

Predictive Controller Using Modal Series for DC/AC converters

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Abstract- The real processes encountered in industry, have nonlinear behavior. For the sake of simplicity, generally a linear model around the operation point is developed in the controller design procedure and then the linear controller is applied to the real process. In this paper, an NMPC controller is designed for a DC/AC converter in state space. First, an MPC controller is designed based on a linearized state space model of the converter. Then using Modal Series the linear controller is modified such that the effects of nonlinear dynamics, which are not considered in the linear model, are compensated. Finally, the proposed method is compared with the EDMC approach. Results show that while the proposed method is comparable with the EDMC in the sense of system performance, its less computational capacity makes it a more attractive option for fast processes.

Keywords- DC/AC converters, EDMC controller, Modal series, Nonlinear controller

I. INTRODUCTION

Because of high performance and their simplicity, controllers based on predictive models, abbreviated as MPC, are widely used in different industries [1]. A predictive model provides an estimation of the system's responses in a prediction horizon k . Indeed, it should be able to predict the k samples of forthcoming responses while having access to the present system states. In other word, it should be able to estimate $\hat{y}(t+k|t)$. After the predictive model was developed, a proper cost function is required such that its minimization results in finding out the optimal control rule. The cost function should be designed to minimize both the control effort and tracking error. Also it is desirable that its minimization results in an explicit control rule. Explicit rules not only alleviate calculation burden but also provide valuable information about the effective parameters on the performance of the system.

The real world systems have nonlinear behavior. However, in many cases, their behavior can be approximated by means of a linear model around their operation point. As there are no limitation on exerting different predictive models, model predictive control (MPC) can be applied on both linear and nonlinear models. Numerical iterative methods usually are used to solve nonlinear optimizations resulted from nonlinear models. These methods can't provide closed form explicit solutions. It increases the required calculation capabilities enormously. Furthermore, the designer has no insight about the parameters which are more effective in the response. As a result, in many cases, when a process can be approximated by using a linear model around the operation point, the controller is designed based on the linear model and then it is applied to the process. Unfortunately, by moving away from the operation point, the accuracy of the linear approximation decreases and nonlinear dynamics, which are

not included in the linear model, results in the decreasing of the controller's efficiency. To include the nonlinear dynamic effects, different methods have been proposed, some of them are cited in the following.

The extended dynamic matrix control (EDMC) is a generalized form of dynamic matrix control (DMC), which can be used to control nonlinear process. In this approach, a nonlinear model of the plant is supposed to be available. However, to determine the control inputs, an instantaneous linear model of the process is applied, which is updated and sampled in sampling intervals. Compared with ordinary DMC, EDMC requires additional calculations. Another issue which should be tackled in EDMC is the minimization of difference between the instantaneous linear model and the exact nonlinear model. To solve this optimization problem, iterative methods should be applied which time are consuming. In general, EDMC has less complexity compared with nonlinear MPC, as calculation requirements are noticeably reduced [2].

The EDMC system has proved to be more proper for processes whose steady state gain is single sing and changes slowly. Otherwise, the applied method in reiterative algorithm will lead to an unacceptable result. Moreover, EDMC, like DMC method, is applicable only to processes which are open loop stable.

Another group of methods can be classified as robust MPC methods. To design a robust MPC controller, the nonlinear model is first transformed to a discrete time one and then is divided to linear and nonlinear parts. The nonlinear part is then dealt as a model uncertainty. As an example, in [3] the authors replace the nonlinear terms with a dominant Lipschitz term as $W^T \cdot W$. Then the required conditions for a stabilizing state feedback gain are presented as linear matrix inequality (LMI). However these methods are so conservative because they are designed based on the worst case conditions.

In this paper, a new method based on Modal Series is suggested to make up for the effects of nonlinear dynamics which are not considered in linear model. Modal series is a new approach to analyze nonlinear systems in space state. This method was first proposed for power system studies [4-5]. Since the method provides the system's response in terms of natural modes of the system, the authors named their method Modal Series. In [6], a new form of Modal Series is presented for the analysis of nonlinear circuits.

By applying Modal Series, we can decompose a nonlinear system to a set of linear subsystems and consequently a large number of linear system concepts can be generalized to nonlinear ones.

In this paper, first an MPC controller is designed for a DC/AC converter based on its linear model in space state. Then Modal Series method is used to compensate the nonlinear dynamic effects which are not considered in the designing of the linear controller.

The proposed method reduces the calculation burden in comparison with EDMC method and also is less conservative in the comparison with RMPMC methods.

The following parts of this paper are organized as follows: in the second section the DC/AC converter model will be discussed and also a new form of Modal Series will be introduced. Using the new form, a corollary is derived which can be used to estimate the difference between nonlinear and linear models. In the third section, first a linear MPC controller is designed in space state. After that, it is modified so that it can compensate for the neglected nonlinear dynamics. Moreover, the suggested method is applied to the DC/AC converter and the results will be compared with those of EDMC. Finally, the conclusion will be the fourth section.

II. THE MAIN RESULTS

Let's consider a DC/AC converter model as follows [7]:

$$\begin{aligned} \frac{d\omega}{dt} &= 5v_1\{V_1 - v_1\omega\} - \frac{V_2^2}{\omega} \\ \frac{dV_2}{dt} &= \frac{V_2}{\omega}\left\{\frac{d\omega}{dt}\right\} - 2V_2 + 2\omega v_2 \end{aligned} \quad (1)$$

The converter schematic is shown in Fig. 1. Like [7], the values of R and R_1 are set to unity. The system's inputs are v_1 , v_2 and V_1 and its outputs are ω and V_2 . Choose v_2 and V_1 as control inputs, let $V_1 = v_2$ and v_1 is supposed to be unity ($v_1 = 1$). The model can be rewritten as follows.

$$\begin{aligned} \frac{d\omega}{dt} &= 5v_1\{V_1 - \omega\} - \frac{V_2^2}{\omega} \\ \frac{dV_2}{dt} &= \frac{-V_2^3}{\omega^2} - 7V_2 + \frac{5V_2}{\omega}V_1 + 2\omega V_1 \end{aligned} \quad (2)$$

Supposing $x_1(t) = \omega(t)$, $x_2(t) = V_2(t)$ and $u(t) = V_1(t)$, the following state space model can be presented.

$$\begin{aligned} \dot{x}_1(t) &= -\frac{x_2^2(t)}{x_1(t)} - 5x_1(t) + 5u(t) \\ \dot{x}_2(t) &= -\frac{x_2^3}{x_1^2} - 7x_2(t) + \frac{5x_2(t)}{x_1(t)}u(t) + 2x_1(t)u(t) \\ y(t) &= x_1(t) \end{aligned} \quad (3)$$

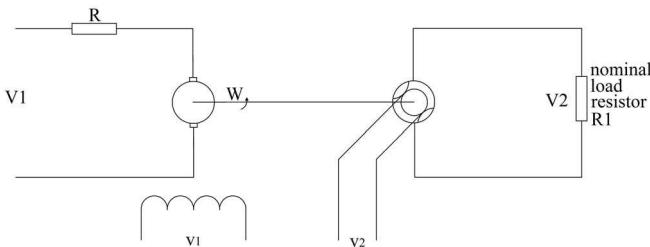


Fig.1. DC/AC converter

To design the controller, the following discrete linear model should be used.

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) \\ y_m(k) &= C_m x_m(k) \end{aligned} \quad (4)$$

The propose is to regulate the output. MPC controller tries to calculate a sequence of control inputs such that the cost function, which is defined as follows, be minimized at the prediction horizon N_p .

$$J = (R_s - Y_{nl})^T (R_s - Y_{nl}) + \Delta u^T \bar{R} \Delta u \quad (5)$$

The R_s is information vector including the operation point information. As only the regulation of operation point is studied in this research, set

$$R_s = \overbrace{[0 \ 0 \ 0 \ \dots \ 0]^T}^{N_p}$$

Define Y_{nl} as follows.

$$\begin{aligned} Y_{nl} &= [y(k_i + 1|k_i) \ y(k_i + 2|k_i) \ \dots \ y(k_i + N_p|k_i)]^T \\ &= [C_m x(k_i + 1|k_i) \ C_m x(k_i + 2|k_i) \ \dots \ C_m x(k_i + N_p|k_i)]^T \end{aligned} \quad (6)$$

The diagonal matrix \bar{R} is defined as $\bar{R} = r_\omega I_{N_C \times N_C}$ ($r_\omega \geq 0$) in which r_ω is used as a regulating parameter to design the closed loop performance. To minimize the cost function, Y_{nl} is needed, while, in the case of using a linear model only Y_{lin} , which is defined as follows, can be derived.

$$Y_{lin} = [y_m(k_i + 1|k_i) \ y_m(k_i + 2|k_i) \ \dots \ y_m(k_i + N_p|k_i)]^T \quad (7)$$

It is evident that the outputs of linear and nonlinear models are not exactly same, therefore, the following definition is presented.

$$D = Y_{nl} - Y_{lin}$$

Vector D which indicates the difference between linear and nonlinear models can be obtained by applying Modal Series.

Notice - Such an idea is used in EDMC as well. In EDMC vector D can be estimated by solving a nonlinear equation using numerical iterative methods. Nevertheless, in the proposed method, vector D is analytically calculable, which increases the speed of algorithm. Moreover, like DMC, EDMC can be only used for the control of stable processes, whereas the proposed method is applicable to both stable and unstable process.

Theorem[6]. Consider the nonlinear system introduced below

$$x(t) = f(x, u), \quad x(0) = x_0 \quad (8)$$

where x represents n-dimension state vector, u shows m-dimensional input vector and $F: R^N \times R^m \rightarrow R^N$ is an analytical function. Without loss of generality, it can be assumed $F(0,0) = 0$. The response of nonlinear system (8) can be written as follows:

$$x(t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} x_{ij}(t) \quad (9)$$

$$\begin{cases} x_{00}(t) = 0 \\ \dot{x}_{10}(t) = Ax_{10}(t), \quad x_{10}(0) = 0 \\ \dot{x}_{ij}(t) = Ax_{ij}(t) + B_{ij}u_{ij}(t) \\ x_{ij}(0) = 0 \quad \text{for: } i, j \geq 0 \ \& \ (i, j) \notin (0,0), (1,0) \end{cases} \quad (10)$$

where the definition of A , B_{ij} and $u_{ij}(t)$ are presented in [6]. Fig. 2 represents the idea of the theorem.

Corollary. Let define the following vectors.

$$\begin{aligned} Y_{01} &= [C_m x_{01}(k_i + 1|k_i) \ \dots \ C_m x_{01}(k_i + N_p|k_i)]^T \\ Y_{10} &= [C_m x_{10}(k_i + 1|k_i) \ \dots \ C_m x_{10}(k_i + N_p|k_i)]^T \\ Y_{02} &= [C_m x_{02}(k_i + 1|k_i) \ \dots \ C_m x_{02}(k_i + N_p|k_i)]^T \\ Y_{20} &= [C_m x_{20}(k_i + 1|k_i) \ \dots \ C_m x_{20}(k_i + N_p|k_i)]^T \\ Y_{11} &= [C_m x_{11}(k_i + 1|k_i) \ \dots \ C_m x_{11}(k_i + N_p|k_i)]^T \\ &\vdots \end{aligned}$$

Where x_{ij} can be derived from the previous calculations. Then

$$D = Y_{nl} - Y_{lin} = Y_{02} + Y_{20} + Y_{11} + \dots$$

Proof – considering the given theorem and equation (6), one can conclude

$$Y_{nl} = Y_{01} + Y_{10} + Y_{02} + Y_{20} + Y_{11} + \dots$$

On the other hand, considering equation (7), will results in

$$Y_{lin} = Y_{01} + Y_{10}.$$

Consequently, the following equation can be derived.

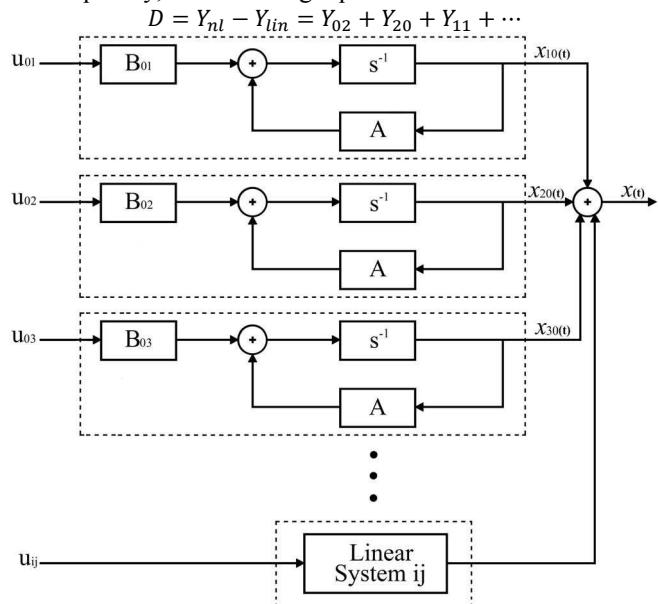


Fig. 1. Modal series or system illustration

III. THE MODIFICATION OF LINEAR CONTROLLER USING THE PROPOSED METHOD

The target of predictive control is to find the best controlling parameter vector Δu , which minimizes the error between the operation point and the predicted output. To obtain the optimal Δu , the cost function (5) should be minimized. Since the control objective is to regulate the output, the cost function is modified as following.

$$J = (Y_{lin} + D)^T(Y_{lin} + D) + \Delta u^T \bar{R} \Delta u$$

Using (4), the following equation can be written for the discrete time DC/AC converter

$$Y_{lin} = Fx(k_i) + \Phi \Delta u.$$

In the above equation

$$\Phi = \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots & CA^{N_p-N_c}B \end{bmatrix}$$

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N_p} \end{bmatrix}$$

Now, the cost function J can be expressed as follows.

$$J = (Fx(k_i) + D)^T(Fx(k_i) + D) - 2\Delta u^T \Phi^T(Fx(k_i) + D) + \Delta u^T(\Phi^T \Phi + \bar{R}) \Delta u$$

Taking the first derivative of the cost function with respect to Δu , the following relation will be obtained.

$$\frac{\partial J}{\partial \Delta u} = -2\Phi^T(Fx(k_i) + D) + 2(\Phi^T \Phi + \bar{R}) \Delta u = 0$$

From the above calculation, the optimal control signal will be obtained as

$$\Delta u = (\Phi^T \Phi + \bar{R})^{-1} \Phi^T(Fx(k_i) + D)$$

of course, provided that $(\Phi^T \Phi + \bar{R})^{-1}$ exists.

Vector D which indicates the difference between linear and nonlinear models can be calculated using the proposed corollary. Therefore, the neglected effects of nonlinear dynamics in the designing of linear controller will be compensated.

To evaluate the proposed method, it is applied to the DC/AC converter model (1). The sampling time is chosen as 0.001 min. Assuming operation point as $x_{ss} = [0.82 \ 1]^T$ and the initial conditions as $x(0) = [0.1 \ 0]^T$, the control signal and system response are shown in Fig. 3 and Fig. 4, respectively. The following parameters are used in the simulation.

$$N_p = 20, \ N_c = 3, \ \alpha = 1, \ \gamma = 0.0025, \ \beta = 0.01, \ \delta = 0.01$$

Fig. 3 and Fig. 4 also show the result of EDMC approach. As the figures reveals, the modal series method has a faster response than EDMC. Moreover, Fig. 5 shows the required times to calculate the control signals for 100 samples using modal series and EDMC methods. It can be obviously observed that, on average, the calculation speed increases by 37.92% using the proposed method.

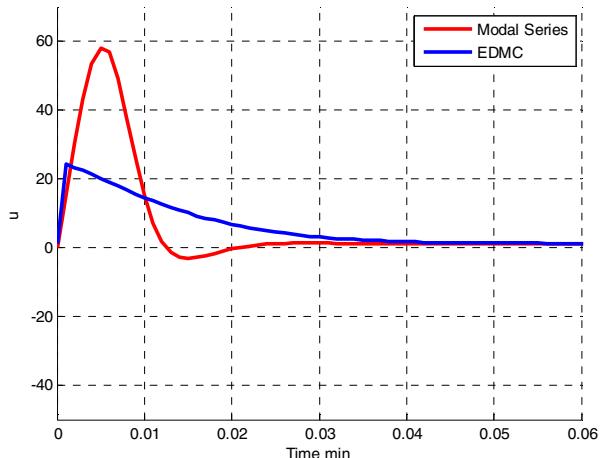


Fig.3. controller signal with modal series and EDMC method

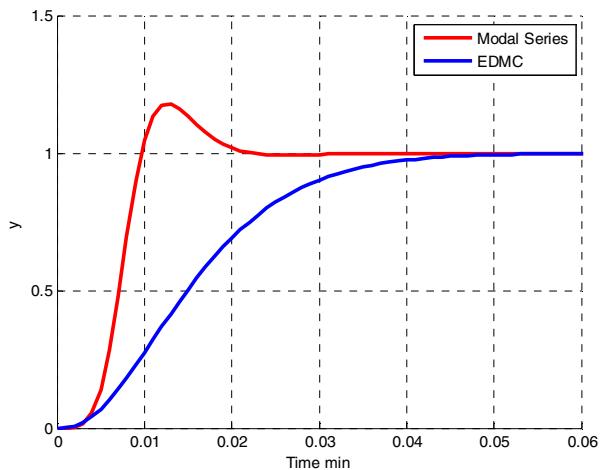


Fig.4. controller response with modal series and EDMC method

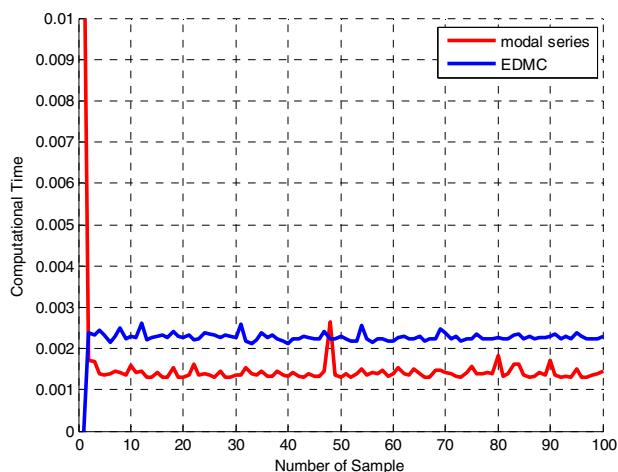


Fig.5 Computation time of modal series method and EDM

IV. CONCLUSION

In many cases, the behavior of practical systems can be approximated around an operation point by using a linear model. Unfortunately, as moving away from the operation point, the accuracy of linear approximation decreases. Consequently, the neglected dynamics in linear model degrade the performance of the linear controller. In this paper, an MPC controller was first designed for a DC/AC converter based on its linearized model. Then using Modal Series, it was modified such that the effects of nonlinear dynamics, which were not considered in the designing of linear controller, are compensated. In comparison with EDMC method, the suggested method needs less computational capacity. Moreover, in spite of EDMC, it can be applied to stable and unstable processes. In addition since the proposed method modifies a linear controller instead of designing a new controller, it can be easily applied in industry to modify the controllers which are designed already based on a linear model.

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