9th International Iranian Operation Research conference
(IORC 2016)

Shiraz University of technology

Iranian Operation Research Society

IORC 00 (2016) 000–000

Mathematical modelling for computation goal oriented duties in the special economic zone

M. Ranjbar[a]*, A. V. Kamyad[a], S. Effati[a]

[a]Department of Applied Mathematics, Ferdowsi University of Mashhad, Mashhad, Iran.

Abstract

In this paper, we propose a new approach to obtain flexible taxation of productive commodities from industrial units in a special economic zone (SEZ) based on mathematical models and optimization, that it is generalizable on other trade free zones. For this purpose, we introduced a mathematical model to obtain a goal oriented duties by using partial differential equation and recursive equation. To analyze this model we make a nonlinear programming problem that by it zone manager can study the effects of the proposed model. Finally, we implement this model on a case study to receive taxation of productive commodities into a special economic zone.
Keywords: Mathematical modelling; Partial differential equation; Nonliner programming.

1- Introduction

A special economic zone, refers to a large geographical delimited area comprising a complex of related economic activities and services, whose objective is to facilitate broad based comprehensive development, in part by attracting foreign direct investment through equity and contractual joint ventures, joint explorations in off-shore oil production, and even wholly owned foreign subsidiaries [4, 6]. Duty is a kind of indirect tax, that has led a number of economists to undertake research on it, and so one might expect the link between the researchers. Smith listed four maxims with regard to taxes in general [7]:

1. Equality: that people tax payment should be in proportion to their income.
2. Certainty: the tax liabilities should be clear and certain, rather than arbitrary.
3. Convenience of payment: that taxes should be collected at a time and in a manner that is convenient for the taxpayer.
4. Economy in collection: that taxes should not be expensive to collect, and should not discourage business.

The idea of equality has been widely discussed and is still a major part of the evaluation of any tax policy proposal. In this paper according to the zone management objectives, we presented a new approach to obtain duty in a SEZ, so that moreover the first maxims, the paid duty should be proportional with other features of commodity, such as volume and weight.

A mathematical model is a description of a system using mathematical concepts and language. Mathematical models are used not only in the natural sciences such as physics, biology, earth science, meteorology and engineering disciplines, e.g. computer science, artificial intelligence, but it can be also in the social sciences such as economics, psychology, sociology and political science[1, 2, 3]. In this paper we propose a partial differential equation model and a recursive equation to achieve the goals.

The paper is organized as follows. In Section 2 the basic framework is proposed. In Section 3 simulation results of the new model in case study are reported. Finally, some concluding remarks are drawn in Section 4.

2- Mathematical Equations

A particular policy proposal for payment duty is difficult, such that having separate criteria. The purpose of this paper is to determine rate paid duty for production commodities in industrial units situated in a SEZ, according to features of

[a] Corresponding author: *E-mail address*: m.ranjbart@gmail.com

commodity. Now, payment duty for any unit of commodity is only based on value of commodity as follows:

$$(1) \qquad U_f = d_f \times x$$

where x is the value of a unit of the commodity, d_f is tariff rate that is fixed and U_f is payment duty for a unit of commodities, that with multiply U_f in the number of commodities (n), we obtain total payment duty of each industrial unit, as follows:

$$(2) \qquad T_f = U_f \times n$$

In this type of reception excluding value of commodities, not to be effective other features of commodity on payment duty. Therefore, according to goals of zone management for importation high-tech to the SEZ, also for satisfaction of investors, in this paper, we want that other features of commodities as volume and weight, be effective on payment duty. For this purpose we achieve the one variable tariff rate d_v, based on features of commodities.
The first we let that d_v is a function of three features, value, volume and weight of commodity as follows:

$$(3) \qquad d_v = f(x,y,z)$$

where, x, y and z are the value (price), volume (m^3) and weight (kg) of a unit of commodity, respectively. The goal is to obtain d_v, such that:

i. If the value of commodity increase , then, d_v decrease and vice versa.
ii. If the volume and weight of commodity increase , then, d_v increase and vice versa.

Also, d_v is bounded as follows:

$$(4) \qquad d_f - a \leq d_v \leq d_f + b$$

where a and b are characterized via evaluation of experts. According to *(i)* and *(ii)* we should have:

$$(5) \qquad \frac{\partial d_v}{\partial x} < 0, \frac{\partial d_v}{\partial y} > 0, \frac{\partial d_v}{\partial z} > 0$$

Differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology [2,5]. Since, d_v is a function of three variables, we presented a partial differential equation (PDE) model as follows, so that solution of it satisfies in (5).

$$(6) \qquad \frac{\partial d_v}{\partial x} = -k\left(\frac{\partial d_v}{\partial y} + \frac{\partial d_v}{\partial z}\right)$$

where $k>0$, by initial conditions as follows:

- $dv=f(x_{min},y_{max},z_{max})=d_f+b$
- $dv=f(x_{mean},y_{mean},z_{mean})=d_f$
- $dv=f(x_{max},y_{min},z_{min})=d_f-a$

where x_{max}, x_{mean} and x_{min} are respectively maximum, mean and minimum amount of value of existent commodities in SEZ.

Theorem 2.1. By using equation (6) we have:

$$(7) \qquad d_v = he^{-k(\lambda+\mu)x+\lambda y+\mu z}$$

where h, λ, μ and k are constant and positive coefficients, that obtain by initial conditions.
Proof:
For solving this PDE model, we use of separation of variables method.

Corollary 2.1. By using Theorem 2.1 we have:

a) $\frac{\partial d_v}{\partial x} < 0$

b) $\frac{\partial d_v}{\partial y} > 0$

c) $\frac{\partial d_v}{\partial z} > 0.$

Proof: According to equation (7), it is clear.

3- Case study

In order to demonstrate the effectiveness and performance of the proposed model, we discuss one case study in a special economic zone in Iran.

Example 3.1. Consider 44 commodities in a special economic zone with three features, value, volume and weight and nominal capacity that are shown in Table 1.

Table1- Value, volume and weight of commodities existent in the SEZ for Example 3.1.

Commodities	*X (Rial)*	*Y* (m^3)	*Z (kg)*
C_1	5000000	0.5	40
C_2	2500000	1	200
. . .	. . .	. . .	. . .
C_{43}	8500000	0.2	15
C_{44}	30000000	0.2	24

Now, tariff rate for all commodities is fixed and equal to 0.75 percent in (1), zone management want that this coefficient be in the maximum amount equal to 1 percent and in the minimum amount equal to 0.5 percent, in other words by considering a=b=0.25 percent in (4), we have:

$$0.5 \leq d_v \leq 1$$

After normalizing the data in the interval [0,1] , with initial conditions as follows

- $dv=f(x_{min},y_{max},z_{max})=1$
- $dv=f(x_{mean},y_{mean},z_{mean})=0.75$
- $dv=f(x_{max},y_{min},z_{min})=0.5$

The goal is to determine payment duty of industrial units situated in the zone based on features.
According to section 2, the first determines tariff rate d_v by using equation (7) of Theorem 2.1, that obtain by initial conditions as follows:

$$d=0.77\, e^{0.4416x+0.0788y+0..1727z}$$

Accordingly, Table 2 lists new tariff rates of commodities that obtained by the new model.

Table1- Variable tariff rate of commodities by the new model for Example 3. 1.

Commodities	***d_v* (percent)**
C_1	0.79
C_2	0.95
. . .	. . .
C_{43}	0.74
C_{44}	0.61

Then we present a model by using a recursive equation for the encouragement of investors, for produce of more products as follows:

$$T_n^v = \frac{u_v}{2mb}\left((2mb+1)n - n^2\right) \quad (8)$$

where, u_v is payment duty for an unit of commodity by variable tariff rate, n is number of the products in a time interval (at one month or one year), b is nominal capacity of an industrial unit and m ≥1 is a controlling factor for increase rate of taxation. Also, for analysing this model by using (8), we make a nonlinear programming problem as follows, that by it zone manager can study the effects of the proposed model.

$$\text{Minimize} \quad z = p$$

$$\text{subject to} \quad \frac{u_v}{2bm}\left((2bm+1)(n+np) - (n+np)^2 + (kb)^2 - kb\right) \geq u_f n$$

$$n + np \leq b$$

$$m \geq 1$$

$$p \geq 0.$$

Where p is to increase of the minimum percentage in the number of products and k can be selected based on current production and evaluation of experts. Fig. 1 and Fig. 2 show behavior of T^f and T_n^v for two commodities C_2 and C_{44} respectively, by controlling factor $m = 2$ and $k = 0.5$ for $0 \leq n \leq b$.

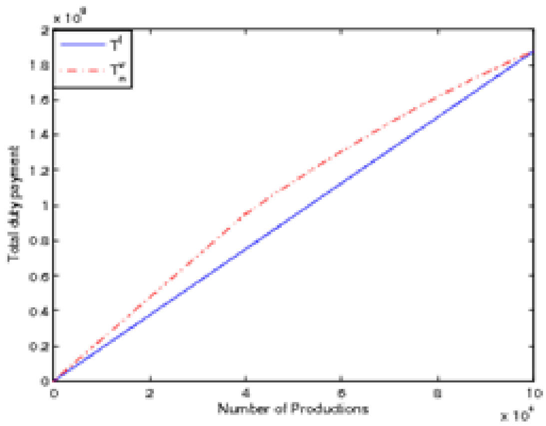

Figure 1: Total duty payment for C_2, that $U_v > U_f$ by T^f and T_n^v.

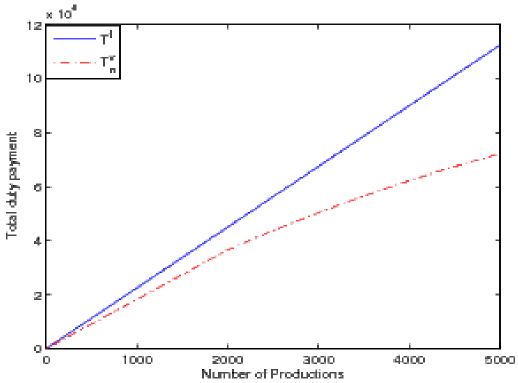

Figure 2: Total duty payment for $\boldsymbol{C_{44}}$, that $\boldsymbol{U_v < U_f}$ by $\boldsymbol{T^f}$ and $\boldsymbol{T_n^v}$.

If controlling factor *m* be unknown, we can achieve it by solving above nonlinear programming problems, that it is dependent to amount of *p*.

5- Conclusion

Research shows that SEZs have an influence on the host country's economy and that the effect can be positive or negative. In this paper, we introduced a new approach for reception goal oriented duties by using mathematical models as partial differential equation and recursive equation, that caused satisfaction of zone management and encouragement of investors to import high-tech industries in the SEZ. To analyze this model we make a nonlinear programming problem that by it zone manager can study the effects of the proposed model. This approach also can be implemented on free trade zones and other locations to achieve payment duty.

References

[1] R. Andersson, A. Samartin, "A model for the economic evaluation of master city plans: a pilot study of Västerås," Sweden, Applied Mathematical Modelling, .Vol7, pp. 345-355, 1983.

[2] F. Guerriero, R. Musmanno, O. Pisacane, F. Rende, "A mathematical model for the Multi-Levels Product Allocation Problem in a warehouse with compatibility constraints, "Applied Mathematical Modelling, .Vol37, .pp4385-4398 ,2013.

[3] L. Li, Y. H. Li, Q. K. Liu, H. W. Lv, "A mathematical model for horizontal axis wind turbine blades," Applied Mathematical Modelling, .Vol 38, .pp2695-2715, 2014.

[4] R. Pastusiak and K. B. Lason, " the influence of special economic zone on local finances, an analysis of the Polish case," Vol. 75, .pp76-89, 2013.

[5] M. A. Pinsky, Partial diferential equations and boundary value problems with applications. Waveland Press Inc., Prospect Heights, Illinois, third edition, 2003.

[6] S. Sikidar and P. Hazarika, "Special Economic Zones: Genesis and Development," The Chartered Accountant, .Vol55, .pp1555-1562, 2008.

[7] A. Smith, An Enquiry into the Nature and Causes of the Wealth of Nations. Cannan edition, London: Methuen, 1776.

[8] W. A. Strauss, Partial diferential equations. An introduction. John Wiley & Sons Ltd., Chichester, second edition, 2008.