

# Uncertainty-based prioritization of road safety projects: An application of data envelopment analysis



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## ABSTRACT

The use of ranking methods in safety retrofit projects, in order to reduce uncertainty to an acceptable level, is a crucial problem. This paper presents a multidimensional method for prioritizing safety retrofit projects, in which uncertainty is taken into account in benefits estimation (accident reduction) and costs. Data Envelopment Analysis (DEA) with uncertainty assessment is described to help decision makers select the most cost effective projects. It is different from other ranking methods in that this approach adds standard errors of crash modification factor and crash costs in selecting process as well as the average values. Furthermore, this model is applied to a sample of intersections that are required to improve safety. Results have revealed that the proposed model is a suitable tool in selecting efficient projects when tolerances in accident reductions and project cost are incorporated. Comparative study between the proposed model and incremental benefit cost analysis and integer programming methods has also indicated that some changes in the list of selected projects considering the uncertainty impacts of data were observed. This analysis allows such safety projects to be identified. This also provides more complete information for safety analysts to allocate a limit budget to more efficient safety projects.

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## 1. Introduction

Prioritizing hazard sites and their countermeasures based on benefit or effectiveness is a significant part of literature in traffic safety research. Benefits of retrofit programs usually are associated with high costs for traffic authorities and society. Safety managers always are under increasing pressure to improve safety and reduce crashes while their budget is limited. Hence, ranking of projects is an inevitable necessity. In the ranking process cost effective projects are often chosen to render the best results from limited available resources (Montella, 2010). Better screening techniques and practices to introduce more efficient projects are needed with an extensive network of transportation, limited financial resources, and some problems such as lack of proper information.

The typical prioritization methods of retrofit projects include:

- Ranking based on economic effectiveness measures (such as net present value)
- Incremental benefit-cost analysis
- Optimization methods.

Ranking of projects by economic effectiveness measures or by the incremental benefit-cost analysis method is performed based on just a chosen criterion. Optimization methods such as linear or integer programming regard the impact of the budget constraints to find an optimized selection. (AASHTO, 2010). While all of these methods for prioritizing of projects have merit, they usually do not consider the multi-criteria nature of the problem and uncertainty of data and predictions.

Some studies have mentioned that uncertainty has an important effect on costs and benefits estimation and prioritization of projects. Elvik (2008a, 2010) has emphasized that safety analysts need to move towards reduction of uncertainty in costs and benefits of road safety treatments. He mentioned that due to resources of uncertainties, finding significant differences between previous estimations and actual outcomes of projects is not uncommon. Highway Safety Manual (AASHTO, 2010) has noted that wrong decision and chance of failure in benefits estimation of safety treatments go along with large variance in safety performance functions (SPFs) and crash modification factors (CMFs). Cafiso and Dagostino, 2015, with introducing an assessment method based on reliability and considering variance of CMF, have shown remarkable variation between results of their method and existent methods. Hermans et al. (2009) has recommended the use of uncertainty and sensitivity analysis in the selection of indicators and their method of weighting in ranking of countries safety situations.

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Uncertainty in estimating the monetary values of crashes or the relative value of accidents based on their severities (fatal, injury or property damage only (PDO)) are another sources of uncertainty (Council et al., 2005). UK Department for Transport (2007); Elvik (2008b) have reported different numbers for the relative importance of accidents. Geurts et al. (2004) have shown different weighting values to accidents based on severity has an important effect on black spots ranking. Also, due to the uncertainty associated with estimation of statistical life value and discounting problems of life and time, Hauer (2011) has shown that cost-benefit analysis cannot be a sufficient tool for prioritizing among projects.

Researchers have used optimization methods for prioritizing projects (Banihashemi, 2007; Harwood et al., 2004). However, the described problems by Hauer (2011) have still remained. Yu and Liu (2012) presented a multi-criteria model for ranking safety projects using the Analytical Hierarchy Process (AHP) method. They add a fuzzy scale level between the criteria level and the alternative level to reduce the uncertainty in judgments of decision makers. However, in this method uncertainty in synthesis of final scores for various alternatives and uncertainty in decision making process are also not considered.

Despite standard error being introduced as a sign of uncertainty in the various parts of safety research (such as calculation of CMFs, value of statistical crash, etc.), it is not commonly used in practical applications up to present.

This research proposes to use some uncertainties in decision making process, such as uncertainty in expected crash reduction, uncertainty in crash ratios based on severity (fatality, injury or PDO) and uncertainty in estimation of retrofit project cost to find a more optimized ranking. This will be done by using DEA with uncertainty assessment method.

In recent years, DEA has been used as an appropriate tool for evaluating and comparing in road safety fields. Cook et al. (2000) applied DEA for prioritization of safety treatment projects. They mentioned that different weights could be dedicated to different accident types from a road section to another by DEA method. Sadeghi et al. (2013) suggested DEA method in identifying and prioritizing accident prone road sections as it can consider the interaction of accidents as well as their casual factors such as traffic, geometric and environmental circumstances. Hermans et al. (2009); Shen et al. (2012) applied DEA to construct a composite index and to compare the safety situation of countries. Also, DEA method was applied for assessing the relative productivity of US states (Egilmez and McAvoy, 2013) and evaluating the efficiency of municipalities in providing traffic safety (Alper et al., 2015). Sala-Garrido et al. (2012) criticized that ranking by DEA methods are highly sensitive to data errors and therefore the role of uncertainty is of great importance.

León et al. (2003) applied the fuzzy mathematical programming for treatment of uncertainty in DEA models. After that, Bonilla et al. (2004) developed DEA model with uncertainty assessment for considering probable tolerances of inputs and outputs. In this method, an interval of efficiency scores is defined and prediction of efficiency will be possible when data are variable. Bosca et al. (2011) suggested a ranking method based on the statistical analysis of possible cases subsequent to computation of efficiency scores with existing tolerance in data.

This paper presents a procedure for prioritizing safety retrofit projects in a budget constrained area and considering uncertainty as tolerance in data inspired by what Bonilla et al. (2004); Bosca et al. (2011) suggested. In the next section, the suggested methodology of project ranking is described and traditional DEA is introduced followed by DEA with uncertainty assessment and prioritizing criteria. Section 3 presents an implicational example of intersections ranking; the results of this method will be compared

with the ranking of incremental benefit-cost analysis and integer programming method. The last section contains some concluding remarks and suggestions for further research.

## 2. Methodology

### 2.1. Definition of problem

Road authorities have to prioritize the sites which require safety treatment due to budget limitations. To achieve this, benefits and costs of each treatment for each site should be determined and a ranking measure or method should be defined. Along with other possible benefits, reduced accidents due to implementation of a treatment are the most important benefits which are estimated as follows:

$$y_i = t_i(1-a_i) \quad (1)$$

where,

$y_i$  = the expected number of reduced accidents by crash severity  $i$ .

$t_i$  = the expected number of accident without the implementing countermeasure.

$a_i$  = crash modification factor of treatment by crash severity  $i$ .

The total benefit can be estimated as

$$B = \sum_{i=1}^K \mu_i y_i \quad (2)$$

where  $\mu_i$  is crash cost by severity  $i$  and  $B$  is monetary value of all reduced crashes. Also, benefits can be calculated by converting different types of accidents to equivalent property damage only accident (PDO) and multiplying PDO crash cost.

There are three types of uncertainty in the process of benefit estimation:

- Uncertainty with respect to the expected number of accidents.
- Uncertainty about CMFs of a countermeasure
- Uncertainty with respect to monetary values of different crashes.

So far, many efforts have been made to estimate the number of accidents. Empirical Bayesian (EB) method is accepted as a reliable method to estimate accident frequency in such a way that the variance of SPF as uncertainty is considered in the calculation process. (AASHTO, 2010; Persaud et al., 2010). Uncertainty of CMFs and crash costs are mentioned as standard errors in the literature (Council et al., 2005; AASHTO, 2010) and considering them in the calculation process may change the benefit values.

On the other hand, there are some uncertainties in the estimation of costs. While the cost of implementing countermeasure being the most important factor, there are additionally some other costs such as inconvenience of users. Occasionally, the estimated costs may significantly change along with fluctuations in price inflation or a considerable lapse between decision and execution times.

Accurate ranking may be affected by uncertainties in benefits and costs, hence more efficient projects may not be selected. Following sections describe the DEA with uncertainty assessment method for considering some uncertainties in the prioritization process and also demonstrate such uncertainty impacts on decision making area.

### 2.2. Classic data envelopment analysis

Data Envelopment Analysis (DEA) introduced by Charnes et al. (1978) is a method for measuring the relative performance or

efficiency of decision-making units (DMU's). In each issue, DMU is a member of a set that ranking should be done among them. In our discussion, DMU can be safety retrofit projects for road sections or their other components (such as intersections roundabouts etc.). Costs and benefits of countermeasure for each DMU will be considered as inputs and outputs. DEA has become a powerful tool for evaluation of performance and ranking of DMUs with multiple outputs and inputs. Suppose each unit has “m” input to produce “s” output, then, the multiplier form of classical DEA model will be:

$$\begin{aligned}
 & \text{MAX } EF_j = \sum_{r=1}^s \mu_r y_{rj} \\
 & \text{s. t.} \\
 & \sum_{r=1}^s \mu_r y_{rj} - \sum_{i=1}^m w_i x_{ij} \leq 0 \\
 & \sum_{i=1}^m w_i x_{ij} = 1 \\
 & \mu_r, w_i \geq 0, r = 1, \dots, s, i = 1, \dots, m
 \end{aligned} \tag{3}$$

where  $EF_j$  is the efficiency of the  $j^{\text{th}}$  unit and other variables are as follows:

- $x_{ij}$ : value of the  $i^{\text{th}}$  input for  $j^{\text{th}}$  unit,  $i = 1, 2, \dots, m$ .
- $y_{rj}$ : value of the  $r^{\text{th}}$  output for  $j^{\text{th}}$  unit,  $r = 1, 2, \dots, s$ .
- $\mu_r$ : weight of the  $r^{\text{th}}$  output.
- $w_i$ : weight of the  $i^{\text{th}}$  input.

The multiplier form is derived from fractional form that is the ratio of the weighted sum of outputs to the weighted sum of inputs. This transformation is done to avoid infinite answers. In the linear programming problems, the dual form may be used because of simplicity of calculation. Equivalent dual form of (1) is:

$$\begin{aligned}
 & \text{Min } h \\
 & \text{s. t. } \sum_j \lambda_j x_{ij} \leq h x_{i0}; \quad \forall i \\
 & \sum_j \lambda_j y_{rj} \leq y_{r0}; \quad \forall r \\
 & \lambda_j \geq 0, \quad \forall j
 \end{aligned} \tag{4}$$

where the  $h$  is the efficiency score of  $DMU_0$  and  $\lambda_j$  is the dual weight given to the  $j^{\text{th}}$  DMU's inputs and outputs. Further explanations of these transformations were described previously (Charnes et al., 1978; Cooper et al., 2000).

In the basic DEA models, there is an assumption that values of inputs and outputs should be non-negative, while in our issue, the outputs may take negative values for some DMUs as some retrofit measures may decrease some crashes (e.g. injury accidents) and increase some other kinds of crash simultaneously (e.g. property damage accidents). To overcome this problem, Emrouznejad et al. (2010) replaced two non-negative variables instead of an output variable  $y_{kj}$  which is positive or negative for  $j^{\text{th}}$  DMU as follows:

$$y_{kj}^1 = \begin{cases} y_{kj} & \text{if } y_{kj} \geq 0, \\ 0 & \text{if } y_{kj} < 0, \end{cases} \quad \& \quad y_{kj}^2 = \begin{cases} 0 & \text{if } y_{kj} \geq 0, \\ -y_{kj} & \text{if } y_{kj} < 0, \end{cases} \tag{5}$$

With- this variable changing, all output values will be non-negative and the equivalent form of (2) can be formulated as follows:

$$\begin{aligned}
 & \text{Min } h \\
 & \text{s. t. } \sum_j \lambda_j x_{ij} \leq h x_{i0}; \quad \forall i \\
 & \sum_j \lambda_j y_{rj}^1 \leq y_{r0}; \quad \forall r \neq k \\
 & \sum_j \lambda_j y_{kj}^1 \geq y_{kj0}^1; \quad \forall r \neq k \\
 & \sum_j \lambda_j y_{kj}^2 \leq y_{kj0}^2; \quad \forall r \neq k \\
 & \lambda_j \geq 0, \quad \forall j
 \end{aligned} \tag{6}$$

Efficiency score ( $h$ ) is between 0 and 1. An efficiency score 1 indicates perfect efficiency compared with all units. Units with higher scores are more efficient and take a higher ranking.

### 2.3. DEA with uncertainty in the data

There are two criticisms leveled at uncertainty associated with the basic DEA models. First, although the fact that multipliers of inputs and outputs are not constant in the basic DEA model is favorable, excessive flexibility in choosing them has been criticized (Cook et al., 2000; Pedraja-Chaparro et al., 1997). In our case, this may lead to inappropriate selection as the property damage accidents may get higher multipliers than those of fatality and injury accidents. To prevent inappropriate choice of multipliers, the upper and lower bounds on ratios of multipliers should be defined. (Charnes et al., 1990) In other methods of ranking, the fixed costs for different types of crash are taken into account regardless of the uncertainty. Constraints for choosing multipliers' ratios can be defined based on average and standard error of crash costs. Standard error of values division is calculated as follows:

$$A/B = C \quad c = C \times \sqrt{(a/A)^2 + (b/B)^2} \tag{7}$$

where  $a$ ,  $b$  and  $c$  are standard errors of  $A$ ,  $B$  and  $C$  respectively (Kastner, 2012).

The second criticism is that it does not consider the uncertainty in inputs and outputs values. In many cases, our estimations for benefits or costs of projects are not precise and employing average values can lead to misleading results. Bonilla et al. (2004) developed DEA model with uncertainty assessment to react to this criticism. This model allows the input or output values to be changed between a maximum and minimum values for each DMUs. If we consider only the minimum, original and maximum values for DMUs and limit the calculation of possible combinations to the unit under consideration ( $DMU_{j_0}$ ) and other units in general value, then the number of input and output combinations include  $3^2 = 81$ .

Suppose the input and output values for calculating the efficiency of the  $DMU_{j_0}$  are as follows:

Inputs of  $DMU_{j_0}$ :  $x_{i0}^m, x_{i0}^o, x_{i0}^M$

Outputs of  $DMU_{j_0}$ :  $y_{r0}^m, y_{r0}^o, y_{r0}^M$

Inputs of  $DMU_{j \neq j_0}$ :  $x_{ij}^m, x_{ij}^o, x_{ij}^M$

Outputs of  $DMU_{j \neq j_0}$ :  $y_{rj}^m, y_{rj}^o, y_{rj}^M$

where superscripts “m”, “o” and  $M$  are representatives of minimum, original and maximum values. For example  $x_{i0}^m$  is the minimum value of the input  $i$  for unit  $j_0$ . A range of efficiency scores are calculated by replacing these values in the Eq. (6) instead of original values. For each DMU, 81 efficiency scores are achieved by substituting 81 cases of data correspondingly. The highest efficiency score is achieved when the inputs and outputs of  $DMU_{j_0}$  are minimum ( $x_{i0}^m$ ) and maximum values ( $y_{r0}^M$ ), respectively, while for the other units the maximum and minimum values are ( $x_{ij}^M, y_{rj}^m$ ). The lowest efficiency score for  $DMU_{j_0}$  is calculated when inputs for  $DMU_{j_0}$  are maximum ( $x_{i0}^M$ ) and for the rest of units are minimum ( $x_{ij}^m$ ) while the outputs for  $DMU_{j_0}$  are minimum ( $y_{r0}^m$ ) and for other units are maximum ( $y_{rj}^M$ ) (Sala-Garrido et al., 2012).

Therefore, a range of efficiency scores between maximum and minimum scores are provided for each DMUs at uncertain assessment conditions.

### 2.4. Ranking DMUs

Although uncertainty is taken into account in the described

DEA model, prioritization of road safety projects is difficult based on a range of efficiency scores and we need other criteria to rank projects. Bosca' et al. (2011) have introduced two criteria to rank DMUs on the basis of their relative level of efficiency scores as follows:

$$R_{j_0}^1 = \frac{e_{j_0}}{\text{card}(\Gamma_{j_0})},$$

$$R_{j_0}^2 = \begin{cases} \frac{S_{j_0} - e_{j_0}}{\text{card}(\Gamma_{j_0}) - e_{j_0}} & \text{card}(\Gamma_{j_0}) \neq e_{j_0} \\ 0 & \text{card}(\Gamma_{j_0}) = e_{j_0} \end{cases} \quad (8)$$

where  $\text{card}(\Gamma_{j_0})$  is the total number of the calculated cases, which based on the descriptions in the previous section, is equal to 81 combinations.  $e_{j_0}$  is the number of times that DMU  $j_0$  is efficient, namely the efficiency score is equal to 1 and  $S_{j_0}$  is the sum of all efficiency scores of DMU  $j_0$ .

The  $R_{j_0}^1$  criterion is representative of the ratio of times that a unit is efficient. The higher the value of  $R_{j_0}^1$ , the more efficient and the higher the rank of DMU  $j_0$ . If  $R_{j_0}^1$  is the same for two DMUs, then the  $R_{j_0}^2$  criterion is used.  $R_{j_0}^1$  and  $R_{j_0}^2$  values are between 0 and 1. In other words, unit  $j$  to unit  $k$  has a higher rank if and only if  $R_j^1 > R_k^1$  or  $R_j^1 = R_k^1$  and  $R_j^2 > R_k^2$ .

### 3. Application to a sample of intersections

#### 3.1. Sample description

With the aim of introducing and demonstrating the DEA model with uncertainty assessment acceptably, an example of safety treatment projects for urban intersections is given. The data consist of a sample of 40 signalized intersections located in Toronto. These intersections were selected as a set of accident prone intersections from more than two thousand intersections of Toronto. Occurred accidents data between 2006–2010 were used for developing SPF and estimation of accident frequency by EB method. Table 1 shows the name of intersections and their occurred crashes based on severity and impact type as well as a suggested countermeasure. Retrofit treatment for each intersection was suggested based on accident patterns and specification of intersection (such as traffic and geometry) and field inspections by the authors as part of the research effort. Each countermeasure for each intersection is a DMU in DEA with uncertainty assessment method. It should be noted that although in this example only one retrofit treatment has been proposed for each intersection, suggesting more countermeasures and selection among them is possible by this method.

In the next section, results of the suggested method are compared with the incremental benefit–cost analysis and integer programming methods. A brief description of input needed for three methods is provided in Table 2. AADT growth rate was considered 2% according to the other project in Toronto (Du, 2012) and discount rate was assumed 4%. In this example, three types of outputs (including fatality, injury and PDO crash reductions) and one input (which is the cost of countermeasure) were considered. It is important to consider the uncertainty in the input and outputs to obtain the reliable results. To do so, first, the accident frequency are estimated by developed SPFs and EB method. Next, the average of crash reductions are calculated by accident frequencies and CMF (Eq. (1)), if suggested countermeasure for each intersection is undertaken. Then the maximum and minimum of estimated crash reductions are achieved by considering standard error of CMF.

Table 3 displays input and outputs of DEA with tolerance model. CMF and its standard error for each suggested countermeasure has been extracted from existing literature (AASHTO, 2010; Zein, 2004) Some of the minimum accident reductions are negative in Table 3 which reflects a possible increase in accidents due to implementing retrofit treatment. Some retrofit treatments may decrease fatality and injury accidents and increase PDO accidents simultaneously. An increase may be the result of a large standard error of CMF. For example, a left–turn lane on two approaches was suggested as retrofit treatment where its CMF and standard error are 0.81 and 0.1, respectively. Estimated PDO accidents were equal to 226.56 for 20 years life service of this countermeasure. Using the 95% confidence interval, minimum estimated reduction of PDO accident is equal to  $226.56 * (1 - 0.81 - 1.96 * 0.1) = -1.35$ .

Table 4 shows the crash costs and their standard errors by severity. It should be noted that crash cost can be calculated by some method such as comprehensive cost, human capital and willingness to pay methods. The standard errors reflect the uncertainty in the value of each type of accidents. Incremental benefit–cost analysis and integer programming methods use average values of crash costs to convert accident reduction benefits to monetary values while the suggested method removes the need for this conversion and instead, different weights are given to accidents by degree of severity (outputs). Present values of benefits calculated from accident reductions and crash costs have shown in last column of Table 1. As described in Section 2.3, the upper and lower bounds on ratios of weights were defined to avoid inappropriate weights selection. Therefore, the weight ratio of fatality to injury accidents and its standard error were 8.6 and 1.25, which were computed by Eq. (7) and crash costs of Table 4. Also, the weight ratio of fatality to PDO accidents and its standard error were equal to 512.48 and 41.28, respectively. Instead of using the average and standard error calculated by a certain method, the relative weight of crashes can be calculated based on minimum and maximum value estimated by different methods.

Once the tolerances of input and outputs are calculated, the next step in our analysis is to apply the DEA model and to compare it to the other methods. DEA with uncertainty assessment, incremental benefit–cost and integer programming methods were coded using VBA in Excel software and the necessary analyses were performed.

#### 3.2. Results and discussion

The application of DEA model with uncertainty for minimum, original and maximum values of inputs and outputs lead to 81 efficiency scores for each intersection as described in Section 2.3. Since it is not possible to show all efficiency scores, only the results of original scenario and minimum and maximum possible efficiency scores have been shown in Table 5. Fig. 1 cogently illustrates the variation intervals between the maximum and minimum efficiency scores of intersections as well as the original values. The difference between the best and worst scenario shows the sensitivity of results related to the data of all intersections. Table 5 shows that 19 intersections can be efficient in the best possible cases, while in the worst case scenario, none of the intersections remain efficient.

After computing the efficiency scores, the values of two ranking criteria were calculated as explained in Section 2.4. The last three columns of Table 5 represent the values of  $R_1$  and  $R_2$  criteria and ranking of intersections. The first criteria ( $R^1$ ) indicates the frequency of being efficient; consequently, for each intersection the higher the value, the higher its rank. The  $R^2$  criteria allow intersections to be ranked when their values of  $R^1$  are the same. For example, intersection numbers 9, 27 and 34 have the same highest values of  $R^1$ ; therefore, based on  $R^2$ , ranks 1, 2 and 3 are assigned

**Table 1**  
Specification, occurred accident and suggested countermeasure of selected dangerous intersection.

	Main route	Side route	5-Year crashes (2006–2010)									Suggested countermeasure	Life service (years)	Present value of benefits (\$)
			Total	Accident class			Impact type							
				Fat	injury	PDO	Head on	Angle	Rear end	Side swipe	Turning			
1	STEELES AVE E	WILLOWDALE AVE	79	0	21	58	2	12	34	9	19	Install left-turn lane on two approaches	20	12,770,955
2	YONGE ST	SHEPPARD AVE E	304	0	61	243	5	44	158	37	47	Increase Pavement friction	5	3,540,612
3	STEELES AVE W	KEELE ST	200	1	41	158	10	39	80	20	44	Install right-turn lane on two approaches	20	26,224,618
4	SHEPPARD AVE E	BAYVIEW AVE	377	0	64	313	12	55	139	67	74	Install right- turn lane on one approach	20	17,928,190
5	STEELES AVE E	BAYVIEW AVE	190	0	50	140	3	17	86	35	40	Install red light camera	10	6,163,691
6	STEELES AVE W	BATHURST ST	184	0	53	131	0	15	92	16	49	Install right- turn lane on one approach	20	14,292,044
7	BATHURST ST	FINCH AVE W	250	0	84	166	1	7	111	31	68	Marking guidance lines	3	380,358
8	BAYVIEW AVE	FINCH AVE E	206	0	44	162	7	24	107	21	35	Marking guidance lines	3	255,564
9	YONGE ST	FINCH AVE E	327	0	64	263	8	30	158	54	53	Install left- turn lane on one approach	20	20,584,284
10	STEELES AVE W	JANE ST	184	0	40	144	1	25	79	22	46	Install right- turn lane on one approach	20	12,309,288
11	DUFFERIN ST	FINCH AVE W	303	0	63	240	3	23	163	44	53	Increase Pavement friction	5	3,595,287
12	STEELES AVE E	MAXOME AVE	48	0	11	37	1	13	13	7	9	Install right- turn lane on minor approach	20	4,283,187
13	STEELES AVE E	LAURELEAF RD S	53	0	13	40	0	10	23	4	7	Marking guidance lines	3	77,815
14	STEELES AVE W	HILDA AVE	76	0	32	44	1	12	26	9	17	Install left-turn lane on two approaches	20	15,756,677
15	STEELES AVE W	CACTUS AVE	48	0	18	30	1	5	21	4	11	Increase Pavement friction	5	601,518
16	STEELES AVE W	VILLAGE GT	42	0	23	19	1	10	12	2	8	Marking guidance lines	3	90,133
17	YONGE ST	STEELES AVE W	227	1	66	160	1	14	92	47	49	Install right- turn lane on one approach	20	17,747,056
18	STEELES AVE W	DUFFERIN ST	280	0	63	217	8	39	115	38	55	Install red light camera	10	7,906,127
19	STEELES AVE E	404 STEELES WOODBINE RAMP	222	0	57	165	4	23	91	31	69	Install right-turn lane on two approaches	20	31,559,744
20	YONGE ST	EGLINTON AVE E	252	0	40	212	5	19	98	68	33	Marking guidance lines	3	237,160
21	DUFFERIN ST	EGLINTON AVE W	140	0	39	101	0	15	67	24	17	Increase Pavement friction	5	1,961,487
22	VICTORIA PARK AVE	MCNICOLL AVE	81	0	16	65	1	21	18	8	23	Install left-turn lane on two approaches	20	13,144,369
23	FINCH AVE W	ARROW RD	247	0	55	192	9	31	78	43	79	Install left-turn lane on one approach	20	15,337,576
24	FINCH AVE W	NORFINCH DR	193	1	42	150	1	11	117	27	24	Increase Pavement friction	5	2,414,985
25	STEELES AVE W	FENMAR DR	121	1	27	93	1	42	35	12	24	Install red light camera	10	4,002,581
26	EGLINTON AVE E	VICTORIA PARK AVE	220	0	49	171	2	31	102	29	44	Install right-turn lane on two approaches	20	26,160,324
27	KEELE ST	WILSON AVE	196	0	52	144	7	40	72	33	29	Install right-turn lane on a minor approach	20	13,744,783
28	FINCH AVE E	LESLIE ST	161	0	42	119	4	26	63	15	34	Marking guidance lines	3	240,391
29	LESLIE ST	YORK MILLS RD	197	0	35	162	11	28	76	27	38	Install right-turn lane on one approach	20	11,311,277
30	LAWRENCE AVE W	BATHURST ST	250	0	40	210	6	31	109	41	23	Increase pavement friction	5	2,590,344
31	KEELE ST	LAWRENCE AVE W	260	0	56	204	4	51	107	33	41	Install right-turn lane on two approaches	20	27,264,701
32	STEELES AVE E	DON MILLS RD	154	0	38	116	4	19	71	14	36	Increase pavement friction	5	2,226,247
33	WILSON AVE	JANE ST	264	1	54	209	8	47	94	42	38	Increase pavement friction	5	2,951,704
34	WILSON AVE	BATHURST ST	288	1	49	238	10	56	95	46	38	Install right-turn lane on a minor approach	20	13,951,098
35	WESTON RD	FINCH AVE W	228	1	43	184	4	22	102	39	50	Install right-turn lane on one approach	20	12,721,320



36	EGLINTON AVE W	BATHURST ST	265	0	48	217	0	7	146	58	36	approach
37	BATHURST ST	SHEPPARD AVE W	270	0	50	220	2	72	99	33	43	Increase pavement friction Install right-turn lane on one approach
38	YONGE ST	LAWRENCE AVE E	224	0	33	191	6	38	96	46	19	Install left-turn lane on one approach
39	EGLINTON AVE E	MOUNT PLEASANT RD	196	1	25	170	6	30	73	36	33	Marking guidance lines
40	DUFFERIN ST	LAWRENCE AVE W	188	0	33	155	3	25	88	27	28	Install right-turn lane on one approach

**Table 2**  
Summary of required initial inputs for the project ranking methods.

Method	Input needs
Incremental benefit- cost analysis	Expected number of accident (for example by EB method) Cost estimate for implementing of countermeasures CMF of countermeasures Estimate of service life of countermeasures Crash costs by severity Discount rate
Integer programming	Expected number of accident (for example by EB method) Cost estimate for implementing of countermeasure CMF of countermeasures Estimate of service life of countermeasure Crash costs by severity Discount rate Available budget
DEA with uncertainty assessment	Expected number of accident (for example by EB method) Cost estimate for implementing of countermeasure Estimate of service life of countermeasure CMF and standard errors of them for countermeasures Crash costs (by severity) and their standard errors (or crash severity ratios and SD of them)

to them, respectively. Furthermore,  $R^2$  indicator allows prioritization of intersections that even in the best case scenario are not efficient (i.e.  $R^1=0$ ). Intersection number 15 has the worst rating due to the lowest value of  $R^2$ .

Comparing the ranks between DEA model with uncertainty assessment and classic DEA (original case scenario) shows some differences in the rankings with regard to tolerance in the data. For example, efficiency scores of intersections number 17 and 19 by classic DEA model (original case scenario in Table 5) are 0.951 and 0.846, respectively, and therefore, the rank of intersection 19 is higher than intersection 17, while it is reversed in the suggested DEA model.

In the real world, costs in design or decision making time may be different with them in implementation time and this is a kind of uncertainty in estimation of costs. Table 6 shows a comparison of results with and without considering uncertainty in the cost of projects. As it can be seen, uncertainly in the estimation of costs can lead to uncertain prioritization. For instance, the rank of intersection number 1 is 34 in the case of not considering uncertainty in costs, whereas it comes 35th when taking into account 20% increase or decrease in the cost by the suggested DEA method.

The results of two commonly ranking methods, i.e. incremental cost-benefit analysis and integer programming method, were compared to those of DEA model with uncertainty assessment. Table 7 shows the selected intersections (retrofit projects) based on the different methods with a limitation of \$1.5 million available budget. Since the available budget should be defined in integer programming method, this assumption was made. Intersection numbers marked with a star are those which are not common in the three methods. While in the incremental benefit-cost analysis intersections numbers 14 and 22 were selected, in integer programming method 10, 29, 22 and in DEA with uncertainty assessment 10, 29, 40 and 11 intersection numbers were selected.

The results of the analysis show that unreliable data can lead to some changes in the list of the selected projects and further and more detailed investigation need to be carried out among doubtful projects. On the other hand, some projects, like intersection 9, will be reliably efficient even though there is uncertainty in the data.

**Table 3**  
Input and outputs of DEA with uncertainty assessment model.

	Expected number of crash reduction (based on expected number of accident EB method multiplied (1-CMF)) (max and min are bounds of 95% confidence interval)									Cost of countermeasure
	Fatality			Injury			PDO			
	Min	Original	Max	Min	Original	Max	Min	Original	Max	
1	0.061	0.080	0.098	12.011	15.611	19.211	-1.359	43.046	87.450	140,000
2	0.034	0.048	0.062	8.973	12.710	16.447	33.371	47.268	61.165	60,000
3	0.221	0.221	0.221	33.273	33.273	33.273	13.619	51.391	89.163	140,000
4	0.013	0.100	0.188	3.149	24.429	45.709	0.972	48.597	96.223	70,000
5	0.060	0.064	0.069	13.450	14.462	15.475	22.575	24.729	26.883	100,000
6	0.011	0.089	0.166	2.690	20.870	39.050	0.433	21.664	42.894	70,000
7	0.006	0.006	0.006	2.375	2.375	2.375	4.936	4.936	4.936	10,000
8	0.006	0.006	0.006	1.345	1.345	1.345	4.592	4.592	4.592	10,000
9	0.053	0.093	0.134	13.562	24.026	34.491	-98.211	102.303	302.818	70,000
10	0.011	0.086	0.160	2.172	16.852	31.531	0.468	23.383	46.298	70,000
11	0.034	0.048	0.063	9.233	13.078	16.923	33.066	46.836	60.606	60,000
12	0.005	0.036	0.067	0.724	5.613	10.503	0.122	6.103	12.084	50,000
13	0.002	0.002	0.002	0.419	0.419	0.419	1.090	1.090	1.090	10,000
14	0.098	0.098	0.098	23.571	23.571	23.571	4.345	16.395	28.445	100,000
15	0.007	0.010	0.013	1.702	2.410	3.119	3.436	4.867	6.298	60,000
16	0.002	0.002	0.002	0.566	0.566	0.566	0.699	0.699	0.699	10,000
17	0.016	0.123	0.230	3.257	25.273	47.289	0.523	26.170	51.816	70,000
18	0.065	0.070	0.075	17.242	18.540	19.837	35.252	38.615	41.979	100,000
19	0.243	0.243	0.243	42.596	42.596	42.596	14.108	53.237	92.367	140,000
20	0.004	0.004	0.004	1.172	1.172	1.172	5.806	5.806	5.806	10,000
21	0.018	0.025	0.032	5.530	7.833	10.135	14.171	20.072	25.973	60,000
22	0.062	0.081	0.099	11.867	15.424	18.980	-1.629	51.577	104.783	140,000
23	0.040	0.071	0.101	10.393	18.412	26.431	-67.009	69.801	206.610	70,000
24	0.028	0.039	0.051	6.118	8.666	11.214	20.315	28.774	37.234	60,000
25	0.050	0.054	0.058	7.986	8.587	9.188	14.882	16.302	17.722	100,000
26	0.160	0.160	0.160	36.631	36.631	36.631	14.479	54.637	94.795	140,000
27	0.011	0.082	0.153	2.578	20.005	37.432	0.465	23.246	46.028	50,000
28	0.005	0.005	0.005	1.358	1.358	1.358	3.609	3.609	3.609	10,000
29	0.010	0.077	0.143	1.946	15.095	28.244	0.516	25.796	51.075	70,000
30	0.024	0.034	0.044	6.162	8.728	11.294	28.384	40.204	52.024	60,000
31	0.136	0.136	0.136	39.226	39.226	39.226	16.860	63.621	110.382	140,000
32	0.025	0.035	0.045	5.950	8.428	10.906	16.554	23.447	30.341	140,000
33	0.027	0.038	0.049	7.527	10.661	13.796	28.140	39.859	51.578	60,000
34	0.011	0.085	0.159	2.406	18.670	34.934	0.731	36.528	72.326	50,000
35	0.011	0.085	0.160	2.196	17.038	31.880	0.575	28.773	56.970	70,000
36	0.022	0.031	0.041	6.859	9.715	12.572	29.030	41.119	53.208	60,000
37	0.009	0.069	0.129	2.427	18.834	35.240	0.678	33.885	67.093	70,000
38	0.038	0.068	0.097	7.970	14.121	20.271	-71.156	74.121	219.399	70,000
39	0.005	0.005	0.005	0.943	0.943	0.943	4.891	4.891	4.891	10,000
40	0.008	0.066	0.123	1.813	14.069	26.324	0.490	24.491	48.493	70,000

**Table 4**  
Crash costs by severity (speed limit <=45 mile/h).

Crash severity	Mean comprehensive cost per crash (\$)	Standard error (\$)
No injury (PDO)	7068	547
Injury	60,900	7441
Fatality	3,622,179	80,996

Therefore, this method seems to provide a more complete view for decision makers to allocate a limit budget to more efficient safety projects.

**4. Conclusion**

In this paper, a procedure for picking out a set of safety retrofit projects considering some uncertainties in data has been offered. Apart from being a multi-dimensional problem, ranking and selecting the best projects suffers from uncertainty. It has been proven that DEA technique is seemingly a suitable tool for evaluating the efficiency of processes with multiple inputs and outputs like the traffic safety field. The DEA with uncertainty assessment

method is applied to this multiple criteria settings using a sample of intersections, each accompanied by a suggested countermeasure. This approach enables the analysis of ranking despite tolerances in data and provides new information as to the effect of uncertainty on the choice of the best projects. Analysis by considering tolerances in data allows safety retrofit projects located on the border of selection to be introduced because small changes in the preliminary data may lead to dramatic changes in the ranking list. Therefore, a more detailed investigation should be performed for low efficiency projects or projects with no agreement in their merit among different ranking methods. More appropriate allocation of funds will be followed by ensuring a proper selection of more efficient projects.

Besides the advantages mentioned, it is noteworthy that the proposed method requires more detailed data and it is more complicated than the existing methods and as a result requires a spreadsheet or software program.

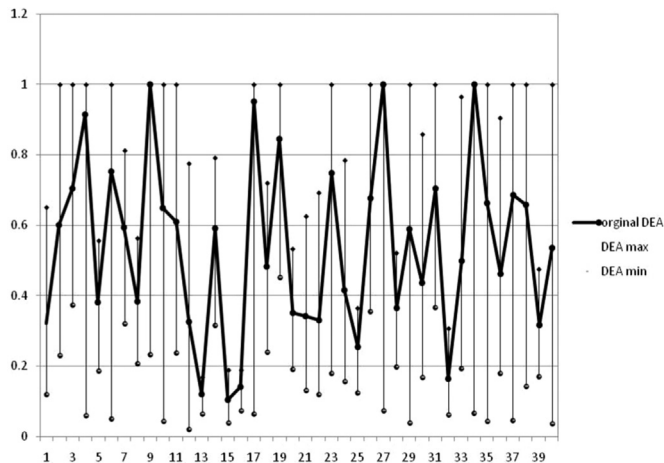
Only three kinds of accident reductions as outputs and initial cost of project as input are used in the illustrative example herein. In other circumstances, other inputs and outputs such as environmental effects or user inconveniences may be considered. Further research is required to include other types of uncertainty which may affect the results of the analysis.

**Table 5**  
Efficiency scores and ranking using DEA with uncertainty assessment model (calculated with 95% confidence interval in crash reductions and without tolerance in costs).

	Original of efficiency score	Maximum of efficiency score	Minimum of efficiency score	R <sup>1</sup>	R <sup>2</sup>	Ranking number
1	0.322	0.65	0.121	0	0.321	34
2	0.6	1	0.232	0.111	0.539	19
3	0.705	1	0.374	0.111	0.63	16
4	0.915	1	0.06	0.333	0.441	7
5	0.381	0.557	0.188	0	0.367	28
6	0.754	1	0.052	0.222	0.453	9
7	0.594	0.812	0.32	0	0.576	20
8	0.383	0.563	0.208	0	0.384	27
9	1	1	0.234	0.556	0.461	1
10	0.648	1	0.044	0.222	0.39	12
11	0.611	1	0.237	0.111	0.548	18
12	0.325	0.776	0.022	0	0.304	35
13	0.12	0.169	0.064	0	0.118	39
14	0.592	0.792	0.317	0	0.557	21
15	0.105	0.189	0.04	0	0.103	40
16	0.142	0.189	0.075	0	0.135	38
17	0.951	1	0.065	0.333	0.458	6
18	0.483	0.721	0.241	0	0.472	24
19	0.846	1	0.452	0.333	0.658	4
20	0.351	0.534	0.192	0	0.359	30
21	0.343	0.626	0.131	0	0.337	31
22	0.33	0.693	0.12	0	0.333	32
23	0.748	1	0.179	0.333	0.474	5
24	0.416	0.784	0.158	0	0.416	26
25	0.254	0.364	0.125	0	0.243	36
26	0.677	1	0.356	0.111	0.613	17
27	1	1	0.073	0.556	0.233	2
28	0.365	0.522	0.198	0	0.362	29
29	0.589	1	0.04	0.222	0.356	13
30	0.436	0.859	0.168	0	0.446	25
31	0.705	1	0.367	0.111	0.639	15
32	0.165	0.307	0.063	0	0.164	37
33	0.5	0.965	0.194	0	0.507	22
34	1	1	0.066	0.556	0.22	3
35	0.662	1	0.045	0.222	0.4	11
36	0.463	0.906	0.181	0	0.473	23
37	0.687	1	0.046	0.222	0.419	10
38	0.658	1	0.143	0.222	0.492	8
39	0.317	0.476	0.172	0	0.322	33
40	0.537	1	0.036	0.222	0.325	14

**Table 6**  
Comparison of ranking based on DEA with uncertainty assessment (95% confidence interval in crash reduction and different uncertainty in cost of countermeasures).

	Without uncertainty in the costs	10% Uncertainty in the costs	20% Uncertainty in the costs
1	34	34	35
2	19	19	19
3	16	16	16
4	7	5	5
5	28	28	30
6	9	9	8
7	20	23	20
8	27	27	29
9	1	1	1
10	12	12	12
11	18	18	18
12	35	35	28
13	39	39	39
14	21	24	23
15	40	40	40
16	38	38	38
17	6	4	6
18	24	25	25
19	4	6	4
20	30	30	32
21	31	31	33
22	32	32	27
23	5	7	7
24	26	26	26
25	36	36	36
26	17	17	17
27	2	2	2
28	29	29	31
29	13	13	13
30	25	22	24
31	15	15	14
32	37	37	37
33	22	20	21
34	3	3	3
35	11	11	11
36	23	21	22
37	10	10	9
38	8	8	10
39	33	33	34
40	14	14	15



**Fig. 1.** Results of DEA with tolerances: maximum, minimum and original scores. (Calculated with 95% confidence interval in crash reductions and without tolerance in costs).

**Table 7**  
Comparison of selected intersections by different methods (\$1.5 million budget).

Selected intersections by incremental benefit cost method	Selected intersections by integer programming method	Selected intersections by DEA with uncertainty assessment (95% confidence interval in crash reductions and without uncertainty in cost of countermeasures)
19	9	9
31	34	27
3	27	34
26	19	19
9	4	23
4	17	17
17	23	4
14*	6	38
23	37	6
6	38	37
34	35	35
27	10*	10*
37	29*	29*
38	31	40*
22*	26	31
35	3	3
	22*	26
		11*



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