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New method for estimation of stage-discharge curves in natural rivers

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ABSTRACT

In the presented paper, a simple and feasible method for estimation of stage-discharge relationship in natural rivers using the concept of discharge estimation based on single point velocity measurement is introduced. Two innovative techniques have been implemented in rating curve estimation. First, discharge at a certain stage can be estimated by velocity measurement at an arbitrary point in the flow cross section. Then a relationship between the parameters obtained by the single point measurement technique and discharge ratios at different stages is presented. For this purpose, a relationship based on dimensional analysis through minimization of the differences between the calculated and analytical discharges at two different stages is established. For the verification, the results of the proposed relationship are compared with the observed stage-discharge data taken from the Severn River in UK, the Cuenca and Tomebamba rivers in Ecuador. The results have shown the high accuracy of the proposed method in such a way that the maximum relative errors for the Severn, Cuenca and Tomebamba rivers are less than 3%, 6% and 3%, respectively. In estimation of stage-discharge relationship, if the referenced stage is taken from higher stages, maximum error will be reduced to much smaller values.

1. Introduction

The role of discharge measurement in rivers, is important for a variety of applications such as water resources management, flood control, river engineering, sediment transport and hydraulic and hydrologic modeling. The knowledge of hydrometry is based on direct measurement of water depth and velocity which are required for setting up the stage-discharge relationship at different stages. A large number of researches regarding discharge calculation, identification of stage-discharge relationship as well as the way of its extension have been carried out. The stage-discharge curve in circular conduct with and without a flat bed have been investigated by Sterling and Knight [1]. The results indicate that the Manning roughness n is invariant with stage and, as a consequence, an empirical relationship based on Manning formula can be established which allows the prediction of a non-dimensional stage-discharge curve.

Leonard et al. [2] by the use of a modified form of the Manning formula have suggested to express flow rate as a function of the hydraulic radius and longitudinal water surface slope rather than water level alone which is conventionally used in the classical stage-discharge relationship. Abril and Knight [3] have introduced a stage-discharge relationship for natural rivers by the use of mean velocity through their model and implementation of finite element approach. In their proposed method three parameters namely the hydraulic coefficient of flow resistance, lateral eddy viscosity and mean velocity should be evaluated prior to any calculations. Sivapragasam and Muttil [4] have extended rating curves using the support vector method, which works on the principle of linear regression on a higher dimensional feature space. The results have shown that using the support vector method to extrapolate a rating curve is more accurate than other methods. Maghrebi [5] has estimated the discharge in rectangular channels using the single point velocity measurement method (SPM) and validated the accuracy of their model with the measured data obtained from the Severn River in UK. The results indicate that the discharge obtained using this approach is highly accurate. Moreover, discharge can be estimated at a higher velocity and lower cost using this technique. Rahimpour and Maghrebi [6] have estimated the discharge in rectangular channels with composite roughness using the SPM. The artificial neural networks (ANNs) have been used to set up stage-discharge relation as an important part of the stream flow processing [7]. Guven and Aytek [8] have introduced a method based on genetic algorithm for estimation of the stage-discharge relationship. They have also mentioned that this relationship is not unique. One of the important factors that need to be accurately addressed is the uncertainties associated with the estimation of stream flow data. Baldassarre and Montanari [9] used 1-D model for

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Nomenclature

A	cross-sectional area of flow
a, b, c, d	constants
A_e	area at estimated water level
A_r	area at referenced water level
C	normalized (or dimensionless) velocity
c_1	a factor that depends on shear velocity, roughness on the wall and turbulent intensity
$d\mathbf{s}$	a finite element of boundary
$d\mathbf{u}$	differential velocity deviation between an element of the boundary and an arbitrary point in the flow field
$f()$	a function of
H	water depth along y-axis at a cross section
N	number of observed or estimated data
$NRMSE$	normalized root mean square error
$NRMSE_C$	normalized root mean square error in circular cross section
$NRMSE_R$	normalized root mean square error in rectangular cross section
$NRMSE_T$	summation of normalized root mean square error for circular and rectangular cross sections
n	manning roughness coefficient
P	total wetted perimeter $P=P_w+T$
P_e	total wetted perimeter at estimated water level
P_r	total wetted perimeter at referenced water level
P_w	wetted perimeter
Q_e	estimated discharge
Q_r	referenced discharge
$\mathbf{r}$	position vector of arbitrary point in field
S_O	slope of the channel bottom
T	width of channel section at the free surface
U	cross sectional mean flow velocity in the streamwise direction (computational)
u	streamwise velocity at a point in the channel section
U_e	mean velocity at estimated water level
U_r	mean velocity at reference water level
V	average velocity of flow
v_m	measured velocity component in the streamwise direction
y	normal distance from boundary
z	distance measured in lateral direction
θ	angle between the positional vector and the boundary element

analyzing and quantifying the uncertainty of river flow data in the Po River (Italy). The results indicate that errors in river flow data are indeed far from negligible.

Domeneghetti et al. [10] have presented the general uncertainty in discharge measurement in rivers and proposed a framework to analyze the uncertainty in rating curves and its effects on the calibration of the hydraulic models. Hassanpour Kashani et al. [11] have investigated the efficiency of the artificial neural network in estimation of stage-discharge relationship in Kizilimark River in Turkey. Their results have shown that ANFIS has a better capability in identification of stage-discharge curve. Singh et al. [12] by the use of Tsallis entropy have predicted the discharge in natural rivers. This method is based on two general conditional probability and mean discharge. Yang et al. [13] have focused on the estimation of stage-discharge relationship. The purpose of their investigations was application of momentum coefficient and its correction factor in the area limited between the flood plain and main channel of the rivers.

In spite of a large number of researches which have been carried out on the estimation of stage-discharge curves and their extensions in a large number of hydraulic conduits, still a single method which can produce the rating curve with minimum efforts on the calibration or without the use of any calibration is out of reach. The objective of this study is to propose an innovative method using single point of velocity measurement to estimate the discharge which in turn is implemented to depict the stage-discharge curve. Considering each pair of discharge and stage as a reference section, one easily produce the whole rating curve. The obtained results show a very good accuracy of the presented method.

2. The governing principles

The proposed new method originates from the concept of discharge estimation using the SPM which was first introduced by Maghrebi [5]. The Biot-Savart law is used in this approach in order to model the field of a current flowing inside a wire as an analogue to the flow of water in a section of a river. In fact the effect of the wall on velocity at one point may be simulated by considering the effects resulting from the electromagnetic forces on a particle with static charge that is placed in the field of a wire carrying an electric current. Fig. 1 shows a river cross section which is covered with triangular meshes. The centroid of each triangular element is where the boundary effects are calculated. In order to increase the accuracy of the method, the number of meshes has been increased along the boundary and water surface. The net coverage shown in Fig. 1 is a schematic diagram and in the actual analysis a fine mesh is applied to the cross section..

The wetted perimeter of the river cross section is divided into infinitesimal elements $d\mathbf{s}$. The influence of $d\mathbf{s}$ from the wetted perimeter on the velocity at an arbitrary point with the coordinates of (y,z) is $d\mathbf{u}$ which can be calculated from a vector equation shown as follows:

$$d\mathbf{u} = f(\mathbf{r}) \times c_1 d\mathbf{s} \tag{1}$$

Therefore the total effects of the boundary on each element can be integrated as

$$u(y, z) = \int_{boundary} c_1 f(r) \sin\theta ds \tag{2}$$

where c_1 is a constant related the boundary shear stress, turbulent intensity and relative roughness, θ is the angle between the positional vector $\mathbf{r}$ and the boundary element vector $d\mathbf{s}$, $f(r)$ is the dominant velocity function in terms of r. In fact the actual value obtained from Eq. (2) does not have any significant meaning. Rather, its normalized value plays an important role in the single point velocity measurement method. Now one can extend the calculations of u all over the cross section based on the mesh given in Fig. 1. Then the mean cross sectional velocity can be obtained from the following equation:

$$U = \frac{\int_A u(y, z) dA}{A} \tag{3}$$

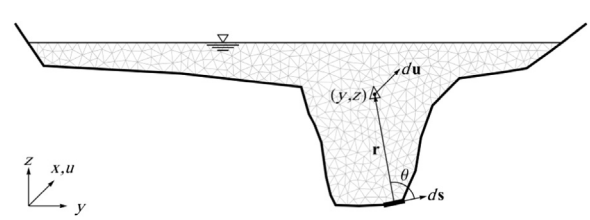

Fig. 1. Illustrative geometry for the effect of boundary on the velocity of an arbitrary point with coordinates of (y,z) at the river cross section.

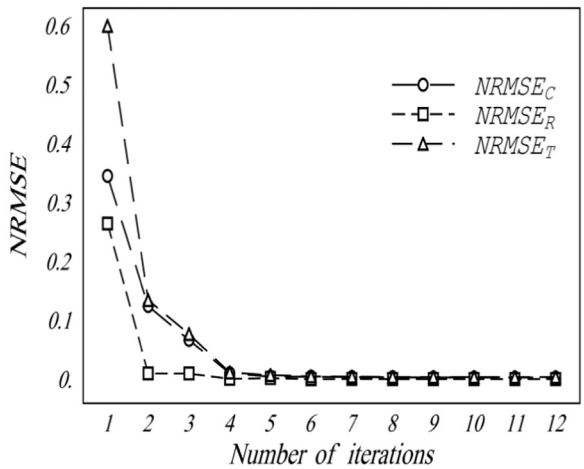

Fig. 2. Variation of NRMSE as a function of iteration steps.

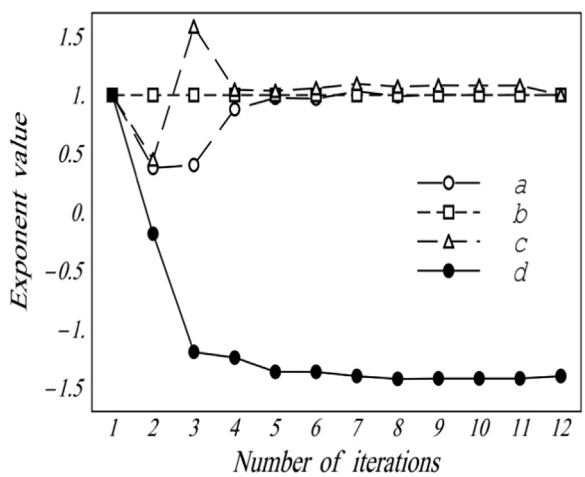

Fig. 3. Variation of a, b, c and d in iteration steps.

where dA is the area of each triangular mesh and A is the total area of the transverse cross-section of flow. If the obtained velocity u at (y,z) from Eq. (2) is divided by the mean cross sectional velocity U, the normalized velocity C at that point will be obtained:

$$C = \frac{u(y, z)}{U} \tag{4}$$

By measuring the streamwise velocity v_m at an arbitrary point with the corresponding value of C, one can easily obtain the mean cross sectional velocity using Eq. (5):

$$V = \frac{v_m}{C} \tag{5}$$

where in the above equations, u, U and C are the computed quantities and v_m is the measured one. In the SPM making a connection between the above mentioned quantities and the geometry of the river cross section is very important to calculate these quantities at different levels. Dimensional analysis is used in order to get a better recognition of the factors that are effective in the determination of the stage-discharge relationship. In the study of phenomena involving discharge, either analytically or experimentally, there are invariably many flow and geometric parameters involved. We may suspect that discharge depends on such variables as the area of the flow section A, the length of the wetted perimeter P_w, the length of the free surface of water T, the Manning roughness coefficient n, the bed slope S_O and the velocity of flow in the cross section U affect the amount of discharge. This could be expressed as:

$$Q = f(A, P_w, U, n, T, S_0) \tag{6}$$

Since the main objective of this study is to find a general relationship between the discharge at two different levels and this can preferably be described in the form of a ratio, the variables which play the main role in this ratio must be distinguished from the variables which stay fixed at all levels and can be omitted from the computational procedure. For example, the effects of roughness are not omitted with an increase in level and such effects are included in the variable U whereas the impact of the slope of the bed does not affect the answer of the problem at hand. Therefore, finally the variables affecting the amount of discharge are chosen according to the following equation:

$$Q \propto A^a P^b U^c P_w^{\,d} \tag{7}$$

where Q is the flow discharge, A is the area of the total flow section, P is the total perimeter of the flow section including the wetted perimeter and the length of the water surface i.e. $P=P_w+T$ and U is the mean velocity obtained from the SPM.

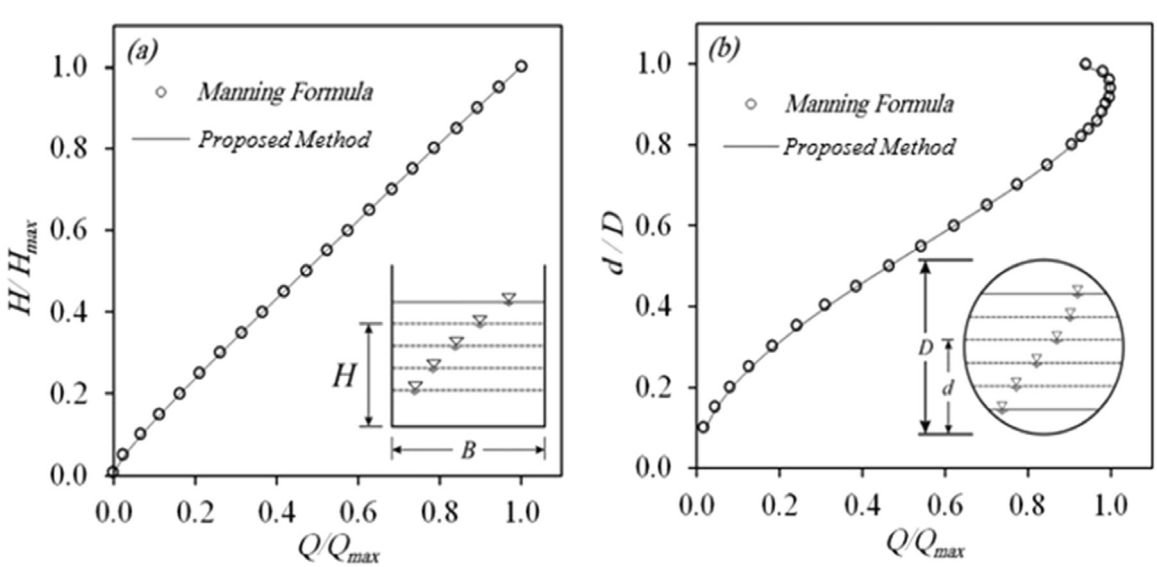

Fig. 4. Stage-discharge curve for the observed and calculated values in the selected sections of (a) rectangular section and (b) circular section.

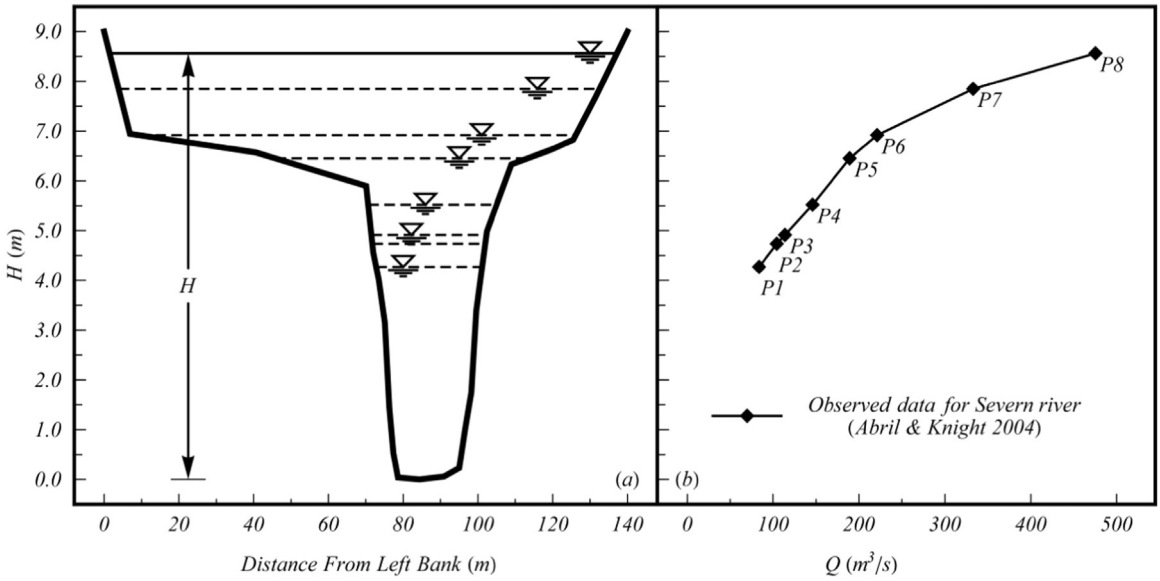

Fig. 5. (a) Cross sectional view of the Severn River in UK and (b) corresponding stage-discharge curve.

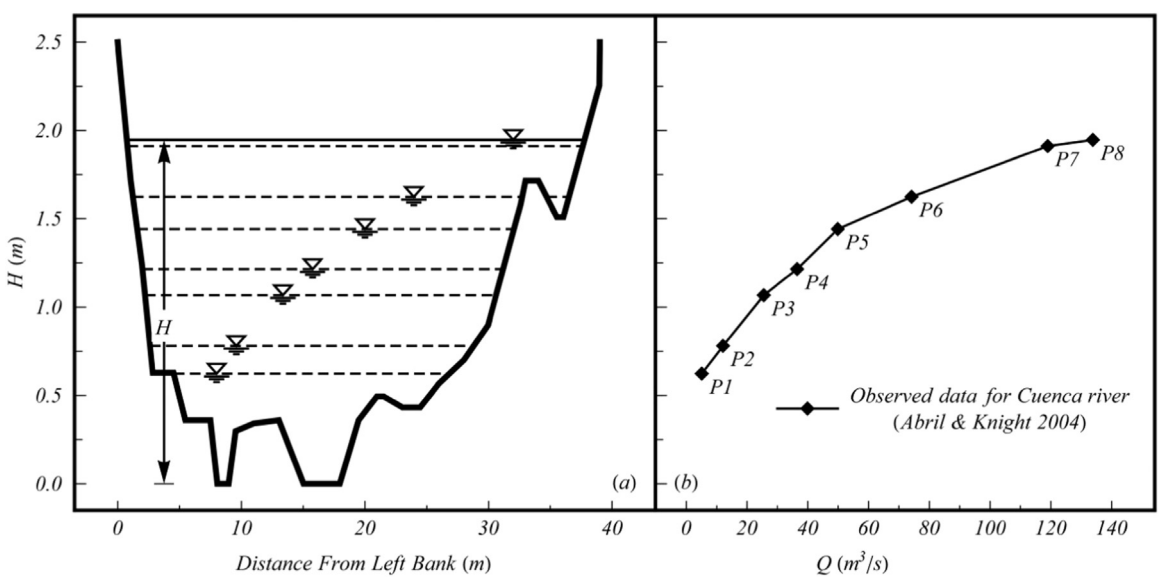

Fig. 6. (a) Cross sectional view of the Cuenca River in Ecuador and (b) corresponding stage-discharge curve.

The following steps are taken to estimate the stage-discharge curves. In the first step, the stage-discharge values are calculated for the rectangular and circular sections at different water levels using Manning formula. The adopted values for the geometry of the cross sections are as the followings: diameter of the circular cross section D=1 m, width of the rectangular cross section B=1 m, maximum level of water over the invert H_{max}=1 m, Manning roughness n=0.01 and bed slope of the conduits S=0.01. It was examined that if some other values were selected for the mentioned parameters, no variation in results would be expected. In the next step, the geometric values of A, P, P_w as well as the velocity parameter of U for each section at each level are calculated. In order to estimate the stage-discharge relationship, a single pair of data of the observed stage and discharge, which are referred as the referenced data, is required. In the current paper, the referenced data are associated with the maximum discharge. Now, by the use of Eq. (8) with exponents a, b, c and d, all of the referenced values such as A_r, P_r, $(P_w)_r$, U_r and Q_r can be replaced into this equation:

$$\frac{Q_e}{Q_r} = \left(\frac{A_e}{A_r}\right)^a \left(\frac{U_e}{U_r}\right)^b \left(\frac{P_e}{P_r}\right)^c \left(\frac{(P_w)_e}{(P_w)_r}\right)^d \tag{8}$$

where the subscripts e and r are referred to the estimated and referenced values, respectively.

Now at each level the values of A_e, P_e, $(P_w)_e$ and U_e can be replaced into Eq. (8). Before being able to estimate the discharge, the exponent values of a, b, c and d in Eq. (8) should be evaluated as a parametric equation with four variables. This can be achieved by the minimization of the statistical measure *NRMSE* which is defined in the following form:

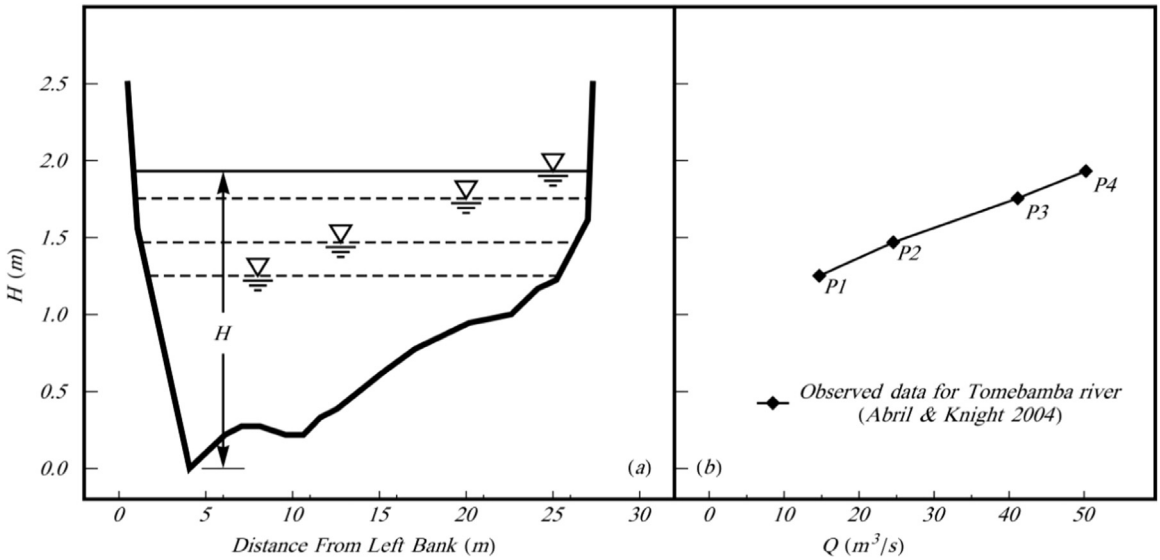

Fig. 7. (a) Cross sectional view of the Tomebamba River in Ecuador and (b) Corresponding stage-discharge curve.

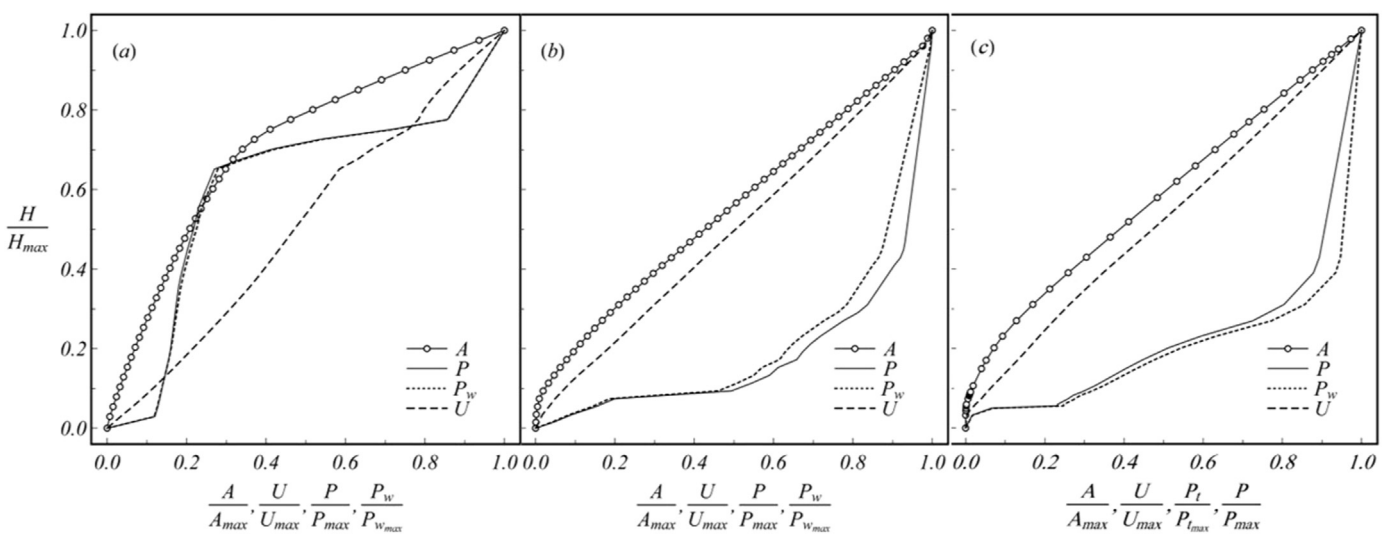

Fig. 8. Variation of the relative values of A, P, P_w, and U (a) Severn River and (b) Cuenca River and (c) Tomebamba River.

$$NRMSE = \frac{\sqrt{\frac{\sum_{i=1}^{N}\left[\left(Q_r\left(\frac{A_e}{A_r}\right)^a\left(\frac{U_e}{U_r}\right)^b\left(\frac{P_e}{P_r}\right)^c\left(\frac{(P_w)_e}{(P_w)_r}\right)^d\right)_i - (Q_o)_i\right]^2}{N}}}{\max[Q_o] - \min[Q_o]} \tag{9}$$

where $(Q_o)_i$ is the observed discharge at the i-th stage. The denominator of this equation is composed of the difference between the maximum and minimum discharges. To take into account the behavior of the two selected sections into the statistical measure, the minimization process is applied to the summation of the statistical measures $NMRSE_T$ in the following form:

$$NRMSE_T = NRMSE_R + NRMSE_C \tag{10}$$

where the subscripts R and C are referred to the rectangular and circular cross sections, respectively. From this equation, by the use of Multivariate Newton's method one will be able to evaluate the unknown exponent values in Eq. (8). Parameter U in Eq. (8) plays the role of the velocity parameter so the power of this parameter was kept constant i.e. b=1.

$NMRSE_T$ is the objective function (f), which should be minimized by an iteration procedure like Multivariate Newton's method. Accordingly, the solution of this function can be achieved by the following equation:

$$\mathbf{x}^{n+1} = \mathbf{x}^n - \frac{f'(\mathbf{x}^n)}{f''(\mathbf{x}^n)} \tag{11}$$

where $\mathbf{x}^n$ and $\mathbf{x}^{n+1}$ are variable vectors in steps n-th and (n+1)-th, respectively which are consisted of m variables where for the current case m=4. The functions $f'(\mathbf{x}^n)$ and $f''(\mathbf{x}^n)$ are the Gradient and Hessian matrices of f which are commonly denoted by$\nabla f(\mathbf{x}^n)$ and $H(\mathbf{x}^n)$ in the literature, respectively.

In order to evaluate the exponent values, we need to assume the initial values. As the starting step, the values of a, c and d are assumed unit. Fig. 2 shows the variation of $NRMSE_T$ as a function of the iteration number..

From Fig. 2 it is observed that the ranges of variation of the exponent values are as the followings: $0.38 < a < 1.03$, $b=1.0$, $0.45 < c < 1.58$ and$-1.47 < d < -0.18$. At the end of minimization

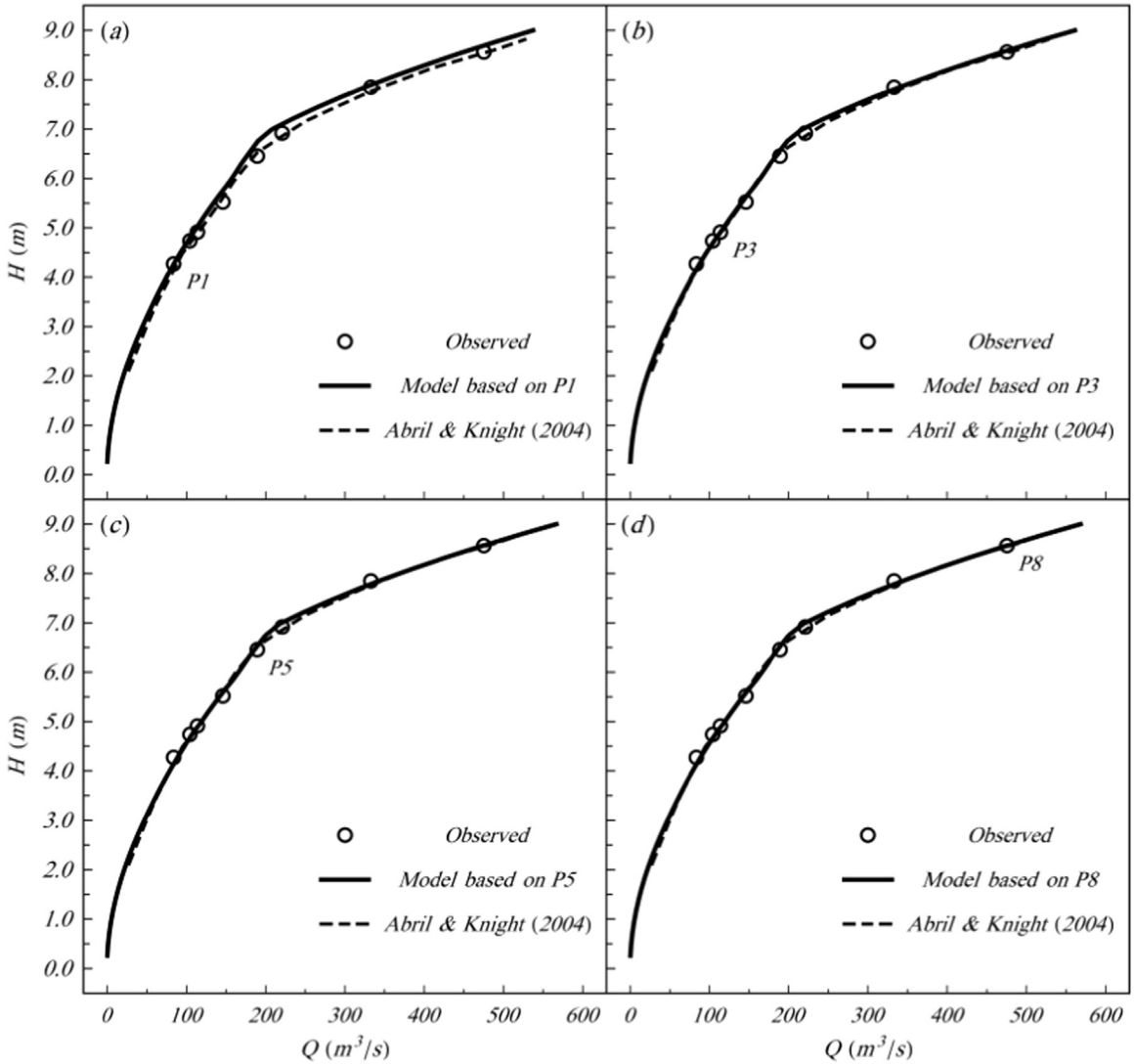

Fig. 9. Estimated stage-discharge curve in the Severn River by the use of different referenced levels (a) *P*1, (b) *P*3, (c) *P*5 and (d) *P*8.

process the final values after eleven iterations are 1 for *a* and *c* and −1.4 for *d*.

Finally, Eq. (8) can be displayed in the following form:

$$Q_e = Q_r\left(\frac{A_e}{A_r}\right)\left(\frac{U_e}{U_r}\right)\left(\frac{P_e}{P_r}\right)\left(\frac{P_{w_e}}{P_{w_r}}\right)^{-1.4} \tag{12}$$

It should be mentioned that in the current paper the SPM is used for the production of *U* and discharge at referenced section is taken from the reported works in the literature (Fig. 3)..

The calculated stage-discharge curve based on Eq. (12) for two cross sections of rectangular and circular shapes in comparison to the Manning formula are shown in Fig. 4. The results indicate high agreement between the proposed method and the Manning formula..

3. Validation of the stage-discharge relationship

In order to evaluate the accuracy of the proposed method, the data which have been taken from the Severn River in UK, Cuenca and Tomebamba Rivers in Ecuador are used for comparison. Abril and Knight [3] by the use of depth-averaged velocity in a finite element numerical model estimated the discharge in natural rivers. In their method, it is required to calibrate the model for each river. Figs. 5(a), 6(a) and 7(a) show the cross sections of the Severn, Cuenca and Tomebamba Rivers, respectively. The observations for the stage-discharge curves for the Severn, Cuenca and Tomebamba Rivers, are shown in Figs. 5(b), 6(b) and 7(b), respectively. It should be noted that the side banks of the rivers are the extensions of the depicted side walls, which are not shown here. Now we would like to estimate the relationship between the stage and discharge using each single pair of observed data as a reference point. For this purpose, the parameters engaged in Eq. (8) should be evaluated using the cross section of the rivers....

Based on Eq. (12), the values of *A*, *P*, P_w and *U* should be calculated at each stage. For this purpose, the variation of each value for the whole range of stage varying between zero and H_{max} in a dimensionless form for three river sections of the Severn, Cuenca and Tomebamba are plotted in Fig. 8. The values of A_{max}, P_{max}, $(P_w)_{max}$ and U_{max} are the maximum values at H_{max} level. In the Severn River the maximum value of stage is H_{max}=9.0 m. The values of A_{max}, P_{max}, $(P_w)_{max}$ and U_{max} are 487.31 m^2, 284.22 m, 144.22 m and 44.41, respectively. Those quantities for the Cuenca River at maximum stage with H_{max}=5.36 m are as the followings: 175.4 m^2, 84.51 m, 45.37 m and 27.65, respectively. In the Tomebamba River at the maximum stage with H_{max}=4 m are as the followings: 87.91 m^2, 31.46 m, 58.96 m and 18.69, respectively. Now

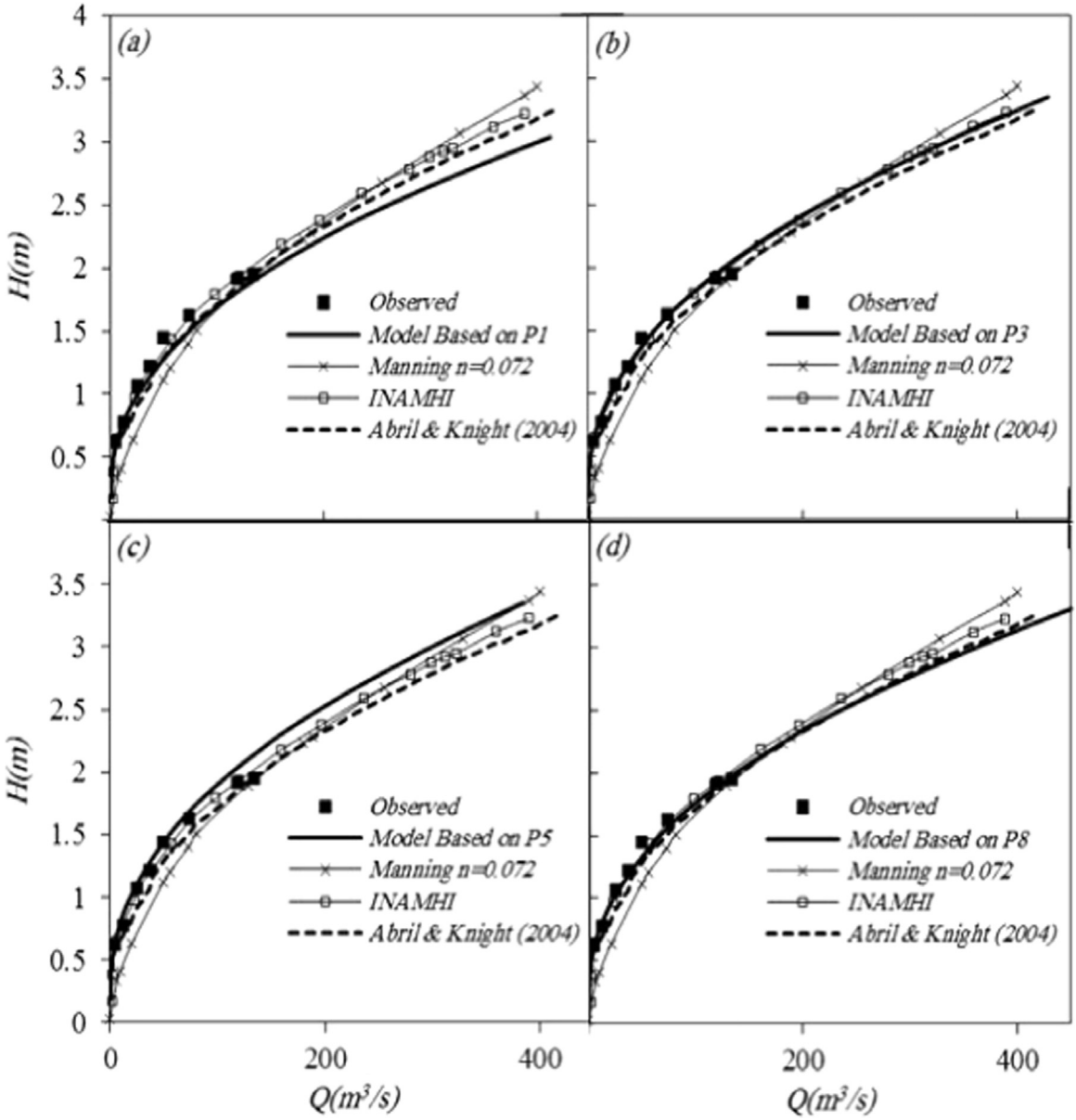

Fig. 10. Estimated stage-discharge curve in the Cuenca River by the use of different referenced levels (a) $P1$, (b) $P3$, (c) $P5$ and (d) $P8$.

at any arbitrary stage, A, P, P_w and U can be calculated. It should be noted that in Fig. 8(a) the diagrams for P and P_w are not coincided on each other and a small difference exists. However, the reason for having a recognizable difference between P and P_w in Fig. 8(b) and (c) is that both the Cuenca and the Tomebamba Rivers do not have a flood plain like the Severn River..

In Figs. 5(b) and 6(b), discharges at eight different levels which are shown by $P1$ to $P8$, have been taken from the Severn and Cuenca rivers, respectively. However, in the Tomebamba River the observational points are limited to four as shown in Fig. 7(b). Considering the discharge at each stage as a reference point on the stage-discharge curve by the use of Eq. (12), discharges at other levels can be calculated. Let's consider the Severn River first. Our intension is focused on the estimation of stage-discharge curve using a single pair of observed data at an arbitrary stage. The reference levels are selected from the observed data namely $P1$, $P3$, $P5$ and $P8$. For example, the reference data of A_r, P_r, $(P_w)_r$ and U_r for $P8$ are 425.57 m^2, 275.32 m, 139.72 m and 41.82, respectively. Now by the use of Fig. 8(a) at any arbitrary stage, discharge can be calculated. In Fig. 9(a), (b), (c) and (d) the stage-discharge curves for the Severn River based on referenced data of $P1$, $P3$, $P5$ and $P8$, are plotted, respectively. In order to evaluate the accuracy of the model, the other observed data except the one which is used for estimation the whole rating curve, are engaged in the statistical measures. The comparisons have also been made with the results obtained by Abril and Knight's [3] method. It should be emphasized that in the proposed method the only required data is related to one reference level and no additional calibration is needed. Although the geometry of the Severn River cross section with a flood plain is rather complicated, the results in Fig. 9 show a very good estimation of the stage-discharge curves for all of the referenced sections..

Fig. 10(a), (b), (c), (d) shows the estimated stage-discharge curves obtained from the presented model considering the flow levels at $P1$, $P3$, $P5$ and $P8$ as the reference discharge in Cuenca River, respectively. Moreover, the obtained results from Abril and Knight [3] with a Manning coefficient of n=0.072 which is mentioned in [3] and the data taken based on the average of two years field observations by the Hydrologic Institute of Ecuador (INAMHI) are compared with each

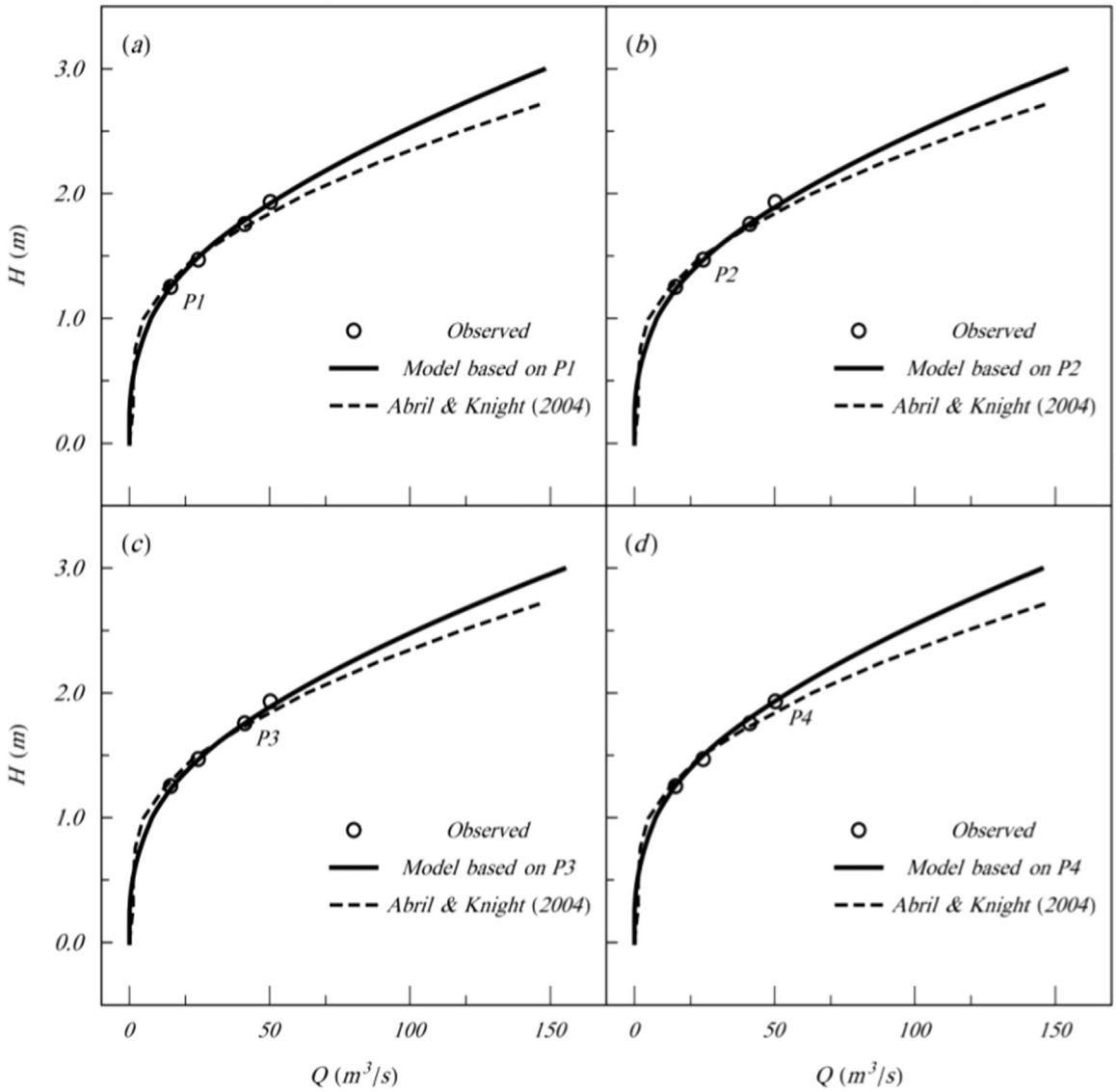

Fig. 11. Estimated stage-discharge curve in the Tomebamba River by the use of different referenced levels (a) *P*1, (b) *P*2, (c) *P*3 and (d) *P*4.

other. According to Fig. 10, it can be observed that the accuracy of the model when the field data are taken from the levels corresponding to *P*3 and *P*8 are higher than those which are taken from the levels corresponding to *P*1 and *P*5. The important point in the discrepancy of the results are related to the way of data acquisition and the accuracy of data collection..

The estimated rating curves for all of the observed data for the Tomebamba River are shown in Fig. 11. A comparison between the porposed model and Abril and Knight model, limitted to the range of observed data, indicates a good consistensy between the two methods. However, for larger dishcharges where no observed data is available, the two methods are diverging from each other..

In order to have more clarifications on the proposed model performance, some of the statistical measures including the percentage of relative error (*Error* %) and the root mean square error (*RMSE*) calculated based on the estimated discharge Q_e and the observed data Q_o are calculated as the following:

$$Error\% = \frac{Q_o - Q_e}{Q_o} \times 100 \tag{13}$$

$$RMSE\ (m^3/s) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Q_{o,i} - Q_{e,i})^2} \tag{14}$$

In order to compare the statistical quantities based on the estimated values using the proposed model and the observed data at different referenced levels in the Severn, Cuenca and Tomebamba rivers a bar chart is depicted in Fig. 13. It can be seen that the maximum error in the Severn, Cuenca and Tomebamba rivers are limited to 3%, 6% and 3%, respectively. The maximum value of *RMSE* in the Severn, Cuenca and Tomebamba rivers are less than 6, 6 and 1 m^3/s, respectively. In Fig. 12 an overall comparison between the estimated and observed results is plotted. All the percentage of errors are laid between the lines of ±5% error. According to the results, it can be said that the performance of the proposed model in setting up the rating curves in natural rivers is very good...

4. Conclusions

Stage-discharge curves are an essential part of surface hydrology, hydraulics, river sediments and open channel computations. The importance of stage-discharge relationship in natural rivers and the

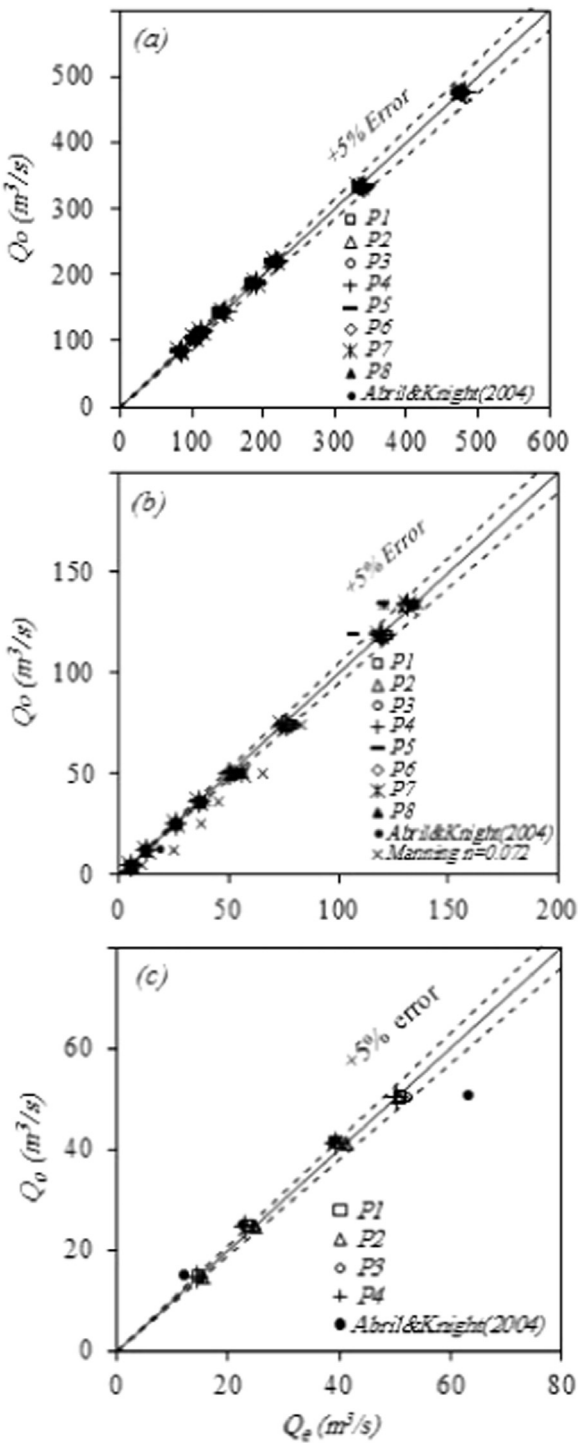

Fig. 12. A comparison between the estimated and observed discharges by Abril and Knight with the proposed model at different reference levels in rivers: (a) Severn, (b) Cuenca and (c) Tomebamba.

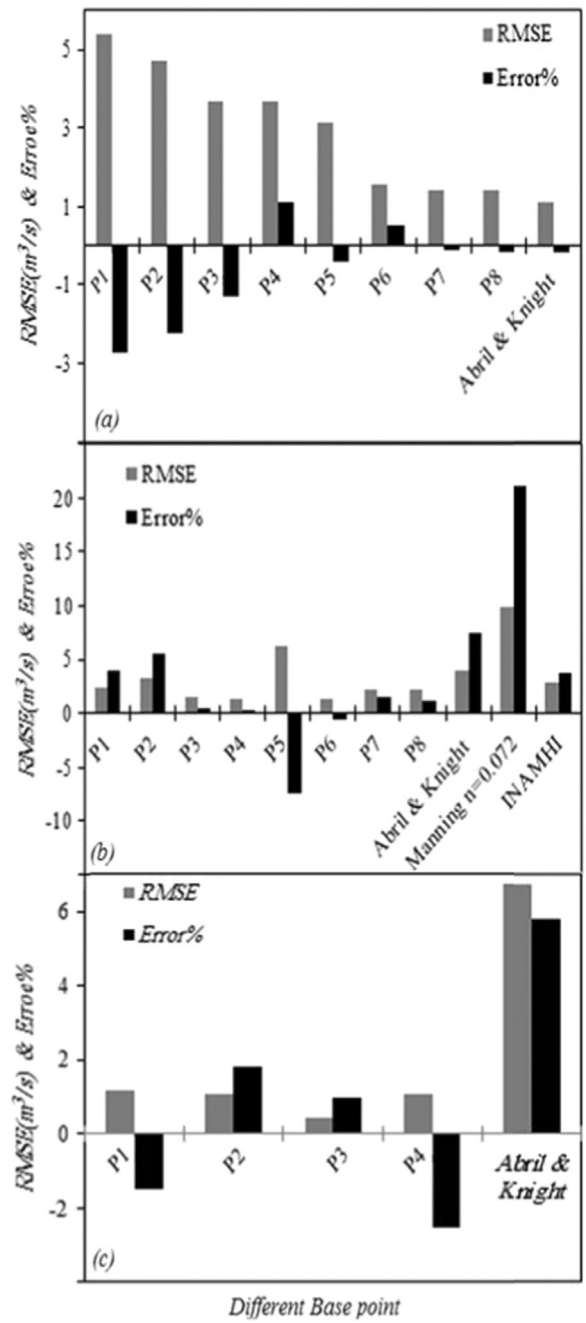

Fig. 13. A comparison between Error% and RMSE obtained from Abril and Knight with the proposed model at different reference levels in rivers: (a) Severn (b), Cuenca and (c) Tomebamba.

difficulties in setting up a relationship, have shown the necessity to implement simpler techniques in obtaining the rating curve relationships. In the present paper a new, practical, rapid and low cost technique to estimate the stage-discharge relationship based on single pair of stage-discharge is introduced. In order to verify the presented relationship, the observed data taken from the Severn River in UK, the Cuenca and Tomebamba rivers in Ecuador are used for comparison. The results have shown the high feasibility of the introduced relationship in estimation of the stage-discharge curves with very low amount of error. The statistical analysis have shown a very low value of error and RMSE especially when the referenced sections are taken from the higher stages of the river.

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