAPE	Median ($200\,\|Y_t - F_t\| / (Y_t + F_t)$)	14%
MASE	Mean ($\|q_t\|$)	0.0074

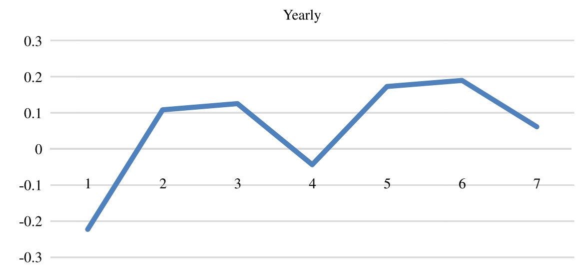

Fig. 4. Non Stationary yearly data from London stock.

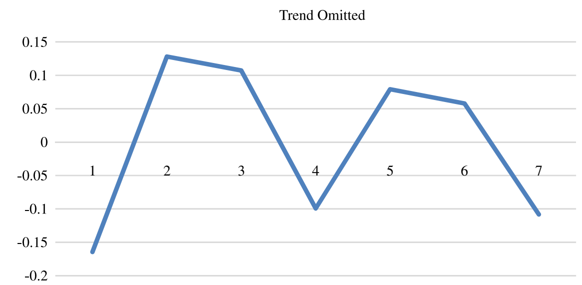

Fig. 5. Stationary yearly data from London stock.

8.2. Model of yearly data from London stock exchange by the ARMA model

According to the registered yearly data from London stock exchange in Fig. 4, an increasing procedure is seen in the data. In order to use the expectancy model according to the time series, we should first eliminate this procedure in the data.
In order to eliminate this procedure, we use the regression method:
In this method, the linear equation is written as the following:

$$Y_i = A + B(X_i - \overline{X}).$$

in which, $\overline{X}$ is the average of independent variable (here, the number of the year), and the coefficients of A and B are calculated as the following:

$$A = \bar{y}, \qquad B = \frac{COV(X, Y)}{VAR(X)}.$$

Calculating the above coefficients for the yearly data of London, the linear equation of the procedure is resulted as the following:

$$Y_i = 0.038X_i - 0.096.$$

Eliminating the procedure from the registered values, the data will become as the following (see Fig. 5).
Now, using the data in which its procedure has been eliminated, we start to model according to ARMA model:
ACF (autocorrelation function) and PACF (partial autocorrelation) graphs of incoming data are included in Fig. 6):
According to these two graphs, the model ARMA (3, 3) should be used for the data. The resulted equation for this model can be displayed as following:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} + a_t \quad a_t \sim N(\mu, \sigma).$$

The resulted coefficients for this model are in Table 5:
In this model, y values are the registered data, and a values are the errors of expectancy.
Finally, to get the main model, the procedure which was eliminated from the data in the first phase, should be added to the model ARMA (3, 3):

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} + 0.038X_i - 0.096 + a_t \quad a_t \sim N(\mu, \sigma).$$

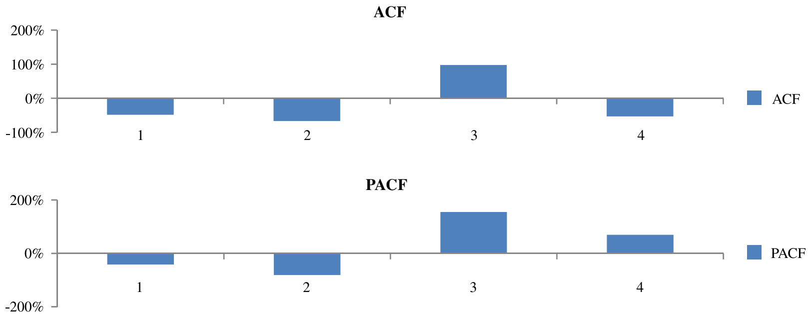

Fig. 6. ACF (autocorrelation function) and PACF (partial autocorrelation) yearly graphs-London stock exchange.

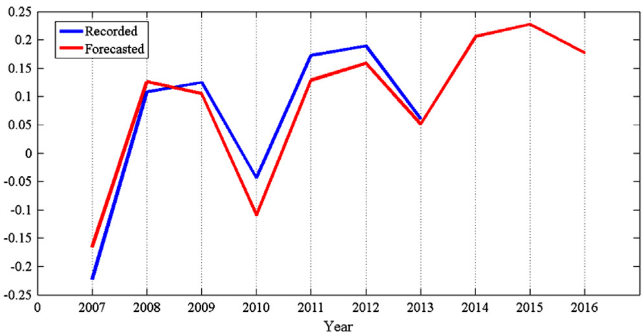

Fig. 7. Forecasted yearly data graph-London stock exchange.

Table 5
Coefficients for yearly data-London stock exchange.

φ_1	−0.28
φ_2	−0.40
φ_3	−0.27
θ_1	−0.05
θ_2	−0.98
θ_3	0.03
μ	0.00
σ	0.06

Table 6
Expected values for 2014–2016 London stock exchange (yearly).

2014	0.2053
2015	0.2274
2016	0.1772

Table 7
Error measures criteria-London stock exchange (yearly).

MAE	Mean ($\lvert e_t \rvert$)	0.035
MAPE	Mean ($\lvert p_t \rvert$)	3.84%
MDAPE	Median ($\lvert p_t \rvert$)	1.7%
SMDAPE	Median ($200 \lvert Y_t - F_t \rvert / (Y_t + F_t)$)	17%
MASE	Mean ($\lvert q_t \rvert$)	0.034

in which, X_t is the number of the year (the year 2007 being the base year).

According to this model, the expected data's curve will be attained as the following, which of course has been compared with the registered data too (see Fig. 7).

The expected values for 2014–2016 is as Table 6:

The error's measures (see Table 7):

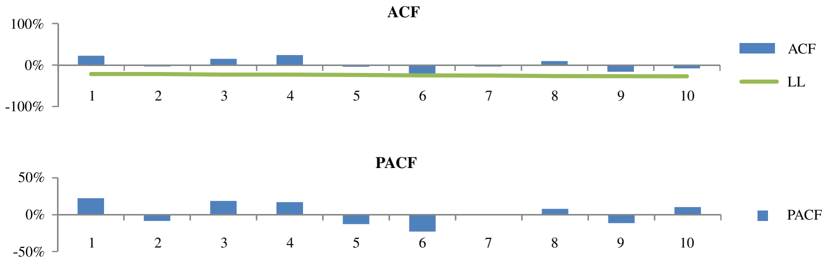

Fig. 8. ACF (autocorrelation function) and PACF (partial autocorrelation) monthly graphs-S&P 500.

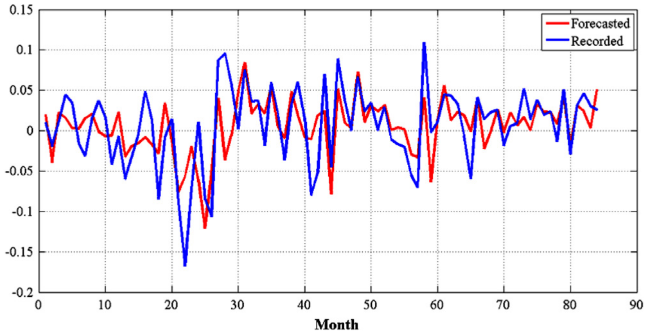

Fig. 9. Forecasted monthly data graph-S&P 500.

Table 8
Coefficients for monthly data-S&P 500.

φ_1	0.2268
φ_2	−0.1024
φ_3	0.1473
φ_4	0.1764
θ_1	0.35
θ_2	0.57
θ_3	0.16
θ_4	−0.20
μ	0.01
σ	0.04

Table 9
Error measures criteria-S&P 500 (monthly).

MAE	Mean ($\lvert e_t \rvert$)	0.0283
MAPE	Mean ($\lvert p_t \rvert$)	24.9%
MDAPE	Median ($\lvert p_t \rvert$)	7.3%
SMDAPE	Median ($200 \lvert Y_t - F_t \rvert / (Y_t + F_t)$)	10.1%
MASE	Mean ($\lvert q_t \rvert$)	0.0071

8.3. Model of monthly data from S&P 500 by the ARMA model

ACF (autocorrelation function) and PACF (partial autocorrelation) graphs of incoming data include (see Fig. 8):
According to these two graphs, should the model ARMA (4, 4) be used for the data:
The resulted equation for this model can be displayed as following:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \theta_4 a_{t-4} + a_t \quad a_t \sim N(\mu, \sigma).$$

The resulted coefficients for this model in Table 8:
In this model, y values are the registered data, and a values are the errors of expectancy. According to this model, the expected data's curve will be attained as the following, which of course has been compared with the registered data too (see Fig. 9).
The error's measures (see Table 9):

8.4. Model of yearly data from S&P 500 by the ARMA model

ACF (autocorrelation function) and PACF (partial autocorrelation) graphs of incoming data include (see Fig. 10):

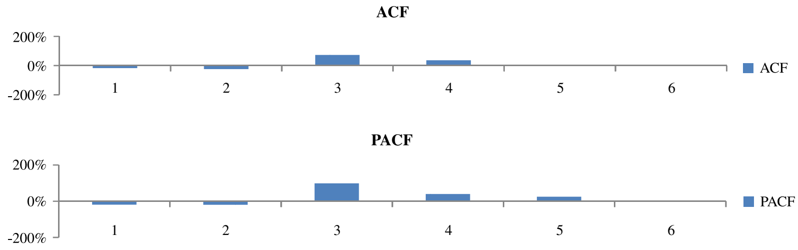

Fig. 10. ACF (autocorrelation function) and PACF (partial autocorrelation) yearly graphs-S&P 500.

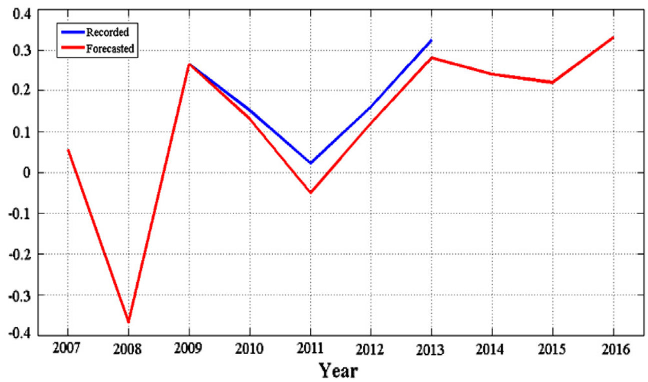

Fig. 11. Forecasted yearly data graph-S&P 500.

Table 10
Coefficients for yearly data-S&P 500.

φ_1	−0.30
φ_2	−0.32
φ_3	−0.14
θ_1	−0.27
θ_2	−0.59
θ_3	−0.14
μ	0.09
σ	0.13

Table 11
Expected values for 2014–2016 S&P 500 (yearly).

2014	24%
2015	22%
2016	33%

According to these two graphs, should the model ARMA (3, 3) be used for the data:
The resulted equation for this model can be displayed as following:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} + a_t \quad a_t \sim N(\mu, \sigma).$$

The resulted coefficients for this model are in Table 10:
In this model, y values are the registered data, and a values are the errors of expectancy.

Finally, to get the main model, the procedure which had been eliminated from the data in the first phase, should be added to the model ARMA (3, 3):

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} + a_t \quad a_t \sim N(\mu, \sigma).$$

In which, X_t is the number of the year (the year 2007 being the base year).

According to this model, the expected data's curve will be attained as the following, which of course has been compared with the registered data too (see Fig. 11).
The expected values for 2014–2016 is as in Table 11:
The error's measures (see Table 12):

The obtained findings indicate that medium and long term forecasting of time series are confirmed in S&P 500 and London stock exchange at the error level of %1. Based on these findings, time series are non-linear and can be forecasted. Given that

Table 12
Error measures criteria-S&P 500 (yearly).

MAE	Mean ($\lvert e_t \rvert$)	0.0807
MAPE	Mean ($\lvert p_t \rvert$)	16.6%
MDAPE	Median ($\lvert p_t \rvert$)	7.3%
SMDAPE	Median ($200 \lvert Y_t - F_t \rvert / (Y_t + F_t)$)	31.7%
MASE	Mean ($\lvert q_t \rvert$)	0.0043

the MASE for monthly and yearly stock returns in London stock exchange are 0.0074, 0.034 respectively and the MASE for monthly and yearly stock returns in S&P 500 are 0.0071, 0.0043 respectively, so we can find both markets are efficient and have Financial Stability during Periods of boom and bust.

9. Conclusion and suggestions

Statistical analysis results show that ARMA model is superior to other methods and using real values it could calculate medium-term and long term stock return for London stock exchange and S&P 500 based on calculations monthly and annually. Also, the statistical analysis of S&P 500 shows that the ARMA model for S&P 500 outperforms the London stock exchange and it is capable for predicting medium or long horizons using real known values. The statistical analysis in London Stock Exchange shows that the ARMA model for monthly stock returns outperforms the yearly. A comparison between S&P 500 and London Stock Exchange shows that both markets are efficient and have Financial Stability during periods of boom and bust.

The recommendation that arises from this study could be determining an efficient process for presenting an appropriate method for complex and volatile series modeling and prediction. On one hand, the result prediction is improved as in this process the ARMA model is confirmed time series and on the other hand as a technical recommendation, researchers could investigate genetics algorithm based model, non-linear regression model and fractal model as useful non-linear model in time series prediction.

References

[1] J. Zahedi, M.M. Rounaghi, Application of artificial neural network models and principal component analysis method in predicting stock prices on Tehran Stock Exchange, Physica A 438 (2015) 178–187.
[2] Z. Nasrollahi, The analysis of Iran stock exchange performance (M.S. thesis of economics), Tarbiat Modares University, 1992.
[3] A. Azizian, Investigating the factors determining price coefficient to the profit of listed companies on Tehran Stock Exchange (M.S. thesis of financial management), Shahid Beheshti University, 2006.
[4] Xia Li, H. Xie, L. Chen, J. Wang, X. Deng, News impact on stock price return via sentiment analysis, Knowl.-Based Syst. 69 (2014) 14–23.
[5] M.M. Rounaghi, M.R. Abbaszadeh, M. Arashi, Stock price forecasting for companies listed on Tehran stock exchange using multivariate adaptive regression splines model and semi-parametric splines technique, Physica A 438 (2015) 625–633.
[6] R.W. Holthausen, Accounting method choice-opportunistic behavior, efficient contracting, and information perspectives, J. Account. Econ. 12 (1990) 207–218.
[7] S.P. Kothari, Capital markets research in accounting, J. Account. Econ. 31 (2001) 105–231.
[8] C.M.C. Lee, Market efficiency and accounting research: a discussion of 'capital market research in accounting' by S.P. Kothari, J. Account. Econ. 31 (2001) 233–253.
[9] A. Ang, G. Bekaert, Return predictability: is it there? Rev. Financ. Stud. 20 (3) (2007) 651–707.
[10] J.Y. Campbell, S.B. Thompson, Predicting the equity premium out of sample:can anything beat the historical average? Rev. Financ. Stud. 21 (2008) 1509–1531.
[11] J.H. Cochrane, The dog that did not bark: a defense of return predictability, Rev. Financ. Stud. 21 (2008) 1533–1575.
[12] L. Pástor, R.F. Stambaugh, Predictive systems: living with imperfect predictors, J. Finance 64 (2009) 1583–1628.
[13] J.H. Binsbergen, R.S. Koijen, Predictive regressions: a present-value approach, J. Finance 65 (4) (2010) 1439–1471.
[14] L.H. Summers, Does the stock market rationally reflect fundamental values? J. Finance 41 (3) (1996) 591–601.
[15] E. Constantinou, R. Georgiades, A. Kazandjian, G.P. Kouretas, Regime switchingand artificial neural network forecasting of the Cyprus Stock Exchange dailyreturns, Int. J. Financ. Econ. 11 (4) (2006) 371–383.
[16] J.C. Hung, A fuzzy GARCH model applied to stock market scenario using agenetic algorithm, Expert Syst. Appl. 36 (9) (2009) 11710–11717.
[17] A.A. Drakos, G.P. Kouretas, L.P. Zarangas, Forecasting financial volatility of theAthens stock exchange daily returns: an application of the asymmetric normalmixture GARCH model, Int. J. Financ. Econ. 15 (4) (2010) 331–350.
[18] J.M. Matías, J.C. Reboredo, Forecasting performance of nonlinear models for intraday stock returns, J. Forecast. 31 (2) (2012) 172–188.
[19] F. Sonney, Financial analysts' performance: sector versus country specialization, Rev. Financ. Stud. 22 (5) (2009) 2087–2131.
[20] J.Y. Campbell, Stock returns and the term structure, J. Financ. Econ. 2 (1987) 373–399.
[21] J.Y. Campbell, R. Shiller, The dividend-price ratio and expectations of future dividends and discount factors, Rev. Financ. Stud. 1 (3) (1988) 195–228.
[22] E.F. Fama, K.R. French, Dividend yields and expected stock returns, J. Financ. Econ. 22 (1) (1988) 3–25.
[23] W.E. Ferson, C.R. Harvey, The variation of economic risk premiums, J. Polit. Econ. 99 (2) (1991) 385–415.
[24] D.B. Keim, R.F. Stambaugh, Predicting returns in the stock and bond markets, J. Financ. Econ. 17 (2) (1986) 357–390.
[25] J. Pontiff, L.D. Schall, Book-to-market ratios as predictors of market returns, J. Financ. Econ. 49 (2) (1998) 141–160.
[26] M. Lettau, S. Ludvigson, Consumption, aggregate wealth, and expected stock returns, J. Finance 56 (3) (2001) 815–849.
[27] C. Polk, S. Thompson, T. Vuolteenaho, Cross-sectional forecasts of the equitypremium, J. Financ. Econ. 81 (1) (2006) 101–141.
[28] B.S. Paye, A. Timmermann, Instability of return prediction models, J. Empir. Finance 13 (3) (2006) 274–315.
[29] M. Lettau, S. Van Nieuwerburgh, Reconciling the return predict ability evidence, Rev. Financ. Stud. 21 (4) (2008) 1607–1652.
[30] S.P. Kotharia, J.S. Sabinob, T. Zachc, Implications of survival and data trimming for tests of market efficiency, J. Account. Econ. 39 (2005) 129–161.
[31] A.E. Bordman, L. Claude, Factors affecting the stock price performance of share issued privatizations, Appl. Econ. 32 (11) (2000).
[32] B. Egeli, M. Ozturan, B. Badur, Stock Market Prediction Using Artificial Neural Networks, Bogazici University, 2003.
[33] D. Olson, C. Mossman, Neural network of canadian stock returns using accounting ratios, Int. J. Forecast. 19 (2003) 453–465.
[34] Yi-Fan Wang, Predicting stock price using fuzzy grey prediction system, Expert Syst. Appl. 22 (2002) 33–38.

[35] T.H. Roh, Forecasting the volatility of stock price index, Expert Syst. Appl. 33 (2007) 916–922.
[36] R. Pincak, E. Bartos, With string model to time series forecasting, Phys. A 436 (2015) 135–146. http://dx.doi.org/10.1016/j.physa.2015.05.013.
[37] I. Vega, Ch. Schütte, T.O.F. Conrad, Finding metastable states in real-world time series with recurrence networks, Phys. A 445 (2016) 1–17. http://dx.doi.org/10.1016/j.physa.2015.10.041.
[38] J.L. McCauley, K.E. Bassler, G.H. Gunaratne, Martingales, nonstationary increments, and the efficient market hypothesis, Physica A 387 (2008) 3916–3920.
[39] P.A. Samuelson, Proof that properly anticipated prices fluctuate randomly, Manag. Rev. 6 (2) (1965) 41–49.
[40] E.F. Fama, Efficient capital markets: A review of theory and empirical work, J. Finance 25 (2) (1970) 383–417.
[41] A. Lo, A.C. MacKinlay, Stock market prices do not follow random walks: Evidence from a simple specification test, Rev. Financ. Stud. 1 (1) (1988) 41–66.
[42] D. Abreu, M.K. Brunnermeier, Bubbles and crashes, Econometrica 71 (1) (2003) 173–204.
[43] E. Immonen, A quantitative description for efficient financial markets, Physica A 433 (2015) 171–181.
[44] D.O. Cajueiro, P. Gogas, B.M. Tabak, Does financial market liberalization increase the degree of market efficiency? The case of the Athens stock exchange, Int. Rev. Financ. Anal. 18 (1) (2009) 50–57.
[45] S.A.R. Rizvi, S. Arshad, Investigating the efficiency of East Asian stock markets through booms and busts, Pac. Sci. Rev. (2015) Available online 20 March 2015.
[46] K.-P. Lim, R.D. Brooks, J.H. Kim, Financial crisis and stock market efficiency: Empirical evidence from Asian countries, Int. Rev. Financ. Anal. 17 (3) (2008) 571–591.
[47] X.I. Chao, C. Cheng-lin, L. Si-min, L. Xiao-hui, L. Zhi-bin, D. Ke-qun, Long-term clock bias prediction based on An ARMA model, Chin. Astron. Astrophys. 38 (2014) 342–354.
[48] R. Hyndman, A. Koehler, Another look at measures of forecast accuracy, Int. J. Forecast. 22 (4) (2006) 679–688.