

## Contribution of orbital angular momentum to the nucleon spin

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We have calculated the orbital angular momentum of quarks and gluons in the nucleon. The calculations are carried out in the next to leading order utilizing the so-called valon model. It is found that the average quark orbital angular momentum is positive, but small, and the average gluon orbital angular momentum is negative and large. We also report on some regularities about the total angular momentum of the quarks and the gluon, as well as on the orbital angular momentum of the separate partons. We have also provided partonic angular momentum,  $L^{q,g}$  as a function of  $Q^2$ .

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## 1. Introduction

Polarized deep inelastic scattering process is the most direct tool to probe the spin content of the nucleon. In such experiments, detailed information can be extracted on the shape and the magnitude of the spin-dependent parton distributions,  $\Delta q_f(x, Q^2)$ . Deep inelastic scattering reveals that the nucleon is a rather complicated object consisting of an infinite number of quarks, antiquarks and gluons. It is a common belief that other strongly interacting particles also exhibit similar internal structure.

The decomposition of nucleon spin in terms of its constituents has been a challenge and an active topic in hadron physics, both from theoretical and experimental points of view. It is now established that quarks carry a small fraction of the nucleon spin. Other sources that might contribute to the nucleon spin are gluon spin and the overall orbital angular momentum (OAM) of the partons. Thus, it is common to write the following spin sum rule for a nucleon:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_{q,g}. \quad (1)$$

Over the past few years, we have studied the first two terms of the above sum rule within the framework of the so-called *valon* model<sup>1</sup> in the next to leading order. The valon model is a phenomenological model for hadrons, introduced first by Hwa about 30 years ago.<sup>2</sup> The model has been quite successful in describing a variety of hadronic phenomena.<sup>3,4</sup> In the polarized deep inelastic scattering domain, the model has successfully reproduced the existing data on  $g_1^{p,n,d}$ , and individual parton contributions,  $\Delta q_f(x, Q^2)$ , to the nucleon spin, while predicting new results yet to be tested. Among other things, the model has predicted that the sea quark polarization is negligible, which is now proven to be the case.<sup>5</sup> This finding is because the valons are generated by perturbative dressing in QCD. In such processes with massless quarks, helicity is conserved and therefore, the hard gluons cannot produce the sea quark polarization perturbatively. So, it turns out that sea polarization is consistent with zero. We have also shown that although  $\delta g(x, Q^2)$  is small, its first moment,  $\Delta G$ , is large and grows as  $Q^2$  increases.<sup>6</sup> This is consistent with QCD and available experimental data.<sup>7-15</sup> Elsewhere, we have reported that with a fixed and almost  $Q^2$  independent value for  $\Delta\Sigma$  and with the growing  $\Delta G$  it is not possible to achieve  $s_z = \frac{1}{2}$  for the nucleon spin. Therefore, there should be other contributing element in order to arrive at spin  $\frac{1}{2}$  of the nucleon. The only possible place would be the OAM of the partons. In Ref. 16, we used Eq. (1) and estimated, but not independently calculated, the magnitude of the overall OAM of the partons inside the nucleon. Our conclusion was that overall OAM of partons is negative and decreases as  $Q^2$  increases. In fact, almost 25 years ago, Ratcliffe<sup>17</sup> suggested that a consistent interpretation of the Dokshitz–Gribov–Altarelli–Parisi evolution equation of helicity weighted parton distributions requires the partons to carry a sizable OAM. Moreover, he concluded that for large  $Q^2$ , the average OAM will be negative.

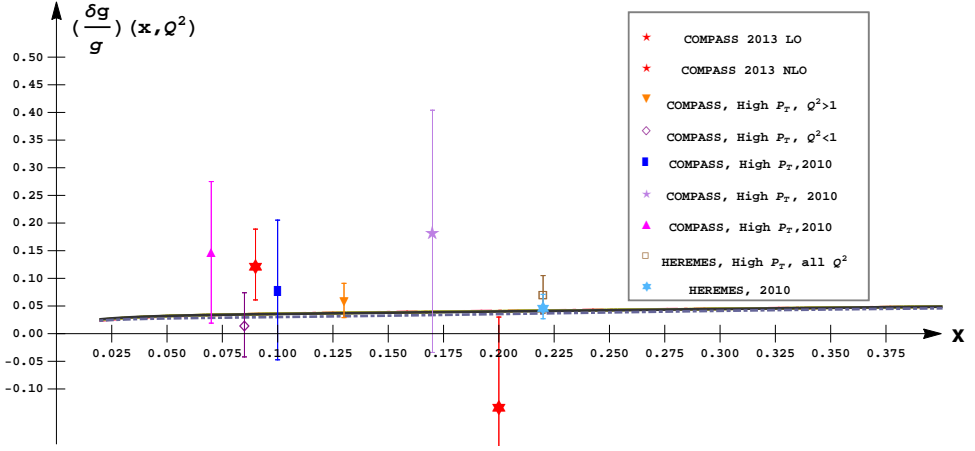


Fig. 1. The ratio  $\frac{\delta g}{g}$  calculated in the valon model and compared with the data. Data points are from Refs. 7–15.

The purpose of this paper is to report the results that we have obtained for the OAM contribution of quarks and gluons to the proton spin. Our calculations are carried out in the next to leading order in perturbation theory and are based on the valon representation of the nucleon. It is important to mention that one could, as well, use other polarized and unpolarized parton distributions, such as those obtained from the available global fits to carry out the same calculation. Our main reason in using the valon model is that, first, it was handy and secondly, its outcomes and the predictions have proven to be consistent with all the experimental data that are available. Hence, providing a reasonable confidence in its physical validity.

## 2. The Experimental Data

Over the past two decades, theoretical framework for the understanding of the spin structure of nucleon has been developed and numerous experiments were carried out. We now have a fairly good understanding of the first term in the sum rule above. Total quark contribution to the nucleon spin is about  $\Delta\Sigma = 0.4$ . Some data also emerged on the gluon polarization<sup>18,19</sup> showing that  $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$  is small. These data are reported at individual kinematics, i.e. at separate  $(x, Q^2)$  points, and lack the same level of precision achieved for the quark sector. The data from COMPASS collaboration<sup>20</sup> may even be able to rule out a negative value for  $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$ , which has been a controversy over the past few years. Nevertheless, the smallness of  $\frac{\delta g}{g}$  by itself cannot rule out the possibility of a large value for the first moment,  $\Delta G$ , of the gluon polarization. In Fig. 1, we present  $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$  that is obtained from our model. We have calculated  $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$  at each kinematical point for which the data exists. The apparent wide band in the figure is actually several closely packed curves corresponding to the several values of  $Q^2$  at which data points are measured. The

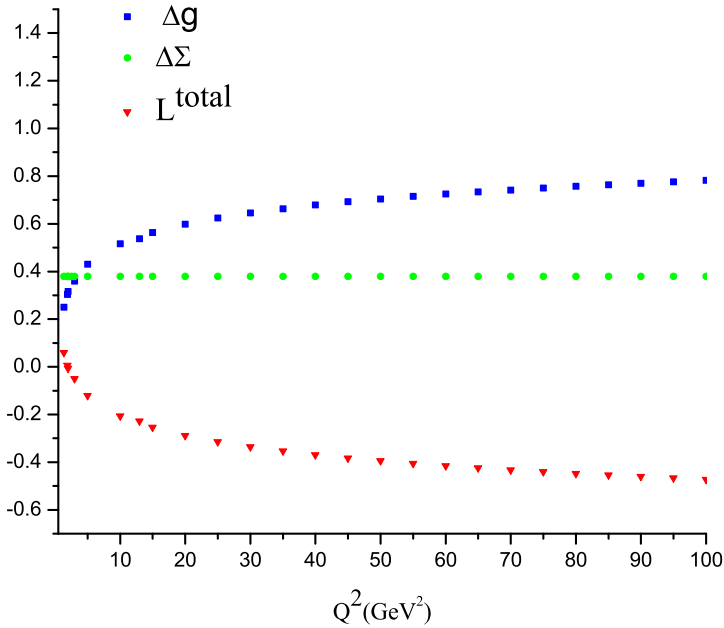


Fig. 2. First moments of polarized gluons and quarks distribution functions and the resulting total OAM obtained from Eq. (1).

details of this calculation can be found in Ref. 6. In Ref. 21, a new method is suggested for measuring the gluon polarization. In Fig. 2, we also show our results on the first moments of the quark,  $\Delta\Sigma$ , and the gluon,  $\Delta G$ , polarization in the nucleon. Substituting these values in Eq. (1), gives a measure of the total overall angular momentum of the partons inside the nucleon, which is also shown in Fig. 2.

### 3. Orbital Angular Momentum

Considering that the spin of quarks accounts for only a part of the nucleon spin, and that of the gluon is still unclear, a substantial fraction of the nucleon spin must be due to the OAM. Unfortunately, in the gauge theories there is no unique decomposition of the nucleon spin into contributions due to spin and the OAM of quarks and gluons. For example, Jaffe and Manohar<sup>22</sup> have used a light-like hypersurface, employed the light-cone framework and light-cone gauge and arrived at the following decomposition:

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \mathcal{L}_q^z + \Delta G + \mathcal{L}_g^z. \quad (2)$$

In this decomposition, each term is defined as the matrix element of the corresponding term in +12 component of the OAM tensor. The first and the third terms are interpreted as the quark and gluon spin, respectively. The second and the fourth terms are identified as the quark and gluon OAM. In this decomposition, except for

the first term, individual terms are not separately gauge invariant. An alternative decomposition is provided by Ji<sup>23</sup>

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L_q^z + J_g^z, \quad (3)$$

where each term is separately gauge invariant. However, the gluon total angular momentum is not decomposed, in a gauge invariant way, into its spin and the OAM. In Ji's decomposition, the total spin of quarks,  $J_q$ , and that of the gluons,  $J_g$ , are related to the generalized parton distribution (GPD) at twist-two level. Other decompositions have also been proposed.<sup>24,25</sup> A thorough analysis of these decompositions is given in Ref. 26. Briefly, it has been established that there are only two types of complete decompositions of the nucleon spin. The first one is the decomposition of canonical type, while the other is the decomposition of mechanical (or kinetic) type. The famous Jaffe–Manohar decomposition belongs to the former, while another complete decomposition proposed in Ref. 25 is of the mechanical type. Since these two quark OAMs are apparently different, the gluon OAMs are also different in the two types of nucleon spin decomposition.

It is now shown that at the twist-three level, once the GPD is integrated over  $x$ , both decompositions given in Eqs. (2) and (3) can be obtained.<sup>27,28</sup>

As mentioned, Ji's decomposition is related to twist-two GPDs, which can be measured in deeply virtual Compton scattering. The quark OAM in terms of GPDs is given by<sup>29</sup>

$$L^q = \int dx \int d^2b (xH^q(x, b) + xE^q(x, b) - \tilde{H}^q(x, b)). \quad (4)$$

The GPDs describe the dynamics of partons in the transverse plane in position space. Complementary information on the dynamics of partons in the transverse plane, but in the momentum space, is obtained from Transverse Momentum Dependent parton distributions (TMD-PDF).<sup>30,31</sup> Therefore, one naturally expects that TMD and GPDs will teach us about partonic OAM.

The OAM of partons plays an important role in hadron physics. It is well known that in order to have a nonzero anomalous magnetic moment, the light-cone wave function of nucleon must have components with  $L_z \neq 0$ .<sup>32</sup>

In the local limit, GPDs reduce to form factors, which are obtained from the matrix elements of the energy–momentum tensor  $\Theta^{\mu\nu}$ . Since one can define  $\Theta^{\mu\nu}$  for each parton, one can identify the momentum fraction and contributions to the OAM of each quark flavor and gluon in a hadron. Spin flip form factor  $B(q^2)$  which is the analog of the Pauli form factor  $F_2(Q^2)$  of the nucleon provides a measure of the OAM carried by each quark and gluon constituent of the nucleon at  $q^2 = 0$ . Similarly, the spin conserving form factor  $A(q^2)$ , the analog of Dirac form factor  $F_1(q^2)$ , allows one to measure the momentum fraction carried by each constituent. This is the underlying physics of Ji's sum rule:<sup>29</sup>

$$J_{q,g}^z = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)], \quad (5)$$

where  $B_{q,g}$  are the second moments of unpolarized spin-flip GPD in the forward limit. It is subject to the constraint that

$$B(0) = \sum_i B_i(0) = 0, \quad (6)$$

that is, when summed over all partons, spin flip form factor vanishes. For composite systems, this has been proven by Brodsky *et al.* in the light-cone representation.<sup>33</sup> In fact, it is a consequence of equivalence principle.

For the quark and gluon sectors, the above equation translates into

$$J_q^z(x) = \frac{1}{2}x[\langle q(x) \rangle + B_q(0)], \quad J_g^z(x) = \frac{1}{2}x[\langle g(x) \rangle + B_g(0)]. \quad (7)$$

Based on some lattice calculations and the model dependent analysis<sup>34</sup> it is expected that  $B_{q,g}$  would be small. In fact, lattice calculations show that the valence quarks give a value between  $-0.077$  and  $0.015$ .<sup>35</sup> Excluding the unlikely possibility for large value due to strange quark and antiquark, we find that the sum of the contributions from the sea quarks and the gluons must be small. We do realize the possibility that gluon and the sea quark contributions could be large, but with opposite sign. Also, they can be large, but have nodes such that their second moments become small. Lattice calculations<sup>36</sup> have verified that indeed the total anomalous gravitomagnetic moment of the nucleon is zero. A more recent lattice calculation<sup>47</sup> has shown that  $B_{q,g} = 0.00(6)$ . They have used a different notation, namely  $T_2(0)$ , and presented their results in Table III of their paper.<sup>47</sup>

Therefore, in the following analysis, we will set  $B_{q,g} = 0$ . With a zero value for  $B_{q,g}$ , the “mechanical orbital angular momentum” of partons,  $L_{q,g}$ , can be determined entirely from polarized and unpolarized parton distributions. Moreover, with such an assumption, the evolution equation for the angular momentum distributions  $J_{q,g}$  is exactly the same as that for the unpolarized quark and gluon distributions.<sup>37</sup> These distributions are evaluated by us in the valon model with good accuracy in a wide range of kinematics  $Q^2 = [0.4, 10^6]$  GeV<sup>2</sup> and  $x = [10^{-6}, 0.95]$  and will be utilized here. The details and the functional form of the unpolarized parton distributions in the valon representation can be found in Ref. 4. Table 1 shows the numerical results for  $\Delta\Sigma$ ,  $\Delta g$ ,  $L_q$ ,  $L_g$  in some values of  $Q^2$ .

Table 1. The numerical results for  $\Delta\Sigma$ ,  $\Delta g$ ,  $L_q$ ,  $L_g$  in the valon model.

$Q^2$	1.9 GeV <sup>2</sup>	5 GeV <sup>2</sup>	10 GeV <sup>2</sup>
$\Delta\Sigma$	0.38	0.38	0.38
$\Delta g$	0.303	0.440	0.516
$L_q$	0.0944	0.0723	0.0616
$L_g$	-0.0848	-0.275	-0.310

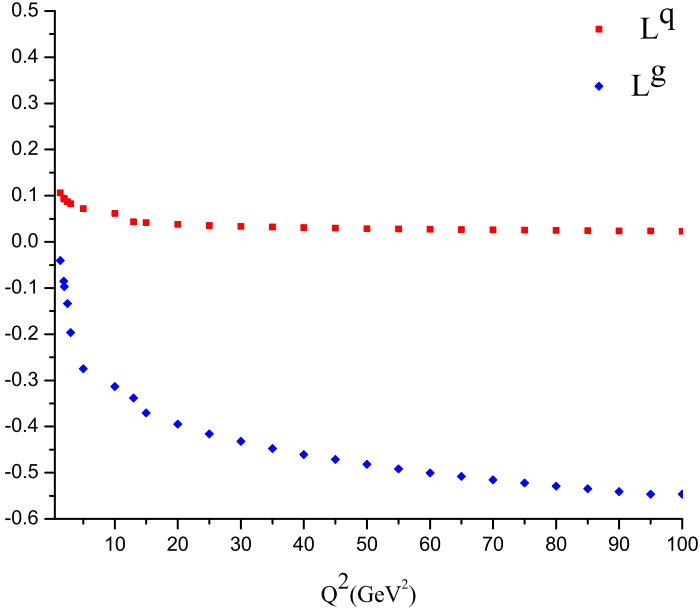


Fig. 3. Orbital angular momentum of quarks and gluons in the valon model as a function of  $Q^2$

In Fig. 3, we show the behavior of  $L_q(Q^2)$  and  $L_g(Q^2)$  at several  $Q^2$  values. It is apparent that while the quark OAM is small and positive, the gluon OAM is negative and decreases as  $Q^2$  increases.

We have checked to make sure that our results reproduce  $J^p = \frac{1}{2} = J_q + J_g$ . The results are shown in Fig. 4. Evidently, this is the case.

In Fig. 5, we present the gluon spin,  $\Delta g$ , the gluon OAM,  $L_g$  and the total angular momentum as a function of  $Q^2$ . This figure indicates that  $J_g$  is independent of  $Q^2$  and contributes an amount of about 0.22 to the nucleon spin.

In Fig. 6, the total angular momentum of individual quark flavors,  $J_u$  and  $J_d$ , is presented. In this figure, we also compare our results for  $J_{u,d}$  with those of Refs. 35, 38–44. We further found that the OAMs of  $u$ -quark,  $L_u$ , and  $d$ -quark,  $L_d$ , have opposite signs and largely cancel each other. Our results indicate that  $L_d$  is positive and  $L_u$  is negative. This is shown in Fig. 7 in comparison with those from Refs. 35 and 38. Their difference is shown in Fig. 8 and seems that the dependence on  $Q^2$  is marginal. In an interesting paper,<sup>38</sup> authors have derived a sum rule for spin-1 system through which they have obtained the total and the OAM for  $u$  and  $d$  quarks in the proton at  $Q^2 = 4 \text{ GeV}^2$ . Our results on total angular momentum carried by quarks, that is,  $J_q = J_u + J_d$  agree with the findings of Ref. 38, amounting to 0.26 at  $Q^2 = 4 \text{ GeV}^2$ . This is interesting because the two approaches are quite different. The two approaches also agree on the sign of  $L_u$  and within the errors, the numerical values are also compatible. Our findings, however, is different from those of Ref. 38 on the total and the OAMs of the  $d$ -quark.

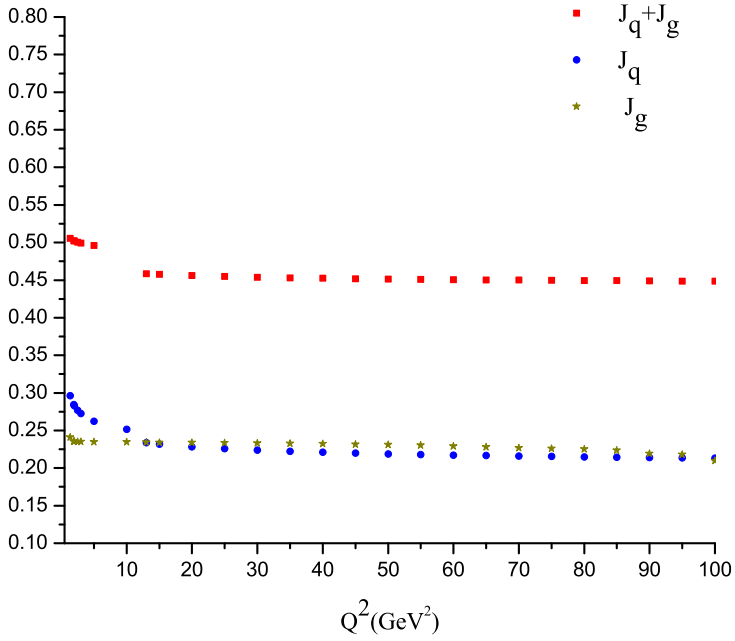


Fig. 4. Total angular momentum of quarks and gluons in the valon model.

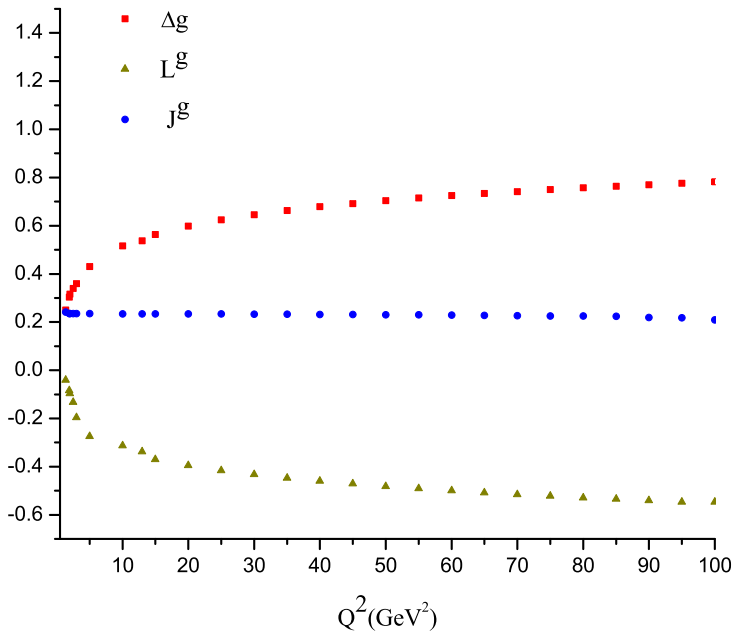


Fig. 5. Spin, OAM and the total angular momentum of gluons in the valon model.



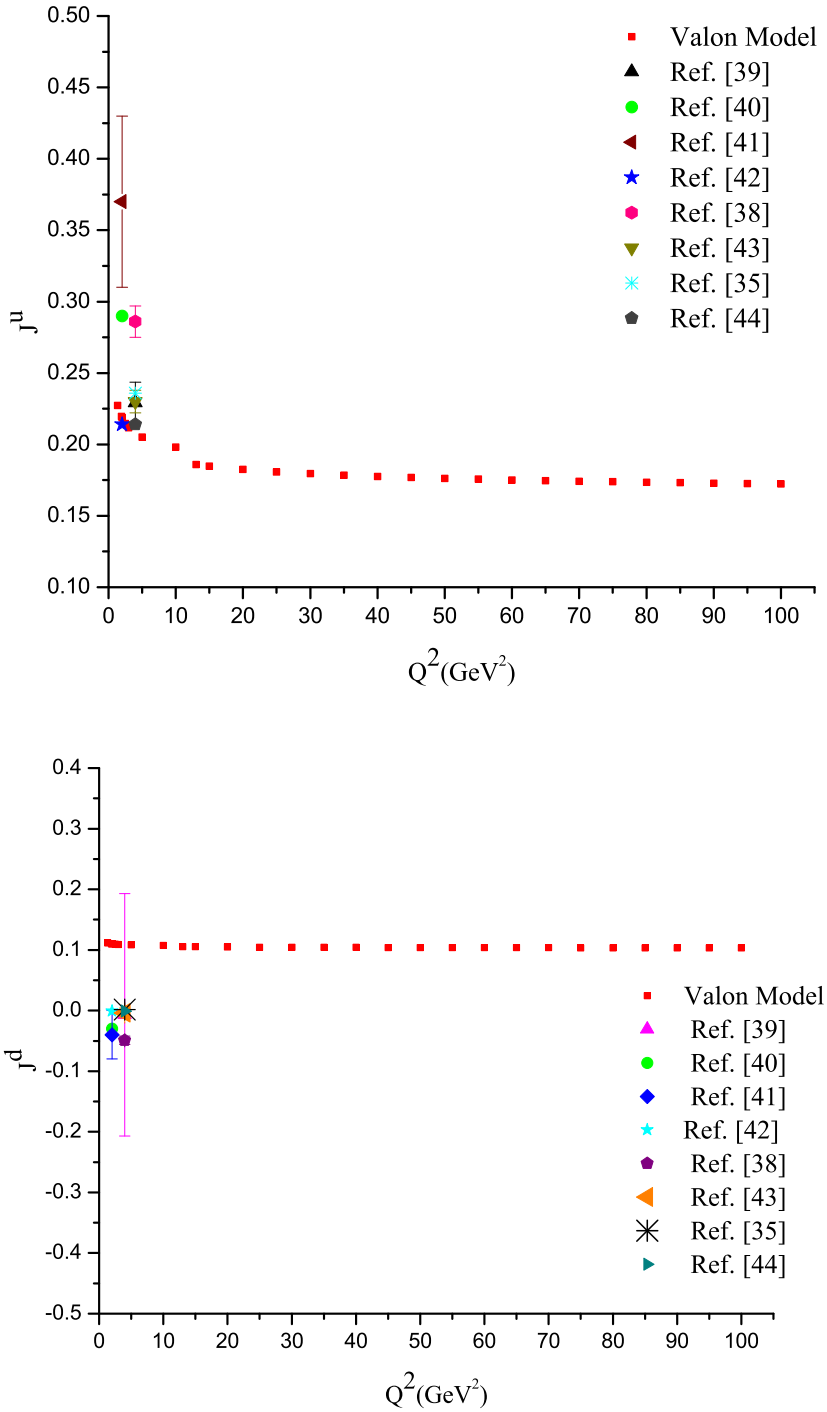


Fig. 6. Total angular momentum for  $u$ -quark and  $d$ -quark and comparison with other models.

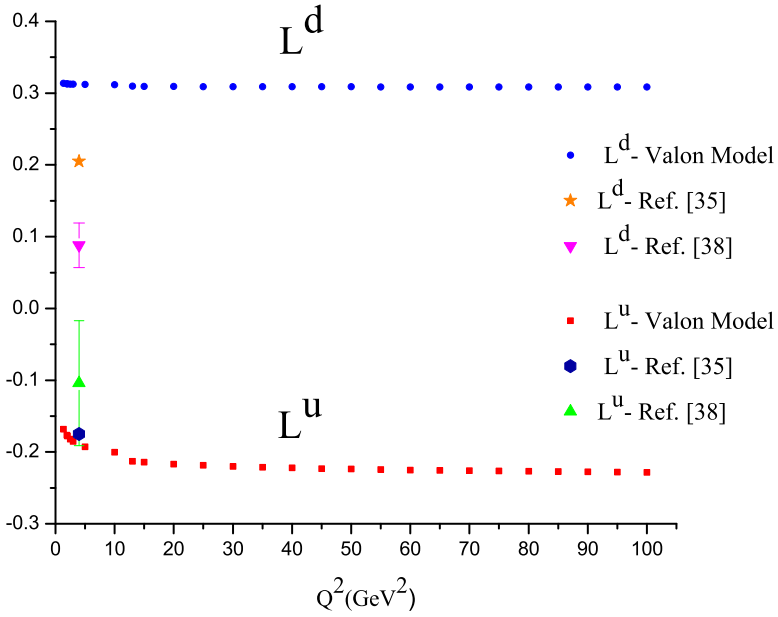


Fig. 7. Orbital angular momentum for  $u$ -quark and  $d$ -quark and comparison with other models.

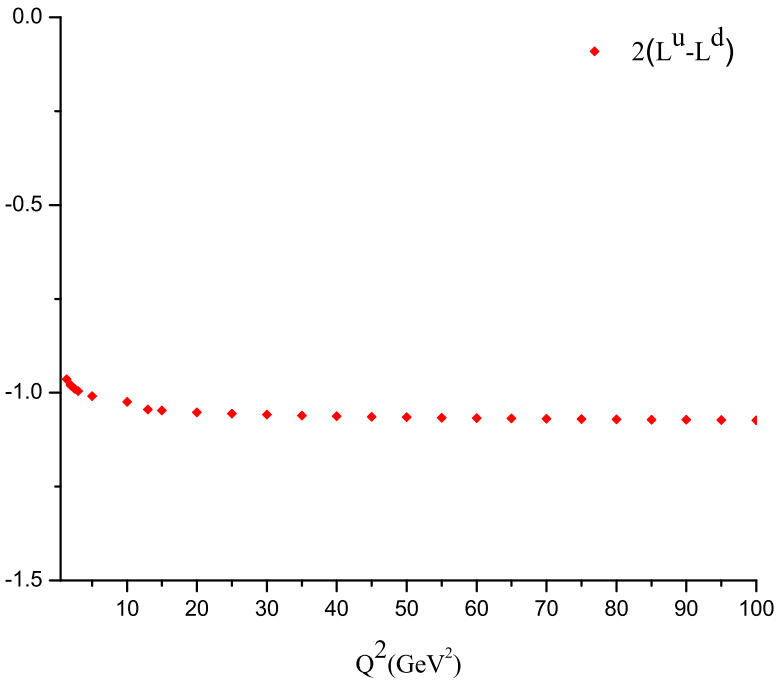


Fig. 8. The difference between OAM of the  $u$ -quark and  $d$ -quark.

Yet, both approaches produce compatible values for the spin component of the  $d$ -quark. The total quark OAM,  $L^Q = L_u + L_d$  in our model gives a value of 0.08 at  $Q^2 = 4 \text{ GeV}^2$ , whereas the result of Ref. 35 is  $-0.016 \pm 0.084$ . An earlier estimate of  $L^Q = 0.05\text{--}0.15$  is also given by Ji and Tang.<sup>29</sup> It is evident that within the quoted errors, the two values are not far apart. In fact, except for  $L_d$ , within the errors, our results are fairly close to those of Ref. 35. We find no crossover between  $L_u$  and  $L_d$  when  $Q^2$  is varied. We also find that  $L_u - L_d$  remains large and negative and this finding is in nice agreement with Ref. 48.

Finally, we note that  $Q^2$  dependence of  $J_g$  is marginal, and the interplay is between  $\Delta g$  and  $L_g$ , the former increases with  $Q^2$ , while the latter decreases. This is evident from Fig. 5. It is also interesting to mention that above  $Q^2$  around  $5 \text{ GeV}^2$  or so, the total angular momentum of quarks and the gluons seems to approach an identical value, indicating that they equally share the spin of the nucleon. This observation is manifestly apparent from Fig. 4.

#### 4. Conclusion

Although the valon representation of hadrons is a simple phenomenological model, it describes the structure of nucleon rather nicely. Within this model, we have investigated the OAM contribution of quarks and gluons to the nucleon spin. It shows that the quarks OAM contribution to the total angular momentum of the nucleon is positive and relatively small. However, the gluon OAM contributes substantially. Thus, we conclude that gluon is a major player in describing the spin structure of nucleon. On the one hand, while  $\frac{\Delta g}{g}$  is small, the first moment of the gluon polarization,  $\Delta G$ , is large and increases as  $Q^2$  grows. On the other hand, its OAM is large and negative, thus compensating the growth of  $\Delta G$ .

Some regularities also have emerged from our study: both orbital and total angular momenta of the  $u$ -quark and the  $d$ -quark seem to be independent of  $Q^2$ , though, some  $Q^2$  dependence for  $J_u$  is observed at low  $Q^2$ , but rapidly disappears. The same is true for the total angular momentum of the gluon, however, its OAM varies. These are evident from Figs. 7 and 5, respectively. Finally, we notice that our calculations seems to be compatible with those that are obtained in Ref. 35. We have also presented various OAM components as a function of  $Q^2$  which may be utilized to gain information on some GPDs.

#### Appendix A. The Valon Model

Our understanding of hadron structure comes from the deep inelastic data and the hadron spectroscopy. In the latter picture, hadrons are bound states of massive particles, loosely called “quarks” or “constituent quarks.” The bound states of those entities describe the static properties of the hadrons. On the other hand, the interpretation of the deep inelastic data relies on the quarks of the QCD Lagrangian with a very small mass. The hadronic structure in this picture is intimately connected with the presence of a large number of partons (quarks and gluons). The quarks

that participate in the bound state problem and the quarks of the QCD Lagrangian differ in other important properties as well. The very obvious example is the color charge of quark field in QCD Lagrangian, which is not gauge invariant and, thus, ill defined; reflecting the color of gluons in an interacting theory, whereas color associated with the quarks of a bound state (constituent quark) is a well-defined entity.

In the bound state problem, we regard a proton as consisting of three quarks and pion, a quark–antiquark pair. These are the constituent quarks. In deep inelastic scattering, a proton is viewed as having valence quarks, sea quarks and gluons, collectively called partons. In fact, even at  $Q^2$  as low as a few  $\text{GeV}^2$ , the gluons carry nearly half of the nucleon momentum. To reconcile the two pictures of hadron, it is necessary to realize that the quarks probed in deep inelastic scattering are current quarks of the QCD Lagrangian and not the constituent quarks of the bound state problem. The failure to recognize this difference can lead to many mistakes.

By definition, a valon is a structureful object consisting of a valence quark plus its associated cloud of sea quarks and gluons. The cloud or the structure of the valon is due to the dressing process in QCD. Indeed, it is shown<sup>45,46</sup> that one can dress a QCD Lagrangian field to all orders in perturbation theory and construct such an object (which we called a *valon*) in conformity with the color confinement. From this point of view, a valon emerges from the dressing of a valence quark with gluons and  $q\bar{q}$  pair in QCD. In a bound state problem, those processes are virtual and a good approximation for the problem is to regard a valon as an integral unit whose internal structures cannot be resolved. Thus, it may be identified as an indivisible, point-like object. As such, in a bound state problem they interact with each other in a way that is characterized by the valon wave function. On the other hand, in a scattering process, the virtual partons in a valon can be excited and put on mass-shell. They respond independently in an inclusive hard collision with a  $Q^2$  dependence that can be calculated in QCD at high  $Q^2$ . The point is that the valons play a dual role in hadrons: on the one hand, they are constituents of bound state problem involving the confinement at large distances. On the other hand, they are quasi-particles whose internal structures are probed with high resolution and are related to the short distance problem of current operators. This picture suggests that the structure function of a hadron involves a convolution of two distributions, namely, the valon distribution in the hosting hadron and the parton distribution in the valon. In an unpolarized situation, one may write the structure function of hadron  $h$  as follows:

$$F_2^h(x, Q^2) = \sum_{\text{valon}} \int_x^1 dy G_{\text{valon}}^h(y) F_2^{\text{valon}}\left(\frac{x}{y}, Q^2\right), \quad (\text{A.1})$$

where  $F_2^{\text{valon}}\left(\frac{x}{y}, Q^2\right)$  is the structure function of the probed valon and can be calculated in perturbative QCD. The function  $G_{\text{valon}}^h(y)$  represents the valon distribution in the hosting hadron carrying momentum fraction  $y$  of the hadron. It is  $Q^2$  independent. These functions are already calculated for a number of hadrons. Details for the proton can be found in Ref. 4.

Similarly, for a polarized hadron, we can write the polarized structure function  $g_1^h$  as

$$g_1^h(x, Q^2) = \sum_{\text{valon}} \int_x^1 dy \delta G_{\text{valon}}^h(y) g_1^{\text{valon}}\left(\frac{x}{y}, Q^2\right), \quad (\text{A.2})$$

where  $\delta G_{\text{valon}}^h(y)$  is the helicity distribution of valon in the hadron with momentum fraction  $y$  of the hadron.  $g_1^{\text{valon}}\left(\frac{x}{y}, Q^2\right)$  is the polarized structure function of the valon. Again, the detailed calculation of these functions are given in Ref. 1.

We have worked in  $\overline{\text{MS}}$  scheme with  $\Lambda_{\text{QCD}} = 0.22$  GeV. The initial scale of energy is  $Q_0^2 = 0.283$  GeV<sup>2</sup>. The motivation for this initial inputs at  $Q_0^2$  comes from the phenomenological consideration that requires us to choose the initial input densities as  $\delta(z - 1)$  at  $Q_0^2$ . This condition means that the internal structure of the valon cannot be resolved at  $Q_0^2$  and at this initial scale, the nucleon can be considered as a bound state of three valence quarks that carry all the momentum and the spin of the nucleon. Increasing the  $Q^2$  values resolved the other partons in the nucleon. Therefore, our initial input densities to solve the DGLAP equations inside the valon are

$$\delta q^{NS}(z, Q_0^2) = \delta q^S(z, Q_0^2) = \delta(z - 1), \quad (\text{A.3})$$

$$\delta g(z, Q_0^2) = 0. \quad (\text{A.4})$$

In the valon picture of hadron, the deep inelastic scattering with high enough  $Q^2$  actually is the structure of a valon that is probed. At low  $Q^2$ , the valon structure cannot be probed and hence behaves as a quark in the bound state problem. This means that if  $Q^2$  of the probe is less than a threshold value of  $Q_0$ , then a valon would appear as a constituent quark. Yet, from the early SLAC days of the deep inelastic scattering on proton, we know that quark distribution in a proton shows precocious scaling for  $Q^2$  in the range as low as one GeV<sup>2</sup>. That is,  $Q^2$  evolution has already run the course. For this reason, if  $Q^2$  is small enough, we may identify valon structure,  $F^{\text{valon}}(z, Q^2)$  as  $\delta(z - 1)$  at some point because we cannot resolve its internal structure at that  $Q^2$  value.

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