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Detectability of Damage by a New Modal Sensitivity Function

at, K. N. Toosi University of Technology, Tehran, Iran

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Detectability of damage by a new modal sensitivity function

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Abstract

In vibration-based damage detection problems, sensitivity-based techniques are applicable and robust approaches for detecting and identifying damage in civil and mechanical engineering systems. A salient characteristic of a well-established sensitivity function is its ability to detect damage by itself without using any mathematical solution methods. This article, therefore, presents a novel modal sensitivity function based on a combination of fundamental dynamic equations. The proposed sensitivity function is associated with the mode shape that is established by combining the eigenvalue problem and orthogonality conditions. The main contribution of the proposed function is its sensitivity to damage and applying a few dynamic and physical characteristics for formulating the sensitivity matrix. A high compatibility with incomplete modal parameters is another innovation of the proposed sensitivity function that makes it as an applicable approach for using in vibration-based applications. To demonstrate the accuracy and capability of the proposed sensitivity function, the numerical model of the ASCE SHM benchmark structure is applied. Results show that the proposed function has a reliable sensitivity to damage and can detect damage without solving the damage equation by any mathematical solution methods.

Keywords: Damage detection; sensitivity analysis; modal parameters.

1. Introduction

Vibration-based damage identification has received considerable attention in the last two decades due to the importance of health and integrity of engineering systems. Structural damage will result in permanent changes in the distribution of structural stiffness, and these alterations can be detected through model-based or data-based approaches [1]. A large number of methods have been developed and applied to detect structural damage using dynamic characteristics. Since modal parameters only depend on the physical properties of the structures regardless of the excitation sources, the motivation of using model-based damage detection methods increases in civil, mechanical, and aerospace engineering communities.

A sensitivity-based damage detection method uses a sensitivity analysis of measurable outputs of the structure such as the modal parameters with respect to structural parameters to determine the rate of changes in the structure as a result of damage [2]. In this area, Zhoa and DeWolf [3] studied the sensitivity coefficients of the natural frequencies, mode shapes and modal flexibilities with respect to the elements of the stiffness matrix for the damage detection problem. Gomez and Silva [4] used a modal sensitivity approach through frequency sensitivity to damage and proposed a genetic algorithm to detect damage. Yang [5] presented a mixed sensitivity method that combines eigenvalue and flexibility sensitivity to identify structural damage. Naseralavi et al. [6] proposed an efficient sensitivity-based method for structural damage detection using natural frequencies. In another study, Esfandiari et al. [7] presented a new sensitivity function of modal frequency as a second order element level function of the stiffness reduction for diagnosing any number of localized damages.

Although the sensitivity of modal frequency has widely been used in many damage detection problems, changes in natural frequencies cannot provide spatial information about the structural damage [8]. Furthermore, damage may cause very small alterations in natural frequencies, particularly for larger structures; therefore, these small changes may not be detected due to the measurement or processing errors. In addition, variations in the mass of the structure or environmental temperatures may introduce uncertainties in the measured frequency changes. Thus, the sensitivity of mode shape may be a better tool in comparison with the sensitivity of modal frequency. Several methods have been developed for the calculation of the sensitivity of mode shape including the model method [9-11], the finite difference [12], Nelson's method [13-15], the modified modal method [16], the algebraic method [17, 18], and the adjoint method [19]. In the vibration-based damage detection problems, a well-established sensitivity function should be sensitive to damage in the sense that it is intuitively able to detect damage without applying any mathematical solution methods. In fact, the detectability of damage by the sensitivity function is a good option that can aid us to have an entire damage identification process and improve damage detection results.

Therefore, the main objective of this article is to propose a new modal sensitivity function based on incomplete modal parameters. The sensitivity function is concerned with the mode shape and its basic idea relies on the combination of the fundamental dynamic equations such as eigenvalue problem and orthogonality conditions. The major contribution of the proposed sensitivity function is its damage detectability without using any mathematical techniques. To verify the accuracy and performance of the proposed function, the numerical model of the ASCE benchmark structure is utilized. Results of the benchmark model demonstrate that the proposed sensitivity of mode shape has a good capability for detecting damage in the presence of incomplete modal parameters.

2. Theory

The basic premise of the proposed sensitivity function relies on using a combination of the fundamental dynamic equations such as the eigenvalue problem and orthogonality conditions. For an undamped dynamic system with N degrees of freedom (DOF), the eigenvalue problem is expressed as follows:

$$\left(\mathbf{K}-\lambda_i\mathbf{M}\right)\boldsymbol{\varphi}_i = 0 \tag{1}$$

where **M** and **K** represent the mass and stiffness matrices of the dynamic system, respectively. In this equation, $\boldsymbol{\varphi}_i$ and λ_i are the vector of modal displacements (mode shape) and natural frequency at the ith measured mode. Assume that the measured mode shapes are normalized; thus, the mass and stiffness orthogonality conditions are written in the following forms:

$$\boldsymbol{\varphi}_i^T\mathbf{M}\boldsymbol{\varphi}_i = 1 \tag{2}$$

$$\boldsymbol{\varphi}_i^T\mathbf{K}\boldsymbol{\varphi}_i = \lambda_i \tag{3}$$

In order to establish a sensitivity function, the first-order derivative of Eqs. (1)-(3) are derived with respect to the parameter p. For the eigenvalue problem, one can express:

$$\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\lambda_i}{\partial p}\mathbf{M}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i+\left(\mathbf{K}-\lambda_i\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}=0 \tag{4}$$

The first-order derivatives of the orthogonality conditions with respect to the parameter, p, are given by:

$$\left(\frac{\partial\boldsymbol{\varphi}_i^T}{\partial p}\right)\mathbf{M}\boldsymbol{\varphi}_i+\boldsymbol{\varphi}_i^T\frac{\partial\mathbf{M}}{\partial p}\boldsymbol{\varphi}_i+\boldsymbol{\varphi}_i^T\mathbf{M}\left(\frac{\partial\boldsymbol{\varphi}_i}{\partial p}\right)=0 \tag{5}$$

$$\left(\frac{\partial\boldsymbol{\varphi}_i^T}{\partial p}\right)\mathbf{K}\boldsymbol{\varphi}_i+\boldsymbol{\varphi}_i^T\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_i+\boldsymbol{\varphi}_i^T\mathbf{K}\left(\frac{\partial\boldsymbol{\varphi}_i}{\partial p}\right)=\frac{\partial\lambda_i}{\partial p} \tag{6}$$

The mass and stiffness matrices are symmetric, therefore, it is reasonable that $\mathbf{M}\boldsymbol{\varphi}_i=\boldsymbol{\varphi}_i^T\mathbf{M}$ and $\mathbf{K}\boldsymbol{\varphi}_i=\boldsymbol{\varphi}_i^T\mathbf{K}$; hence, the above equations can be compressed as:

$$\boldsymbol{\varphi}_i^T\frac{\partial\mathbf{M}}{\partial p}\boldsymbol{\varphi}_i+2\left(\boldsymbol{\varphi}_i^T\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}=0 \tag{7}$$

$$\boldsymbol{\varphi}_i^T\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_i+2\left(\boldsymbol{\varphi}_i^T\mathbf{K}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}-\frac{\partial\lambda_i}{\partial p}=0 \tag{8}$$

By expanding Eq. (4), one can obtain:

$$\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_i-\frac{\partial\lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_i-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\boldsymbol{\varphi}_i+\mathbf{K}\frac{\partial\boldsymbol{\varphi}_i}{\partial p}-\lambda_i\mathbf{M}\frac{\partial\boldsymbol{\varphi}_i}{\partial p}=0 \tag{9}$$

which can be rewritten as:

$$-\frac{\partial\lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_i+\left(\mathbf{K}-\lambda_i\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}=-\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i \tag{10}$$

or

$$\left(\mathbf{K}-\lambda_i\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}=\frac{\partial\lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_i-\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i \tag{11}$$

The ith natural frequency (λ_i) is multiplied by Eq. (7) and the new obtained equation is summed by Eq. (8) to yield:

$$\boldsymbol{\varphi}_i^T\frac{\partial\mathbf{M}}{\partial p}\boldsymbol{\varphi}_i\lambda_i+2\left(\boldsymbol{\varphi}_i^T\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}\lambda_i+\frac{\partial\lambda_i}{\partial p}-\boldsymbol{\varphi}_i^T\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_i-2\left(\boldsymbol{\varphi}_i^T\mathbf{K}\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p}=0 \tag{12}$$

which can be arranged as follows:

$$\frac{\partial\lambda_i}{\partial p}=\boldsymbol{\varphi}_i^T\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i+2\boldsymbol{\varphi}_i^T\left(\mathbf{K}-\mathbf{M}\lambda_i\right)\frac{\partial\boldsymbol{\varphi}_i}{\partial p} \tag{13}$$

It is evident that the expression regarding the $\left(\mathbf{K}-\mathbf{M}\lambda_i\right)\partial\boldsymbol{\varphi}_i/\partial p$ in Eq. (13) corresponds to the right-hand side of Eq. (11); thus, this equation can be rewritten as follows:

$$\frac{\partial\lambda_i}{\partial p}=\boldsymbol{\varphi}_i^T\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i+2\boldsymbol{\varphi}_i^T\left(\frac{\partial\lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_i-\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i\right) \tag{14}$$

By expanding this equation, obtains:

$$\frac{\partial \lambda_i}{\partial p}\left(1-2\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\mathbf{M}\boldsymbol{\varphi}_{\mathrm{i}}\right)+2\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial p}-\frac{\partial \mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_{\mathrm{i}}=\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial p}-\frac{\partial \mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_{\mathrm{i}} \tag{15}$$

Taking the mass orthogonality condition into account, Eq. (15) is rewritten as follows:

$$\frac{\partial \lambda_{\mathrm{i}}}{\partial p}=\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial p}-\frac{\partial \mathbf{M}}{\partial p}\lambda_{\mathrm{i}}\right)\boldsymbol{\varphi}_{\mathrm{i}} \tag{16}$$

In order to derive the new sensitivity function of mode shape, Eq. (16) is inserted into Eq. (11) that leads to:

$$\left(\mathbf{K}-\lambda_{\mathrm{i}}\mathbf{M}\right)\frac{\partial \boldsymbol{\varphi}_{\mathrm{i}}}{\partial p}=\left(-\frac{\partial \mathbf{K}}{\partial p}+\frac{\partial \mathbf{M}}{\partial p}\lambda_{\mathrm{i}}\right)\boldsymbol{\varphi}_{\mathrm{i}}+\left(\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial p}-\frac{\partial \mathbf{M}}{\partial p}\lambda_{\mathrm{i}}\right)\boldsymbol{\varphi}_{\mathrm{i}}\right)\mathbf{M}\boldsymbol{\varphi}_{\mathrm{i}} \tag{17}$$

Eventually, the first-order derivative of the mode shape as a new sensitivity function is proposed as follows:

$$\frac{\partial \boldsymbol{\varphi}_{\mathrm{i}}}{\partial p}=\left(\mathbf{K}-\lambda_{\mathrm{i}}\mathbf{M}\right)^{+}\left(-\frac{\partial \mathbf{K}}{\partial p}+\frac{\partial \mathbf{M}}{\partial p}\lambda_{\mathrm{i}}\right)\boldsymbol{\varphi}_{\mathrm{i}}+\left(\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial p}-\frac{\partial \mathbf{M}}{\partial p}\lambda_{\mathrm{i}}\right)\boldsymbol{\varphi}_{\mathrm{i}}\right)\mathbf{M}\boldsymbol{\varphi}_{\mathrm{i}} \tag{18}$$

where "+" denotes the pseudo inverse. In most cases, damage leads to a reduction in the stiffness and the mass of the structure often remains invariant. Therefore, the derivative of mass matrix with respect to the structural parameter, *p*, can be omitted from Eq. (18). On this basis, the proposed sensitivity function of the mode shape regarding the damage detection problem is rewritten as:

$$\frac{\partial \boldsymbol{\varphi}_{\mathrm{i}}}{\partial p}=\left(\mathbf{K}-\lambda_{\mathrm{i}}\mathbf{M}\right)^{+}\left(-\frac{\partial \mathbf{K}}{\partial p}\boldsymbol{\varphi}_{\mathrm{i}}+\left(\boldsymbol{\varphi}_{\mathrm{i}}{}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial p}\boldsymbol{\varphi}_{\mathrm{i}}\right)\mathbf{M}\boldsymbol{\varphi}_{\mathrm{i}}\right) \tag{19}$$

A great advantage of the proposed sensitivity function is that it only depends on initial dynamic characteristics such as the measured modal parameters and inherent physical properties of the structure. In Eq. (19), $\partial\mathbf{K}/\partial p$ refers to the derivative of the global stiffness matrix with respect to the structural parameter *p*, which can be obtained as:

$$\frac{\partial \mathbf{K}}{\partial p}=\sum_{\mathrm{t}=1}^{ne}\frac{\partial \mathbf{k}_{\mathrm{t}}}{\partial p} \tag{20}$$

where **k** denotes the local stiffness matrix and *ne* is the number of elements. It should be pointed out that the first-order derivative of the local and global stiffness matrices can be determined by the well-known finite difference method. In this study, the forward difference method is used and the first-order derivative of the stiffness matrix is computed as follows:

$$\frac{\partial \mathbf{K}}{\partial p}=\sum_{\mathrm{t}=1}^{ne}\frac{\mathbf{k}_{\mathrm{t}}\left(p+\Delta p\right)-\mathbf{k}_{\mathrm{t}}\left(p\right)}{\Delta p} \tag{21}$$

3. Numerical verifications

3.1 A simple mass-spring discrete system

In order to demonstrate the correctness and capability of the proposed sensitivity function for the damage detectability, particularly the identification of damage location and estimation of damage severity, a simple mass-spring discrete system is modelled as shown in Fig. 1. This model consists of 6 degrees of freedom (DOFs) as a simulation of 6-story shear building model. The initial inherent physical properties of the system include 1500 KN/m stiffness and 10000 Kg mass at each DOF.

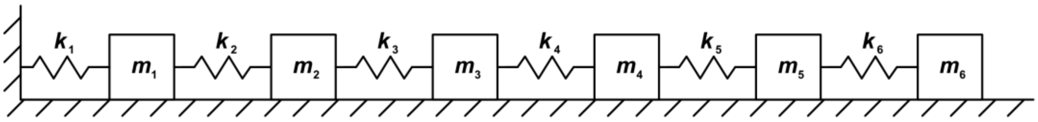

Figure 1. The 6-DOF mass and spring system

In this model, damage is simply simulated as loss of spring stiffness. Two different damage scenarios are listed in Table 2. The eigenvalue problem is applied to determine the modal parameters of the system. Once the modal parameters and inherent physical properties of the system have been obtained, the matrices of modal sensitivity are determined using complete and incomplete modal parameters

Table 1. The damage patterns in the numerical mass-spring system

Case No.	DOF No.					
	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
1	0.2	0.3	0.4	0.5	0.6	0.7
2	0.2	0.7	0.9	-	-	-

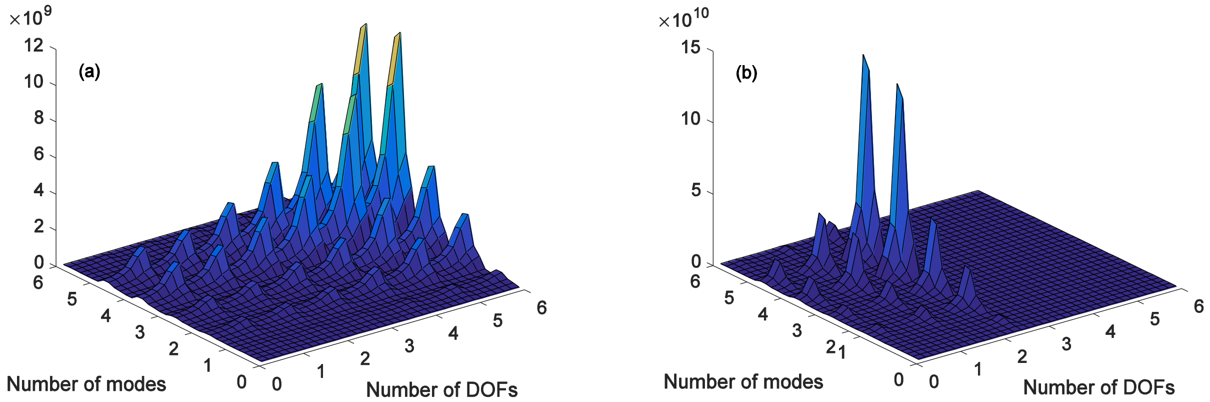

Figure 2. Damage detectability by the proposed sensitivity function: (a) case #1, (b) case #2

The results of damage detectability by the proposed sensitivity function are illustrated in Figs. 2(a) and 2(b) for both damage scenarios. As Fig. 2(a) indicates, the amounts of the sensitivity matrix at the 6^{th} DOF are more than the other DOFs. In this figure, the values of sensitivity function increase from the first DOF to the sixth DOF. This is a reasonable result since the extent of damage increases as presented in Table 1. On this basis, the damage severity at the first DOF includes the lowest level of the stiffness reduction factor. For This case one can simply observe at the first DOF in Fig. 2(a). Furthermore, Fig. 2(b) demonstrates the results of damage detectability in the damage case #2. It can be discerned that most of the changes in the values of sensitivity function have occurred at the first, second, and third DOFs. This observation confirms that the proposed sensitivity function is able to identify the location of damage and estimate the extent of damage. In Fig. 2(b), the amounts of sensitivity function at the third DOF is the maximum value in the sense that this area shows the highest level of damage.

3.2 The ASCE benchmark structure: Phase I

In the previous section, the capability of the proposed sensitivity of mode shape has been examined for the damage localization and damage quantification processes. In this section, one attempts to perceive whether this sensitivity function is capable of detecting early damage. Accordingly, ASCE benchmark structure is used to evaluate the sensitivity of mode shape in global damage detection. The numerical study of ASCE benchmark model, known as Phase I, provides several simulated data with diverse damage patterns for the applications of structural health monitoring. The structure is a 4-story, 2-bay by 2-bay steel frame, which has a 2.5 m by 2.5 m plan and is 3.6 m tall [20]. The members are hot rolled grade 300 W steel with a nominal yield stress 300 MPa. The columns are B100×9 sections and the floor beams are S75×11 sections. There is one floor slab per bay per floor. Two finite element models based on this structure were developed to generate the simulated response data such as a 12-DOF and a 120-DOF models. Fig.3 shows the simulated model of ASCE benchmark structure.

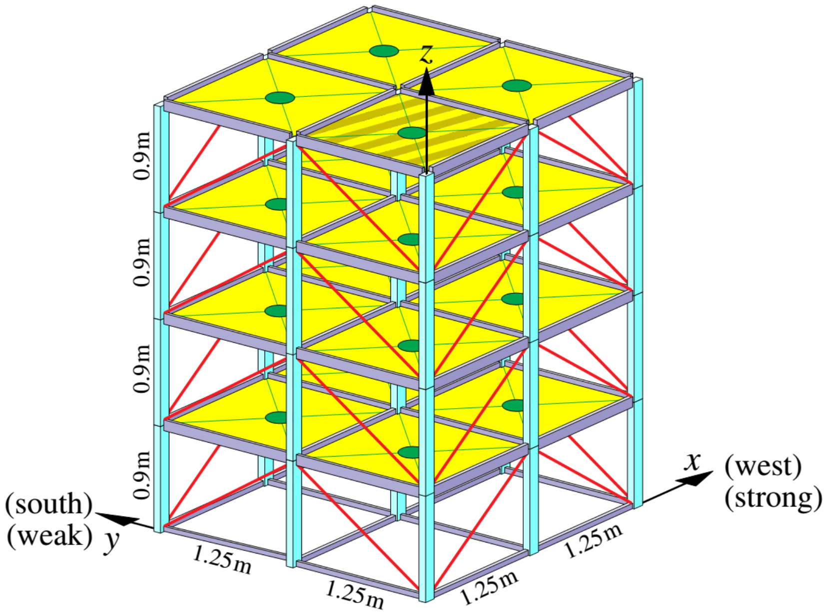

Figure 3. Diagram of analytical model with strong and weak directions

The numerical model of the structure involves six damage patterns as reductions of the model stiffness by removing braces. The damage patterns were not intended to directly represent the complexity of damage mechanisms; however, they provide proper capability to examine the various damage detection methods. Table 2 presents the first four damage patterns of the structure, which are utilized in this article.

Table 2. The damage patterns in the ASCE benchmark structure

Case No.	Condition	Description
1	Damaged	Removing all braces from the first floor
2	Damaged	Removing all braces from the first and third floors
3	Damaged	Removing one brace from the first floor
4	Damaged	Removing one brace from the first floor and one brace from the third floor

In order to establish the sensitivity matrix of mode shape, one needs to make the mass and stiffness matrices of the structure. Although, the main goal of the numerical ASCE benchmark structure is to simulate acceleration time histories at simulated sensor locations, it is possible to access the numerical mass and stiffness matrices of the 12-DOF model [20].

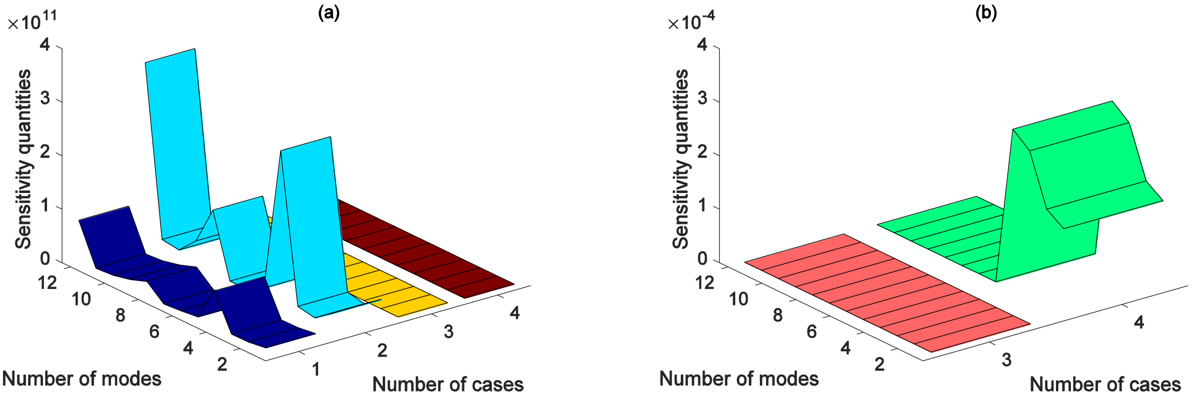

Figure 4. Damage detectability by the proposed sensitivity function in the ASCE benchmark structure: (a) all cases, (b) cases 3 and 4

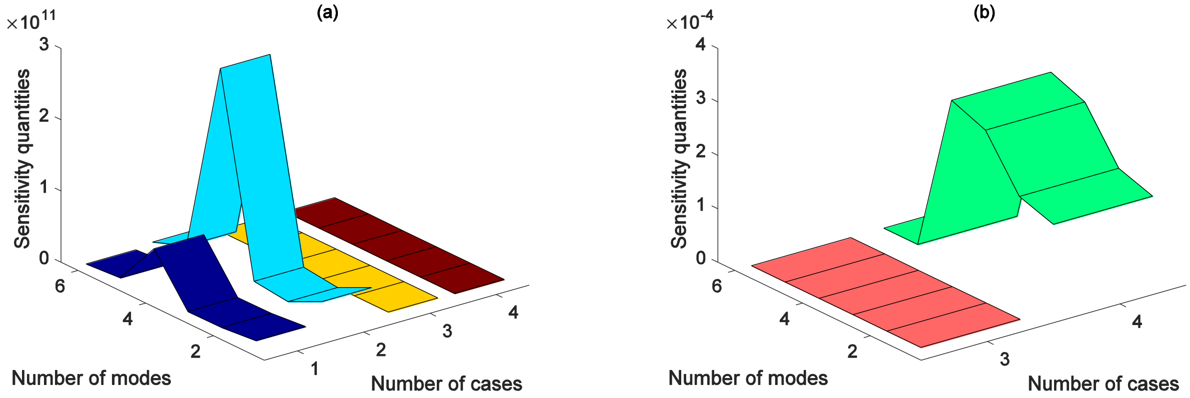

Figure 5. Damage detectability by the proposed sensitivity function in the ASCE benchmark structure using incomplete modes: (a) all cases, (b) cases 3 and 4

Figs. 4 and 5 show the quantities of the proposed sensitivity function in the presence of complete and incomplete modes, respectively. Fig. 4(a) illustrates the sensitivity values in all cases using the complete modes. From this figure, it can be inferred that the case 2 has the highest level of damage since the quantities of the sensitivity matrix in this case are more than the other cases, particularly the cases 3 and 4. The same conclusion can be obtained when the incomplete modal parameters are used to establish the sensitivity matrix as shown in Fig. 5(a). All of the obtained results are accurate because the level of damage in the case 2 is the highest one. The values of sensitivity in the case 1 are larger than the corresponding values in the cases 3 and 4 in Figs. 4(a) and 5(a). Another observation is related to evaluating the sensitivity quantities in the cases 3 and 4. As Figs. 4(b) and 5(b) appear, the amounts of the sensitivity matrix in the case 4 are more than the case 3 in both complete and incomplete measured modes. This is an accurate result on the basis of the level of damage in these cases as presented in Table 2. All of the results lead to the conclusion that the proposed sensitivity of mode shape has an acceptable and reliable capability for the recognition of the global condition of the structure.

4. Conclusion

The main aim of this article is to evaluate the capability of a new modal sensitivity function in the damage detection problem. This function is a new sensitivity of mode shape established by using the eigenvalue problem and orthogonality conditions. The main innovation of the proposed function is its sensitivity to damage and applying a few dynamic and physical characteristics for establishing the sensitivity matrix. On this basis, the capability of the proposed function in early damage detection, damage localization, and damage severity identification have been assessed by a 6-DOF mass-spring system and the ASCE model-scale benchmark structure. In the mass-spring system, it was seen that the proposed sensitivity function has a reliable ability to identify the location of damage and estimate the level of damage severity. Furthermore, the results of the benchmark structure demonstrated that this function is potentially able to detect early damage and can be used to evaluate the global condition of the structures.

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