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دکتر حمید رضا ابراهیمی ویشکی (دانشگاه فردوسی مشهد)

Function algebras on locally compact quantum group

(Ebrahimi Vishki.pdf)

دکتر رسول نصر اصفهانی (دانشگاه صنعتی اصفهان)

Strong amenability of representations

(See abstract)

دکتر امیر خسروی (دانشگاه خوارزمی)

Frames, wavelets and some generalizations

(Khosravi.pdf)

دکتر علی غفاری (دانشگاه سمنان)

Gamma-invariant operators associated with locally compact groups

(See abstract)

دکتر حسن پورمحمود آقابابا (دانشگاه تبریز)

Pseudomeasures and pseudofunctions on inverse semigroups

Function Algebras on Locally Compact Quantum Groups

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Abstract

Some of the most famous function algebras on a locally compact group G are $\mathrm{WAP}(G)$, $\mathrm{AP}(G)$, $\mathrm{LUC}(G)$ and $\mathrm{C}_0(G)$. In this talk we recast these for a locally compact quantum group $\mathbb{G}$ and investigate the analogous properties for these spaces. In particular, we examine when some of them are C^*-subalgebras of $L^\infty(\mathbb{G})$ and we also study their inclusion relations from the quantum group compactification point of view.

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1. Introduction and Preliminaries

During the last years there has been a noticeable increase of interest in the theory of Hopf von Neumann algebras, especially, locally compact quantum groups introduced by J. Kustermans and S. Vaes [6].

A Hopf von Neumann algebra is a pair $(\mathfrak{M}, \Gamma)$, where $\mathfrak{M}$ is a von Neumann algebra and $\Gamma : \mathfrak{M} \to \mathfrak{M}\bar{\otimes}\mathfrak{M}$ is a coproduct, that is, a normal unital $*-$homomorphism satisfying

$$(\Gamma \otimes \iota) \circ \Gamma = \iota \otimes \Gamma) \circ \Gamma.$$

The coproduct Γ induces a product Γ_* on the predual $\mathfrak{M}_*$ of $\mathfrak{M}$ defined by

$$\langle \Gamma_*(\omega, \omega'), x\rangle = \langle \Gamma(x), \omega \otimes \omega'\rangle \quad (\omega, \omega' \in \mathfrak{M}_*, x \in \mathfrak{M}),$$

making it into a Banach algebra.

A locally compact quantum group $\mathbb{G}$ is a quadruple $\mathbb{G} = (\mathfrak{M}, \Gamma, \varphi, \psi)$, in which, $(\mathfrak{M}, \Gamma)$ is a Hopf von Neumann algebra and φ and ψ are left and right Haar weights on $\mathfrak{M}$, respectively; that is, they are normal semifinite faithful weights on $\mathfrak{M}$ satisfying the following identities:

$$\varphi((\omega \otimes \iota)(\Gamma(x))) = \omega(1)\varphi(x),\ \psi((\iota \otimes \omega)(\Gamma(x))) = \omega(1)\psi(x)\ (\omega \in \mathfrak{M}_*, x \in \mathfrak{M}_\varphi, y \in \mathfrak{M}_\psi),$$

where, $\mathfrak{M}_\varphi = \mathrm{span}\{x \in \mathfrak{M}^+ : \varphi(x) < \infty\}$.

In the realm of abstract harmonic analysis, the main examples are the locally compact quantum groups $\mathbb{G}_a = (L^\infty(G), \Gamma_a, \varphi_a, \psi_a)$ and $\mathbb{G}_s = (VN(G), \Gamma_s, \varphi_s, \psi_s)$ on a locally compact group G, whose coproducts $\Gamma_a : L^\infty(G) \to L^\infty(G \times G)$ and $\Gamma_s : VN(G) \to VN(G \times G)$ are given by

$$(\Gamma_a(\phi))(x, y) = \phi(xy) \ \ (\phi \in L^\infty(G), x, y \in G), \ \Gamma_s(\lambda(x)) = \lambda(x) \otimes \lambda(x) \ (x \in G),$$

respectively, where λ denotes the left regular representation of G. The weights φ_a, ψ_a are left and right Haar measures on G, respectively, and $\varphi_s = \psi_s$ can be chosen as Plancherel weight [10, Definition VII.3.2].

It is worthwhile mentioning that for $\mathbb{G}_a$ the product Γ_* imposed on $\mathrm{L}^1(G)$ is just the usual convolution on $\mathrm{L}^1(G)$ whereas for $\mathbb{G}_s$ it yields the usual pointwise product on $\mathrm{A}(G)$.

A somewhat less well-known example of a commutative Hopf von Neumann algebra is $M(G)^* = C_0(G)^{**}$ whose coproduct is expertly described [1].

For a locally compact quantum group $\mathbb{G}$, using the GNS construction, the left Haar weight φ induces a representation (π_φ, H_φ) for $\mathfrak{M}$. Associated with the pair $(\mathbb{G}, \varphi)$ there exists a unique unitary operator $W \in B(H_\varphi \bar{\otimes} H_\varphi)$ – the so-called multiplicative unitary – which presents the coproduct Γ via $\Gamma(x) = W^*(1 \otimes x)W, \ (x \in \mathfrak{M})$. More information on the general theory of locally compact quantum groups can be found in [4], [5], [6] and [10]. As usual, for a locally compact quantum group $\mathbb{G}$, we usually write: $L^\infty(\mathbb{G})$ for $\mathfrak{M}$, $L^1(\mathbb{G})$ for $\mathfrak{M}_*$ and $L^2(\mathbb{G})$ for H_φ.

2. Some well-known function algebras on $\mathbb{G}$

For a Banach algebra A, the space of almost periodic functionals $\mathrm{AP}(A)$ and weakly almost periodic functionals $\mathrm{WAP}(A)$ are defined, respectively, as follows:

$$\mathrm{AP}(A) = \{f \in A^* : a \mapsto a \cdot f : A \to A^* \text{ is compact}\} \text{ and}$$

$$\mathrm{WAP}(A) = \{f \in A^* : a \mapsto a \cdot f : A \to A^* \text{ is weakly compact}\}.$$

We define $\mathrm{LUC}(A)$ and $\mathrm{RUC}(A)$, respectively, by $\mathrm{LUC}(A) = \langle A^* \cdot A \rangle$ and $\mathrm{RUC}(A) = \langle A \cdot A^* \rangle$. It is easy to verify that $\mathrm{AP}(A)$ and $\mathrm{WAP}(A)$ are closed $A-$submodules of A^*, $\mathrm{LUC}(A)$ is a left $A-$submodule while $\mathrm{RUC}(A)$ is a right $A-$submodule of A^*.

It might be excepted that the above mentioned spaces have extra structures when A is sufficiently rich. For instance, in [1] Daws used a technique, based on developing the theory of corepresentations on reflexive Banach spaces, to show that, if $\mathfrak{M}$ is a a commutative Hopf von Neumann algebra then both of $\mathrm{AP}(\mathfrak{M}_*)$ and $\mathrm{WAP}(\mathfrak{M}_*)$ are C^*-subalgebras of $\mathfrak{M}$; see also [9] where Runde mildly improved Daws' result for subhomogeneous Hopf von Neumann algebras. As a consequence, for a locally compact group G, $\mathrm{AP}(\mathrm{M}(G))$ and $\mathrm{WAP}(\mathrm{M}(G))$ are C^*-algebras of $\mathrm{M}(G)^*$.

As usual, for a locally compact quantum group $\mathbb{G}$ we usually write $\mathrm{WAP}(\mathbb{G})$, $\mathrm{AP}(\mathbb{G})$, $\mathrm{LUC}(\mathbb{G})$ and $\mathrm{RUC}(\mathbb{G})$, respectively, for $\mathrm{WAP}(L^1(\mathbb{G}))$, $\mathrm{AP}(L^1(\mathbb{G}))$, $\mathrm{LUC}(L^1(\mathbb{G}))$ and $\mathrm{RUC}(L^1(\mathbb{G}))$. Some substantial properties of $\mathrm{WAP}(\mathbb{G})$, e.g. its inclusion relation with some other subspaces of $L^\infty(\mathbb{G})$ are investigated in[8]; see also [7], FV and [2]. It has shown that under certain conditions, which are always satisfied when $\mathbb{G}$ is a locally compact group, LUC($\mathbb{G}$) is a C^*-subalgebra of $L^\infty(\mathbb{G})$ [8].

Following the above mentioned results, in this talk we are mainly deal with the following questions:

1. When is WAP($\mathbb{G}$) a C^*-algebras? How about AP($\mathbb{G}$) and LUC($\mathbb{G}$)?
2. How are the inclusion relations among WAP($\mathbb{G}$), AP($\mathbb{G}$), LUC($\mathbb{G}$), RUC($\mathbb{G}$) and $C_0(\mathbb{G})$?

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