ENTROPY GENERATION ANALYSIS OF FREE CONVECTION FROM A CONSTANT TEMPERATURE VERTICAL PLATE USING SIMILARITY SOLUTION

by

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This paper presents a similarity solution analysis of entropy generation due to heat transfer and fluid flow which has been carried out for laminar free convection from a constant temperature vertical plate in an infinite quiescent fluid. The governing partial differential equations are transformed into a set of ordinary differential equations using similarity variables. An analytical expression, in terms of entropy generation, entropy generation number, Bejan number, and irreversibility distribution ratio are derived using velocity and temperature similarity (exact) solution. The rate of entropy generation is investigated and discussed in details. The results presented by the similarity solution are compared with integral method results. The similarity solution presents more appropriate and correct distribution of entropy generation in boundary layer because of more accuracy than integral method. It shows true position of maximum entropy generation and value of it. Also, the result shows that the exact solution minimizes the rate of total entropy generation in the boundary layer compared to integral solution. By introducing group parameter which is the ratio of friction entropy to thermal entropy generation, one can recognize that one of the thermal entropy and friction entropy generation is dominated in the boundary layer.

Key words: *entropy generation, laminar free convection, similarity solution, integral method, group parameter number*

Introduction

The optimal design criteria for thermodynamic systems can be achieved by analyzing entropy generation in the systems. Entropy generation minimization or thermodynamic optimization is not an old technique. It is also components of second law of thermodynamics analysis. Entropy generation has recently been the topic of great interest in fields such as heat exchangers, turbomachinery, electronic cooling, porous media, and combustion. The entropy generation is also a measure of the destruction of available work in a system. Therefore, it is important and useful focus on the irreversibility of heat transfer processes and understanding function of the entropy generation.

The usage of the method of the second law of thermodynamics as a measure of system performance was introduced by Bejan [1]. Then, Bejan [2] showed that the entropy generation

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for convective heat transfer is due to temperature gradient and viscous effect in the fluid. Many different studies have been published on the second law of thermodynamics and entropy generation. Bejan [3] showed that the natural shape of the velocity and temperature profiles of the 2-D turbulent jet is the one that minimizes the total entropy generation rate. Esfahani and Jafarian [4] studied steady-state boundary layer equations over a horizontal flat plate with a constant wall temperature by various solution methods and showed that the similarity exact solution similarity is the one that minimizes the rate of total entropy generation in the boundary layer. Then, Esfahani and Fayegh [5] compared the integral solution of 1-D heat conduction in a semi-infinite wall with constant temperature at its surface with the exact solution for three temperature profiles. They introduced an average normalized entropy generation, to find the error of the integral solution. They also concluded that the normalized entropy generation rate behaves similar to the average error. Andreozzi *et al.* [6] investigated numerical predictions of local and global entropy generation rates in natural convection in air in a vertical channel symmetrically heated at uniform heat flux. Ozkol *et al.* [7] studied entropy generation for laminar natural convection from vertical plate at a constant surface temperature using an approximate solution. They applied integral method and used a third order velocity profile and second order temperature profile to analysis of entropy generation. Weigand and Birkefeld [8] studied similarity solutions of the entropy transport equation. They investigated two cases as examples: laminar flow over a flat plate and Jeffery-Hamel flows. Mukhopadhyay [9] investigated entropy generation due to natural convection in an enclosure heated locally from below with two isoflux sources and showed minimum entropy generation rate was achieved for the same condition at which the minimum peak heater temperature was obtained. Sun [10] simulated mixed convection of air between vertical isothermal surfaces, numerically to determine the optimum spacing corresponding to the peak heat flux transferred from an array of isothermal, cooled parallel plates. Ibanez *et al.* [11] studied effects of slip flow on heat transfer and entropy generation by considering the conjugate heat transfer problem in microchannels, analytically. They derived an optimum value of the slip length that maximizes the heat transfer. Esfahani *et al.* [12] estimated the accuracy of a predicted velocity profile which can be gained from experimental results, in comparison with the exact ones by the methodology of entropy generation.

Some fundamental studies on entropy generation in convective heat transfer have been performed in recent years such as Amani and Nobari [13], Chen *et al.* [14], and Atashafrooz *et al.* [15]. Also, a useful and brief review on entropy generation in natural and mixed convection heat transfer for energy systems has been done by Oztop and Al-Salem [16]. They summarized the recent works on entropy generation in buoyancy-induced flows in cavity and channels. Many searches have been done in area of entropy generation analysis due to heat transfer and fluid friction, inclusive of the latest works of Esfahani and Modirkhazeni [17], Mahian *et al.* [18], Kargaran *et al.* [19], Mwesigye *et al.* [20], Ghasemi *et al.* [21], El-Maghlany *et al.* [22], and Atashafrooz *et al.* [15].

In the present work, analysis of entropy generation of boundary layer over a vertical flat plate with a constant wall temperature has been performed by similarity solution (exact solution) and results presented by it are compared with integral method results. The integral method results has been reproduced after Ozkol *et al.* [7]. This comparison shows that unlike integral method results, similarity solution results show that maximum entropy due to heat transfer generate at a small distance from the wall and not on the wall. While this maximum amount due to friction contribution occur on the wall, exactly. Also, the result shows that the exact solution minimizes the rate of total entropy generation in the boundary layer compared to integral solution. Dimensionless number group parameter (*GP*) number is introduced as an important parameter for irreversibility distribution ratio and contribution of entropy generation due to fluid friction *vs.* thermal entropy generation. Here, because of small *GP* number, it has shown negligible contribution of entropy generation due to flow friction compared to the contribution of entropy generation due to heat transfer and by increasing *GP* number this contribution is increased.

Governing equations

Natural convection is the motion that results from the interaction of gravity with density differences within a fluid. The vertical plate is one of the most important flow geometries in engineering processes. A constant temperature vertical flat plate immersed in an infinite fluid is illustrated in fig. 1. Because of plate is hot ($T_w > T_\infty$) so heat is transferred from the plate of length, L, to a fluid and flow rises parallel to the plate in the x-direction.

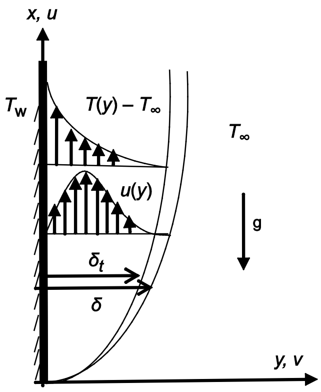

Figure 1. Laminar boundary layer on a hot vertical flat plate in free convection

The governing equations of this physical problem are the continuity, momentum and energy boundary layer equations that simplify for 2-D flow with fixed properties which is presented, respectively:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}+\mathrm{g}\beta(T-T_\infty) \tag{2}$$

$$u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}=\alpha\,\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Because of the buoyancy force in the momentum equation that is due to temperature gradient, flow and temperature fields are depended together. So, eqs. (1)-(3) are coupled, and must be solved simultaneously.

Similarity solution

According to the similarity solution for free convection, the velocity and temperature master profiles along the wall, with increasing x does not change. By choosing dimensionless similarity variable as [23]: $\eta=(y/x)\mathrm{Ra}_x^{1/4}$ and defining dimensionless velocity profile and dimensionless temperature profile, respectively:

$$F\,'(\eta,\,\mathrm{Pr})=-u\,\frac{x}{\alpha}\,\mathrm{Ra}_x^{-0,5}$$

$$\theta(\eta,\,\mathrm{Pr})=\frac{T-T_\infty}{T_\mathrm{w}-T_\infty}$$

where Ra_x is the dimensionless Rayleigh number that is expressed:

$$\mathrm{Ra}_x=\frac{\mathrm{g}\beta\Delta T x^3}{\alpha\nu},\quad \Delta T=T_\mathrm{w}-T_\infty$$

The momentum and energy similarity equations given below been procured, for more details refer to Bejan [23]:

$$\frac{3}{4}F\theta' = \theta'' \tag{4}$$

$$\frac{1}{\Pr}\left(\frac{1}{2}F'^{2} - \frac{3}{4}FF''\right) = -F''' + \theta \tag{5}$$

where Pr is the Prandtl number. Boundary conditions of eqs. (4) and (5) introduced:

$$y = 0,\ (\eta = 0) \quad \begin{cases} v = 0 & F = 0 \\ u = 0 & F' = 0 \\ T = T_{\mathrm{w}} & \theta = 1 \end{cases}$$

$$y \to \infty,\ (\eta = \infty) \quad \begin{cases} u = 0 & F' = 0 \\ T = T_{\infty} & \theta = 0 \end{cases}$$

Entropy generation

Convective heat transfer in a homogeneous medium is characterized by thermodynamic irreversibility due to two distinct effects: fluid friction and heat transfer in the direction of finite temperature gradients. Assuming a finite size open thermodynamic system (control volume) subjected to mass fluxes, energy transfer, and entropy transfer interactions that penetrate the fixed control surface formed by the dx-dy rectangle, and applying the second law of thermodynamics, the mixed entropy generation per unit time and per unit volume S'''_G [Wm^{-3}K^{-1}] can be estimated:

$$S'''_G = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\} \tag{6}$$

where u and v denote to the velocity components, T – the temperature, k and μ – the conductivity and dynamic viscosity of the fluid, respectively. Equation (6) confirms that locally the irreversibility is due to two effects, heat transfer, k, and fluid friction, μ. For a detailed derivation see Bejan [2, 24]. As seen, entropy generation depends on the determining of flow field velocity and temperature gradients. On the other hand, velocity and temperature profiles depend on the method of solution.

The mixed entropy generation rate per unit volume, S'''_G, can be written as a function of similarity variable in the form:

$$S'''_G = S'''_{G,\mathrm{ch}}\left\{\left[\frac{\frac{\eta^2}{2}}{8\,\mathrm{Ra}_x^{1/2}} + 1\right]\theta'^2 + \frac{\mathrm{Br}}{\Omega}\left[\frac{\left(2\frac{f'}{f''} - \eta\right)^2}{8\,\mathrm{Ra}_x^{1/2}} + 1\right]F''^2\right\} \tag{7a}$$

where

$$\mathrm{S}'''_{G,\mathrm{ch}} = \frac{\left(\dfrac{q}{T}\right)^2}{k}\mathrm{Ra}_x^{1/2}, \quad q = k\frac{\Delta T}{x}, \quad \mathrm{Br} = \frac{\mu U^2(x)}{k\Delta T}, \quad U(x) = \frac{\alpha}{x}\mathrm{Ra}_x^{1/2}, \quad \Omega = \frac{\Delta T}{T} \tag{7b}$$

In the previous equations, $\mathrm{S}'''_{G,\mathrm{ch}}$, q, T, and ΔT are the characteristic entropy generation, the heat flux, the local absolute temperature, and the reference temperature difference ($T_\mathrm{w} - T_\infty$), respectively. Brinkman number, Br, determining the relative importance between dissipation effects and fluid conduction affects, U and Ω are reference velocity in free convection and the dimensionless temperature difference, respectively.

The entropy generation in eq. (7a) can be simplified by scale analysis for flow over the plate with length, L. Because for most fluid flows in free convection around the vertical plate (the present problem), RaL $\gg$ O (1), and η, F', $F' \approx$ O (1), so:

$$\frac{\dfrac{\eta^2}{2}}{8\,\mathrm{Ra}_x^{1/2}} \ll 1 \quad \text{and} \quad \frac{\left(2\dfrac{F'}{F''} - \eta\right)^2}{8\,\mathrm{Ra}_x^{1/2}} \ll 1$$

Thus entropy generation can be simplified:

$$\mathrm{S}'''_G = \mathrm{S}'''_{G,\mathrm{ch}}(\theta'^2 + GP\,F''^2) \tag{7c}$$

where GP is the dimensionless ratio Br/Ω. This dimensionless number shows the relative importance of viscous effects on heat transfer irreversibility. The first term on the right hand side of equation is called thermal entropy generation and the second term is called friction entropy generation. On the other hand, velocity and temperature profiles depend on the method of solution. In the result section, similarity solution and integral methods are evaluated by entropy generation and the accuracy of them is discussed.

As seen in eq. (7c), entropy generation depends on GP number, $\mathrm{S}'''_{G,\mathrm{ch}}$, flow field velocity, F', and temperature distribution, θ'. According to eq. (7b) GP and $\mathrm{S}'''_{G,\mathrm{ch}}$ are depend on properties of flow (k, α, …), boundary condition, ΔT, and geometry, x. The velocity, F', and temperature gradients, θ', are function of Prandtl number, according to the set of eqs. (4) and (5). In here, we just investigate influence of geometry, x, and GP number changes.

It should be mentioned that all the results discussed here are for air near vertical flat plate with the following characteristics unless the characteristics expressed are in the text.

$$\mu = 1.86 \cdot 10^{-5}\ \mathrm{kg/ms}, \qquad \nu = 1.59 \cdot 10^{-5}\ \mathrm{m^2/s}$$

$$\mathrm{Pr} = 0.72,\ k = 2.6 \cdot 10^{-2}$$

$$T_\mathrm{w} = 320\ \mathrm{K},\ T_\infty = 300\ \mathrm{K}$$

Non-dimensional mixed entropy generation can be written in the form:

$$N_G = \frac{\mathrm{S}'''_G}{\mathrm{S}'''_{G,\mathrm{ch}}} = \theta'^2 + GP\,f''^2 \tag{8.a}$$

The total entropy generation number can be written as a summation of the heat transfer contribution on local entropy generation, N_H, which is the first term on the right-hand side and fluid friction contribution on local entropy generation, N_F, which is the second term on the right-hand side of eqs. (8a):

$$N_G = N_H + N_F \tag{8.b}$$

During the calculations, it may be possible to calculate these terms separately, and then compare them to notice which entropy generation mechanisms dominate. The irreversibility distribution ratio, Φ, is an important dimensionless parameter in the entropy generation analysis of convective heat transfer problems that was introduced for first time by Bejan [2]. This ratio shows the ratio of the entropy generation due to fluid friction ($S'''_{G,\text{fluid friction}}$ or N_F) to thermal entropy generation ($S'''_{G,\text{heat transfer}}$ or N_H). For the present problem, this ratio can be expressed:

$$\phi = \frac{S'''_{G,\text{fluid friction}}}{S'''_{G,\text{heat transfer}}} = \frac{N_\text{F}}{N_\text{H}} = GP\left(\frac{f''}{\theta'}\right)^2 \tag{9}$$

As seen, *GP* number is an important parameter for irreversibility distribution ratio. A large *GP* number shows that friction entropy generation is more than the thermal entropy generation in the boundary layer and vice versa for a small *GP* number.

Bejan number is another important parameter in irreversibility problem. It is defined as ratio of the entropy generation due to heat transfer, N_H, to mixed entropy generation, N_G:

$$\text{Be} = \frac{N_\text{H}}{N_\text{G}} = \frac{1}{1+\phi} = \frac{\theta'^2}{\theta'^2 + GP\, f''^2} \tag{10}$$

Integral method

This method is based on the overall balances of mass, momentum, and energy and on the use of approximate representations of the local velocity and temperature fields. Following the integral analysis developed by von Karman [25], the method was first used by Squire [26] for the analysis of the buoyancy-driven boundary layer flow adjacent to a heated isothermal vertical surface.

The velocity and temperature profiles are then chosen to satisfy the boundary conditions at the surface and in the ambient medium. If a third order velocity and second order temperature profiles are assumed, as used by Squire [26], the distributions for velocity and temperature are obtained (if the velocity and thermal boundary layer thickness are taken as equal and denoted by δ) [27]:

$$u = U\eta(1-\eta)^2 \qquad \text{and} \qquad \theta = \frac{t - t_\infty}{t_0 - t_\infty} = (1-\eta)^2$$

where $\eta = y/\delta$, U, and δ are functions of x which are obtained from the governing integral equations.

Based on the previous model, Ozkol *et al.* [7] wrote the second law of thermodynamics to estimate the volumetric rate of entropy generation in Cartesian co-ordinates for constant temperature vertical plate in an infinite quiescent fluid, that we reproduced after them to compare with similarity solution results.

Results and discussions

As expressed, eqs. (4) and (5) are coupled ordinary differential equation. Therefore, they should be solved simultaneously. A convenient method for solution these equation is fourth order Runge-Kutta numerical method with shooting method and iterating of solving process. By this method, purpose of problem is finding $F''(0)$ and $\theta'(0)$. Thereupon initial conditions $F''(0)$ and $\theta'(0)$ have been superseded for boundary condition $F'(\infty) = 0$ and $\theta'(\infty) = 0$.

As seen in set of eqs. (4) and (5), they just depend on Prandtl number. Initially, to investigate how variations of velocity and temperature in boundary layer and conformity of them with physics of problem, velocity and temperature profiles for different Prandtl numbers, have been plotted in figs. 2 and 3. As expected, with increasing boundary layer thickness (at higher value of x), velocity and temperature gradients decrease.

Figure 2 shows dimensionless velocity profiles at different Prandtl numbers. The velocity profiles have theirs maximum value near the plate surface and approach ambient conditions far from the plate. It has been perceived that with higher Prandtl number, maximum of velocity increases. Also, it has been prospected at higher Prandtl number; penetration of velocity profile into quiescent fluid will be more.

Figure 3 shows dimensionless temperature profiles at different Prandtl numbers. With increasing Prandtl number, dimensionless temperature decrease. Also, when Pr $\to \infty$, dimensionless temperature profiles tend to profile at Pr = 1000.

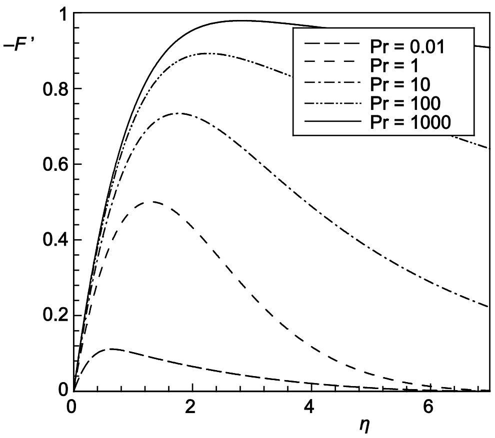

Figure 2. Dimensionless velocity profiles at different Pr

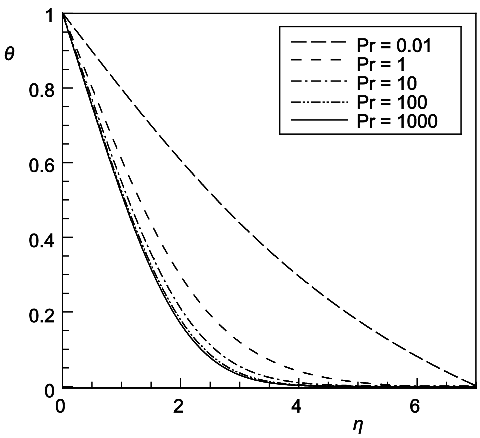

Figure 3. Dimensionless temperature profiles at different Pr

In fig. 4, it has been perceived $F''(0)$ as a function of Prandtl number. Considering the relationship between shear stress and velocity gradient:

$$\tau_{\mathrm{w}} = -\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \to F''(0) = \tau_{\mathrm{w}} \frac{x^2}{\mu\alpha} \mathrm{Ra}_x^{-3/4}$$

As seen previously, dimensionless parameter $F''(0)$ is proportional to the amount of shear stress on the wall. Therefore, by increasing the Prandtl number, the wall friction increases.

Figure 5 shows the $-\theta'(0)$ as a Prandtl number. This coefficient is related local Nusselt number:

$$\mathrm{Nu}_x = \frac{hx}{k} \to \theta'(0) = -\,\mathrm{Nu}_x\ \mathrm{Ra}_x^{-1/4}$$

As seen previously dimensionless parameter $-\theta'(0)$ is proportional to the amount of local Nusselt number. Therefore, increasing Prandtl number, local Nusselt number increased and thus the rate of heat transfer is also increased.

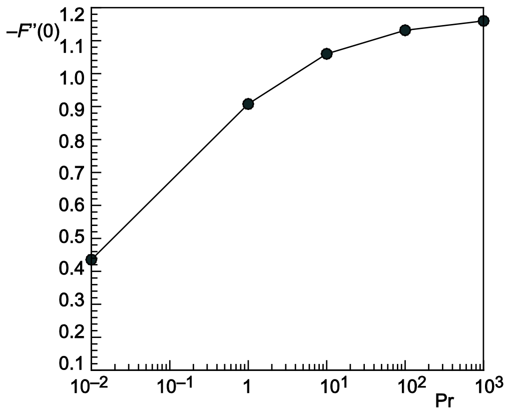

Figure 4. Dimensionless shear stress on the plate as a function of Prandtl number

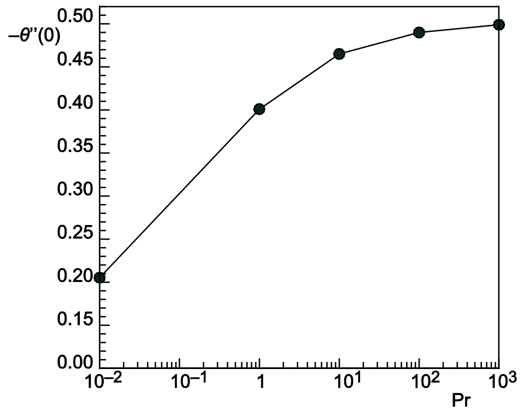

Figure 5. Local Nusselt number as a function of Prandtl number

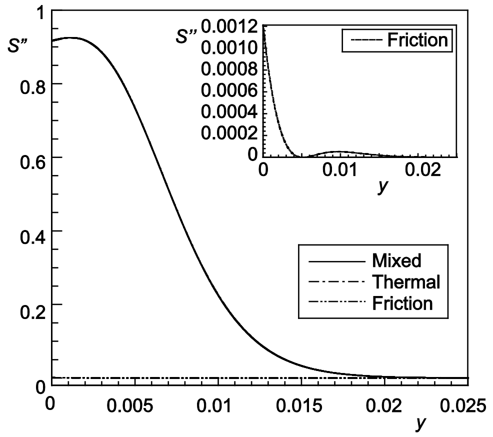

Figure 6. Entropy generation as a function of η for $x = 0.5$

Figure 6 shows the distribution of mixed, friction and thermal entropy generation in the boundary layer for $x = 0.5$. As seen, entropy generation becomes significant in the region close to the plate. The plate has a strong effect on entropy generation because of the sudden changes in temperature and velocity profile. So, entropy generation decreases sharply with increasing y. Although the friction contribution profile, after reaching zero at a distance from the wall (in here $y = 0.005$), increases and then again decreases. This is because at this point, velocity is maximum so velocity gradient is zero. Hence friction entropy generation will be zero.

In this physical problem, the value of the friction term is very small (about 0.1% near the wall). So, thermal and mixed entropy generation functions seem to be coincident with each other. Also, it is seen that the ratio of friction to thermal entropy generation is about 0.001 which show the magnitude of the Bejan number in boundary layer to be about one.

It can also be seen that the maximum of entropy due to heat transfer generate at a small distance from the wall and not on the wall. While this maximum amount due to friction contribution occur on the wall exactly. Because of small value of the friction contribution in entropy generation, maximum of mixed entropy generation profile is an almost coincided to maximum point in thermal entropy generation profile.

Mixed entropy generation function for different plate lengths has been plotted in fig. 7. Because of boundary layer thickness is proportional to x, with increasing the velocity and temperature gradients decrease. As a result, entropy generation decreases with increasing x.

It can be seen that with increasing x, maximum value is closer to value on the wall. As seen, position of inflection point in curves is almost fixed (about $\eta = 1.8$).

The *GP* number used in the previous calculations was for air in temperature 300 K and atmosphere pressure, $GP = \mathrm{O}\ (10^{-4})$. It is a very small value that as shown makes small friction contribution in entropy generation. Figure 8 shows the distribution of mixed, friction and thermal entropy generation functions in the boundary layer for a *GP* number that is 100 times higher than air. As seen, still the thermal profile has maximum value at a point near the wall (not on the wall). But maximum value for mixed profile is on the wall, because friction term is increased in comparison with fig. 7.

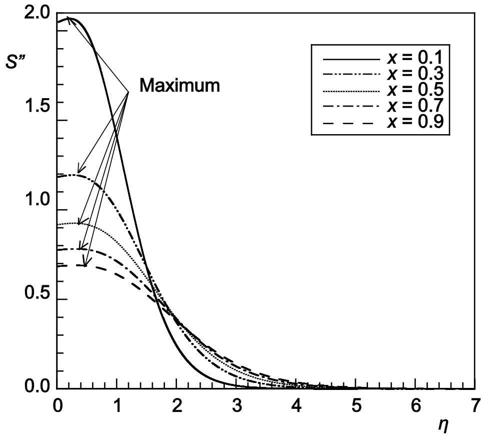

Figure 7. Entropy generation as a function of *η* for different plate lengths

The influence of the *GP* number on the entropy generation is presented in fig. 9. As seen, *GP* number does not affect thermal entropy generation, eq. (7c), but with increasing *GP*, friction entropy generation and thus mixed entropy generation increase.

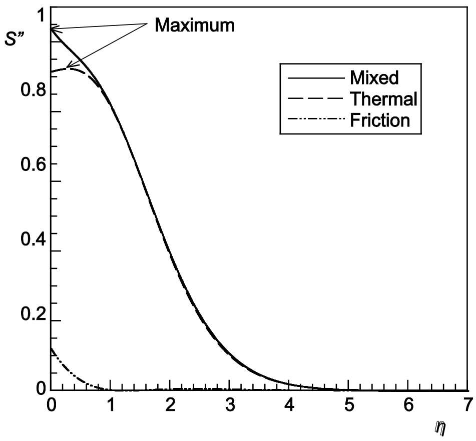

Figure 8. Entropy generation as a function of *η* for $GP = \mathrm{O}\ (10^{-2})$, ($x = 0.5$)

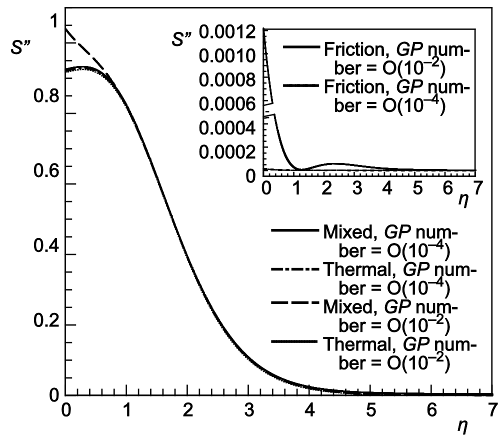

Figure 9. Distribution of entropy generation in boundary layer for different *GP* numbers, ($x = 0.5$)

One of the most important parameters in irreversibility problems is the Bejan number, which is the ratio of entropy generation due to heat transfer to the total entropy generation. Clearly, the Bejan number varies from 0 to 1, eq. (10). When the value of Bejan number is greater than 0.5, the irreversibility due to heat transfer dominates, whereas $\mathrm{Be} < 0.5$ refers to irreversibility due to viscous dissipation. When $\mathrm{Be} = 0.5$, the contribution of the heat transfer and fluid friction entropy generation are equal.

As expressed, in here, the irreversibility due to heat transfer dominates, so $\mathrm{Be} > 0.5$. Also, it is seen from fig. 6 that the ratio of thermal to mixed entropy generation is about 1.0 which show $\mathrm{Be} \approx 1$ in boundary layer.

Figure 10 plots the variation of the distribution of Bejan number in boundary layer. The Bejan number first increases to one and then decreases. This point is maximum of Bejan number. Because in this position velocity profile gradient is zero, so entropy generation due to fluid friction is zero in this location of boundary layer.

Figures 11 and 12 show the integral solution distribution of entropy generation functions in the boundary layer (for different plate lengths) and comparison of it with similarity solution results, respectively. As seen, there is no entropy generation occurring at places of zero velocity and temperature gradients, outside the velocity and temperature boundary layer thicknesses.

Comparing figs. 11 and 12, in generally, show integral solution has a similar trend to similarity solution except in near wall. The similarity solution results show that maximum thermal entropy generation is occurred in near the wall (not on the wall) but in integral solution this maximum point is on the wall exactly, that it is not true. Because of similarity solution gives us an exact solution. So, similarity solution result is more reliable.

For a designer who designs any energy conversion system, it is important to know maximum point and value of entropy generation to minimize it in system. So, similarity solution results give designer a good and better insight in comparison to other methods.

Finally, fig. 13 shows the flux of entropy generation function in different plate lengths which is determined as follows for various solutions: $S''_G = \Sigma_{y=0}^{\max(\delta,\delta_t)} S'''_G$.

As expressed, with increasing boundary layer thickness (at higher value of x), velocity and temperature gradients decrease. So, entropy generation decreases along the wall.

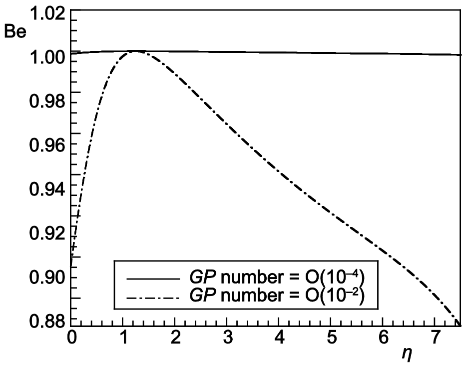

Figure 10. The Be as a function of η for x = 0.5

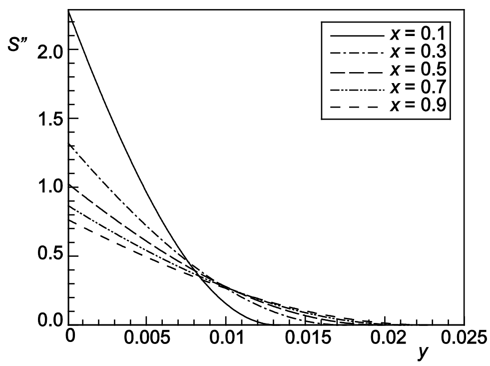

Figure 11. Integral solution of entropy generation as a function of η for different plate lengths

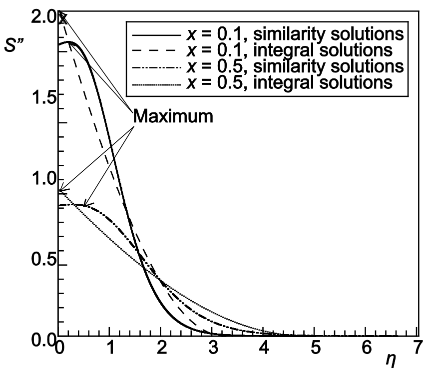

Figure 12. Comparison similarity solution results (thermal entropy generation function) with integral solution results for two different plate lengths

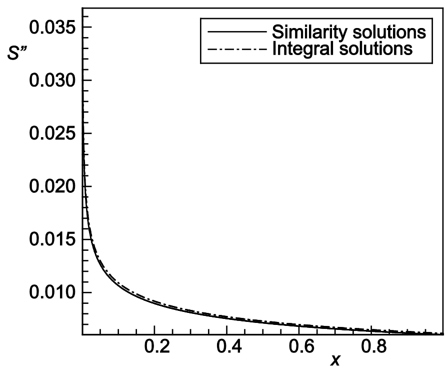

Figure 13. Entropy generation flux in boundary layer as a function of x for similarity and integral solution

It is seen that the total entropy generation of the similarity solution is smaller than the integral solution. It shows that the exact velocity and temperature profiles (similarity solution) minimize the rate of total entropy generation compare to using approximate profiles.

Conclusions

The entropy generation analysis of a hot constant temperature vertical plate in an infinite quiescent fluid was carried out using similarity solution method and some result was compared with integral method. In this study, entropy generation was represented as a function of $S'''_{G,\mathrm{ch}}$, *GP* number, flow field velocity, F'', and temperature, $-\theta'$, distribution. Based on this study the following conclusions can be drawn.

- Because existence of hot vertical plate in quiescent fluid, and as a result creating free convection flow and heat transfer, the wall (plate) is main agent in entropy generation. So, maximum of entropy generation is occurred in near the wall (not necessary on the wall).
- Unlike integral method results, similarity solution results show that maximum entropy due to heat transfer generate at a small distance from the wall and not on the wall. While this maximum amount due to friction contribution occur on the wall, exactly.
- The *GP* number was introduced as an important parameter for irreversibility distribution ratio and contribution of entropy generation due to fluid friction *vs.* thermal entropy generation. By dimensionless number *GP*, one can recognize that one of the thermal entropy and friction entropy generation is dominated in the boundary layer.
- Because of small *GP* number, it has shown negligible contribution of entropy generation due to flow friction compared to the contribution of entropy generation due to heat transfer and by increasing *GP* number this contribution is increased. For fluids like air at 300 K temperature, *GP* number is order of 10^{-4}, so entropy generation due to heat transfer dominates. As a result, for minimization of total entropy generation, a designer should attend to control rate of heat transfer by changing geometrical and physical properties of the system.
- The comparing result of similarity and integral solution, show that the exact velocity and temperature profile (similarity solution) minimizes the rate of mixed entropy generation in boundary layer.
- Using similarity profile for analysis of the entropy transport equation is very helpful in examining the individual terms in the entropy transport equation and helps therefore to better understand their behavior. Also, similarity (exact) solution presents more appropriate and correct distribution of entropy generation in boundary layer because more accuracy than other methods (like integral method). Also, it shows true position of maximum entropy generation and value of it.

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