# A New Strength Model for FRP Confined Circular Concrete Columns

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ABSTRACT: Extensive researches in the past decades have demonstrated that FRP Jackets increase load carrying capacity and ductility of concrete columns which are one of the most important structural elements from the stand point of retrofitting. In addition, FRP confinement increases concrete strength. Therefore, applicable relations for computation of confined concrete strength are a necessity in design and analysis of concrete structures. Several studies have been performed on FRP wrapped circular and rectangular concrete columns and various models are proposed for calculation of confined concrete strength. Most of these existing models, which are derived from experiments on cylindrical concrete samples, are modifications of the well-known formula of Richart et al. It is proved that the FRP retrofitting is less effective in rectangular columns due to non-uniform distribution of confinement pressure. Consequently, many relations of the existing strength models for circular columns are reviewed and their accuracy compared. The comparison is conducted based on experimental results of various researchers. Moreover, two new strength models for short confined circular column are proposed using nonlinear regression techniques. Numerical evaluations verify the suggested relation accuracy in comparison with the existing models.

## 1. *Introduction*

Nowadays, utilizing fiber reinforced polymers (FRP) is one of the most efficient and useful methods for strengthening concrete structures. First works in this field started in the early 80 decade. But the severe earthquakes in California and Kobe (at 1990 and 1995 respectively) were the true motivations which leads to extensive researchesonFRP as a means for strenghtening and retrofiting of reinfirced concrete structures.These studies which are accelerated and extended every day, provid the present knowledge for practical purposes.

In early stages, metalic tubes were used to strengthen existing concrete structures. This method increased cross section of structural members which was inappropriate from the architectural standpoint. Wrapping FRP layers around members is an acceptable alternative method from the mentiond viewpoint. In addition, other positive characteristics of FRPs (such as high tensile strength, corrosion resistanse, durability, light weight, electromagnetic neutrality and ...) have introduced this method as the most practical and efficient technique for strenghtening structures. FRP wraps increase strength and ductility of concrete structural members such as columns. One of the main functions of columns is to transfer structural forces to the base. Although the major internal force in columns is axial compressive force, a pure compressive column is rare. In other words, columns are axial-fluxeral members due to moments at rigid connections or eccentric axial loads. An accurate relation to estimate confined concrete strength is necessary for reliable and economic design of FRP layers. Most studies about FRP effects on concrete confinement is conducted on circular culumns. Chaallal et al(2000) proved that in recatngular columns, the effects of FRP layers on concrete strength is less than circular columns due to non-uniform distribution of confinement pressure. Many studies are performed on behavior of FRP wrapped circular columns and different relation and models are proposed. These models can be categorized into two groups, namely design oriented and analysis-oriented models. The first strength model for confined concrete is suggested by Richart (1928) .Then, Fardis et al(1982) stated that the mentiond model can be used for FRP confined conctrete. Later studies by Mirmiran et al(1997), Samaan et al(1998) and Saafi et al(1999) indicated that the general model proposed by Richart(1928) and Mander et al(1988) leads to unconservative results for FRP confined concrete, because they assume confinement pressure to be constant, which is not true for FRP confinement. Richart (1928) model which has the following simple form is considered the basic strength model, because many of the other relations are developed based on it.

$$f'_{cc} = f'_{co}(1 + k_1\frac{f_l}{f'_{co}}) \quad (1)$$

In this equation, $f_l$, $f'_{co}$ and $f'_{cc}$ are confinement pressure of FRP layers, strength of unconfined and confined concrete respectively and $k_1$ is the confinement effectiveness coefficient. $f_l$ is computed by the following equation:

$$f_l = \frac{2f_{frp}t_{frp}}{d} \quad (2)$$

In equation 2 parameters $d$,$f_{frp}$,and $t_{frp}$are column diameter,tensile strength and thickness of the FRP layers.Investigators such as Karbhari et al(1997) ,Samaan et al(1998), Saafi et al(1999), andMiyauchi et al(1999) have modified Richart(1928) model by taking advantage of different values for $k_1$(which was taken equal to 4.1 by Richart(1928)) in order to remove assumption of constant confinement pressure. In addition, some of the existing models computes $k_1$ as a function of confinement pressure and concrete strength.

## 2. *Existing strength models for FRP confined circular columns*

As mentioned in the previous section, many relations are proposed to estimate confined concrete strength. Most of the relations are suggested for circular columns without considering eccentricity effects. These relations are presented in Table 1.

Table 1. Strength Models for FRP Confined Circular Concrete Columns

| Name | Strength model for FRP confined circular concrete column |
|---|---|
| Richart (1928) | $f'_{cc} = f'_{co}(1 + 4.1\frac{f_l}{f'_{co}})$ |
| Fardis et al (1982) | $f'_{cc} = f'_{co} + 3.7(\frac{f_l}{f'_{co}})^{0.86}$ |
| Mander et al (1988) | $f'_{cc} = f'_{co}(-1.254 + 2.254\sqrt{1 + \frac{7.94f_l}{f'_{co}}} - 2\frac{f_l}{f'_{co}})$ |
| Karbhari et al (1997) 1st equation | $f'_{cc} = f'_{co}(1 + 2.1(\frac{f_l}{f'_{co}})^{-0.13} \times \frac{f_l}{f'_{co}})$ |
| Karbhari et al(1997) 2nd equation | $f'_{cc} = f'_{co} + 3.1f'_{co}\nu_c\frac{2t}{D}\frac{E_{frp}}{E_c} + \frac{2f_{frp}t_{frp}}{D}$ |
| Mirmiran et al(1997) | $f'_{cc} = f'_{co}(1 + 4.269 \times \frac{f_l^{0.587}}{f'_{co}})$ |
| Kono et al(1998) | $f'_{cc} = f'_{co}(1 + 0.0579 \times f_l)$ |

| | |
|---|---|
| Samaan et al (1998) 1st equation | $f'_{cc} = f'_{co}(1 + 6(f_l)^{-0.3} \times \frac{f_l}{f'_{co}})$ |
| Samaan et al(1998) 2nd equation | $f'_{cc} = f'_{co} + 6.14 f_l^{0.75}$ |
| Razvi et al(1999) | $f'_{cc} = f'_{co}(1 + 6.7 \times \frac{f_l^{0.83}}{f'_{co}})$ |
| Spoelstra et al(1999) | $f'_{cc} = f'_{co}(0.2 + 3 \times (\frac{f_l}{f'_{co}})^{0.5})$ |
| Miyauchi et al(1999) | $f'_{cc} = f'_{co}(1 + 2.98 \times \frac{f_l}{f'_{co}})$ |
| Toutanji (1999) | $f'_{cc} = f'_{co}(1 + 3.5(\frac{f_l}{f'_{co}})^{-0.15} \times \frac{f_l}{f'_{co}})$ |
| Saafi et al(1999) | $f'_{cc} = f'_{co}(1 + 2.2(\frac{f_l}{f'_{co}})^{-0.16} \times \frac{f_l}{f'_{co}})$ |
| Karabinis et al (2001) | $f'_{cc} = f'_{co}(1 + 2.1 \times (\frac{f_l}{f'_{co}})^{0.87})$ |
| Shehata et al (2002) | $f'_{cc} = f'_{co}(1 + 1.25 \times \frac{f_l}{f'_{co}})$ |
| Lam et al (2001) | $f'_{cc} = f'_{co}(1 + 2.15 \times \frac{f_l}{f'_{co}})$ |
| Lam et al (2003) | $f'_{cc} = f'_{co} + 3.3 \times f_l$ |
| CSTR (2004) | $f'_{cc} = f'_{co}(1 + 0.05 \times 2t \times \frac{E_{frp}}{d \times f'_{co}})$ |
| CNR-DT200 (2004) | $f'_{cc} = f'_{co}(1 + 2.6 \times (\frac{f_l}{f'_{co}})^{\frac{2}{3}})$ |
| Matthys et al(2005) | $f'_{cc} = f'_{co}(1 + 2.3(\frac{f_l}{f'_{co}})^{-0.15} \times \frac{f_l}{f'_{co}})$ |
| Robert et al (2006) | $f'_{cc} = f'_{co}(1 + 3.57 \times \frac{f_l}{f'_{co}})$ |
| Guralnick (2006) | $f'_{cc} = f'_{co}(0.616 + \frac{f_l}{f'_{co}} + 1.57 \times (\frac{f_l}{f'_{co}} + 0.06)^{0.5})$ |
| Kumutha et al (2007) | $f'_{cc} = f'_{co}(1 + 0.93 \times (\frac{f_l}{f'_{co}})$ |
| Youssef (2007) | $f'_{cc} = f'_{co}(1 + 2.109 \times (\frac{f_l}{f'_{co}})^{0.783})$ |
| Wu et al(2009) | $f'_{cc} = f'_{co}(1 + 2.2 \times (\frac{f_l}{f'_{co}})^{0.94})$ |
| Wu et al(2010) | $f'_{cc} = f_l + f'_{co} \times \sqrt{\frac{16.7}{f'^{0.42}_{co}} - \frac{f'^{0.42}_{co}}{16.7}\ \frac{f_l}{f'_{co}} + 1}$ |

It is evident that many of the relations are constructed based on the form of the one proposed by Richart (1928), which is appropriate for confined reinforced concrete columns.

It can be seen that almost all of the mentioned relations are functions of the same parameters, namely column diameter (*d*), thickness, tensile strength and modulus of elasticity of the FRP layers ($\boldsymbol{t_{frp}}$, $\boldsymbol{f_{frp}}$&$\boldsymbol{E_{frp}}$) and concrete compressive strength ($\boldsymbol{f'_{co}}$).

3. *Evaluation of the strength models*

Taking advantage of the results derived from experimental results performed by various researchers (Watanable et al. (1997), Xiao et al. (2000), Berthet et al. (2005), Akogbe et al. (2011), Liang et al. (2012), Wu et al. (2013)).The above mentioned relations are evaluated to determine the most accurate ones. To evaluate accuracy of the models, SRSS method is used to compute error of the estimated strength. The following relation is used to compute the error:

$$Error = \sqrt{\frac{\sum_{i=1}^{n}\left(f_{ccT} - f_{ccE}\right)^2}{n}} \tag{3}$$

In this equation, $f_{cc_T}$ is the estimated strength using models and $f_{cc_E}$ is the experimental result and n is the number of tested specimens. Table 2 presents the obtained results for aforementioned strength models.

Table 2 . Relative error of the existing strength models

| Model | Error | Model | Error | Model | Error |
|---|---|---|---|---|---|
| Richart(1928) | 4.54 | Razvi(1999) | 3.93 | CSTR(2004) | 3.20 |
| Fardis(1982) | 4.43 | Spoelstra(1999) | 2.26 | CNR-DT200(2004) | 3.33 |
| Mander(1988) | 3.64 | Miyauchi(1999) | 2.85 | Matthys(2005) | 2.30 |
| Karbhari(1997)1st equation | 2.03 | Toutanji(1999) | 4.13 | Robert(2006) | 2.20 |
| Karbhari(1997)2nd equation | 1.95 | Saafi(1999) | 2.20 | Guralnik(2006) | 2.33 |
| Mirmiran(1997) | 1.87 | Karabinis(2001) | 2.03 | Youssef(2007) | 2.21 |
| Kono(1998) | 2.49 | Lam(2002) | 1.89 | Kumutha(2007) | 2.02 |
| Samaan(1998)1st equation | 1.88 | Shehata(2002) | 1.76 | Wu(2009) | 3.12 |
| Samaan(1998)2nd equation | 2.35 | Lam(2003) | 2.10 | Wu(2010) | 1.70 |

Table 3 lists the models with lower errors that would be used to produce a new more accurate model numerically.

Table 3. The most accurate relations

| Model | Error | Model | Error |
|---|---|---|---|
| Shehata | 1.76 | Karbhari(a) | 1.95 |
| Mirmiran | 1.87 | Kumutha | 2.02 |
| Samaan | 1.88 | Karabinis | 2.03 |
| Lam2002 | 1.89 | Karbhari(b) | 2.03 |

*4. Derivation of new strength models*

To proposed a new accurate relation, at first step more than 10000 sample data are produced using MATLAB. These sample data consist of major parameters that were used in previous strength model. These parameters are column diameter (*d*), thickness, tensile strength and modulus of elasticity of the FRP layers ($\boldsymbol{t_{frp}}, \boldsymbol{f_{frp}} \& \boldsymbol{E_{frp}}$) and unconfined concrete compressive strength ($\boldsymbol{f'_{co}}$). The next step is compute strength of confined concrete for the sample data using the most accurate relation presented in Table 3. Average of the outcomes is taken as dependent variable and the independent ones are the sample parameters. Finally, nonlinear regression using the variables leads to the proposed relations that are presented in Table 4. One of the relations does not consider modulus of elasticity of FRP layers as independent variables.

Table 4. The proposed strength models

| Model Name | Strength model for FRP confined circular concrete column |
|---|---|
| AGT1 | $f'_{cc} = f'_{co}(1 + 84250.71 \times \frac{{f_{frp}}^{2.671} \times t^{2.758} \times {E_{frp}}^{0.108}}{d \times {f'_{co}}^{3.206}})^{0.2}$ |
| AGT2 | $f'_{cc} = f'_{co}(1 + \frac{65.64 \times {f_{frp}}^{1.301} \times t^{1.342}}{d \times {f'_{co}}^{1.472}})^{0.5}$ |

*5. Evaluation of the proposed relations*

To evaluate the suggested relations, behavior of the models at limit states must be analyzed. It is expected that when the column diameter increases, effects of the confinement on concrete strength would be negligible. It is evident that this property is preserved in the suggested equations, because when *d* tends to infinity, the confined concrete strength approaches to the concrete strength. Provided that the thickness of the FRP layers approaches zero, FRP effects would not be considerable. Also when FRP strength is low, it would not lead to significant increase in confined concrete strength. These two conditions are also maintained in the suggested models.

Now that the limit state behavior of the proposed relations assured, the accuracy of them must be investigated. To evaluate accuracy, average error of the models is computed using experimental results and Eq. 3. The average errors are presented in Table 5.

Table 5.Relative error of the proposed models

| Model Name | Relative Error |
|---|---|
| AGT1 | 1.5888 |
| AGT2 | 1.5982 |

Comparing Tables 2 and 5, it is evident that the proposed strength models are in better agreement with the experimental results. This proves that the suggested models are more accurate than the existing ones. Finally, behavior of the suggested relations regarding variation of different independent variables is studied. Figs 1 to 4 demonstrate the trend of the models with respect to different variables. They show that the proposed models exhibit same behavior as the existing relation. However, it was expected due to their form similarities.

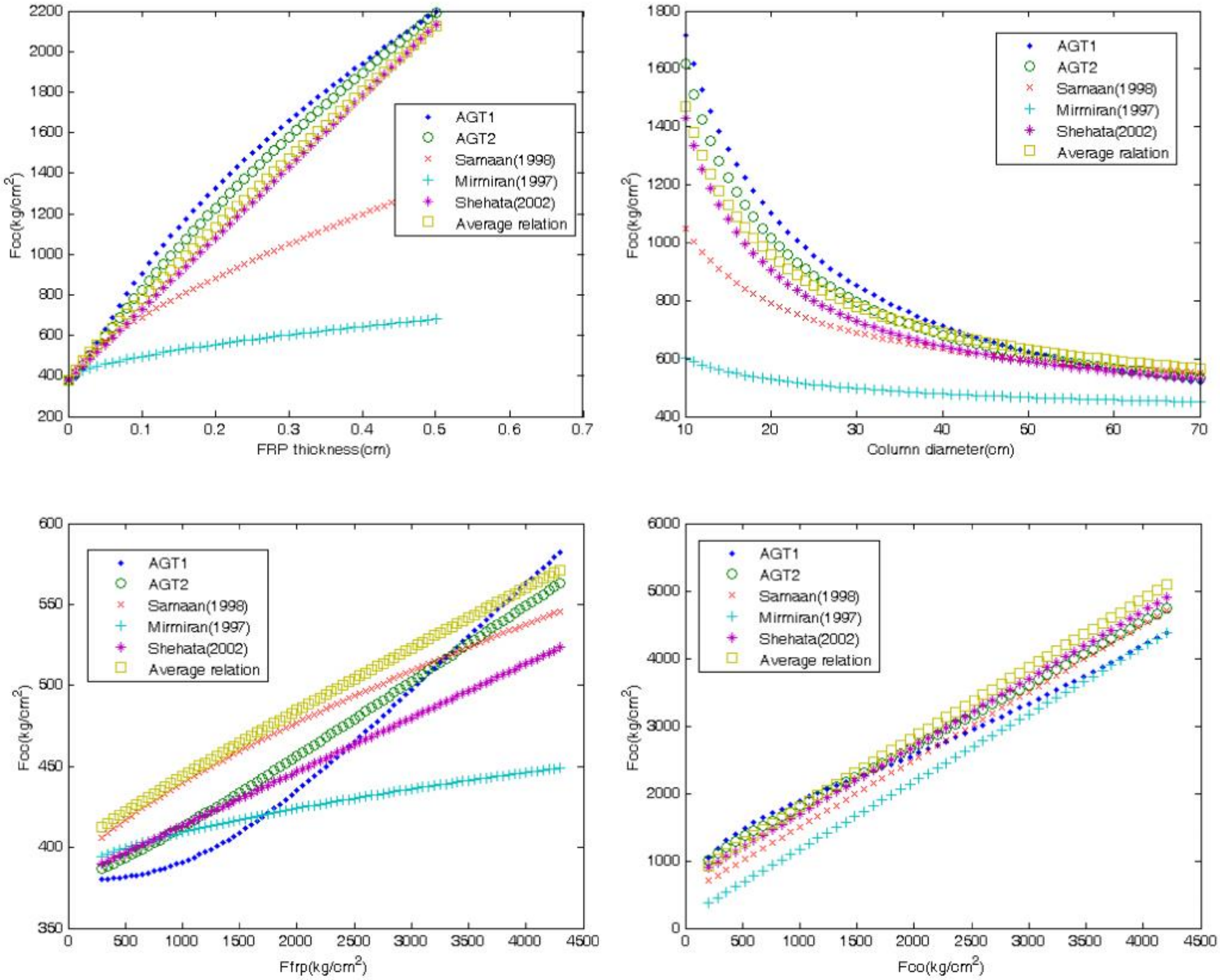


Figure 1.comparison between AGT equations and exists equations

## *6. Conclusion*

In this paper, after presenting existing strength models for circular FRP confined concrete columns, their accuracy estimated based on experimental results and SRSS method. Then the most accurate ones are selected. Taking advantage of the average of the accurate models and through nonlinear regression, two new strength models which exhibit better accuracy than the existing ones were formulated. These new models consider same parameters as those used in older versions. The difference between two suggested models is that one of them does not take account of FRP modulus of elasticity, while the other does. However, their accuracy does not differ considerably. Therefore, it can be concluded that $E_{frp}$ is not very effective parameter on behavior of FRP confined concrete members. Table 5 demonstrates that there is still error in the presented models in comparison with experimental results. This error did not remove completely despite hard mathematical and statistical efforts. At the end, it should be mentioned that the only disadvantage of the proposed models is their relative complicated form, which is the cost of their higher accuracy.

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