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Homotopy Perturbation Method to Study Flow and Heat Transfer of a Modified Second Grade Fluid over a Porous Plate

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Abstract
In this study flow and heat transfer of a modified second grade fluid over a porous plate is considered. The modified second grade fluid model is a combination of power-law and second grade fluid models. This model is suitable for studies where shear dependent viscosity (shear-thinning and shear-thickening) may happen, and has capability to predict normal stress differences too that are seen in die-swelling and rod-climbing behaviors in non-Newtonian fluids. Equations of motion and heat transfer for this flow are derived in dimensionless form. Homotopy perturbation method (HPM) is employed to solve non-linear coupled differential equations governing the problem. The obtained solutions show a very good accuracy in comparison with numerical results. The temperature profiles for power-law and second grade parameters are also given and depicted in figures.

Keywords: *Non-Newtonian fluid, Modified second grade fluid, Homotopy perturbation method, Porous plate*

Introduction
Modified second grade fluid model that was first proposed by Sun and Man [1], is capable of representing shear dependent viscosity as well as behaviors which are due to normal stress in some non-Newtonian fluids. In this model, Cauchy stress tensor is given by:

$$\mathbf{T} = -p\mathbf{I} + \mu \Pi^{\frac{m}{2}} \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2 \qquad (1)$$

witch

$$\mathbf{A}_1 = \mathbf{L} + (\mathbf{L})^T \qquad (2)$$

$$\mathbf{A}_2 = \frac{d}{dt}\mathbf{A}_1 + \mathbf{A}_1\mathbf{L} + (\mathbf{L})^T \mathbf{A}_1 \qquad (3)$$

$$\mathbf{L} = \nabla \mathbf{v} \qquad (4)$$

here m is material parameter, α_1 and α_2 are material moduli, $\mathbf{v}$ represents velocity and Π is defined as follows:

$$\Pi = \frac{1}{2}\,\mathrm{tr}\,(\mathbf{A}_1^2) \qquad (5)$$

Man [2] considered the unsteady channel flow of ice as a modified second grade fluid and showed that this model could be a good alternative to represent creep of ice. Stability and nonexistence of the model was considered by Franchi [3].

Perturbation method is one of the well-known methods for solving nonlinear equations. The basis of the common perturbation method is upon the existence of a small parameter, its use for some problems is difficult or impossible. Different methods have been introduced to solve non-linear equations analytically such as homotopy perturbation method that introduced by He [4]. This method does not depend on a small parameter and is successfully applied to many works with strongly nonlinear governing differential equations.

The boundary layer equations and stretching sheet solutions of modified second grade fluid was studied by Aksoy and Pakdemirli [5]. Hayat and Khan [6] solved equations of motion for this flow by using HAM and solved equations of motion for integer values of material parameter.

In this undertaking we shall study the flow and heat transfer for the modified second grade fluid past a porous plate. The governing equations of motion and energy are derived and made dimensionless. Velocity and temperature distributions are obtained by using HPM.

Governing Equations
The governing equations are the conservation of mass, momentum and energy equations which are as follows

$$div\, \mathbf{v}^* = 0 \qquad (6)$$

$$\rho \frac{d\,\mathbf{v}^*}{dt} = div\mathbf{T}^* + \rho\, \mathrm{b}^* \qquad (7)$$

$$\rho \frac{d\varepsilon}{dt} = \mathbf{T.L} - div\,\mathbf{q} + \rho r \qquad (8)$$

In the above equations, ε is specific internal energy, r is the radiant heating and $\mathbf{q}$ is the heat flux vector given by Fourier's conduction law**.** For fully developed flow, we try to find velocity and temperature fields with the following forms

$$\mathbf{v} = u(y)\,\mathbf{i} + v(y)\,\mathbf{j} \qquad (9)$$

$$\theta = \theta(y) \qquad (10)$$

By substituting Eq. (9) in the continuity equation (6) and with the assumption that the fluid is incompressible, we have

$$v = -v_0 = \text{constant} \qquad (11)$$

here, v_0 states suction velocity for positive values or injection for negative values of v_0. Substituting Equations (1)-(5) and (9), (11) into equation of momentum, Eq. (7), and neglecting body forces after some simplifications and nondimensionalization we have

$$v_0 \frac{du^*}{dy^*} + \varepsilon(m+1)\left(\frac{du^*}{dy^*}\right)^m \frac{d^2u^*}{dy^{*2}} - \varepsilon_1 v_0 \frac{d^3u^*}{dy^{*3}} = 0 \qquad (12)$$

$$u^*(0) = 0,\ u^*(\infty) = 1,\ \frac{du^*}{dy^*}(\infty) = 0 \qquad (13)$$

ε and ε_1 are power-law and second grade parameters, respectively and v_0 is porosity number. The third boundary condition is needed because of third-order derivatives that appear in the equations of motion. For unbounded domains like the present problem, vanishing shear stress at infinity is selected.

Now for the energy equation, we assume that the effect of radiant heating is negligible. Substituting Eq. (1)-(5) and (10)

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into (8) and considering Fourier's conduction law we obtain energy equation as follows:

$$Ec\left[\varepsilon\left(\frac{du^{*}}{dy^{*}}\right)^{m+2}-\varepsilon_1 v_0^{*}\frac{du^{*}}{dy^{*}}\frac{d^2u^{*}}{dy^{*2}}\right]+v_0^{*}\frac{d\theta^{*}}{dy^{*}}+Pe\frac{d^2\theta^{*}}{dy^{*2}}=0 \tag{14}$$

$$\theta^{*}(0)=1,\theta^{*}(\infty)=0 \tag{15}$$

In which Pe stands for Peclet number and Ec is Eckert number.

Solutions

In order to solve momentum and energy equations with HPM, first we rewrite equations when material parameter is a constant integer, therefore the HPM solution of motion and energy when m=1 becomes:

$$u=1-e^{-y}+\frac{\left((-4\varepsilon_1 v_0-\varepsilon+v_0)e^{-\frac{y}{\sqrt{\varepsilon_1}}}+(4\varepsilon_1 v_0-v_0)e^{-y}+\varepsilon e^{-2y}\right)}{v_0(4\varepsilon_1-1)} \tag{16}$$

$$\theta=\frac{\left\{\begin{array}{c}\left[(-3Ec\varepsilon_1+6)v_0^2+4Pe(\varepsilon Ec+9Pe)\right]e^{-\frac{v_0 y}{Pe}}\\+(9\varepsilon_1 EcPe-2\varepsilon Ec-30Pe)v_0\\-9Ec\left[\frac{4}{9}\varepsilon\left(Pe-\frac{1}{2}v_0\right)e^{-3y}+v_0\varepsilon_1 e^{-2y}\left(Pe-\frac{1}{3}v_0\right)\right]\end{array}\right\}}{36Pe^2-30Pev_0+6v_0^2} \tag{17}$$

for other values of material parameters, we can find a similar solution in form of above equations.

Result and Discussion

In order to validate the results, comparison between analytical and numerical solutions are shown in Fig. 1.

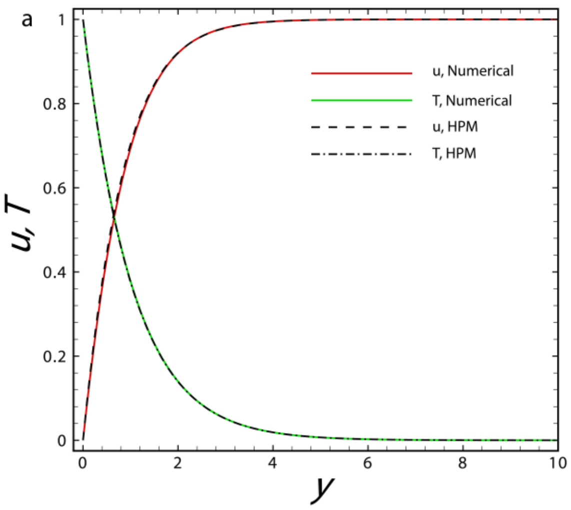

Fig. 1 Comparison of HPM solutions with numerical solutions

As it can be seen in Figs. 2 and 3 temperature profiles and thermal boundary layer thickness increase with increasing either of power-law and second grade parameters slightly as a result, gradient of temperature decreases and causes a reduction in heat transfer rate of flow.

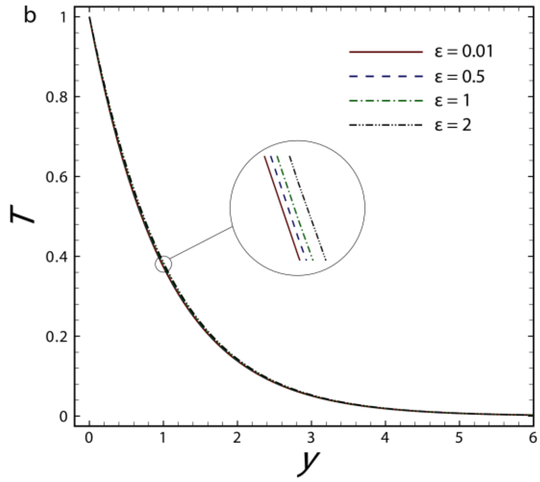

Fig. 2 Impact of power-law parameter on temperature profiles

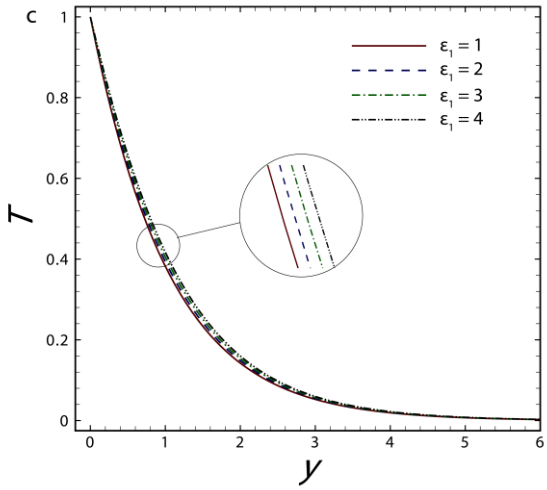

Fig. 3 Impact of second grade parameter on temperature profiles

Concluding Remarks

In this study, the flow and heat transfer of a modified second grade fluid over a porous plate have been considered. The governing equations of momentum and energy have been solved using the homotopy perturbation method. The results show that HPM provides good approximations to the solution of nonlinear system with high accuracy.

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