

# Exact analytical solutions for ADAFs

Asiyeh Habibi,<sup>1</sup> Shahram Abbassi<sup>1,2★</sup> and Mohsen Shadmehri<sup>3</sup>

<sup>1</sup>*Department of Physics, School of Sciences, Ferdowsi University of Mashhad, Mashhad 91775-1436, Iran*

<sup>2</sup>*School of Astronomy, Institute for Research in Fundamental Sciences (IPM), PO Box 19395-5531 Tehran, Iran*

<sup>3</sup>*Department of Physics, Faculty of Sciences, Golestan University, Gorgan 49138-15739, Iran*

Accepted 2016 October 29. Received 2016 October 29; in original form 2016 August 8

## ABSTRACT

We obtain two-dimensional exact analytic solutions for the structure of the hot accretion flows without wind. We assume that the only non-zero component of the stress tensor is  $T_{r\phi}$ . Furthermore, we assume that the value of viscosity coefficient  $\alpha$  varies with  $\theta$ . We find radially self-similar solutions and compare them with the numerical and the analytical solutions already studied in the literature. The no-wind solution obtained in this paper may be applied to the nuclei of some cool-core clusters.

**Key words:** black hole physics – hydrodynamics.

## 1 INTRODUCTION

Hot accretion flows such as advection-dominated accretion flow (ADAF; Narayan & Yi 1994, 1995a,b, hereafter NY94, NY95a,b; Abramowicz et al. 1995) are of great interest because they are likely operating in low-luminosity active galactic nuclei, which are the majority of galaxies at least in the nearby universe, and the hard/quiescent states of black hole X-ray binaries (see Yuan & Narayan 2014 for the latest review). Many numerical simulations have been carried out to study the structure of hot accretion flows (e.g. Igumenshchev & Abramowicz 1999, 2000; Stone, Pringle & Begelman 1999, Stone & Pringle 2001; Machida, Matsumoto & Mineshige 2001; Hawley & Balbus 2002; Pang et al. 2011; Yuan, Wu & Bu 2012a).

One of the most important progresses in this field is the discovery of strong wind launched from the accretion flow by numerical simulations (Narayan et al. 2012; Yuan, Bu & Wu 2012b; Bu et al. 2013, 2016a,b; Li, Morris & Baganoff 2013; Sadowski & Narayan 2016). This result is confirmed by the three million seconds *Chandra* observations of the accretion flow around the supermassive black hole in the Galactic Center, combined with the modelling to the detected iron emission lines (Wang et al. 2013). Begelman (2012) and Gu (2015) analytically address the question why wind exists in hot accretion flow.

Despite the existence of wind found by simulations of hot accretion flow, observations of the nuclei in the centres of cool-core clusters which are hot accretion flows show that outflow may be weak (Hlavecek-Larrondo & Fabian 2011). Allen et al. (2006) studied eight massive nearby elliptical galaxies where the gas properties close to the Bondi radius can be observed or reasonably extrapolated. In all the sources, the Bondi accretion rate  $\dot{M}_B$  can be determined by the gas properties at the Bondi radius. The jet power

$P_j$  in these sources can also be determined by observations of bubbles inflated by the jet in the surrounding gas. In this case, one may write  $P_j = \eta \dot{M}_B c^2$ , where  $\eta$  is the jet production efficiency. It is found that  $\eta \simeq 2$  per cent. This is rather a large efficiency. On the other hand, it is required that almost all the gas captured at the Bondi radius by the black hole needs to go into the black hole. There is almost no gas lost when the gas goes towards the black hole. Based on the above observations, it is still necessary to find hot accretion flow solution with no wind. Many authors have found one-dimensional (e.g. NY94; Zhang & Dai 2008; Bu, Yuan & Xie 2009) or two-dimensional (e.g. NY95a; Xue & Wang 2005; Ghosh & Mukhopadhyay 2009; Jiao & Wu 2011; Shadmehri 2014; Gu 2015; Zeraatgari & Abbassi 2015) analytical solutions for hot accretion flow by assuming radial self-similarity. In all the analytical solutions mentioned here, the authors assume that the viscosity coefficient  $\alpha$  is a constant. However, three-dimensional numerical simulations of hot accretion flow show that  $\alpha$  is not a constant, but varies with  $\theta$  (Penna et al. 2013).

By adopting modified  $\alpha$ -prescription for viscosity, Zeraatgari & Abbassi (2015) improved solution presented by Gu (2015). In fact, they have assumed a specific latitudinal dependent form for the viscosity while keeping the  $\alpha$  parameter constant. In this case, the advection parameter  $f$  will also vary in the latitudinal direction. However, in this paper we present analytical solutions following the same methodology as described in Shadmehri (2014). In this paper, we construct a two-dimensional solution of hot accretion flow without wind. NY95a studied two-dimensional solutions of hot accretion flow without wind by assuming radial self-similarity. However, in their paper, the authors assume that the viscosity has nine components. However, numerical simulations show that the  $(r, \phi)$  component of viscosity dominates other components. Therefore, it is necessary to re-study the hot accretion flow structure without wind in which only the  $(r, \phi)$  component is presented. The solutions of hot accretion flows without wind in this paper may be applied to nuclei of cool-core clusters mentioned above.

★ E-mail: [abbassi@um.ac.ir](mailto:abbassi@um.ac.ir)

The outline of this paper is as follows. In Section 2, the governing equations in the spherical polar coordinates for a steady-state flow with zero latitudinal velocity are presented. In Section 3, we obtain a set of radially self-similar solutions and discuss about their physical behaviour. We then conclude with possible astrophysical implications.

## 2 BASIC EQUATIONS

We solve the standard hydrodynamic equations in the spherical polar coordinates  $(r, \theta, \phi)$ , where  $r$  is the radial distance,  $\theta$  and  $\phi$  are the polar and the azimuthal angles, respectively. In this study, the disc is taken to be axisymmetric (with respect to the rotational axis and the equatorial plane) and steady state (i.e.  $\partial/\partial\phi = \partial/\partial t = 0$ ). Besides, the gravitational potential of the central black hole is assumed to be Newtonian, i.e.  $\psi(r) = -GM/r$ . This assumption is convenient for finding radial self-similar solutions. Of course this assumption means that we are studying distances which are far enough from the centre. Otherwise one has to take into account the general relativistic effects. Therefore, it is worth mentioning that we are using non-relativistic hydrodynamics and gravity. Moreover, we assume that the flow is not self-gravitating. The basic equations are the continuity equation, Euler's equation and the energy equation. The continuity equation is

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho v_\theta) = 0, \quad (1)$$

three components of the Euler's equation are

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \left( \frac{\partial v_r}{\partial \theta} - v_\theta \right) - \frac{v_\phi^2}{r} = -\frac{GM}{r^2} - \frac{1}{\rho} \frac{\partial p}{\partial r}, \quad (2)$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right) - \frac{v_\phi^2}{r} \cot \theta = \frac{1}{\rho r} \frac{\partial p}{\partial \theta}, \quad (3)$$

$$v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r} (v_r + v_\theta \cot \theta) = \frac{1}{\rho r^3} \frac{\partial}{\partial r} (r^3 T_{r\phi}), \quad (4)$$

and the energy equation of the gas is given by

$$\rho \left( v_r \frac{\partial e}{\partial r} + \frac{v_\theta}{r} \frac{\partial e}{\partial \theta} \right) - \frac{p}{\rho} \left( v_r \frac{\partial \rho}{\partial r} + \frac{v_\theta}{r} \frac{\partial \rho}{\partial \theta} \right) = f_{T_{r\phi} r} \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right). \quad (5)$$

In above equations,  $\rho$  is the gas mass density;  $v_r$ ,  $v_\theta$  and  $v_\phi$  are the three components of velocity;  $p$  stands for the gas pressure;  $f$  refers to the advection parameter defined by NY94 and also  $e$  denotes the specific internal energy of the gas which can be expressed as

$$e = \frac{p}{\rho(\gamma - 1)}, \quad (6)$$

where  $\gamma \equiv c_p/c_v$  is the ratio of specific heats. As we mentioned before, it is assumed that only  $r\phi$  component of anomalous stress tensor is dominated. In this case, it is convenient to write  $T_{r\phi} = -\alpha p$ . Following Lovelace, Bisnovatyi-Kogan & Rothstein (2009), we adopt the modified  $\alpha$  description of viscosity as a function of  $\theta$ .

## 3 SELF-SIMILAR SOLUTIONS AND ANALYSIS

Following NY95a, we propose the following radially similar solutions

$$\rho(r, \theta) = r^{-3/2} \rho(\theta), \quad (7)$$

$$p(r, \theta) = r^{-5/2} GM p(\theta), \quad (8)$$

$$v_r(r, \theta) = \sqrt{\frac{GM}{r}} v_r(\theta), \quad (9)$$

$$v_\theta(r, \theta) = 0, \quad (10)$$

$$v_\phi(r, \theta) = \sqrt{\frac{GM}{r}} v_\phi(\theta). \quad (11)$$

Substituting the above self-similar solutions into equations (1)–(5), we obtain the following algebraic equations:

$$5p(\theta) + \rho(\theta)[v_r(\theta)^2 - 2 + 2v_\phi(\theta)^2] = 0, \quad (12)$$

$$\frac{dp(\theta)}{d\theta} = \rho(\theta)v_\phi(\theta)^2 \cot \theta, \quad (13)$$

$$\alpha(\theta)p(\theta) + \rho(\theta)v_r(\theta)v_\phi(\theta) = 0, \quad (14)$$

$$(3\gamma - 5)v_r(\theta) - 3\alpha(\theta)f(\gamma - 1) = 0. \quad (15)$$

From equation (14), we obtain

$$p(\theta) = -\frac{v_r(\theta)v_\phi(\theta)\rho(\theta)}{\alpha(\theta)}. \quad (16)$$

Also from equation (15), we have

$$v_r(\theta) = -\frac{\alpha}{\epsilon} v_\phi(\theta). \quad (17)$$

In the above equation,  $\epsilon$  is a constant defined by NY95a as follows

$$\epsilon = \frac{1}{f} \left( \frac{5/3 - \gamma}{\gamma - 1} \right). \quad (18)$$

Using these equations and from equation (12), the rotational velocity becomes

$$v_\phi(\theta) = \frac{\sqrt{2\epsilon}}{\sqrt{2\epsilon^2 + 5\epsilon + \alpha(\theta)^2}}. \quad (19)$$

Then, the radial velocity is obtained from equation (17), i.e.

$$v_r(\theta) = -\frac{\sqrt{2\alpha(\theta)}}{\sqrt{2\epsilon^2 + 5\epsilon + \alpha(\theta)^2}}. \quad (20)$$

Thus, from equation (13), one can easily obtain the density of the flow,

$$\rho(\theta) = \frac{\rho(\frac{\pi}{2}) \sin \theta^\epsilon (2\epsilon^2 + \alpha(\theta)^2 + 5\epsilon)}{2\epsilon^2 + \alpha(\frac{\pi}{2})^2 + 5\epsilon}. \quad (21)$$

In this case, using equation (16) we can obtain the pressure profile

$$p(\theta) = \frac{2\epsilon}{2\epsilon^2 + \alpha(\theta)^2 + 5\epsilon} \rho(\theta). \quad (22)$$

Interestingly, if the functional form of  $\alpha(\theta)$  is known, then equations (12)–(15) can be integrated. In other words, the functionality of  $\rho(\theta)$  can be exactly obtained for a given  $\alpha(\theta)$ .

Finally, the net mass accretion rate can be obtained from the volume integral of equation (1) as follows

$$\dot{M} = - \int 2\pi r^2 v_r(r, \theta) \rho(r, \theta) \sin \theta d\theta. \quad (23)$$

Also by putting the self-similar solution into the mass accretion rate, a dimensionless form is defined as

$$\dot{m} = - \int \rho v_r \sin \theta d\theta, \quad (24)$$

where  $\dot{m} = \dot{M}/(2\pi\sqrt{GM})$ . Since the radial self-similarity has been employed in this study, the radial dependence has been omitted in

the above equation. Then, pressure at the equatorial plane can be determined uniquely as

$$p\left(\frac{\pi}{2}\right) = \frac{\dot{m}}{I(\alpha, \epsilon)}, \quad (25)$$

where,  $I(\alpha, \epsilon)$  is defined as

$$I(\alpha, \epsilon) = \int_0^\pi \sin \theta^{\epsilon+1} \alpha(\theta) \sqrt{2\epsilon^2 + 5\epsilon + \alpha(\theta)^2} d\theta. \quad (26)$$

It is necessary to mention that using equation (22), one can find the integration constant  $\rho(\frac{\pi}{2})$  as

$$\rho\left(\frac{\pi}{2}\right) = \frac{2\epsilon^2 + 5\epsilon + \alpha(\frac{\pi}{2})^2}{2\epsilon} \frac{\dot{m}}{I(\alpha, \epsilon)}. \quad (27)$$

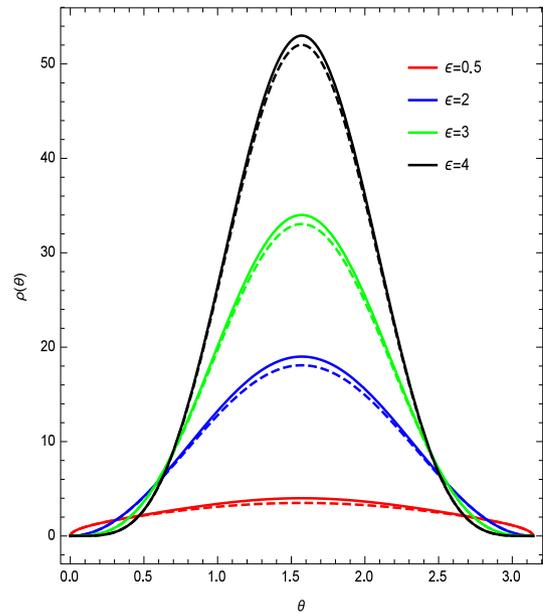
We can integrate equations (12)–(15) for a given  $\alpha(\theta)$ . However, we have found several functions for  $\alpha(\theta)$  for which this integral can be solved analytically. In the following equation, we introduce one of them which seems more realistic and discuss its physical implications:

$$\alpha(\theta) = \alpha_0 \sin(\theta).$$

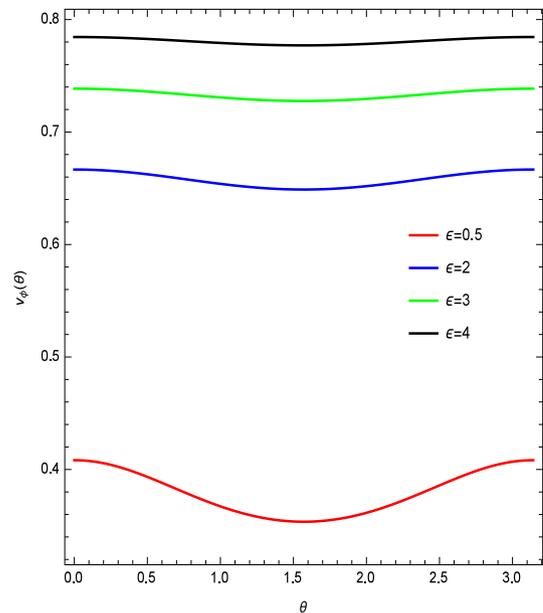
Just as an illustrative example, we have chosen  $\alpha = \alpha_0 \sin \theta$ , where  $\alpha_0$  is a constant. In fact, among our solutions, this form recovers the main features of the ADAFs presented in NY95a. On the other hand, this exact solution does not have the problems of the exact solutions introduced in Shadmehri (2014). This particular form of the dimensionless stress parameter  $\alpha(\theta)$  becomes zero at  $\theta = 0$  and reaches to a maximum at the equatorial plane of the disc. Obviously, there are more functional forms for this parameter with a similar behaviour as a function of the polar angle; however, this particular choice is the simplest case and it is also a reminiscent of the model studied by Lovelace et al. (2009), where the magneto-rotational instability turbulence plays a key role in the dynamics of the disc. We note that Lovelace et al. (2009) have been proposed dependence of the stress parameter on the vertical coordinate of a disc in a completely different system, i.e. protoplanetary disc. But it is a good motivation to speculate dependence of the stress parameter on the spatial coordinates in ADAFs, though our knowledge about source of this possible dependence is not adequate and it needs further investigation.

We have plotted the relevant quantities, i.e.  $\rho$ ,  $v_r$  and  $v_\phi$ , in Figs 1–3. In fact, in these figures we have chosen different values for advection parameter  $f$  and a fixed value  $\dot{m} = 1$ . Furthermore, in these figures we have set the adiabatic index  $\gamma$  to 1.5. Consequently, in order to check the model's behaviour for different values of  $\epsilon$ , we have adopted different values for advection parameter. Fig. 1 shows that smaller values for the advection parameter lead to denser disc at  $\theta = \pi/2$ .

The above-mentioned functionality for  $\alpha$  seems interesting in the sense that the density and velocity profiles are more or less similar to those obtained in Shadmehri (2014) for a constant  $\alpha$ . Also there are some interesting features for this kind of viscosity. It is clear from Fig. 2 that the azimuthal velocity has a minimum at the plane  $\theta = \pi/2$ . This is consistent with the fact that the viscosity is maximum at  $\theta = \pi/2$ . Also it is clear that for a fixed accretion rate, the average circular velocity increases when the advection parameter decreases. More interestingly, when the advection parameter is small, effect of viscosity on the azimuthal velocity becomes negligible. As it is clear from Fig. 2, for small advection parameter, the azimuthal velocity  $v_\phi$  does not change significantly with respect to the angle  $\theta$ . On the other hand for a larger  $f$ , there is an explicit minimum in  $v_\phi$  at  $\theta = \pi/2$ .



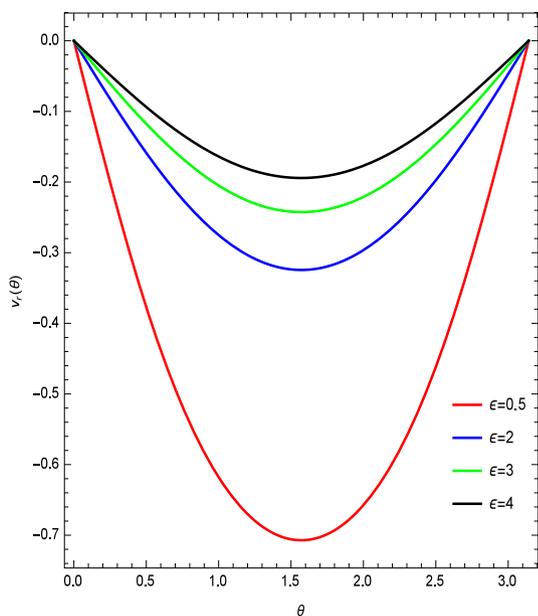
**Figure 1.** Density profile as a function of  $\theta$ . Solid curves are corresponding to  $\alpha(\theta) = \alpha_0 \sin \theta$ . Also dashed curves represent Shadmehri (2014) solutions. It is interesting that this choice for  $\alpha$  leads to density profile reminiscent to that of NY94.



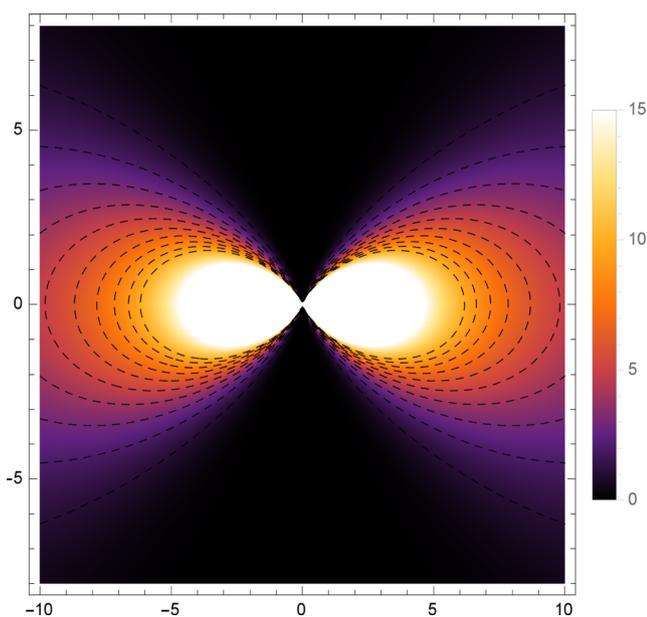
**Figure 2.** Behaviour of  $v_\phi$  as a function of  $\theta$  for different values of  $\epsilon$ .

Although the circular velocity for small advection parameter is almost constant with variations of  $\theta$ , the radial velocity  $v_r$  is modified significantly even for small advection parameter  $f$ . Fig. 3 shows that the average radial velocity increases when the advection parameter increases.

One may require more evidence from relevant hydrodynamic simulations or observations in order to justify the viability of this sinusoidal  $\alpha$ . As we have already shown, there is a broad variety of exact solutions. Also in order to provide a better understanding of our main exact solutions, we have plotted the iso-density profile for presented model in Fig. 4.



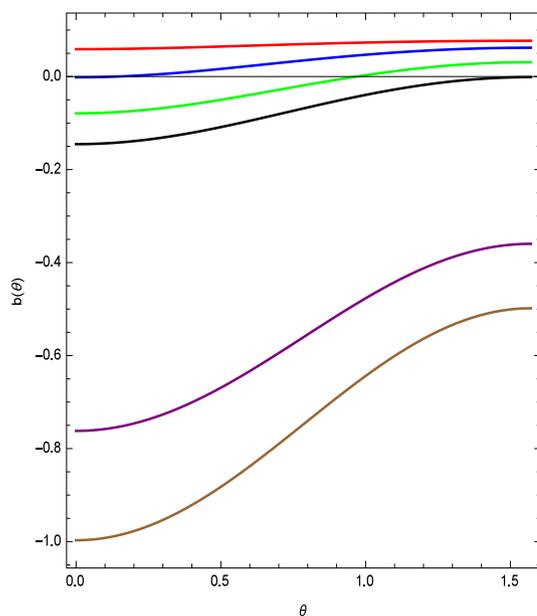
**Figure 3.** Behaviour of  $v_r$  as a function of  $\theta$  for different values of  $\epsilon$ .



**Figure 4.** Iso-density plot for different values of  $\rho$  when  $\alpha(\theta) = \alpha_0 \sin(\theta)$  (Section 3).

It is instructive to find the Bernoulli parameter ( $Be$ ) for our solution and explore the range of the advection parameter for which the Bernoulli parameter becomes positive. As it has been mentioned by NY95a, when  $Be$  parameter is positive, one may expect existence of the outflows in the system. Albeit it does not mean that if  $Be > 0$  then necessarily there is outflow. More specifically, it can be a necessary condition for the existence of outflow and not a sufficient one. Following NY95a, let us introduce the dimensionless parameter  $b$  as

$$b = \frac{Be}{\Omega_K r^2} = \frac{1}{2} v_r^2 + \frac{1}{2} (v_\phi \sin \theta)^2 - 1 + \frac{\gamma}{\gamma - 1} c_s^2, \quad (28)$$



**Figure 5.** Bernoulli parameter  $b$  with respect to  $\theta$  for different values of  $\epsilon$ .  $f$ , down to up in the plot, is chosen as 0.0033, 0.033, 0.33, 0.44, 0.66, 1.

where  $\Omega_K$  is the Keplerian angular velocity and it is defined as  $\Omega_K = \sqrt{\frac{MG}{r^3}}$ . By substituting our exact solution into this equation for a given  $\alpha(\theta)$ , we obtain

$$b(\theta) = \left( \frac{\epsilon}{\sqrt{2\epsilon^2 + 5\epsilon + \alpha(\theta)^2}} \right)^2 (\sin^2 \theta - 2 + 3f). \quad (29)$$

The sufficient condition for positive  $b > 0$  is  $f > 2/3 \simeq 0.66$ . It is surprising that this constraint is independent of the form of  $\alpha(\theta)$ . This result is in agreement with the solutions presented in Shadmehri (2014), where parameter  $b$  is obtained for a constant  $\alpha$ . It is also important to mention that for having a positive  $b$  the system should be highly advective. In Fig. 5, we have plotted  $b$  for different values of  $f$ . In fact, we have chosen the same values used in NY95a. It is clear that there is an obvious deviation between our analytic results and the numerical solutions of NY95a. The parameter  $b$  is an increasing function with respect to  $\theta$  and has its maximum at the plane  $\theta = \frac{\pi}{2}$ . On the other hand, in NY95a this parameter decreases as the angle  $\theta$  increases. The origin for this behaviour may be directly related to the behaviour of the velocity components. We recall again that, as expected, the azimuthal velocity has a minimum at the plane and the magnitude of the radial component has a maximum there. In fact, the maximum shear viscosity on this plane may cause a maximum radial velocity towards the centre. On the other hand, the slope of the radial component is much larger than the azimuthal case. Therefore, although  $v_\phi$  decreases with  $\theta$  and so does the Bernoulli parameter, the radial velocity increases more rapidly. Finally the overall behaviour is that  $b$  increases with  $\theta$ .

#### 4 DISCUSSION

Our goal is to provide some exact solutions for the vertical structure of the ADAFs. We assume that the only non-zero component of viscosity is  $T_{r\phi}$ . Moreover, the latitudinal velocity is assumed to be negligible which greatly simplifies the model. Then, we obtained a set of self-consistent radially similar solutions with constant accretion rate. This can also be interpreted as solutions without outflows.

Moreover, unlike analytical and semi-analytical solutions which have already presented, we assume that viscosity coefficient  $\alpha$  has a prescribed  $\theta$  dependence rather than being a constant. This assumption is consistent with numerical simulations (Penna et al. 2013). We showed that our solutions reveal most of the features of ADAF models. Also, these solutions do not have the previous problem in flattening of rotational velocity component  $v_\phi$  (Shadmehri 2014). Our solutions may be applied to the nuclei of some cool-core clusters where observations show that wind is weak. Our results are summarized as follows.

(1) We showed that for every functionality of  $\alpha(\theta)$ , mass density profile of ADAFs can be calculated exactly. However, the accretion rate cannot be obtained for arbitrary models. For illustration, we studied a toy model which is totally integrable. For this model, we found all relevant quantities and discussed their physical interpretation. Our solutions recover all general features of the ADAFs.

(2) We have generalized the results of the recent paper by Shadmehri (2014). Unlike analytical solutions presented by Shadmehri (2014), our solutions include structures with latitudinal dependent azimuthal velocity. Variation of this component with respect to  $\theta$  may provide more realistic situations to understand the physics of ADAFs.

(3) For our toy model, we calculated the Bernoulli parameter  $b$  (introduced by NY95a) and studied the conditions under which the Bernoulli parameter is positive. In this case, one may expect outflows in the system. We proved that, surprisingly, independent of the form of  $\alpha(\theta)$ , the Bernoulli parameter would be positive if  $f > 0.66$ . This result is consistent with that of Shadmehri (2014). Using the same advection parameters as in NY95a, we realized that  $b$  parameter decreases with  $\theta$ . This result is in a gross disagreement with NY95a in which the Bernoulli parameter increases with  $\theta$ . Albeit it is necessary to mention that our assumptions are different from NY95a and one may naturally expect such a deviation.

## ACKNOWLEDGEMENTS

Authors would like to thank De-Fu Bu and Mahmood Roshan for valuable and constructive comments on this paper. SA acknowledges support from the International Center for Theoretical Physics (ICTP) for a visit through the regular associateship scheme. The anonymous referee is thanked for his/her helpful comments.

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