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# Comparison of Systems Ageing Properties by Gini-type Index

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**Abstract**

The investigation of systems ageing properties is vitally important to distinguish the optimum strategy for connecting the system components. Here, Gini-type index is introduced as a useful tool to study the ageing properties of a system and is utilized to compare two complicated systems models under different ageing assumptions of their consisting components.

**Keywords:** Ageing properties, cumulative hazard, Gini-type index, systems comparison.

## 1 Introduction

In reliability and risk analysis the terms of ageing or rejuvenating are used for describing the behaviour of a repairable or non-repairable system and many authors studied this concept of various viewpoints; for instance see Lai and Xie (2003) and Lawless (2003). Karminskiy and Krivtsov (2010) introduced Gini-type index as a methodological tool for investigation the degree of ageing or rejuvenation of repairable or non-repairable systems.

In Section 2 the Gini-type (GT) index is introduced. Section 3 discusses about two complicated systems and compares them based on their corresponding GT indexes. The effect of ageing property of the components on the system is also studied in different cases.

## 2 Gini-type index

Let $T$ be a non-negative random variable denoting the lifetime of a component or system and $D = \{t \in R^+ : \bar{F}(t) < 1\}$. Suppose that $\bar{F}(t) = P(T \geq t)$ and $H(t) = -\log \bar{F}(t)$, respectively, represent the survival function and cumulative hazard rate function of $T$. Then the GT index which was introduced by Karminskiy and Krivtsov Karminskiy and Krivtsov (2010) is as follows

**Definition 2.1.** *The GT index for a non-negative absolutely continuous random variable $T$ in time interval $(0, t]$ is*

$$GT(t) = 1 - \frac{2\int_0^t H(u)du}{tH(t)}.$$

It is shown that $GT(t)$ satisfies the inequality $-1 < GT(t) < 1$ for all $t \in D$; it is positive for increasing failure rate (IFR) distributions; it is negative for decreasing failure rate (DFR) distributions and equal to zero for constant failure rate (CFR) distribution, *i.e.* the exponential distribution. In the following

*Speaker: parsa.motahareh@stu-mail.um.ac.ir sections we consider special complicated systems and compare the behaviour of their corresponding GT index.

# 3 Comparison of two systems strategies

Let $T_1, \ldots, T_n$ be non-negative iid random variables which denote the lifetime of $n$ components working in the same system. In the following subsections two strategies for connecting the components are considered.

## 3.1 Parallel-series system with shared components

In this system, suppose $Y_j = \min_{j \le i \le j+k-1} T_i$, $j = 1, \ldots, n-k+1$, $1 \le k < n$, which are locally dependent, and the lifetime of the system is given by $T_{sys1} = \max_{1 \le j \le n-k+1} Y_j$. The cumulative distribution function of the system lifetime is reached as follows

$$G_{n-k+1}(t) = P\left[T_{sys1} \le t\right] = P\left[Y_1 \le t, \ldots, Y_n \le t\right].$$

By conditioning on the failure of the first $k$ components and some calculations we get

$$G_{n-k+1}(t) = F(t)\sum_{i=1}^{k} \bar{F}^{i-1}(t) G_{(n-k+1)-i}(t),\ t \in D. \tag{1}$$

To find the $G_{n-k+1}(t)$ one needs to solve the difference equation of order $k$ given in (1) which requires numerical methods.

**Example 3.1.** *Let $k = 2$. If iid random variables $T_i$'s have Weibull distribution function $F(t) = 1 - e^{-(\lambda t)^m}$, where $\lambda > 0, m > 0$, then*

$$\begin{aligned}
G_{n-1}(t) = (&[\frac{1}{2} - e^{-2(\lambda t)^m} + \frac{1}{2}e^{-(\lambda t)^m} \\
&+ \frac{1}{2}\sqrt{(1-e^{-(\lambda t)^m})^2 + 4(1-e^{-(\lambda t)^m})e^{-(\lambda t)^m}}] \\
&\times (\frac{1}{2} - \frac{1}{2}e^{-(\lambda t)^m} \\
&+ \frac{1}{2}\sqrt{(1-e^{-(\lambda t)^m})^2 + 4(1-e^{-(\lambda t)^m})e^{-(\lambda t)^m}})^n / \\
&\sqrt{(1-e^{-(\lambda t)^m})^2 + 4(1-e^{-(\lambda t)^m})e^{-m\lambda x}} \\
&+1 - (\frac{1}{2} - e^{-2(\lambda t)^m} + \frac{1}{2}e^{-(\lambda t)^m} \\
&+ \frac{1}{2}\sqrt{(1-e^{-(\lambda t)^m})^2 + 4(1-e^{-(\lambda t)^m})e^{(\lambda t)^m}} / \\
&\sqrt{(1-e^{-(\lambda t)^m})^2 + 4(1-e^{-(\lambda t)^m})e^{-(\lambda t)^m}} \\
&(\frac{1}{2} - \frac{1}{2}e^{-(\lambda t)^m} \\
&- \frac{1}{2}\sqrt{(1-e^{-(\lambda t)^m})^2 + 4(1-e^{-(\lambda t)^m})e^{-(\lambda t)^m}}]^n.
\end{aligned} \tag{2}$$

The values and alterations of GT index for the system of Example 3.1 are illustrated in Figure 3.1.

Figure 1: The alterations of GT index for parallel-series system with shared components versus time while $\lambda = 1$.

Obviously the GT index value decreases during the time and as the number of components is grown this this behaviour is more severe. By Figure 3.1 it is also included that the higher number of components increases the value of GT index; which expresses the more IFR property of this system as more components are put in the system. By the Weibull distribution properties one knows that as the shape parameter satisfies $0 < m < 1$, $m = 1$ and $m \geq 1$ the corresponding distribution is DFR, CFR and IFR respectively. Considering Figure 3.1, it is concluded that when the components are DFR and we have a limited number of components, the system will be DFR as well. But CFR and IFR components always make an IFR parallel-series system.

### 3.2 Series-parallel system with shared components

Under the assumptions of Subsection 3.1, here the second strategy is expressed. Suppose that $Y_j = \max_{j \leq i \leq j+k-1} T_i$, $j = 1, \ldots, n-k+1$, $1 \leq k < n$, which are locally dependent, and the lifetime of the system is given by $T_{sys2} = \min_{1 \leq j \leq n-k+1} Y_j$. Following the similar steps we previously did, the survival function of this system is given by

$$\bar{G}_{n-k+1}(t) = \bar{F}(t) \sum_{i=1}^{k} F^{i-1}(t) \bar{G}_{(n-k+1)-i}(t), \tag{3}$$

Finding the $\bar{G}_{n-k+1}(t)$ requires numerical methods to solve the difference equation of order $k$ given in (3).

**Example 3.2.** *Assume that $k = 2$. Let that the iid random variables $T_i$'s have Weibull distribution function as was expressed in Example 3.1. Then we have*

$$\begin{aligned}
\bar{G}_{n-1}(t) =& [1 - (1 - e^{-(\lambda t)^m})^2 - \frac{1}{2} e^{-(\lambda t)^m} \\
&+ \frac{1}{2}\sqrt{e^{-2(\lambda t)^m} + 4(1 - e^{-(\lambda t)^m}) e^{-(\lambda t)^m}}] \\
&[\frac{1}{2} e^{-(\lambda t)^m} + 0.5\sqrt{e^{-2(\lambda t)^m} + 4(1 - e^{-(\lambda t)^m}) e^{-(\lambda t)^m}}]^n )/ \\
&\sqrt{e^{-2(\lambda t)^m} + 4(1 - e^{-(\lambda t)^m}) e^{-(\lambda t)^m}} \\
&+ [1 - (1 - (1 - e^{-(\lambda t)^m})^2 - \frac{1}{2} e^{-(\lambda t)^m} \\
&+ \frac{1}{2}\sqrt{e^{-2(\lambda t)^m} + 4(1 - e^{-(\lambda t)^m}) e^{-(\lambda t)^m}} / \\
&\sqrt{e^{-2(\lambda t)^m} + 4(1 - e^{-(\lambda t)^m}) e^{-(\lambda t)^m}}] \\
&[\frac{1}{2} e^{-(\lambda t)^m} - \frac{1}{2}\sqrt{(e^{-2(\lambda t)^m} + 4(1 - e^{-(\lambda t)^m}) e^{-(\lambda t)^m}}]^n
\end{aligned} \tag{4}$$

The values and alterations of GT index for this system in Example 3.2 are illustrated in Figure 3.2; and the behaviour of GT index of system versus the changes of number of components is shown in Figure 3.2.

Figure 2: The alterations of GT index for series-parallel system with shared components versus time while $\lambda = 1$.

The GT index value decreases during the time but increasing the number of components in this system seems to have no considerable influence on GT index. Also it is shown that when the components are DFR the system is DFR as well. But CFR and IFR components always make an IFR series-parallel system.

## Conclusion

The GT index can be a highly applicable concept to give information about the ageing properties of components and systems. One needs to be aware that the complicated systems may not save the ageing property of their consisting components and it is important to consider the ageing property of whole structure as a system to choose the optimum strategy of components connection.

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