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g-noncommuting graph of finite groups

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Abstract. Let $Z(G)$ be the center of a non-abelian group G and g be a fixed element of G. The g-noncommuting graph of G, denoted by Δ_G^g, is an undirected graph with vertex set $G \setminus Z(G)$ and two distinct vertices x and y join by an edge if $[x, y] \neq g$ and g^{-1}. In this talk, we survey some graph theoretical properties like connectivity and planarity.

1 Introduction

Let $Z(G)$ be the center of a group G. Associate a graph Γ_G to G as follows: Take $G \setminus Z(G)$ as vertex set of Γ_G and join two distinct vertices x and y whenever $xy \neq yx$. This graph is called the non-commuting graph of G. Now, we are going to consider the new generalization of non-commuting graph called g-noncommuting graph which is associated to a fixed element g of a group G given by Tolue et al. in [4] as the following.

Definition 1. For any non-abelian group G and fixed element g in G, the g-noncommuting graph of G is the graph with vertex set G and two distinct vertices x and y join by an edge if $[x, y] \neq g$ and g^{-1}.

In this article, we consider an induced subgraph of Γ_G^g whose vertices are all non-central elements of G. It is called the g-noncommuting graph of G and is denoted by Δ_G^g.

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* Speaker

It is clear that Δ_G^e coincides with the non-commuting graph.
Let $K(G) = \{[x, y]|x, y \in G\}$ be the set of all commutators of G and set $G' =< K(G) >$, where G' is the commutator subgroup of G. The g-noncommuting graph Δ_G^g is a complete graph if $g \notin K(G)$. Therefore, we always assume that g is a non-identity element, $g \in K(G)$ and G is a finite group.

2 Main Results

In this section, we may investigate some graph theoretical properties of Δ_G^g. Let us start with to mention some relations between the new graph Δ_G^g and commuting graph.

Lemma 1. The commuting graph of group G is a spanning subgraph of Δ_G^g.

Lemma 2. If $K(G) = \{e, g\}$ or $\{e, g, g^{-1}\}$ then Δ_G^g is equal to commuting graph.

In the following theorems, we determine connectivity and diameter of the g-noncommuting graph.

Theorem 1. $\Delta_{D_{2n}}^g$ is connected if and only if $n \neq 3, 4$ and 6.

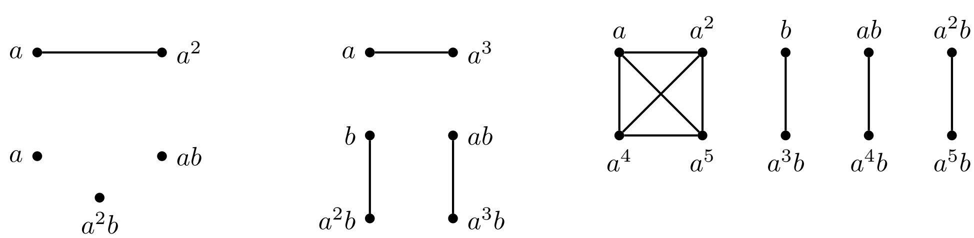

Figure 1: $\Delta_{D_6}^{a^2}$, $\Delta_{D_8}^{a^2}$ and $\Delta_{D_{12}}^{a^2}$

Theorem 2. Let g be a non-central element of G. Then

1. If $|g| \neq 3$, then $diam(\Delta_G^g) = 2$.

2. If $|g| = 3$, then the following cases occur:

   (a) Let $[C(g) : Z(G)] = 3$. If there exists a vertex x such that $d(x, g) > 2$, then $\frac{G}{Z(G)} \cong S_3$ and $\Delta_G^g = K_{2|Z(G)|} \cup 3K_{|Z(G)|}$, otherwise $diam(\Delta_G^g) \leq 4$.

   (b) Let $[C(g) : Z(G)] > 3$. Then $diam(\Delta_G^g) \leq 4$.

Theorem 3. Let g be a central element of G. If there are two vertices such that their distance is greater than 5, then Δ_G^g is disconnected and the following cases occur:

(i) If $|g| \geq 3$, then $\frac{G}{Z(G)} \cong \mathbb{Z}_3 \times \mathbb{Z}_3$ and $\Delta_G^g = 4K_{2|Z(G)|}$,

(ii) If $|g| = 2$, then $\frac{G}{Z(G)} \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ and $\Delta_G^g = 3K_{|Z(G)|}$.

Otherwise Δ_G^g is the connected graph and $diam(\Delta_G^g) = 2$.

Corollary 1. If G is a group of odd order and Δ_G^g is connected, then $diam(\Delta_G^g) = 2$.

Theorem 4. The girth of g-noncommuting graph is 3, unless $G \cong S_3, D_8$ or Q_8. Moreover, g-noncommuting graph of groups S_3, D_8 and Q_8 are forest.

In the following theorem, we classify all groups that their g-noncommuting graph is planar.

Theorem 5. Let G be a finite non-abelian group. Then Δ_G^g is planar if and only if G is isomorphic to one of the following groups:

(1) $S_3, D_8, Q_8, D_{10}, D_{12}, D_8 \times Z_2, Q_8 \times Z_2$,

(2) $< a, b : a^3 = b^4 = e, ab = a^{-1} > \cong Z_3 \rtimes Z_4$,

(3) $< a, b : a^4 = b^4 = e, ab = a^{-1} > \cong Z_4 \rtimes Z_4$,

(4) $< a, b : a^8 = b^2 = e, ab = a^{-3} > \cong Z_8 \rtimes Z_2$,

(5) $< a, b : a^4 = b^2 = (ab)^4 = [a^2, b] = e > \cong (Z_4 \times Z_2) \rtimes Z_2$,

(6) $< a, b, c : a^2 = b^2 = c^4 = [a, c] = [b, c] = e, [a, b] = c^2 > \cong (Z_4 \times Z_2) \rtimes Z_2$,

References

[1] Nasiri, M., Erfanian, A., Ganjali, M. and Jafarzadeh, A., g-noncommuting graph of some finite groups, *J. Prime Res. Math.* **12** (2016), 16–23.

[2] Nasiri, M., Erfanian, A., Ganjali, M. and Jafarzadeh, A., Isomorphic g-noncommuting graphs of finite groups, *Submitted*.

[3] Nasiri, M., Erfanian, A. and Mohammadian, A., Connectivity and Planarity of g-noncommuting graph of finite groups, *Submitted*.

[4] Tolue, B., Erfanian, A. and Jafarzadeh, A., A kind of non-commuting graph of finite groups, *J. Sci. Islam. Repub. Iran* **25(4)** (2014), 379–384.