د چلدمین کنفرانس مل و دومین کنفرانس بین الللی شوحش بای کلابردی در مهندی برق، کلنگ و کلارونیک که توسط دانشکاه صنعتی ماک اثسر در مورخ ۶۶، بس ماه ۲۶۵۵ بدین وسیله کوابی می شود متاله با عنوان: مر سر المر على جلارشيدى كم اين شاله در ISC و CIVILICA نايه نواحد شد. توفق دوز افزون شارا در عرصه بلى على وابرايي كثور غزينان ايران آرزومنديم. ، میں کنوانی کنترل توزیح شده سیستم های چند عاملی با انداز گیری های نویزی بركزارشد، باحضور ايشان به صورت تخمراني ارانه كرديد. برنويندي باعلى كرميم يور، ميلادشاه ولى لوامينامه يذيرش وارانه مقاله المراجع ISC کد اختصاصی ۳۰۶۱۷۰۶۱۵ رکتر مصطفی مانی (ديرابزاني كخطبل Code: EMME16-10110211105 المجار من المحالية المسالحة المحالية المسالحة المحالية المح محالية المحالية محالية محالي WWW. EL WINEE . W D: 1.11



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Distributed Control for Multi-Agent Systems with Noisy Measurments

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ABSTRACT

In this paper a distributed leader-follower model-based control approach is proposed for a class of heterogeneous nonlinear multi-agent systems under directed graph. The dynamic of each follower agent is modeled by a first-order continues-time nonlinear state equation with process noises. The states are measured by the restriction of measurement sensors and the contamination of independent noises. A suitable observer to estimate unmeaured states is designed for each follower agent via employing bucy extended Kalman filter. Finally, the simulation results are provided to show the validity of the proposed control scheme.

Keywords: Multi-Agent systems, Distributed control, Process noise and noisy measurement, Bucy extended Kalman filter, Directed graph.

1. INTRODUCTION

In recent years, multi-agent systems (MASs) have been widely developed due to their various features such as reliability, robustness, redundancy and etc. As a result, the problem of MASs control has received much attention. MASs control can be studied in diffrent aspect such as distributed consensus [1] and [2], formation control [3], rendezvous [4], flocking [5] and so on. One of the essential problems in the distributed control design of MASs is to ensure that the states of all follower agents to reach an agreement based on neighbors' information which is the named *synchronization* or *consensus*.

Various control methodologies have been focused on the consensus problem of multiple systems with single-integrator or double-integrator dynamics [6] and [7]. Moreover, many interesting results via employing the graph theory and the matrix theory have been studied, [8], [9], and [10]. For linear forms, [11] investigated the problem of consensus for a general linear MASs. Further outcomes have been developed in [12] and [13] on consensus control for first- and second-order nonlinear MASs. Then, the distributed consensus controller was designed for network of Lagrangian systems in [14]. In most of the proposed control approaches with the distributed structure, it is assumed that all the states of follower agents are available for the measurement. However, this assumption is not appropriate in the realistic situations because of measurement sensors are restricted and the observation is taken with measurement noises. On the other hand, it should be noted that dynamics of the above mentioned controlled MASs are free of the process noise. As we know, in many real applications it is impossible to model agents without process noise. Hence, the investigations on the distributed control design for MASs with both of the independent measurement and process noises become one of the attractive research fields in control community.

Recently, some contributions have been made to deal with the distributed control design for MASs in presence of the measurement or process noises. In [15], a leader-less consensus of networked first-order linear multi-agent systems have been considered based on the directed graph where each follower agent has only noisy measurements of its neighbors' states. The mean square consensus of linear multi-agent systems with communication noises has been proposed in [16]. [17] investigated the average-consensus problem for first-order discrete-time multi-agent networks in stochastic communication noises. It is obvious that the proposed control approaches in [15] to [17], only investigated random communication environments. Moreover, the control approaches are only valuable for network of multi-agent systems without system functions, which restrict the applicability of the proposed control methodologies. On the other hand, [18] focuses on the leader-following consensus control problem of first-order continues-time multi-agent systems in random vibration environment by employing forth-order Lyaponov function. In [19], the control problem of distributed output tracking consensus was discussed for a class of high-order stochastic nonlinear multi-agent systems (multi-agent systems with process noise) under a directed graph topology. In [20], the distributed output tracking consensus in [18] to [20] are only considered the multi-agent systems with stochastic dynamics.

Motivated by the above-mentioned discussion, a distributed model-based controller for first-order continues-time multiagent systems with independent process and measurement noises has been studied by employing Bucy Extended



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Kalman Filter (BEKF). The BEKF technique provides a recursive optimal solution to the continues-time nonlinear filtering problem rooted in the state-space dynamical systems [21] and [22]. More recently, in [23] by emplying EKF, an adaptive fuzzy backstepping control was proposed for uncertain discrete-time systems with a set of noisy measurments. However, the control approach in [23] is established for single agent without considering gragh topology. The rest of this paper is organized as follows. In Section 2, we give some preliminary knowledge and the agent description is presented. In Section 3, the controller structure is proposed based on the BEKF, feedback linrarization techniqe. Then simulation results show the effectiveness of the proposed method in Section 4. Finally, this paper is concluded in Section 5.

2. PROBLEM FOURMULATION AND PRELIMINARIES

2.1. Gragh Theory

To solve the coordination problem and model the information exchange between agents, according to [2] a brief introduction of graph theory is presented here.

Let $G = \{v, E\}$ be a directed weighted graph of order N, $v = \{v_1, ..., v_N\}$ denotes the set of agents, $E \subseteq v \times v$ denotes the set of edges and $e_{ji} = (v_j, v_i) \in E$ if and only if there exists an information exchange from agent j to agent i. The adjacency matrix represents topology of directed graph as $A = [a_{ji}] \in R^{N \times N}$ and $a_{ji} > 0$ if $(v_i, v_j) \in E$; otherwise $a_{ji} = 0$. The value a_{ji} in adjacency matrix A associated with the edges e_{ij} denotes the communication quality from the *i*-th agent to *j*-th agent. Throughout this paper, it is assumed that $a_{ii} = 0$ and graph topology associated with communication among agents may not change over time. In other words, the adjacency matrix is time invariant. Laplacian matrix is defined as L = D - A, where $L \in R^{N \times N}$. $D = diag(d_1, ..., d_N)$ is the weighted degree of node j, where $d_j = \sum_{i=1}^N a_{ji}$. A directed graph has directed spanning tree if there exists agent called root such that a directed path from this agent to every other agents. Finally, define the leader adjutancy matrix as $B = diag(b_j) \in R^{N \times N}$, where $b_j > 0$ if only if *j*-th agent has access to leader information; otherwise $b_j = 0$.

2.2. Agent description and control problem

The *j*-follower agent can be described by a stochastic dynamic model with unknown nonlinear dynamics:

$$\dot{x}_j = f_j(x_j) + u_j + w_j, \qquad j = 1, ..., N,$$
 (1)

where $x_j \in \Re$ is the state vector of *j*-th follower agent and $u_j \in \Re$ is the control input of *j*-th follower agent. $f_j(\cdot): \Re \to \Re$ (j = 1, ..., N) is a known smooth nonlinear function which satisfies $f_j(0) = 0$. The *i*-th follower agent can receive information from its neighbors

$$y_j = x_j + v_{ij}, \qquad i \in N_j \tag{2}$$

where y_j denotes the measurement of the *j*-th agent's state x_j by the *i*-th agent; and $\{v_{ji}, i, j = 1, ..., N\}$ is the measurment (or communication) noise. It is assumed that w_j and v_{ij} are uncorrelated, zero-mean Gaussian noises with covariances

$$E\left\{w_{j}w_{j}^{T}\right\} = Q_{j}$$
$$E\left\{v_{ij}v_{ij}^{T}\right\} = R_{j}$$

where E denotes the mathematical expectation.

Control objective: the control objective is to design the distributed mobel-based controller for a network of the nonlinear follower agents (1) under both of measurmaent and process noises, such that all closed-loop network signals remain cooperatively bounded whereas consensus performance is satisfied.

Assumption 1 [8] The graph G consists of N follower agents and a leader, which contains a spanning tree rooted at the leader at all times.



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3. MAIN RESULTS

In this paper, two cases are considered for a network first-order nonlinear MASs: (i) the states of network system are safely available for the measurement; and (ii) the states of network system are not measurable accurately, namely the measurements of states are taken with process and measurement noises. For the both cases, our main targets are to study the problems of controller design. To these ends, at first, a feedback linearzation design procedure is employed to construct a distributed controller. Then, when states cannot be safely measured, the BEKF is constructed to estimate them and ensure the control performance.

3.1. Distributed controller development with accurate measurments

According to [6], define the distributed error surface for j^{th} the follower agent as:

$$z_j = b_j(x_j - x_0) + \sum_{i=1}^N a_{ji}(x_j - x_i),$$
(3)

where
$$a_{ji}$$
, b_j are definded in graph theory, z_j is the distributed error surface, and x_0 is the state of leader.
Now, by time differentiating (3), one gets

$$\dot{z}_{j} = b_{j}(\dot{x}_{j} - \dot{x}_{0}) + \sum_{i=1}^{N} a_{ji}(\dot{x}_{j} - \dot{x}_{i}).$$
(4)

Substituting (1) into (4), note to $d_j = \sum_{i=1}^{N} a_{ji}$ and L = D - A, we have

$$\dot{z}_j = (l_j + b_j)(u_j + f_j(x_j) - \dot{x}_0),$$
(5)

or

$$\dot{z} = (L+B)(u+f(x)-I\dot{x}_0),$$
 (6)

where $f(x) = [f_1(x_1), ..., f_N(x_N)]^T \in \mathbb{R}^N$, $z = [z_1, ..., z_N]^T \in \mathbb{R}^N$, $u = [u_1, ..., u_N]^T \in \mathbb{R}^N$, $I = [1, ..., 1]^T \in \mathbb{R}^N$, and L, B are definded in graph theory.

Design the control function as follows:

$$u_{j} = \frac{-c_{j}z_{j}}{(l_{j} + b_{j})} - f_{j}(x_{j}) + \dot{x}_{0},$$
(7)

or colectively

$$u = -(L+B)^{-1}Cz - f(x) + I\dot{x}_0,$$
(8)

where $C = diag(c_1,...,c_N)$ is a positice design matirx. Via (5) and (7), one has

$$\dot{z}_j = -c_j z_j. \tag{9}$$

Choose the Lyapunov function candidate as

$$V = \frac{1}{2} \sum_{j=1}^{N} z_j^2.$$
 (10)

Then, we have

$$\dot{V} = \sum_{j=1}^{N} -c_j z_j^2 < 0.$$
⁽¹¹⁾

Therefore, the closed-loop network system is cooprativly stabe.

3.2. Distributed controller development with noisy measurments

Due to noisy measurments, the distributed error surface (3) is modified as follows:

$$\hat{z}_j = b_j(\hat{x}_j - x_0) + \sum_{i=1}^N a_{ji}(\hat{x}_j - \hat{x}_i),$$
(12)

where \hat{z}_j is a new distributed error surface, and \hat{x}_j is the estimate of x_j which is obtained by BEKF, (13). For completeness, a brief review of the EKF theory [23], in its continues form, is summarized in this subsection.



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The bucy extended Kalman filter algorithm for (1) can be rewritten as follows:

$$\dot{\hat{x}}_{j} = F_{j}(\hat{x}_{j}) + u_{j} + K_{j}(y_{j} - \hat{x}_{j}),$$
(13)

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where $F_j(\hat{x}_j) = \frac{\partial f_j(x_j)}{\partial x_j} \bigg|_{x_j = \hat{x}_j}$ and K_j is the gain of BEKF observer and it is ontained by

$$\begin{cases} K_{j} = p_{j}(R_{j} + p_{j})^{-1}, \\ \dot{p}_{j} = F_{j}(\hat{x}_{j})(p_{j} - K_{j}p_{j})F_{j}(\hat{x}_{j}) + Q_{j}. \end{cases}$$
(14)

By time differentiating (12), we have

$$\dot{\hat{z}}_{j} = b_{j}(\dot{\hat{x}}_{j} - \dot{x}_{L}) + \sum_{i=1}^{N} a_{ji}(\dot{\hat{x}}_{j} - \dot{\hat{x}}_{i}).$$
(15)

Substituting (13) into (15) results in

$$\dot{\hat{z}} = (L+B)(u+F(\hat{x})+Ke-I\dot{x}_0),$$
(16)

where $e_j = y_j - \hat{x}_j$ for j=1,...,N is the observer error, $F(\hat{x}) = [F_1(\hat{x}_1),...,F_N(\hat{x}_N)]^T \in \mathbb{R}^N$, $u = [u_1,...,u_N]^T \in \mathbb{R}^N$, $e = [e_1,...,e_N]^T \in \mathbb{R}^N$, e = $..., e_N]^T \in \mathbb{R}^N, \quad \hat{z} = [\hat{z}_1, ..., \hat{z}_N]^T \in \mathbb{R}^N, \quad I = [1, ..., 1] \in \mathbb{R}^N, \text{ and } K = diag(K_1, ..., K_N) \in \mathbb{R}^{N \times N} \text{ is a positive design matirx.}$ Design the distributed control function as follows:

$$u_{j} = \frac{-c_{j}\hat{z}_{j}}{(l_{j} + b_{j})} - F_{j}(\hat{x}_{j}) - K_{j}e_{j} + \dot{x}_{0},$$
(17)

or colectivey

$$u = -(L+B)^{-1}C\hat{z} - F(\hat{x}) - Ke + I\dot{x}_0$$
(18)

where $C = diag(c_1, ..., c_N) \in \mathbb{R}^{N \times N}$ is a positice design matirx.

4. SIMULATON RESULTS

In this example, a gragh structure containing four follower agents (indicated by 1 up to 4) and one leader agent (indicated by 0) is shown in Fig. 1, and we can set $a_{ji} = 1$ on $(v_i, v_j) \in G$ and otherwise $a_{ji} = 0$, where j = 1, ..., 4, and i = 1, ..., 4. The dynamics of the each follower agent is described by the following equations:

$$\begin{cases} \dot{x}_{j} = f_{j}(x_{j}) + u_{j} + w_{j}, \\ y_{j} = x_{j} + v_{ji}, \end{cases}$$
(19)

where j = 1, ..., 4, i = 1, ..., 4 and $j \neq i$.

Nonlinear functions in the dynamics follower agents are chossen as $f_j(x_j) = -0.9x_j + x_j^2$, for j = 1, ..., 4. w_j is a Gaussian white noise (GWN) as a process noise with zero mean and variance 0.1, and v_{ji} is GWN as a measurment noise with zero mean and variance 1 for j = 1, ..., 4, i = 1, ..., 4 and $j \neq i$. We suppose that the initial conditins of the agents and BEKFs are selected as $[x_0(0), x_1(0), x_2(0), x_3(0), x_4(0)]^T = [0, 2, 1, -1, 1]^T$, and $[\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0), \hat{x}_4(0)]^T = [0, 2, 1, -1, 1]^T$ $[1, -1, 1, 2]^T$. Moreover, choose the design parameters as $c_1 = 20, c_2 = 10, c_3 = 10$, and $c_4 = 10$.

It is hoped that the proposed distributed controller (17) can steer the follower agents to keep leader-following consensus with a predesigned leader agent 0:



Fig. 1. Communication topology.



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Fig. 4. Bucy extended Kalman filters outputs.

The simulation results are shown in Figs. 2-4. Fig. 2 shows that the entire multiagent systems synchronize to the leader agent with both of process and measurment noises, moreover, it can be concluded that the state of each agent synchronizes to that of the leader despite only using the measured output information of each agent. The bounded control input corresponding to each follwer agent is shown in Fig. 3. Fig. 4 verifies that the estimated state of follower agent by Bucy extended Kalman filters also synchronizes to the leader.

5. CONCLUTIONS

This paper has been studied a distributed control design for a class of nonlinear MASs with process and measurment noises under directed graph. With the help of garph theory, matrix theory and feedback linearization technique, a distributed control scheme has been proposed for nonlinear dynamics of MASs. Moreover, BKEF has been used to eliminate undesirable effects of process and measurment noises. Finally, the simulation results have been provided to show the validity of the proposed control scheme.

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