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Early damage detection in structural health monitoring by a sensitivity method and DBSCAN clustering

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Abstract

Early damage detection is an important and initial step in structural health monitoring (SHM) that aims to evaluate the global state of a structure and finds whether damage is available throughout the structure. To do this, a new sensitivity function regarding modal strain energy (MSE) is proposed to use as a novel damage-sensitive feature. The sensitivity function is based on proposing a new equation of modal sensitivity and using the presented equation in the global formulation of MSE. In order to apply this sensitivity function as the damage-sensitive feature, the matrix of sensitivity of MSE is converted into a vector by the vectorization procedure. A new density-based clustering method named DBSCAN is also presented here to detect early damage using the vector of the sensitivity of MSE. To demonstrate the performance and reliability of the proposed methods, the ASCE benchmark structure (Phase I) is employed as a numerical example. Results show that the proposed sensitivity function is sensitive to damage and can be a reliable damage-sensitive feature in the applications of SHM. Furthermore, numerical results demonstrate that the proposed DBSCAN approach is a robust tool for detecting damage.

Keywords: Early damage detection; sensitivity analysis; modal strain energy; clustering; DBSCAN.

1. Introduction

Structural damage detection is an important process in most civil engineering infrastructures such as long span bridges, high-rise buildings, large space structures, offshore structures, large dams, and cultural heritage buildings due to their applicability to human life quality, their role in national economic and serviceability and importance of satisfactory structural performance. Material deterioration, geometric alterations, fault in boundary conditions and damage caused by catastrophic phenomena such as earthquake and destructive wind and thunders threaten their health and integrity. Therefore, there is a great necessity to evaluate health of such infrastructures to avoid any irrecoverable and dramatic events. Structural health monitoring (SHM) is an implementing process that aims to assess integrity and safety of the structures using vibration data [1]. The SHM systems record various structural responses (acceleration, strain, and frequency domain data), evaluate the conditions of the structures, and detect any probable structural damage [2-4]. On this basis, a SHM system performs a damage detection process in four main steps: (i) early damage detection, (ii) damage localization, (iii) damage quantification, and (iv) damage prognosis [5].

The general methodology for the damage identification problems is to extract meaningful features from the measured vibration data [3]. Monitoring the evolution of the extracted features over time allows civil engineers to evaluate the global state of the structure or detect any local damaged areas and their extents. The most important note is that the extracted features from the measured dynamic responses should be sensitive to structural damage and not to operational and environmental variability [6]. In addition to the damage-sensitive features, a SHM system needs robust and efficient methods to implement the damage detection problems. Nowadays, most of SHM systems exploit data-driven methods on the basis of statistical pattern recognition paradigm because these methods do not require finite element models of the structure and only use measured vibration responses or features extracted from them [7]. The statistical techniques used in the data-driven methods are generally categorized into: (i) unsupervised learning class, including outlier detection and clustering approaches and (ii) supervised learning class, containing classification and regression approaches [3, 8]. Among these classes, the algorithms related to the unsupervised learning class provide more feasible approaches since they only need features from the undamaged (known) condition of the structure to train a model, whereas the approaches associated with the supervised learning class require features of the undamaged and damaged (unknown) conditions to learn a model.

Despite the use of various types of damage-sensitive features in time, frequency, and time-frequency domains, the modal parameters are still applicable vibration characteristics in the SHM community since they are sensitive to damage and depend directly on the global and local stiffness of the structure as the main index of damage occurrence in the vibration-based applications. Among the features from the modal parameters, modal strain energy (MSE) can take into account as the most sensitive feature to damage because it is simultaneously composed of the stiffness and mode shape. Both of them are related to damage; hence, the sensitivity of MSE can be proposed as a novel damage-sensitive feature. The main objective of this article is to propose a sensitivity function of MSE and introduce a novel density-based clustering method presented for the first time in the SHM community. The accuracy and ability of the proposed methods are numerically demonstrated by the ASCE benchmark structure. Results show that the proposed sensitivity of MSE is sensitive to damage and the new clustering approach is a robust damage detection classifier.

2. Theory

2.1 A sensitivity formulation of modal strain energy

The modal strain energy (MSE) is an energy stored in a structure when the mode shapes are equivalent to the nodal displacements in each element of the structure [9, 10]. Mathematically speaking, the MSE is a combination of the stiffness matrix and mode shapes. The MSE in i^{th} vibration mode of a structure is expressed as follows:

$$\mathrm{MSE}_i = \frac{1}{2}\boldsymbol{\varphi}_i^T \mathbf{K} \boldsymbol{\varphi}_i \tag{1}$$

where $\mathbf{K}$ is the global stiffness matrix of the structure; $\boldsymbol{\varphi}_i$ represents i^{th} mode shape (the modal displacement). The sensitivity of MSE is based on computing the first derivative of the stiffness and mode shapes of each element with respect to a parameter. Thus, the simple formulation of the sensitivity of MSE in the i^{th} mode with respect to the parameter p is written as follows:

$$\frac{\partial \mathrm{MSE_i}}{\partial \mathrm{p}} = \frac{1}{2}\left(\left(\frac{\partial \boldsymbol{\varphi}_i}{\partial \mathrm{p}}\right)^{\mathrm{T}} \mathbf{K}\boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{K}\frac{\partial \boldsymbol{\varphi}_i}{\partial \mathrm{p}} + \boldsymbol{\varphi}_i^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial \mathrm{p}}\boldsymbol{\varphi}_i\right) \tag{2}$$

This sensitivity function can numerously be applied to many vibration-based applications such as system identification, sensitivity design analysis, finite element model updating, structural design optimization, and structural damage detection; however, determining the first derivative of the mode shape is a limitation of the sensitivity of MSE. The first derivative of mode shape cannot be directly determined due to the fact that it needs to overcome the singular problem. Additionally, it is intensively sensitive to any perturbation (noise) in the vibration response data. Therefore, the direct or indirect uses of the derivative of mode shape may not lead to a robust and compact sensitivity formulation for the modal strain energy.

The basic premise of the proposed sensitivity function is to eliminate the first derivative of mode shape and establish a formulation by combining the fundamental dynamic equations. For an undamped dynamic system with N degrees of freedom (DOF), the eigenvalue problem is expressed as follows:

$$\left(\mathbf{K} - \lambda_i \mathbf{M}\right)\boldsymbol{\varphi}_i = 0 \tag{3}$$

where $\mathbf{M}$ represents the mass matrix of the dynamic systems and λ_i is the natural frequency at the i^{th} measured mode. Assume that the measured mode shapes are normalized; thus, the mass and stiffness orthogonality conditions are written in the following forms:

$$\boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{M}\boldsymbol{\varphi}_i = 1 \tag{4}$$

$$\boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{K}\boldsymbol{\varphi}_i = \lambda_i \tag{5}$$

In order to establish a sensitivity function, the first-order derivative of Eqs. (3)-(5) are derived with respect to the parameter p. For the eigenvalue problem, one can express:

$$\left(\frac{\partial \mathbf{K}}{\partial p} - \frac{\partial \lambda_i}{\partial p}\mathbf{M} - \frac{\partial \mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i + \left(\mathbf{K} - \lambda_i\mathbf{M}\right)\frac{\partial \boldsymbol{\varphi}_i}{\partial p} = 0 \tag{6}$$

The first-order derivatives of the orthogonality conditions with respect to the parameter, p, are given by:

$$\left(\frac{\partial \boldsymbol{\varphi}_i^{\mathrm{T}}}{\partial p}\right)\mathbf{M}\boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^{\mathrm{T}}\frac{\partial \mathbf{M}}{\partial p}\boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{M}\left(\frac{\partial \boldsymbol{\varphi}_i}{\partial p}\right) = 0 \tag{7}$$

$$\left(\frac{\partial \boldsymbol{\varphi}_i^{\mathrm{T}}}{\partial p}\right)\mathbf{K}\boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial p}\boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{K}\left(\frac{\partial \boldsymbol{\varphi}_i}{\partial p}\right) = \frac{\partial \lambda_i}{\partial p} \tag{8}$$

The mass and stiffness matrices are symmetric, therefore, it is reasonable that $\mathbf{M}\boldsymbol{\varphi}_i=\boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{M}$ and $\mathbf{K}\boldsymbol{\varphi}_i=\boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{K}$; hence, the above equations can be compressed as follows:

$$\boldsymbol{\varphi}_i^{\mathrm{T}}\frac{\partial \mathbf{M}}{\partial p}\boldsymbol{\varphi}_i + 2\left(\boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{M}\right)\frac{\partial \boldsymbol{\varphi}_i}{\partial p} = 0 \tag{9}$$

$$\boldsymbol{\varphi}_i^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial p}\boldsymbol{\varphi}_i + 2\left(\boldsymbol{\varphi}_i^{\mathrm{T}}\mathbf{K}\right)\frac{\partial \boldsymbol{\varphi}_i}{\partial p} - \frac{\partial \lambda_i}{\partial p} = 0 \tag{10}$$

By expanding Eq. (6), one can obtain:

$$\frac{\partial \mathbf{K}}{\partial p}\boldsymbol{\varphi}_i - \frac{\partial \lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_i - \frac{\partial \mathbf{M}}{\partial p}\lambda_i\boldsymbol{\varphi}_i + \mathbf{K}\frac{\partial \boldsymbol{\varphi}_i}{\partial p} - \lambda_i\mathbf{M}\frac{\partial \boldsymbol{\varphi}_i}{\partial p} = 0 \tag{11}$$

which can be rewritten as:

$$-\frac{\partial \lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_i + \left(\mathbf{K} - \lambda_i\mathbf{M}\right)\frac{\partial \boldsymbol{\varphi}_i}{\partial p} = -\left(\frac{\partial \mathbf{K}}{\partial p} - \frac{\partial \mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_i \tag{12}$$

or

$$\left(\mathbf{K}-\lambda_i\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p}=\frac{\partial\lambda_i}{\partial p}\mathbf{M}\boldsymbol{\varphi}_\mathbf{i}-\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_\mathrm{i} \tag{13}$$

The i$^{\mathrm{th}}$ natural frequency (λ_i) is multiplied by Eq. (9) and the new obtained equation is summed by Eq. (10) to yield:

$$\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\frac{\partial\mathbf{M}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}\lambda_i+2\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p}\lambda_i+\frac{\partial\lambda_i}{\partial p}-\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}-2\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\mathbf{K}\right)\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p}=0 \tag{14}$$

which can be arranged as follows:

$$\frac{\partial\lambda_i}{\partial p}=\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_\mathrm{i}+2\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\mathbf{K}-\mathbf{M}\lambda_\mathrm{i}\right)\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p} \tag{15}$$

It is evident that the expression regarding the $\left(\mathbf{K}-\mathbf{M}\lambda_i\right)\partial\boldsymbol{\varphi}_i/\partial p$ in Eq. (15) corresponds to the right-hand side of Eq. (13); thus, this equation can be rewritten as follows:

$$\frac{\partial\lambda_\mathrm{i}}{\partial \mathrm{p}}=\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial \mathrm{p}}-\frac{\partial\mathbf{M}}{\partial \mathrm{p}}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i}+2\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\lambda_\mathrm{i}}{\partial \mathrm{p}}\mathbf{M}\boldsymbol{\varphi}_\mathrm{i}-\left(\frac{\partial\mathbf{K}}{\partial \mathrm{p}}-\frac{\partial\mathbf{M}}{\partial \mathrm{p}}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i}\right) \tag{16}$$

By expanding this equation, one can obtain:

$$\frac{\partial\lambda_i}{\partial p}\left(1-2\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\mathbf{M}\boldsymbol{\varphi}_\mathrm{i}\right)+2\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_\mathrm{i}=\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_i\right)\boldsymbol{\varphi}_\mathrm{i} \tag{17}$$

Taking the mass orthogonality condition into account, Eq. (17) is rewritten as follows:

$$\frac{\partial\lambda_\mathrm{i}}{\partial p}=\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i} \tag{18}$$

In order to derive the sensitivity function, Eq. (18) is inserted into Eq. (13) that leads to:

$$\left(\mathbf{K}-\lambda_\mathrm{i}\mathbf{M}\right)\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p}=\left(-\frac{\partial\mathbf{K}}{\partial p}+\frac{\partial\mathbf{M}}{\partial p}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i}+\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i}\right)\mathbf{M}\boldsymbol{\varphi}_\mathrm{i} \tag{19}$$

Eventually, the first-order derivative of the mode shape is derived as follows:

$$\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p}=\left(\mathbf{K}-\lambda_\mathrm{i}\mathbf{M}\right)^{+}\left(-\frac{\partial\mathbf{K}}{\partial p}+\frac{\partial\mathbf{M}}{\partial p}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i}+\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\left(\frac{\partial\mathbf{K}}{\partial p}-\frac{\partial\mathbf{M}}{\partial p}\lambda_\mathrm{i}\right)\boldsymbol{\varphi}_\mathrm{i}\right)\mathbf{M}\boldsymbol{\varphi}_\mathrm{i} \tag{20}$$

where "+" denotes the pseudo inverse. In most cases, damage leads to a reduction in the stiffness and the mass of structure often remains invariant. Therefore, the derivative of mass matrix with respect to the damaged parameter, *p*, can be omitted from Eq. (20). On this basis, the sensitivity function of the mode shape is rewritten as:

$$\frac{\partial\boldsymbol{\varphi}_\mathrm{i}}{\partial p}=\left(\mathbf{K}-\lambda_\mathrm{i}\mathbf{M}\right)^{+}\left(-\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}+\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}\right)\mathbf{M}\boldsymbol{\varphi}_\mathrm{i}\right) \tag{21}$$

Substituting Eq. (21) into Eq. (3), the proposed sensitivity of MSE is formulated without directly using the derivative of the mode shape in the following form:

$$\frac{\partial\mathrm{MSE}_\mathrm{i}}{\partial \mathrm{p}}=\frac{1}{2}\left(\begin{array}{l}\left(\left(\mathbf{K}-\lambda_\mathrm{i}\mathbf{M}\right)^{+}\left(-\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}+\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}\right)\mathbf{M}\boldsymbol{\varphi}_\mathrm{i}\right)\right)^\mathrm{T}\mathbf{K}\boldsymbol{\varphi}_\mathrm{i}+\\ \boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\mathbf{K}\left(\left(\mathbf{K}-\lambda_\mathrm{i}\mathbf{M}\right)^{+}\left(-\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}+\left(\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial p}\boldsymbol{\varphi}_\mathrm{i}\right)\mathbf{M}\boldsymbol{\varphi}_\mathrm{i}\right)\right)+\boldsymbol{\varphi}_\mathrm{i}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial \mathrm{p}}\boldsymbol{\varphi}_\mathrm{i}\end{array}\right) \tag{22}$$

A key benefit of the proposed sensitivity of MSE is that it only depends on the initial dynamic characteristics such as the measured modal parameters and inherent physical properties of the structure. In this study, the proposed sensitivity of MSE is used as a damage-sensitive feature. To do this, the *n*-by-*m* matrix of sensitivity transforms into a vector with *nm* elements so that the first column of the sensitivity matrix constitutes the first *nm* elements of the vector and this process continues.

2.2 A clustering approach: DBSCAN

Density-based spatial clustering of applications with noise (DBSCAN) is a new data clustering approach proposed by Ester et al. [11] in 1996. It is a density-based clustering that partitions a set of points into some space in such a way that similar points insert into a space named as nearby neighbours or core points, and outlier points make low-density regions. Consider a set of points in some space to be clustered. For the purpose of DBSCAN clustering, the samples are classified as the core points, density (reachable) points, and outliers, which are graphically depicted in Fig. 1. In the following, a concise description about the algorithm of DBSCAN is presented:

- The point *A* is a core point if at least the minimum numbers of points are within a distance of it. Those points are known to be directly reachable from *A*. No points are directly reachable from a non-core point.
- The points *B* and *C* are reachable from *A* if there is a path $A_1,...,A_n$ with $A_1=A$ and $A_n=B$ or $A_n=C$, where each A_{i+1} is directly reachable from A_i; therefore, all the points on the path should be core points with the possible exception of *A*.
- All points not reachable from any other point are outliers.

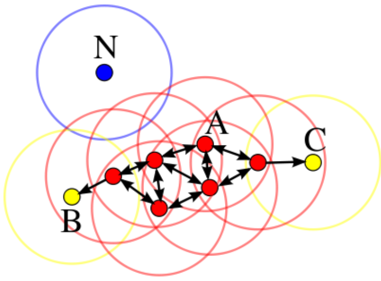


Figure 1. The simple schematic of DBSCAN method

In this figure, the minimum numbers of points is 4 including *A* (the red points), *B* (the yellow point), *C* (the yellow point), and *N* (the blue point). The point *A* and the other red points are core points because the area surrounding these points in a radius contains at least 4 points (including the point itself). Since they are all reachable from one another, they form a single cluster. The points *B* and *C* are not core points, but are reachable from *A* through other core points; thus, belong to the cluster as well. The point *N* is a noise point that is neither a core point nor density-reachable.

DBSCAN requires two parameters such as a radius distance (ε) and the minimum number of points required to form a dense region. This density-clustering approach starts with an arbitrary starting point that has not been observed. This point's ε-neighbourhood is retrieved, and if it contains sufficiently many points, a cluster is started. Otherwise, the point is labelled as noise. Note that this point might later be found in a sufficiently sized ε-environment of a different point; hence, it is made part of a cluster. If a point is found to be a dense part of a cluster, its ε-neighbourhood is also part of that cluster. Therefore, all points that are found within the ε-neighbourhood are added, as is their own ε-neighbourhood when they are also dense. This process continues until the density-connected cluster is completely found. In the following, a new unvisited point is retrieved and processed, leading to the discovery of a further cluster or noise. Further information and details regarding DBSCAN method can be found in [12-14].

3. Numerical verification: The ASCE benchmark structure

In the previous section, the capability and accuracy of the proposed methods have been examined for the early damage detection by the ASCE benchmark structure as shown in Fig. 1. This benchmark model provides several simulated data with diverse damage patterns for the applications of SHM. The structure is a 4-story, 2-bay by 2-bay steel frame, which has a 2.5 m by 2.5 m plan and is 3.6 m tall [15]. The members are hot rolled grade 300 W steel with a nominal yield stress 300 MPa. The columns are B100×9 sections and the floor beams are S75×11 sections. There is one floor slab per bay per floor. Two finite element models based on this structure were developed to generate the simulated response data such as 12-DOF and 120-DOF models.

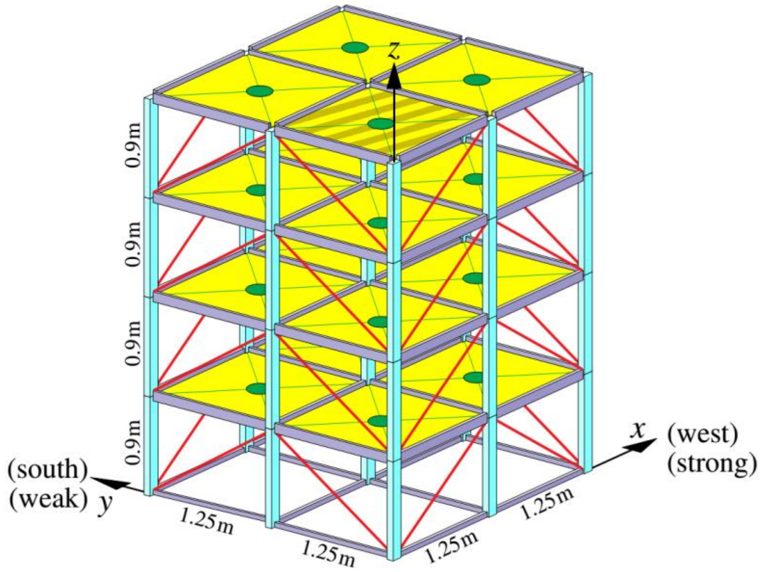


Figure 2. Diagram of analytical model with strong and weak directions

The numerical model of the structure involves six damage patterns as reductions in the model stiffness by removing braces. Table 1 presents the four damage cases (the patterns 3-6) of the structure since they incorporate the same damage pattern with different levels, which can be used to find whether the proposed methods are capable of estimating the level of damage severity.

Table 1. The damage patterns in the ASCE benchmark structure (Phase I)

Case No.	Condition	Pattern No.	Description
1	Damaged	3	No stiffness in one first floor brace
2	Damaged	4	No stiffness in one first and third floor braces
3	Damaged	5	Pattern 4 as well as the beam-column connection weakened
4	Damaged	6	2/3 stiffness in one first floor brace

Based on the numerical model of the ASCE benchmark model, the mass and stiffness matrices of the 12-DOFs model are used to constitute the undamaged (without removing the braces) and damaged conditions. The modal parameters of the frame can be simply determined by the standard eigenvalue problem. For a simulation of real application, the first six modes are applied to the sensitivity function of MSE. As a sample, Fig. 3 indicates the values of the sensitivity of MSE (S_{MSE}) in the damage pattern #3 for all the measured modes and the first six modes, respectively.

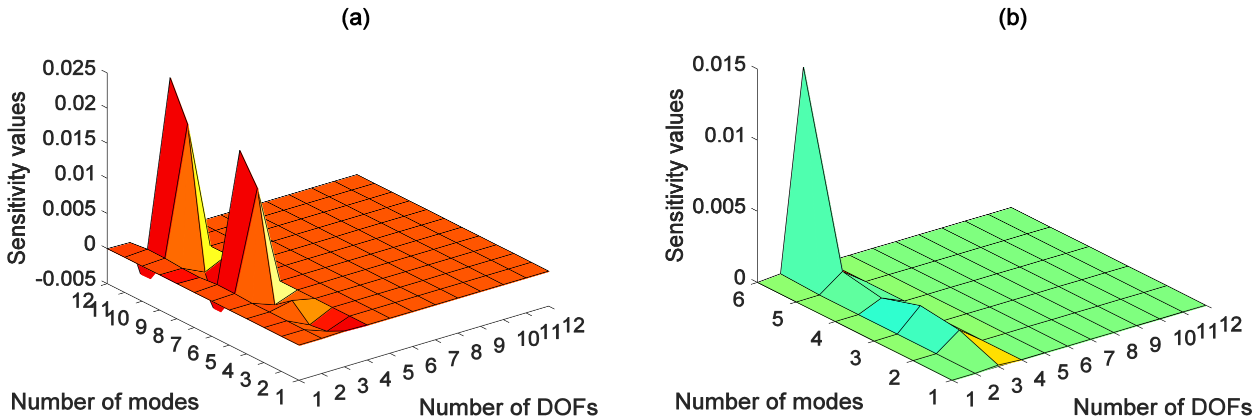
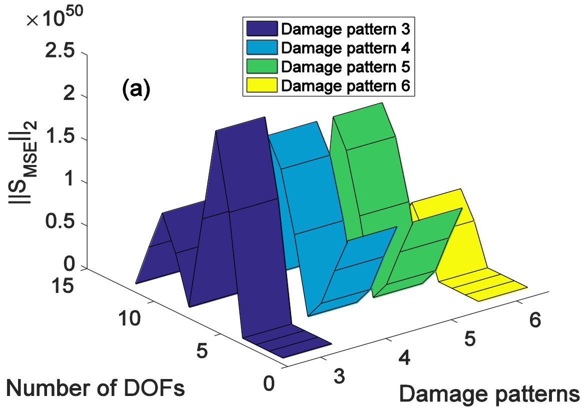


Figure 3. The values of the sensitivity of MSE (S_{MSE}) in the damage pattern #3: (a) all measured modes, (b) the first six modes

As can be observed in this figure, most of the variations in the sensitivity matrix have occurred at the first three DOFs, while the variations of the other DOFs are approximately zero. Before explaining the current observation, it is worthwhile remarking that the first three DOFs including the translational DOFs in X and Y directions and the rotational DOF belong to the first story. Therefore, the obtained results in Fig. 3 demonstrate that the structural damage as the stiffness reduction due to removing the braces have occurred at this story. This conclusion also confirms that the proposed sensitivity of MSE is sensitive to damage and possesses a reliable capability for detecting early damage in the structure.



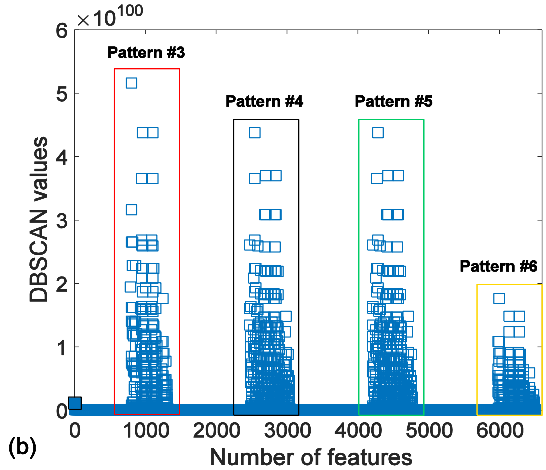


Figure 4. Early damage detection: (a) the l_2-norm of the S_{MSE} quantities, (b) DBSCAN values

The results of the early damage detection in the ASCE benchmark structure are shown in Fig. 4 in such a way that Fig. 4(a) presents the l_2-norm values of the vector of sensitivity function and Fig. 4(b) illustrates the quantities of DBSCAN for detecting the structural damage. In Fig. 4(a), the damage pattern #6 possesses the smallest norm value, whereas the corresponding norm quantity in the pattern #3 is more than the other patterns. Furthermore, the patterns 4 and 5 have the same norm values. These results imply that the level of damage depends directly on the norm of the sensitivity values so that as the level of damage increases, the norm value of the sensitivity function (S_{MSE}) increases. In other words, the highest level of damage has the largest norm value and vice versa. Fig. 4(b) also indicates the same conclusion. As can be seen from this figure, the quantities of DBSCAN in the pattern #3 are more than the other ones, whereas the pattern #6 shows the lowest level of damage. Eventually, it can be concluded that the proposed sensitivity function can be used as a reliable damage-sensitive feature and the proposed DBSCAN method is a robust clustering approach for early damage detection.

4. Conclusion

In this article, a new sensitivity function of modal strain energy and a novel density-based clustering technique have been proposed to detect the early damage and evaluate the global condition of the structures. The main idea behind the proposed sensitivity function is to avoid using the first derivative of mode shape used in the sensitivity of MSE and combining the fundamental dynamic equations for establishing a new and reliable sensitivity formulation associated with the MSE. This sensitivity function has been utilized as the damage-sensitive feature by converting the matrix of sensitivity to a vector through the procedure of vectorization. The clustering approach is known as density-based spatial clustering of applications with noise (DBSCAN), which has been presented for the first time in the SHM community. The ASCE benchmark structure has been applied to numerically verify the capability and accuracy of the proposed methods.

In this model, it was seen that the proposed sensitivity of MSE is able to detect damage on the basis of the sensitivity values; thus, it can be concluded that the proposed sensitivity function is sensitive to damage. In another result, the l_2-norm quantities of the vector of the MSE sensitivity as the new damage-sensitive feature could accurately estimate the level of damage. The results obtained from the clustering approach indicated that the DBSCAN method can detect the early damage that occurred in the structure and estimate the level of damage severity.

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