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TECHNICAL PAPER

Optimum design of straight bevel gears pair using evolutionary algorithms

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Abstract Straight bevel gear is a type of gear which is widely used in mechanical systems to transmit power between perpendicular rotating axes. Designing straight bevel gears with the least possible volume is of great importance in industry since it results in a decrease in energy consumption and the material requirement in manufacturing. In this paper, employing two powerful optimization algorithms, simulated annealing algorithm (SA) and genetic algorithm (GA), techniques for advanced optimization, coupled with American Gear Manufacturers Association (AGMA) instructions the volume of straight bevel gears pair is minimized and the corresponding design variables are obtained. These variables include majors, including teeth number, module and face width. Using a traditional technique, recommended values of the design variables by AGMA, a design example was performed and the values were obtained. Then, the suggested techniques were utilized to get the values. The comparison between the results of all techniques shows that proposed optimization algorithms are considerably capable of minimizing the volume. It indicates that improvement in the attained volume varies between 1.56 and 17.40% for SA and 9.28 and 23.15% for GA.

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Keywords Straight bevel gear · Optimum design · Simulated annealing algorithm · Genetic algorithm · AGMA

List of symbols

A_{iG}	Gear inner cone distance (mm)
A_m	Mean cone distance (mm)
b	Face width (mm)
b_{ilp}	Pinion limit inner dedendum (mm)
b_{ip}	Pinion inner dedendum (mm)
b_p	Pinion mean dedendum (mm)
b_{min}/b_{max}	Lower/upper limit range of face width (mm)
d	Pitch diameter (mm)
e	Base of natural logarithm
E_i/E_{i-1}	Energy level of the system at current/previous position
$f(X)$	Equality constraint
$F(X)$	Objective function
$g(X)$	Inequality constraint
i	Number of steps in searching procedure
K_A	Overload factor
K_V	Dynamic factor
$K_{H\beta}$	Load distribution factor
K_θ	Temperature factor
k	Boltzmann constant
m	Module (mm)
m_{et}	Outer transverse module (mm)
n_1	Input speed (rpm)
P_i	Acceptance probability of the found solution
P_r	A random generated number ($0 \le P_r \le 1$)
Q_v	Transmission accuracy number
r	Cone pitch (mm)
S_H/S_F	Safety factor for contact/bending stress

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$T_i/T_{(i-1)}$	System temperature of the system at current/previous position
$V_{\text{straight bevel gear}}$	Volume of straight bevel gear (mm^3)
Vol	Total volume of pinion and gear
W^t	Transmitted load (N)
X	Vector of design variables
Y_{NT}	Stress-cycle factor for bending strength
Y_x	Reliability factor for bending strength
Y_z	Size factor for bending
Y_β	Length-wise curvature factor for bending strength
z	Teeth number
z_1/z_2	Teeth number of pinion/gear
Z_E	Elastic coefficient for pitting resistance ($[N/mm^2]^{0.5}$)
Z_I	Contact geometry factor
Z_J	Bending geometry factor
Z_{NT}	Stress-cycle factor for pitting resistance
Z_x	Size factor for pitting resistance
Z_{xc}	Crowning factor for pitting
Z_w	Hardness-ratio factor
Z_z	Reliability factor for pitting
α	Cooling rate of the system
γ	Pinion pitch angle (rad)
λ	Cone angle (rad)
$\sigma_{Hlim}/\sigma_{Flim}$	Allowable contact/bending stress number (N/mm^2)
δ_1/δ_2	Cone angle of pinion/gear (rad)
δ_G/δ_P	Gear/pinion dedendum angle (rad)
Ψ_{iG}	Inner gear spiral angle (rad)
φ	Normal pressure angle at pitch surface (rad)
φ_{Ti}	Inner transverse pressure angle (rad)

Abbreviations

AGMA	American Gear Manufacturers Association
GA	Genetic algorithm
min	Minimum
N	No
SA	Simulated annealing
TA	Traditional algorithm
Y	Yes

1 Introduction

Gears, a common way to transmit mechanical power between two axes, especially when there is a short distance between axes, are hired in an extensive range of systems such as transportation, automation and machine tools. The design method of these components has a considerable impact on performance and eventually the system operation. Complicated physical shape, numerous design variables, several tables, graphs, empirical formulas and factors that must be considered have made the design an extremely intricate art [1, 2]. This, alongside the demands for gears with higher quality, less cost and lower weight has done research on these types of machine elements as ongoing activities. Production of gears with minimum volume renders a significant decrease in energy consumption [3] for the consumer and it might cause a reduction in the production costs for the producer due to a reduction in the used materials. The need for lighter gears arises from the fact that the low volume of a gear means a lower weight. In other words, optimizing the volume results in a lower weight of the gear. To make the design more optimal, numerous methods, from traditional techniques basis to meta-heuristics ones, have been engaged by the researcher during recent decades. Modified Iterative Weighted Tchebycheff (MIWT) [4], quasi-Newton method [5], and interactive physical programming as conventional techniques were used to solve multi-objective spur gear deign [6]. From the latest application of these methods in this topic, Marjanovic et al. [7] developed a practical approach to optimize spur gears trains, or Golabi et al. [8] applied a non-linear programming to obtain parameters regarding to the optimum design of gear box. As far as evolutionary techniques of the optimization are considered in the traditional class of methods, a two-phase evolutionary algorithm was employed to optimize the design of a two-stage helical gear reducer [9]. Recently, in a similar manner, Padmanabhan et al. [10] applied selective breeding algorithm, as one of these types of techniques, to optimize the volume of a two-stage gear reducer. The risk to be trapped into local minima, slow convergence and the performance reduction, particularly when the number of design parameters increase or solution space extends are some weaknesses of these techniques. Furthermore, dealing with variables or constraints, which are discrete and hard to express as an explicit function, is another severe limitation in using these methods [11–14]. Thus, in recent years, modern stochastic optimization techniques, owing to the lack of stated disadvantages, have become more popular to get proper solutions of these kinds of complex design problems. In these kinds of techniques, genetic algorithm (GA) has been used more extensively by authors to deal with gears design problems. GA was employed to optimize spur gear design [15]. Later, Marcelin [16] evaluated it, as one of the mentioned techniques, to solve a multi-objective optimization problem of a gear pair design. The author demonstrated GA was able to find the solution with more efficiency in comparison with traditional ones. GA employed to optimize gears design by researchers in a number of papers such as [13, 15, 17–22]. From the most commonly applied methods, simulating annealing (SA) and particle swarm optimization (PSO) used in the design of spur gears [23]. However, as seen above, GA is utilized widely to solve design problems optimization; the insufficiencies of

premature convergence were reported as a possible weakness of the technique. The stated difficulty effects largely on search capability and eventually the performance of the algorithm. Similarly, PSO suffers also the same challenge [24, 25]. Moreover, in some applications under particular conditions such as problems in global structural optimization with multiple local optima, SA shows better robustness and global searching capability [26].

Since the production of gears with least possible volume results in reduction in considerable energy consumption and production cost, as mentioned before, it has been of great significance to achieve lighter gears. The gear volume has a direct relationship with its weight and minimizing the volume means minimizing the gear weight. For this reason, as states above, numerous studies have been done to optimize the gears volume. One of the types of gears, which are utilized widely in mechanical transmission systems, while the axes are intersected at 90°, is straight bevel gear. In the case of low speed and production cost, these types of gear become more desirable for power transmission between two intersected axes. Despite of the gear importance in diverse applications, the number of studies in optimizing the volume of this kind of gear is rare in the literatures.

In this paper, to optimize the volume of pair of straight bevel gears, an approach combining AGMA instructions and simulated annealing algorithm (SA) and genetic algorithm (GA), as two advanced random search tools, is developed. The aim of the approach is optimization of volume of the gears and obtaining the proper values of main design variables. The major design variables include module, teeth number and face width. The capability of the method is evaluated through a design sample. Comparison between the results of the proposed technique and traditional design reveals that the new techniques are able to improve the found solution from traditional technique between 1.56–17.40% for SA and 9.28–23.15% for GA. The traditional method is only based on AGMA suggested values of the major design variables.

The rest of the paper is organized as follows. In the second section, optimization algorithms are discussed. Modeling and optimizing the bevel gears pair is done in the third section. In section four, a designing example is considered and the paper is concluded in the fifth section.

2 Optimization methods

2.1 Simulated annealing algorithm

Simulated annealing (SA) algorithm, as one of the advanced optimization methods proposed by Kirkpatrick et al. [27], is a meta-heuristic and random search technique. The method is inspired by the annealing procedure used in material science and engineering. According to the process, a melted solid at high temperature is allowed to gradually and carefully cool down to room temperature within a planned timeline. As a consequence, the material reaches a complete, crystalline status with minimum energy and free of defect. In the process, if the cooling rate is too large or the initial temperature is not sufficient, it will not reach the minimum energy and local defects will arise.

SA begins with a randomly generated answer which satisfies all constraints of the problem. Then, one of effective parameters of the answer, which is also randomly selected, is randomly modified to produce another suitable solution in the near of the current solution. The new solution is compared to the previous one regarding the objective function. As long as it shows an improvement over the previous one, it will be kept. If not, probability of acceptance will be attained from (1). The mentioned equation depends on the current temperature, which is obtained from a certain cooling function, (2) as an example, as well as energy of the system or obtained answers. If the obtained probability is lower than a randomly generated number, it will be also accepted and maintained. Then, the previous answer is replaced by the kept one and new solution will be generated based on this new obtained solution. The sequence goes on generating new answer in the proximity of the replaced answer with until the condition for stopping the algorithm is met.

The mentioned probability function is in such a way that the acceptance probability of the new answers with no improvement over the previous answers will decrease with an increase in the number of iterations. The best answer from the deposited answers will be the final answer. Figure 1 shows the algorithm in a flow chart.

$$P_i = \min(1, e^{((E_i - E_{i-1})/kT_i)}), \quad i = 1, 2, 3, \tag{1}$$

Regulating parameters of the algorithm include initial temperature, type of cooling rate function and its parameters as well as stopping criterion. Among different proposed cooling rate functions, (2) has found more applicable than others. This equation is also used in this paper. Before changing in the temperature and acceptance probability, SA generally set an inner stopping circumstance. But in the algorithm presented here, this criterion did not considered in order to increase convergence speed.

For the stopping criterion, some criteria may be employed such as number of iterations, percentage of improvement of the answer in regards to the initial one and reaching to a certain execution time. Reaching to a specific iteration number as the stop criterion was used in this paper. Thus, in this research, the initial temperature, cooling rate and number of iterations are the regulating parameters of the algorithm. Increase in the initial temperature,

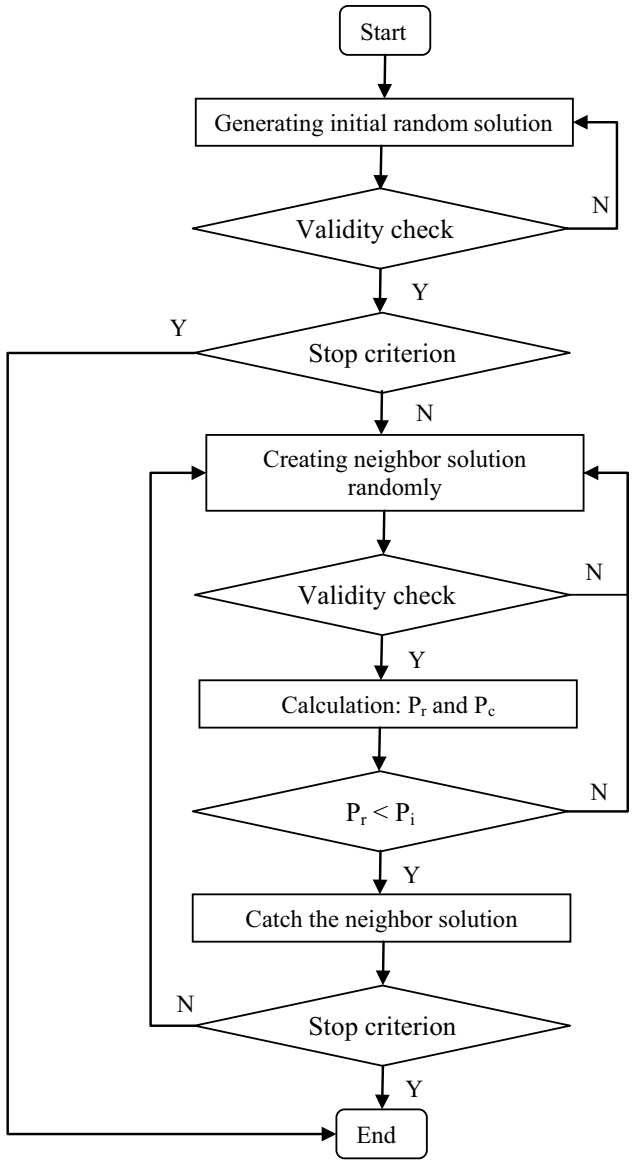

Fig. 1 Flowchart of simulated annealing algorithm

number of iterations and cooling rate result in the improvement of answers, but it will also increase the execution time. However, after a certain amounts, the mentioned rise in the amounts has no effects on the enhancement of the obtained solution and only leads to large execution time. The appropriate values for the above stated parameters are determined through trial and error. These values for each of the parameter have a significant effect on the quality of the obtained answer and also on the execution time.

$$T_i = \alpha T_{(i-1)}, \quad i = 1, 2, 3, \tag{2}$$

In this research, solutions are considered as vectors with three components. These elements are teeth number, module and face width. Moreover, objective function is volume of the straight bevel gears pair. At first, the designed algorithm generates a valid random solution with completely random elements. Since next solution has to be generated near the generated solution (neighbor solution), one of the elements of the solution is chosen randomly. Thus, it might be teeth number, module or face width. Then, value of the chosen component is replaced by another value randomly. After evaluation, if the next created solution is valid regarding the constraints, it will be the next solution and the algorithm builds another solution based on it. In terms of the regulating parameters, after several trials, the appropriate values for these parameters are obtained as below.

- No. of iterations: 1000,
- Initial temperature: 9×10^4,
- Cooling rate: 0.99.

2.2 Genetic algorithm

Genetic algorithm (GA) is a special type of evolutionary algorithm which is derived from the nature's principle of gradual evolution of the generations, Darwin's principle of natural selection; it has found numerous applications in the context of optimization problems such as GMAW welding [28] and cylindrical grinding [29]. According to this principle, next generations created from the previous generations possess members with improved genetic and evolved features.

To carry out GA, first each answer in the solution space is turned into a coded string of values of the variables (each member of the population) by considering the constraints of the problem. Each of the answer of the problem is called chromosome. Each of the variables in the complete string which comprises a chromosome is called a gene. This search method, unlike the SA procedure, evaluates the answers in a batch. It is done in this way that in each execution of the algorithm, a group of answers from the problem answers (generation) is produced. Then, according to their level of merit, which is a criterion for the goodness of each chromosome based on the objective function and the constraints of the problem, the answers are evaluated, ranked and the best answers are determined. According to the best answers, as long as the answer is acceptable, the algorithm stops and the answer is reported. If not, new generation is typically generated using selection, cross over and mutation operation. The flowchart of this algorithm is shown in Fig. 2. The assigned variables to implement the method in MATLAB are available which are the same as those of the simulated annealing algorithm. Table 1 shows parameters of GA utilized in this research. The objective function of this algorithm will be discussed in the next section.

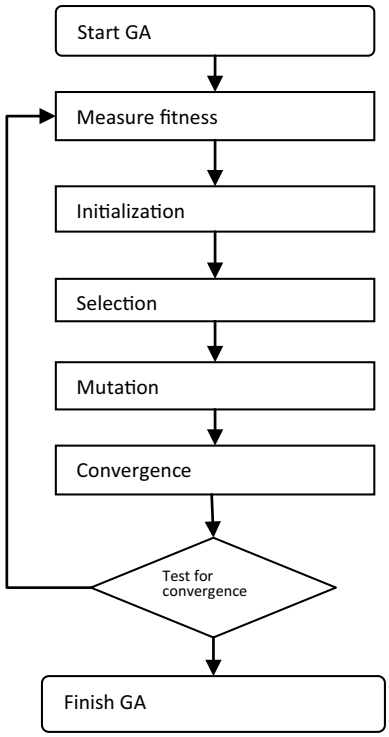

Fig. 2 Biological genetic algorithm process flow

2.3 Model formulation

Volume of pair of straight bevel gears pair, as shown in Fig. 3, is considered to be minimized using SA and GA. Then, values of design variables regarding the minimum volume are determined.

2.4 Objective function

To calculate the volume, accepting some simplicities, a formula which is suggested in [19] as shown in (3), is used. Employing (3), the objective function, will be obtained as (4).

$$V_{\text{straight bevel gear}} = (\pi/3)b\cos\lambda\Big[(mz/2)^2 + (mz/2)(((r-b)/2)(mz/2)) + (((r-b)/2)(mz/2))\Big]^2, \tag{3}$$

$$\begin{aligned} F(X):\ \text{Vol} = {} & (\pi/3)b\cos\delta_1[(mz_1/2)^2 \\ & + (mz_1/2)(((r-b)/2)(mz_1/2)) \\ & + (((r-b)/2)(mz_1/2))^2] \\ & + (\pi/3)b\cos\delta_2[(mz_2/2)^2 \\ & + (mz_2/2)(((r-b)/2)(mz_2/2)) \\ & + (((r-b)/2)(mz_2/2))^2]. \end{aligned} \tag{4}$$

Table 1 parameters used in current genetic algorithm process

Parameters	Value
Crossover function	Heuristic
Crossover fraction	1
Elite number	2
Initial penalty	10
Mutation function	0.1
Penalty factor	100
Population initial range	[−1,1]
Population size	100
Population type	Bit string
Selection function	Stochastic uniform

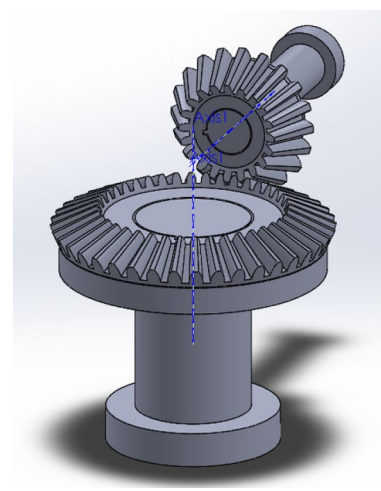

Fig. 3 Pair of straight bevel gears

2.5 Design variables

Design variables, which are gained from the optimization procedure, with upper and lower boundaries are listed in Table 2 in detail.

2.6 Constraints

According to AGMA instructions, upper limits of contact and bending stress as well as linear velocity of the gear on the outer diameter are required to be no excess permissible values. Therefore, the corresponding constraints define as (5)–(8).

$$g_1(X): (Z_E W^t K_A K_V K_{H\beta} Z_X Z_{XC})/(bdZ_I) \le (\sigma_{Hlim} Z_{NT} Z_W)/(S_H K_\theta Z_Z), \tag{5}$$

$$g_2(X): (W^t K_A K_V Y_X K_{H\beta})/(bm_{et} Y_\beta Z_J) \le (\sigma_{F\lim} Y_{NT})/(S_F K_\theta Y_Z), \tag{6}$$

$$g_3(X) : 5.236(10^{-6})mZ_1n_l \leq (50 + 56(1 - 0.25(12 - Q_v)^{2/3}) + (Q_v - 3))^2/200, \quad (7)$$

$$g_4(X) : b \leq \{0.3A_m,\ 10m_{et}\}. \quad (8)$$

All constants which are appeared in the above equations and inequalities are described in detail at [2, 30]. In the design of bevel gears, prevention of undercutting occurrence is considered as one of the most important factors. To taking this into account, based on [30], for the straight bevel gear, Eqs. (9)–(14) have to be considered. Furthermore, teeth number of pinion is planned to be twelve or higher to avoid the undercutting.

$$f_1(X) : A_{iG} = A_m - 0.5b, \quad (9)$$

$$f_2(X) : \Psi_{iG} = 0, \quad (10)$$

$$f_3(X) : \delta_p = \delta_G, \quad (11)$$

$$f_4(X) : \tan\varphi_{Ti} = \tan\varphi, \quad (12)$$

$$f_5(X) : b_{ilp} = (A_m - 0.5b)\tan\gamma\sin^2\varphi, \quad (13)$$

$$f_6(X) : b_{ip} = b_p. \quad (14)$$

3 Designing example

In the design of bevel gears pair based on AGMA instructions, a number of design parameters concerning working conditions and requirements of the design are necessary to be determined. Table 3 shows these parameters and corresponding values for a sample design.

Table 2 Design parameters and working conditions

Major design variables	Type	Explanations
Teeth number	Discrete	To be chosen from available values
Module	Discrete	To be chosen from available values
Face width	Continuous	To be chosen from available range ($b_{min} \leq b \leq b_{min}$)

Table 3 Design parameters and working conditions

Power to be transmitted (kW)	5	Reduction ratio	1:1
Input speed (rpm)	900	Life (number of cycles)	10^8
Load on prime mover	Medium shock	Load on driven machine	Medium shock
Working temperature (°C)	50	Reliability	0.99
Contact safety factor	1.2	Bending safety factor	1.2
Pressure angle	20°	Installation condition	Both members straddle-mounted
Material	Steel (grade II)	Gear/Pinion shape	properly crowned teeth
Gear teeth hardness (HB)	350	Pinion teeth hardness (HB)	350
Transmission accuracy number	11	Gear teeth form	full-depth, coniflex,
Maximum of face width (mm)	10	Maximum of face width (mm)	26.1
Available modules (mm)	1, 1.125, 1.25, 1.375, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3, 3.5, 4, 4.5, 5, 5.5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 25, 28, 32, 36, 40, 45, 50		
Available teeth number for gear/pinion	15, 20, 25, 30, 35,40, 45, 50, 60, 70, 80, 90, 100		

Table 4 The comparison between SA, GA and TA

Transmitted power (kW)	5			10.65			30.65		
Design Variables	TA	SA	GA	TA	SA	GA	TA	SA	GA
Teeth number	25	25	20	30	20	15	35	30	20
Module (mm)	3.5		2.75	4.5	4.5	5.5	5.5	5.5	5.5
Face width (mm)	19.2617	18.5555	27.33	23.1277	22.8520	26.6542	26.9823	38.2841	46.0279
Volume (dm^3)	69.813	68.718	63.341	206.87	170.89	158.97	502.07	453.81	453.621
% improvement	–	1.5685	9.2705	–	17.3926	23.1546	–	9.6122	9.6498

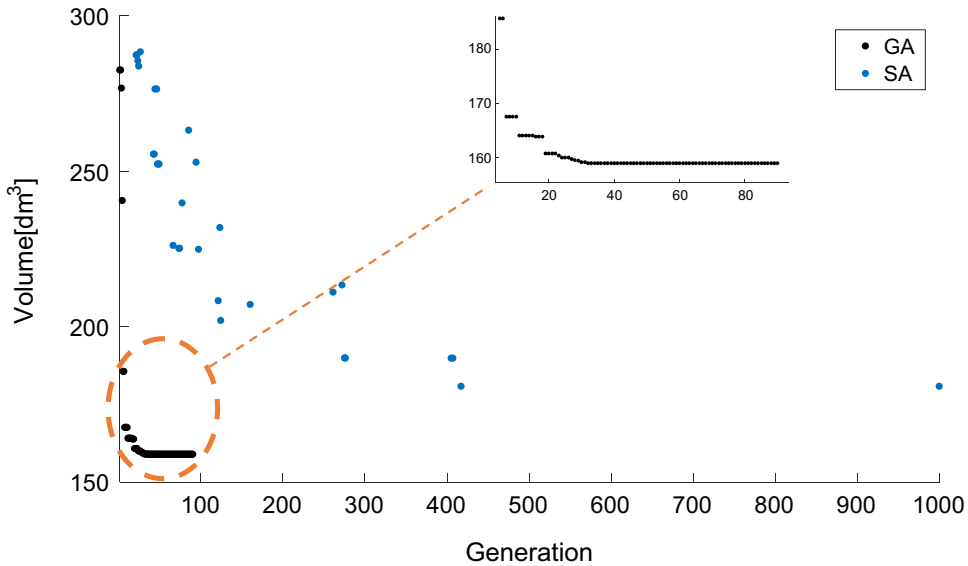

Fig. 4 Convergence curve of the proposed algorithms for 10.65 kW power

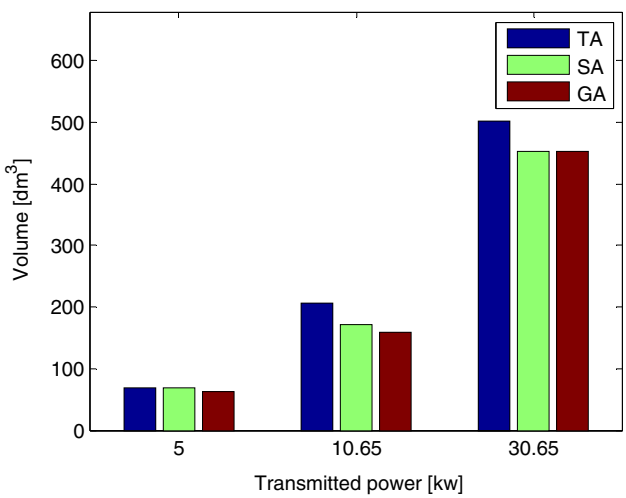

Fig. 5 Optimized gears volume compared with traditional, simulated annealing and genetic algorithms

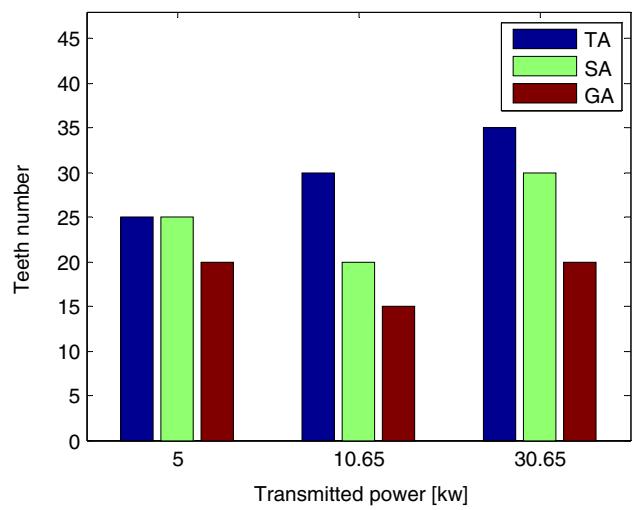

Fig. 6 Optimized gear teeth number compared with traditional, simulated annealing and genetic algorithms

In order to study capability of the simulated annealing algorithm and genetic algorithm in minimizing the volume of the straight bevel gears, the comparisons between obtained results of the two proposed algorithms and the traditional design centered on the AGMA are presented in Table 4. The table illustrates the effect of changes in transmitting power on the calculated volume of the above sample. Using the parameters written in Table 3, execution iteration of the two algorithms is shown in Fig. 4 which demonstrate the convergence curves. According to this table, SA and GA improve considerably the found volume. To show an informative comparison between three traditional, simulated annealing and genetic algorithms, four graphs were drawn in Figs. 5, 6, 7 and 8 which show optimized volume, teeth number, module and face width, respectively.

As shown in Fig. 5 and Table 4, genetic algorithm is able to improve gear volume to 9.27% for 5 kW, 23.15% for 10.65 kW and 9.65% for 30.65 kW while simulated annealing could minimize the gears volume from 1.56 to 17.39% in all of its simulations. Genetic algorithm used a decreasing trend for the number of teeth. However, it increased the face width in all of transmitted power, especially in the 30.65 kW where the face width was achieved to 46 mm. Gear module decreased in 5 kW, increased in 10.65 kW and did not change in 30.65 kW.

4 Conclusion

Volume of pair of straight bevel gears was minimized using two powerful optimization algorithms, simulated

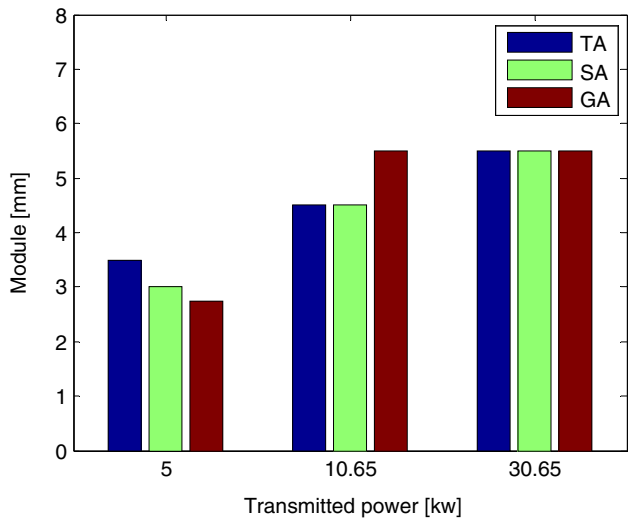

Fig. 7 Optimized Gear module compared with traditional, simulated annealing and genetic algorithms

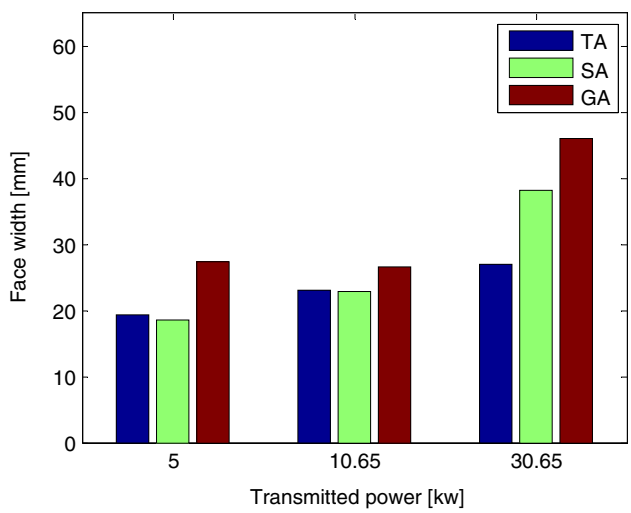

Fig. 8 Optimized gear face width compared with traditional, simulated annealing and genetic algorithms

annealing and genetic algorithm. The results found through a design example showed that the gears volume which is obtained by simulated annealing and genetic algorithm is 68.718 and 63.341 dm^3, respectively. While the corresponding obtained volume utilizing traditional technique is 69.813 dm^3. Thus, GA had a better performance from SA. From the corresponding major design variables, although the attained teeth number of pinion utilizing SA and GA is generally smaller than TA, SA proposed smaller or equal values of module in comparison with TA. In terms of face width, GA suggested bigger values. Change in the power transmission of the design example demonstrates that the volume reduction utilizing the algorithm varies between 9.2705–23.1546% for GA and 1.56–17.39% for SA. Thus, the proposed algorithms are useful tools to optimize considerably the volume of this type of gears. The mentioned optimization will result in a reduction in the material requirement of manufacturing. Moreover, it will enhance the energy efficiency of the system which contains these kinds of gears due to the decrease in the gears weight.

References

1. Radzevich SP (2012) Dudley's handbook of practical gear design and manufacture, 2nd edn. CRC Press, Boca Raton
2. Budynas RG, Nesbit JK (2011) Shigley's mechanical engineering design, 9th edn. McGraw-Hill, New York **[Part 13 (624-732) and part 15 (786-824)]**
3. Vahabi H, Panahi MS, Shirazinezhad RP, Imenabadi A (2015) A neuro-genetic approach to the optimal design of gear-blank lightening holes. J Braz Soc Mech Sci Eng. doi:10.1007/s40430-015-0362-0
4. Wang H, Wang HP (1994) Optimal engineering design of spur gear set. Mech Mach Theory 29:1071–1080. doi:10.1016/0094-114X(94)90074-4
5. Thompson DF, Gupta S, Shukla A (2000) Tradeoff analysis in minimum volume design of multi-stage spur gear reduction units. Mech Mach Theory 35:609–627. doi:10.1016/S0094-114X(99)00036-1
6. Huang H, Tian Z, Zuo MJ (2005) Multiobjective optimization of three-stage spur gear reduction units using interactive physical programming. J Mech Sci Technol 19(5):1080–1086. doi:10.1007/BF02984029
7. Marjanovic N, Isailovic B, Marjanovic V, Milojevic Z, Blagojevic M, Bojic M (2012) A practical approach to the optimization of gear trains with spur gears. Mech Mach Theory 53:1–16. doi:10.1016/j.mechmachtheory.2012.02.004
8. Golabi S, Fesharaki JJ, Yazdipoor M (2014) Gear train optimization based on minimum volume/weight design. Mech Mach Theory 73:197–217. doi:10.1016/j.mechmachtheory.2013.11.002
9. Tudose L, Buiga O, Stefanache C, Sobester A (2010) Automated optimal design of a two-stage helical gear reducer. Struct Multidisc O 42:429–435. doi:10.1007/s00158-010-0504-z
10. Padmanabhan S, Raman VS, Chandrasekaran M (2014) Optimisation of gear reducer using evolutionary algorithm. Mater Res Innov 18(6):378–382. doi:10.1179/1432891714Z.000000000983
11. Yang X (2008) Introduction to mathematical optimization—from linear programming to mataheuristics. Cambridge International Science Publishing, Cambridge
12. Zhao S, Li J, Zhang C, Zhang W, Lin X, He X, Yao Y (2015) Thermo-structural optimization of integrated thermal protection panels with one-layer and two-layer corrugated cores based on simulated annealing algorithm. Struct Multidisc O 51:479–494. doi:10.1007/s00158-014-1137-4
13. Mendi F, Baskal T, Boran K, Boran FE (2010) Optimization of module, shaft diameter and rolling bearing for spur gear through genetic algorithm. Expert Syst Appl 37:8058–8064. doi:10.1016/j.eswa.2010.05.082
14. Bangert P (2012) Optimization for industrial problems. Springer, Berlin doi: 10.1007/978-3-642-24974-7
15. Yokota T, Taguchi T, Gen M (1998) A solution method for optimal weight design problem of the gear using genetic

algorithms. Comput Ind Eng 35:523–526. doi:10.1016/S0360-8352(98)00149-1
16. Marcelin JL (2001) Genetic optimization of gears. Int J Adv Manuf Tech 19:910–915. doi:10.1007/s001700170101
17. Fonseca DJ, Shishoo S, Lim TC, Chen DS (2005) A genetic algorithm approach to minimize transmission error of automotive spur gear sets. 19(2):153–179. doi:10.1080/08839510590901903
18. Gologlu C, Zeyveli M (2009) A genetic approach to automate preliminary design of gear drives. Comput Ind Eng 57:1043–1051. doi:10.1016/j.cie.2009.04.006
19. Zhang X, Rong Y, Yu J, Zhang L, Cui L (2011) Development of optimization design software for bevel gear based on integer serial number encoding genetic algorithm. JSW 6:915–922. doi:10.4304/jsw.6.5.915-922
20. Dhafer G, Jérôme B, Philippe V, Michel O, Mohamed H (2012) Robust optimization of gear tooth modifications using a Genetic Algorithm. In: Haddar M (ed) Design and modeling of mechanical systems. Springer, Heidelberg, pp 189–197
21. Blanco JC, Gobbi M, Muñoz LE, (2014). Gear train optimization of a hybrid electric off-road vehicle. In: International design engineering technical conferences and computers and information in engineering conference, Buffalo, New York, USA, 1–9
22. Tudose L, Buiga O (2014) Optimal mass minimization design of a two-stage coaxial helical speed reducer with Genetic Algorithms. Adv Eng Soft 68:25–32. doi:10.1016/j.advengsoft.2013.11.002
23. Savsani V, Rao RV, Vakharia DP (2011) Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms. Mech Mach Theory 45:531–541. doi:10.1016/j.mechmachtheory.2009.10.010
24. Shi Y, Eberhart R (1998) A modified particle swarm optimizer. In: IEEE world congress computational intelligence, Anchorage, Alaska, pp 69–73
25. Shieh H, Kuo C (2011) Modified particle swarm optimization algorithm with simulated annealing behavior and its numerical verification. Appl Math Comput 218:4365–4383. doi:10.1016/j.amc.2011.10.012
26. Zhao S, Li J, Zhang C, Zhang W, Lin X, He X, Yao Y (2015) Thermo-structural optimization of integrated thermal protection panels with one-layer and two-layer corrugated cores based on simulated annealing algorithm. Struct Multidisc Optim 51:479–494. doi:10.1007/s00158-014-1137-4
27. Kirkpatrick S, Gelatt CD, Vecchi MP (1983) Optimization by simulated annealing. Science 220(4598):671–680
28. Correia DS, Gonçalves CV, Sebastião S, Junior C, Ferraresi VA (2004) GMAW welding optimization using genetic algorithms. J Braz Soc Mech Sci Eng. doi:10.1590/S1678-58782004000100005
29. Thiagarajan C, Sivaramakrishnan R, Somasundaram S (2012) Modeling and optimization of cylindrical grinding of Al/SiC composites using genetic algorithms. J Braz Soc Mech Sci Eng 34(1):32–40
30. ANSI/AGMA 2003-B97 (2003) Design Manual for Bevel Gears. Alexandria, Virginia. USA: American Gear Manufacturers Association (AGMA), 1–32