Joint Image Registration and Super-Resolution Based on Combinational Coefficient Matrix

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Abstract—**In this paper we propose a new joint image registration (IR) and super-resolution (SR) method by combining the three principal operations of warping, blurring and down-sampling. Unlike previous methods, we neither calculate the Jacobian matrix numerically nor derive the Jacobian matrix by treating the three principal operations separately. We develop a new approach to derive the Jacobian matrix analytically from the combination of the three principal operations. Experimental results show that our method has better Peak Signal-to-Noise Ratio (PSNR) than the recently proposed Tian's joint method of IR and SR. Computational complexity also has been decreased in our proposed method.**

Keywords— Super-Resolution, Jacobian matrix, Combinational operation

I. INTRODUCTION

Super-resolution is a class of techniques that integrate the information of a Low Resolution (LR) image sequence captured from a scene to produce a High Resolution (HR) image with better quality. Each LR image frame is required to have partial unique information or it will not have any positive effect on the final HR image. This partial unique information can be obtained in a number of ways such as camera movement or zooming. There are various SR techniques which have been developed and reviewed in the literature including [1]-[4]. Generally SR techniques include the following three phases: 1) image registration, 2) image interpolation and 3) image deblurring and denoising [1]. In a small category of the techniques such as interpolation-based methods, these phases are performed separately [5]-[8]. To overcome the intensive presence of error propagation in these methods, a majority of other SR techniques attempt to perform the last two phases in an integrative phase called image reconstruction. However, an important source of error propagation is the inaccuracy of registration parameters. Therefore, to further prevent the propagation error, a large group of SR techniques have been developed recently that deal with the inaccuracy of registration parameters. Some of these techniques utilize median estimator to reduce the artifacts caused by errors and outliers of registration parameters [9], [10]. Some others use Bayesian methods in which the unknowns (including registration parameters) are treated as stochastic variables [11]-[13]. In Tipping's method [11], marginalization is applied to HR image but in Pickup's method [12], it is applied to registration and blurring parameters. Nevertheless, the latter provides a wide range of various priors (regularizations) to select, but in both of them, IR and image reconstruction are implemented in relatively separate steps without persistent interaction. It seems that such values are not reliable enough. Babacan [13] extends the Pickup's method to consider hyperparameters (such as the regularization parameter) as stochastic variables, and as an AM method, establishes persistent interaction between the estimation of the reconstructed HR image, registration parameters and hyperparameters. AM methods are a class of iterative SR techniques in which the HR image and registration parameters are improved in two consecutive steps at each iteration [14]. A group of AM methods use Expectation-Maximization (EM) to estimate the HR image (in the Expectation phase) and registration parameters (in the Maximization phase) iteratively [15],[16]. The AM methods, nevertheless, may lead to suboptimal solutions [17].

There is another category of SR techniques which are also iterative similar to AM methods, but in this category, HR image and registration parameters are not calculated separately at each iteration [18]-[21]. A nonlinear cost function was used by Chung et al. [18] to estimate HR image and registration parameters. Using Euler–Lagrange necessary conditions for the cost function, they derived a nonlinear system of equations, proposing three methods for its solution. Their first method (called decoupled) resembled an AM method, but their second method (called partially coupled) was a kind of Variable Projection (VP) method [17]. A similar method was also proposed by Robinson et al. [19] where they used a similar non-linear cost function to derive the Maximum Likelihood (ML)/ Maximum a Posteriori (MAP) solution for the HR image. After substituting this solution of HR image into the cost function, the reduced cost function [18] was minimized with respect to the reminder of unknowns, i.e. registration parameters. Finally, these registration parameters were used to obtain the final HR image. Chung et al. [18] attempted to solve the non-linear system of equations through Gauss-Newton algorithm in their third method. This led to the development of a new class of methods called fully coupled. In these techniques, which are referred to as joint methods in this paper, the incremental values of the HR image and registration parameters are jointly calculated in only one system of equations. He et al. [20] used a similar cost function by linearizing it at existing current values for HR image and

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registration parameters using Taylor series approximation. In this linearization, they obtained the Jacobian matrix analytically (in contrast to some methods like [11], [12] where it is calculated numerically). Finally, this linear system of equations (in terms of incremental values of HR image and registration parameters) has been solved through Conjugate Gradient (CG) optimization algorithm. They have used Euclidean motion model and it has been extended to the similarity motion model by Tian et al. [21].

In all of these joint methods [18]-[21] the blurring is assumed to be the same for all LR images, which is impractical in many applications. In reality, when the motion model is more complex than Euclidean, the size of blurring function is no longer the same for all LR images [22]. This has not been considered in [18] and [21], where the motion model has not been restricted to Euclidean. However, in the methods proposed in [20] and [21], the convergence to the global solution is more probable (especially when the initial values are close enough to optimal values) but the derivation of Jacobian matrix is based on bilinear interpolation of warped pixels [18], [20], [21]. This may introduce a restriction in which only four neighboring pixels are effective in determining the values of warped pixels [20].

As an inverse problem, SR dictates the three operations that should be applied to the original HR image to produce the corresponding LR images. These operations are image warping, image blurring and image down-sampling (hereafter referred to as principal operations). In many approaches, these steps are treated separately and in a few ones they are combined into a unit operation [2], [11], [12], [22]. This combination has some advantages including the possibility of incorporating the pixels in any arbitrary neighboring radius to obtain the warped pixels without changing the framework of problem and employing a new interpolation method. Another advantage is the reduced propagation error because the three principal operations are performed in one stage. Moreover, it allows having an adaptive kernel for blurring (blurring is treated as a function of zooming, which may be different for each LR image)

This paper focuses on the joint methods [20], [21] and treats the three principal operations in the inverse problem in a combinational form as [11]. Then, a new joint method based on this combinational form is proposed. In contrast to [11] and [12], the Jacobian matrix is not calculated numerically and unlike the common joint methods [20], [21], the Jacobian matrix is not derived by treating the three principal operations separately. In the proposed method, the bilinear interpolation is not used in the warp operation and its derivative. Moreover, the same blurring for all LR images [18] - [21] is not considered. We develop a new approach to derive the Jacobian matrix analytically based on the combinational form of the three principal operations. In this regard, a Gaussian kernel blur (as is more realistic) is adopted the radius of which is adaptive to each LR image. We also use a bilateral total variation (BTV) regularization [10], which incorporates the eight neighbors of each pixel in the cost function. In this paper, the similarity motion model is used (which consists of translation, rotation and zooming) similar to [21].

The rest of this paper is organized as follows. In Section II problem formulation including notation of SR problem, Gaussian kernel blur and combinational coefficient matrix, is introduced. The proposed iterative joint method is developed in Section III. In Section IV experimental results on simulated and real life image sequences are presented. Conclusion and future works are discussed in Section V.

II. PROBLEM FROMULATION

A. Super Resolution Notations

Let us consider a series of K discrete LR images g_k of size $M_g \times N_g$ where $1 \le k \le K$. The lexicographically ordered LR images are denoted by column vectors $\mathbf{g}_k$ and all these vectors are stacked in one column vector, i.e. $\mathbf{g} = [\mathbf{g}_1^{\mathrm{T}}, ..., \mathbf{g}_K^{\mathrm{T}}]^{\mathrm{T}}$. The purpose of the SR technique is to reconstruct the original HR image f of size $M_f \times N_f$ using existing LR images as well as some prior information about the original HR image. The lexicographically ordered original HR image is presented by column vector $\mathbf{f}$. Here, it is assumed that the decimation factor is the same in both vertical and horizontal directions ($\rho = M_f / M_g = N_f / N_g$). Each LR image $\mathbf{g}_k$ is obtained by applying the three principal operations to the original HR image $\mathbf{f}$ as follows:

$$\mathbf{g}_k = \mathbf{D}\mathbf{H}_k\mathbf{S}(\boldsymbol{\alpha}_k)\mathbf{f} + \mathbf{n}_k \tag{1}$$

Where $\mathbf{n}_k$ is the column vector of additive white Gaussian noise (AWGN), $\mathbf{D}$ is the down-sampling operator (which is realized as a $M_g N_g \times M_f N_f$ matrix), $\mathbf{H}_k$ is the blurring operator (which is realized as a $M_f N_f \times M_f N_f$ matrix) and $\mathbf{S}(\boldsymbol{\alpha}_k)$ is the warping operator (which is realized as a $M_f N_f \times M_f N_f$ matrix) [13],[20],[21]. $\boldsymbol{\alpha}_k$ is the vector of unknown registration parameters used for warping the grid of original HR image $\mathbf{f}$ (called reference grid) onto the up-scaled grid of k^{th} LR image. Practically one of the existing LR images (the first LR image in this paper) is selected as the reference image and the up-scaled grid of the reference image is considered as the reference grid. Generally, in all simultaneous IR and SR methods, including AM and joint methods, it is assumed that initial and imprecision values of registration parameters can be provided by some IR techniques. In this paper, Enhanced Correlation Coefficient (ECC) method [23] is used for the IR.

The combination of the three principal operations can be considered as a unit combinational operation which is realized by a matrix $\mathbf{W}^k(\boldsymbol{\alpha}_k)$ of size $M_g N_g \times M_f N_f$ as follows:

$$\mathbf{g}_k = \mathbf{W}^k(\boldsymbol{\alpha}_k)\mathbf{f} + \mathbf{n}_k \tag{2}$$

The similarity motion model has four degrees of freedom (zooming, rotation, vertical and horizontal translation). Hence, $\boldsymbol{\alpha}_k = [h_{1k}, ..., h_{4k}]$ where $h_{1k}, ..., h_{4k}$ are the main elements of the 3x3 homogenous matrix [24] as follows:

$$M_k = \begin{bmatrix} h_1^k & -h_2^k & h_3^k \\ h_2^k & h_1^k & h_4^k \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Equation (2) shows a series of relations between the original HR image and each LR image. These relations can be written in one equation as:

$$\mathbf{g} = \mathbf{Wf} + \mathbf{n} \quad (4)$$

where $\mathbf{W} = [\mathbf{W}^1(\boldsymbol{\alpha}_1)^{\mathrm{T}},...,\mathbf{W}^K(\boldsymbol{\alpha}_K)^{\mathrm{T}}]^{\mathrm{T}}$ and we call it combinational coefficient matrix and $\mathbf{n} = [\mathbf{n}_1^{\mathrm{T}},...,\mathbf{n}_K^{\mathrm{T}}]^{\mathrm{T}}$.

B. Blur with Gaussian Kernel

Consistent with the literature, in this paper Gaussian kernel, which is more realistic, has been used to model the blurring caused by the atmosphere turbulence and camera lens, and the motion blurring has been excluded. Usually, the blurring is assumed to be isotropic in the imaging plane. When the motion model is similarity, the kernel of back-projected blurring into the scene plane will be isotropic too. However, the greater the distance of a scene plane from the image plane (or less zoom is applied) the more extensive is the area encompassed in the scene to contribute in blurring. Therefore, when the LR images are registered to the reference image, they have isotropic Gaussian blur, but possibly with different radiuses in the reference grid [22].

C. Combinational Coefficient Matrix

The elements of i^{th} row of the $\mathbf{W}^k(\boldsymbol{\alpha}_k)$ are the coefficients of linear combination of $\mathbf{f}$ required to generate the gray scale value of i^{th} pixel of $\mathbf{g}_k$. The elements of $\mathbf{W}^k(\boldsymbol{\alpha}_k)$ are calculated as [2], [11],[12]:

$$W_{ij}^k(\boldsymbol{\alpha}_k) = \frac{\widetilde{W}_{ij}^k(\boldsymbol{\alpha}_k)}{\sum_{j'=1}^{M_f N_f} \widetilde{W}_{ij'}^k(\boldsymbol{\alpha}_k)} \quad (5)$$

where the new elements $\widetilde{W}_{ij}^k(\boldsymbol{\alpha}_k)$ are obtained as:

$$\widetilde{W}_{ij}^k(\boldsymbol{\alpha}_k) = \exp\{-\frac{1}{2\sigma^2|M_k|}\left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right)^{\mathrm{T}}\left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right)\} \quad (6)$$

here $\mathbf{v}_j = [v_j^x \quad v_j^y]^{\mathrm{T}}$ is the position of j^{th} pixel of the original HR image in the reference grid, $\mathbf{u}_i = [u_i^x \quad u_i^y]^{\mathrm{T}}$ is the position of i^{th} pixel of the LR image k and $\mathbf{s}_k(\mathbf{u}_i) = [s_k^x(u_i) \quad s_k^y(u_i)]^{\mathrm{T}}$ is its transformation through motion model, which is characterized by $\boldsymbol{\alpha}_k$, with respect to reference grid:

$$s_k^x(\mathbf{u}_i) = h_1^k u_i^x - h_2^k u_i^y + h_3^k \quad (7)$$

$$s_k^y(\mathbf{u}_i) = h_2^k u_i^x + h_1^k u_i^y + h_4^k \quad (8)$$

Actually $\mathbf{s}_k(u_i)$ is center of isotropic Gaussian kernel after projection to the reference grid. σ^2 is the variance of isotropic Gaussian kernel of the blur and $|\bullet|$ is determinant operator.

III. DEVELOPMENT OF NEW ITRATIVE JOINT METHOD OF SR

A. Cost Function of the new joint method

As super resolution is an ill-posed problem there are infinite or instable solutions which can satisfy Equation (4). Therefore, to make the solution unique and stable, prior information is necessary. In the joint methods, both original HR image and registration parameters are unknowns. Total variation (TV) of the HR image is an important regularization which is often used in SR techniques as prior information. For the registration parameters, a simple Tikhonov regularization, i.e. the minimum energy has been used. Given the generative model of LR images expressed in (4) and considering the mentioned regularizations, the framework of the cost function for the joint method is [18], [20], [21]:

$$E(\mathbf{f},\boldsymbol{\alpha}) = \|\mathbf{g} - \mathbf{Wf}\|^2 + \lambda T_V(\mathbf{f}) + \beta R(\boldsymbol{\alpha}) \quad (9)$$

where $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_1^{\mathrm{T}},...,\boldsymbol{\alpha}_K^{\mathrm{T}}]^{\mathrm{T}}$, λ and β are regularization parameters, $T_V(\mathbf{f})$ is TV of the HR image. Similar to Farsiu's [10] and Tian's [21] methods, BTV is used in our cost function, but here it encompasses all eight neighboring pixels to reduce edge penalization in any directions:

$$T_V(\mathbf{f}) = \sum_{\substack{m=-1 \\ |m|+|n|\neq 0}}^{1} \sum_{n=-1}^{1} \sum_{ij} \frac{|f(i+m,j+n) - f(i,j)|}{\sqrt{m^2+n^2}} \quad (10)$$

Because $T_V(\mathbf{f})$ is a nonlinear function of $\mathbf{f}$ using half-quadratic scheme [25] and fixed-point techniques [26] it can be written as:

$$T_V(\mathbf{f}) = \sum_{\substack{m=-1 \\ |m|+|n|\neq 0}}^{1} \sum_{n=-1}^{1} \sum_{ij} \frac{|f(i+m,j+n) - f(i,j)|^2}{\mu_{m,n}} \quad (11)$$

Where $\mu_{m,n}$ is calculated in the previous iteration as follows:

$$\mu_{m,n} = \sqrt{(m^2+n^2)\{(f(i+m,j+n) - f(i,j))^2 + \gamma} \quad (12)$$

and γ is small positive value to ensure $\mu_{m,n}$ is nonzero. $T_V(\mathbf{f})$ can be expressed in a matrix-vector form as [21]:

$$T_V(\mathbf{f}) = \mathbf{f}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{L}\mathbf{f} = \mathbf{f}^{\mathrm{T}}\mathbf{T}\mathbf{f} = \|\mathbf{L}\mathbf{f}\|^2 \quad (13)$$

$R(\boldsymbol{\alpha}) = \|\boldsymbol{\alpha} - \overline{\boldsymbol{\alpha}}\|^2$ is the Tikhonov regularization for registration parameters. $\overline{\boldsymbol{\alpha}}$ is a vector containing the average values of registration parameters during all previous iterations. Hence the cost function (9) can be expressed as:

$$E(\mathbf{f},\boldsymbol{\alpha}) = \|\mathbf{r}(\boldsymbol{\alpha},\mathbf{f})\|^2 + \|\sqrt{\lambda}\mathbf{L}\mathbf{f}\|^2 + \|\sqrt{\beta}(\boldsymbol{\alpha}-\overline{\boldsymbol{\alpha}})\|$$

$$= \left\| \begin{matrix} \mathbf{r}(\boldsymbol{\alpha},\mathbf{f}) \\ \sqrt{\lambda}\mathbf{L}\mathbf{f} \\ \sqrt{\beta}(\boldsymbol{\alpha}-\overline{\boldsymbol{\alpha}}) \end{matrix} \right\|^2 \tag{14}$$

where $\mathbf{r}(\boldsymbol{\alpha},\mathbf{f}) = \mathbf{g} - \mathbf{W}\mathbf{f}$ is called residual vector.

To estimate the unknown HR image and registration parameters, the cost function should be minimized. Although this optimization problem is convex with respect to $\mathbf{f}$, it is nonconvex in terms of $\boldsymbol{\alpha}$ because the term $\mathbf{r}(\boldsymbol{\alpha},\mathbf{f})$ is nonlinear with respect to $\boldsymbol{\alpha}$. To alleviate this difficulty, linear approximation has been used for $\mathbf{r}(\boldsymbol{\alpha},\mathbf{f})$. Linear approximation requires initial values for unknowns. As mentioned earlier, initial values for registration parameters may be obtained using registration techniques such as ECC algorithm [23]. Given these initial values for registration parameters, initial value for HR image can be obtained using a simple interpolation-based method [5]. If $\Delta\mathbf{f}$ and $\Delta\boldsymbol{\alpha}$ are incremental values for $\mathbf{f}$ and $\boldsymbol{\alpha}$ then $\mathbf{r}(\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha},\mathbf{f}+\Delta\mathbf{f})$ can be approximately linearized with respect to $\Delta\mathbf{f}$ and $\Delta\boldsymbol{\alpha}$ as follows:

$$\mathbf{r}(\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha},\mathbf{f}+\Delta\mathbf{f}) \approx \mathbf{r}(\boldsymbol{\alpha},\mathbf{f}) + \begin{bmatrix} \frac{\partial \mathbf{r}(\boldsymbol{\alpha},\mathbf{f})}{\partial \boldsymbol{\alpha}} & \frac{\partial \mathbf{r}(\boldsymbol{\alpha},\mathbf{f})}{\partial \mathbf{f}} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\mathbf{f} \end{bmatrix} \tag{15}$$

The two derivative terms in the above equation are calculated in the two following relations:

$$\frac{\partial \mathbf{r}(\boldsymbol{\alpha},\mathbf{f})}{\partial \boldsymbol{\alpha}} = \frac{\partial (\mathbf{g}-\mathbf{W}\mathbf{f})}{\partial \boldsymbol{\alpha}} = -\frac{\partial \mathbf{W}\mathbf{f}}{\partial \boldsymbol{\alpha}} = -\mathbf{J}(\boldsymbol{\alpha},\mathbf{f}) \tag{16}$$

$$\frac{\partial \mathbf{r}(\boldsymbol{\alpha},\mathbf{f})}{\partial \mathbf{f}} = \frac{\partial (\mathbf{g}-\mathbf{W}\mathbf{f})}{\partial \mathbf{f}} = -\mathbf{W} \tag{17}$$

Substituting (16) and (17) in (15) yields:

$$\mathbf{r}(\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha},\mathbf{f}+\Delta\mathbf{f}) \approx \mathbf{r}(\boldsymbol{\alpha},\mathbf{f}) - \mathbf{J}(\boldsymbol{\alpha},\mathbf{f})\Delta\boldsymbol{\alpha} - \mathbf{W}\Delta\mathbf{f} \tag{18}$$

Where $\mathbf{J}(\boldsymbol{\alpha},\mathbf{f})$ is the Jacobian matrix, the calculation of which presents a challenging issue, as will be discussed in the next section (III-B).

Substituting (18) in (14) and rewriting the content of the norm as a linear combination of the incremental values of unknowns leads to

$$E(\mathbf{f}+\Delta\mathbf{f},\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha}) = \left\| \begin{matrix} -\mathbf{r}(\boldsymbol{\alpha},\mathbf{f}) + \mathbf{J}(\boldsymbol{\alpha},\mathbf{f})\Delta\boldsymbol{\alpha} + \mathbf{W}\Delta\mathbf{f} \\ \sqrt{\lambda}\mathbf{L}\mathbf{f} + \sqrt{\lambda}\mathbf{L}\Delta\mathbf{f} \\ \sqrt{\beta}(\boldsymbol{\alpha}-\overline{\boldsymbol{\alpha}}) + \sqrt{\beta}(\Delta\boldsymbol{\alpha}) \end{matrix} \right\|^2$$

$$= \left\| \begin{pmatrix} \mathbf{J}(\boldsymbol{\alpha},\mathbf{f}) & \mathbf{W} \\ \mathbf{0} & \sqrt{\lambda}\mathbf{L} \\ \sqrt{\beta}\mathbf{I} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\mathbf{f} \end{pmatrix} + \begin{pmatrix} -\mathbf{r}(\boldsymbol{\alpha},\mathbf{f}) \\ \sqrt{\lambda}\mathbf{L}\mathbf{f} \\ \sqrt{\beta}(\boldsymbol{\alpha}-\overline{\boldsymbol{\alpha}}) \end{pmatrix} \right\|^2 \tag{19}$$

Now, instead of minimizing the nonlinear cost function in (14) with respect to the main unknowns, i.e. $\mathbf{f}$ and $\boldsymbol{\alpha}$, the linear cost function in (19) is minimized with respect to their incremental values, i.e. $\Delta\mathbf{f}$ and $\Delta\boldsymbol{\alpha}$ respectively [20]. The incremental values are used to update the main unknowns. Obtaining the optimal solution for the cost function in (14) is not guaranteed, but starting from initial values close to optimal values and continuing the optimization of the linear cost function in (19) may yield the optimal solution [20].

B. Proposed Method for Derivation of Jacobian Matrix $\mathbf{J}(\boldsymbol{\alpha},\mathbf{f})$

Jacobian matrix in the existing joint methods is derived from the three principal operations separately [18], [20], [21]. In this paper, however, we propose derivation of Jacobian matrix using combinational operation (5), and in contrast to [12], it is not calculated numerically, but rather analytically through our proposed method. As mentioned in (16) it can be written as follows:

$$\mathbf{J}(\boldsymbol{\alpha},\mathbf{f}) = \frac{\partial \mathbf{W}\mathbf{f}}{\partial \boldsymbol{\alpha}} = \frac{\partial \mathbf{W}\mathbf{f}}{\partial [h_1^1,\ldots,h_4^1,h_1^2,\ldots,h_4^K]} \tag{20}$$

$\mathbf{J}(\boldsymbol{\alpha},\mathbf{f})$ is a $KM_gN_g \times 4K$ block diagonal matrix [20] of matrices $\mathbf{J}(\boldsymbol{\alpha}_k,\mathbf{f})$ where

$$\mathbf{J}(\boldsymbol{\alpha}_k,\mathbf{f}) = \frac{\partial \mathbf{W}^k(\boldsymbol{\alpha}_k)\mathbf{f}}{\partial \boldsymbol{\alpha}_k} = \frac{\partial \mathbf{W}(\boldsymbol{\alpha}_k)\mathbf{f}}{\partial [h_1^k,\ldots,h_4^k]} \tag{21}$$

Naturally $\mathbf{J}(\boldsymbol{\alpha}_k,\mathbf{f})$ is a $M_gN_g \times 4$ matrix and the n[th] column of which can be obtained as follows:

$$\mathbf{J}_n(\boldsymbol{\alpha}_k,\mathbf{f}) = \frac{\partial \mathbf{W}^k(\boldsymbol{\alpha}_k)\mathbf{f}}{\partial h_n^k} = \frac{\partial \mathbf{W}^k(\boldsymbol{\alpha}_k)}{\partial h_n^k}\mathbf{f} \tag{22}$$

where $1 \le n \le 4$. Using the equations (5) and (6) each entry of matrix $\partial \mathbf{W}^k(\boldsymbol{\alpha}_k)/\partial h_n^k$ is calculated as follows:

$$\frac{\partial W_{ij}^k(\boldsymbol{\alpha}_k)}{\partial h_n^k} = \frac{\sum_{j'=1}^{M_fN_f}(\nabla_n Q_{i,j,k} - \nabla_n Q_{i,j',k})\widetilde{W}_{ij'}^k(\boldsymbol{\alpha}_k)}{\sum_{j'=1}^{M_fN_f}\widetilde{W}_{ij'}^k(\boldsymbol{\alpha}_k)} W_{ij}^k(\boldsymbol{\alpha}_k) \tag{23}$$

where the operator ∇_n is derivative with respect to h_n^k and

$$Q_{i,j,k} = -\frac{1}{2\sigma^{-2}|M_k|}\left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right)^{\mathrm{T}}\left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right) \tag{24}$$

therefore

$$\nabla_n Q_{i,j,k} = \frac{1}{\sigma^2 |M_k|} \nabla_n \mathbf{S}_k(\mathbf{u}_i)^{\mathrm{T}} \left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right) - \frac{\nabla_n |M_k|^{-1}}{2\sigma^2} \left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right)^{\mathrm{T}} \left(\mathbf{v}_j - \mathbf{S}_k(\mathbf{u}_i)\right) \quad (25)$$

Where $\nabla_n |M_k|^{-1}$ and $\nabla_n \mathbf{S}_k(\mathbf{u}_i)$ can be calculated simply using Equations (3),(7) and (8).

The proposed method can be implemented in the framework of the following algorithm:

Our Proposed Algorithm:

Step 1: Calculating initial values for registration parameters ($\hat{\boldsymbol{\alpha}}$) using ECC algorithm.

Step 2: Calculating initial value for HR image using Delaunay triangulation-based interpolation method of SR, given the LR images and estimated $\hat{\boldsymbol{\alpha}}$.

Step 3: (At the iteration i) Calculating the combinational coefficient matrix $\mathbf{W}(\boldsymbol{\alpha})$, Jacobian Matrix $\mathbf{J}(\boldsymbol{\alpha},\mathbf{f})$, residual vector $\mathbf{r}(\boldsymbol{\alpha},\mathbf{f})$ and BTV matrix $\mathbf{T}$ as discussed earlier.

Step 4: Solving the linear system of equations (26) using the Conjugate Gradient (CG) algorithm:

$$\begin{bmatrix} \mathbf{J}^{\mathrm{T}}\mathbf{J} + \beta\mathbf{I} & \mathbf{J}^{\mathrm{T}}\mathbf{W} \\ \mathbf{J}\mathbf{W}^{\mathrm{T}} & \mathbf{W}^{\mathrm{T}}\mathbf{W} + \lambda\mathbf{T} \end{bmatrix} \begin{pmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\mathbf{f} \end{pmatrix} = \begin{bmatrix} \mathbf{J}^{\mathrm{T}}\mathbf{r} - \beta(\boldsymbol{\alpha} - \bar{\boldsymbol{\alpha}}) \\ \mathbf{W}^{\mathrm{T}}\mathbf{r} - \lambda\mathbf{T}\mathbf{f} \end{bmatrix} \quad (26)$$

This linear system of equations is result of minimizing the cost function expressed in (19), which is calculated by taking derivative of the cost function with respect to desired unknowns and equating the result to zero.

Step 5: Updating the unknown variables using the estimated incremental values

$$\begin{pmatrix} \boldsymbol{\alpha} \\ \mathbf{f} \end{pmatrix}^{i+1} = \begin{pmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\mathbf{f} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\alpha} \\ \mathbf{f} \end{pmatrix}^{i} \quad (27)$$

Where $(\bullet)^i$ means column vector of unknowns in i^{th} iteration.

Step 6: Updating $\bar{\boldsymbol{\alpha}}$ according to the following relation:

$$\bar{\boldsymbol{\alpha}}^{i+1} = \frac{i\bar{\boldsymbol{\alpha}}^{i} + \boldsymbol{\alpha}^{i+1}}{i+1} \quad (28)$$

Step 7: If the following condition of HR image, (29), is satisfied for a specified threshold (Thr), which is assumed 10^{-6} here, or a maximum number of iteration is reached Then stop Else go to step 3.

$$\frac{\left\|\mathbf{f}^{i} - \mathbf{f}^{i-1}\right\|^2}{\left\|\mathbf{f}^{i-1}\right\|^2} < \mathrm{Thr} \quad 29)$$

III. EXPERIMENTAL RESULTS

In this section, the performance of our proposed method is discussed and compared it with a recently proposed joint method (Tian's method [21]). Because of the space limitations in this paper, we have provided only two experiments, i.e. a synthetic image sequence and a real-life images sequence. The synthetic sequence, which is produced by warping a test image through different motion parameters, is used to evaluate the performance of methods according to the following metrics: Normalized Mean Square Error (NMSE) for the estimated registration parameters vector and PSNR for the reconstructed HR image, which are defined as follows [20],[21]:

$$\mathrm{NMSE}(\tilde{\boldsymbol{\alpha}}) = 100 \frac{\left\|\boldsymbol{\alpha} - \tilde{\boldsymbol{\alpha}}\right\|^2}{\left\|\boldsymbol{\alpha}\right\|^2} \quad (30)$$

$$\mathrm{PSNR}(\tilde{\mathbf{f}}) = 10\log_{10}\left(\frac{M_f N_f}{\left\|\mathbf{f} - \tilde{\mathbf{f}}\right\|^2}\right) \quad (31)$$

where "~" denotes the currently estimated values of the unknowns. Lower values of NMSE and higher values of PSNR are preferred in a method.

A. Experimental Results on Degraded Test Images

The "Castle" test image [27] (Fig. 1(a)) is a 128x128 pixels image. Six LR images are produced by zooming factor, rotation angle in terms of degree and translation in both vertical and horizontal direction randomly chosen from the ranges [0.8,1.2], [-14,14] and [-2,2] respectively. Then they are blurred by an isotropic Gaussian kernel with a variance equal to 0.35 LR pixel size. These images are degraded by AWGN to have 30 dB SNR and finally are down-sampled by decimation factor of 2. One of the degraded LR images has been shown in Fig. 1(d). The reconstructed HR images along with their PSNR and the NMSE of estimated registration parameters using Tian's method [21] and the proposed method have also been indicated in Fig. 1. It can be observed, although the two methods have approximately the same NMSE, the PSNR of the proposed method has been improved. Nevertheless, the reconstructed image using the proposed method has better quality than Tian's method. This can be viewed in the areas located in the ellipses.

B. Experimental Results on Real-life Images

A sequence of four images (15^{th} frame to 18^{th} frame) has been extracted out of the "adyoron_rotation" video [28] (a video with a total number of 20 frames) for this experiment. The sequence contains 66x76 pixels images of a chart (Fig. 2(a)). The increasing factor (decimation factor) has been assumed equal to 2. The motion model between these images is Euclidian. As can be seen in these images, the relative rotation between these images is large (in contrast to the Tian's experiments reported in [21]). The reconstructed images using Tian's method and the proposed method have been indicated in Figs. 2(b) and 2(c). It is clearly observed that Tian's method produces some artifacts (white lines) in the margins of up-scaled registered images. This can be result of propagation errors between the three principal operations, when they are treated separately (especially in the margins of the LR images). Moreover, in this experiment, Tian's method is trapped into a suboptimal point. Bottom of the reconstructed image (inside the circle) shows a detailed view of this event.

In all of experiments, the run time of the proposed method is less than Tian's method. For example the run time of this experiment is 26s for the proposed method and 39s for Tian's method using DELL/Vostro notebook with 4 GB RAM and a 2.5 GHz dual core processor. The major reason for this computational complexity reduction of the proposed method is that the three principal operations are merged into one operation.

IV. CONCLUSUION AND FUTURE WORKS

In this paper we proposed a new joint method that combined the three principal operations in one operation. The application of this combinational operation in the calculation of Jacobian matrix is one of the most important contributions of this paper. The proposed joint method reduced propagation errors, was less likely to be trapped into suboptimal solutions, especially when the relative zooming between frames was considerable, and finally increased the quality of the reconstructed HR images. Additionally, the experiments showed the reduction of computational complexity in our method. In future works, we will extend the motion model to the affine and finally to the homography, and the size of blur kernel in the image plane will be refined during the iterations.

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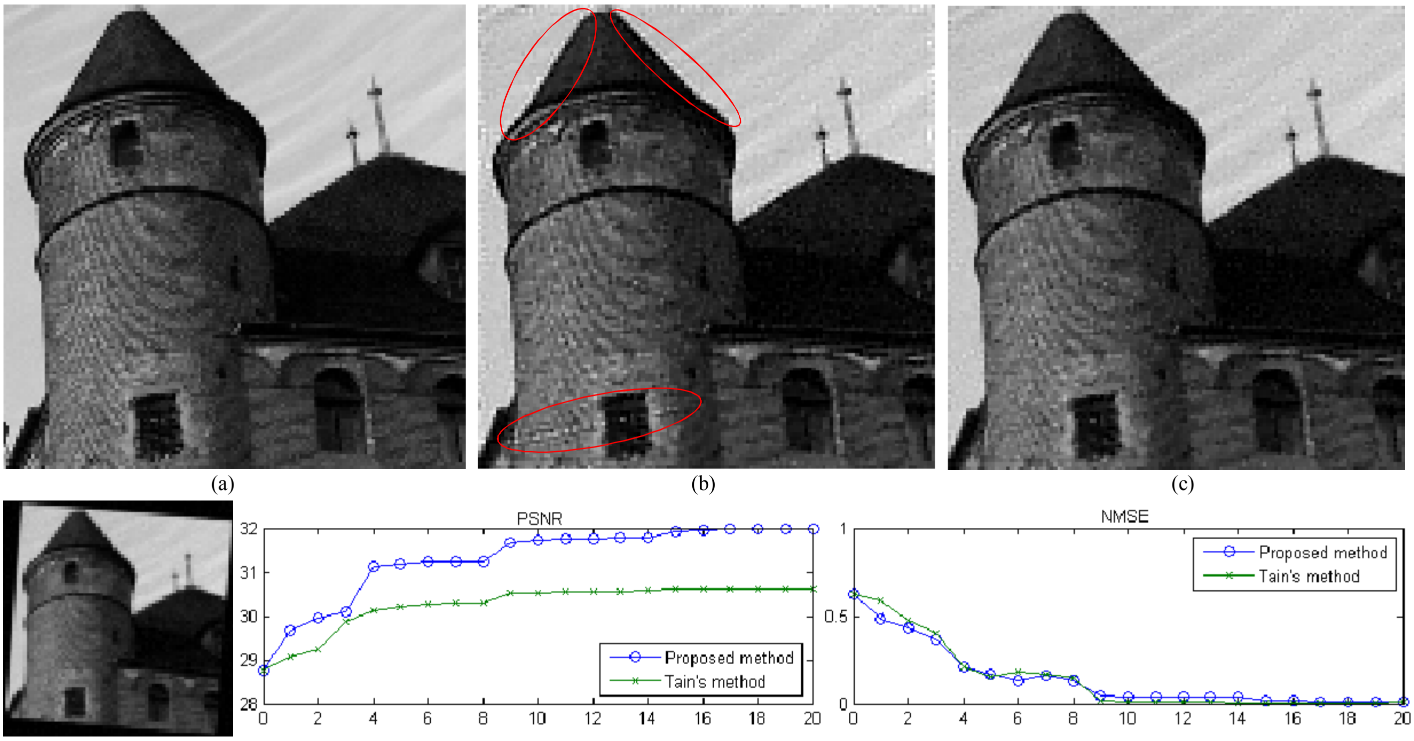
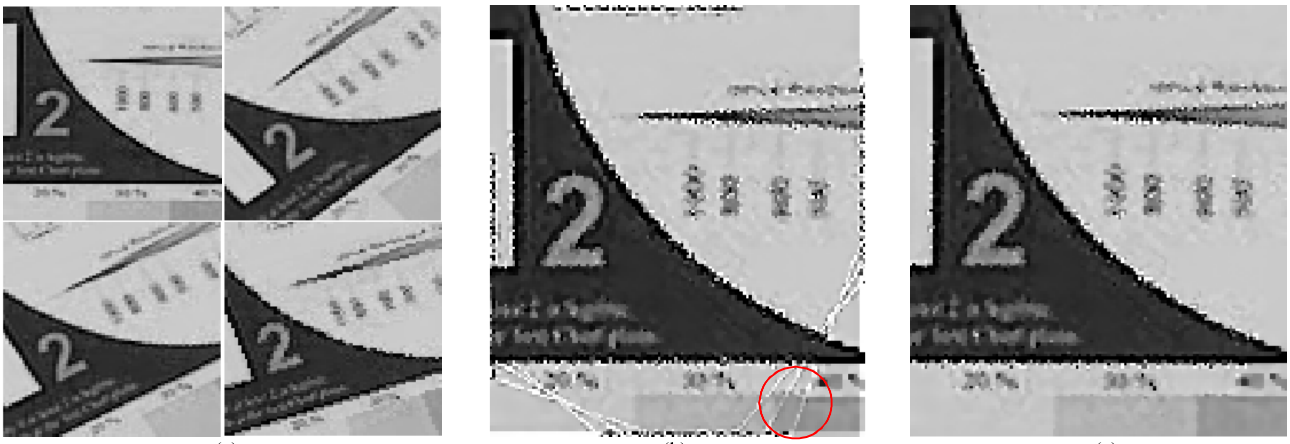


(a) (b) (c)

(d) (e) (f)

Fig. 1 (a) test image, (d) one of the degraded down-sampled images, (b) the reconstructed image using Tian's method, (c) the reconstructed image using our proposed method,(e) PSNR of the reconstructed images, (e) NMSE of the estimated registration parameters using Tian's method and our proposed method.



(a) (b) (c)

Fig. 2 (a) The sequence of real-life LR images, (b) the reconstructed image using the Tian's method, (c) the reconstructed using the proposed method.

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