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Hani Raouf Sheybani <hani.raoof@gmail.com> Fri, Sep 8, 2017 at 6:09 PM
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Impacts of Premium Bounds on the Operation of Put Option and Day-ahead Electricity Markets

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Abstract: In this paper, the impacts of premium bounds of put option contracts on the operation of put option and day-ahead electricity markets are studied. To this end, first a comprehensive equilibrium model for a joint put option and day-ahead markets is presented. Interaction between put option and day-ahead markets, uncertainty in fuel price, impact of premium bounds, and elasticity of consumers to strike price, premium price, and day-ahead price are taken into account in this model. Then, a new method for put option pricing is proposed. By applying the presented model to a test system, the impacts of premium bounds on equilibrium of joint put option and day-ahead markets are studied.

Key words: Equilibrium of joint put option and day-ahead markets, Option market modeling, Supply function competition, Put option pricing.

چکیده: در این مقاله، تأثیر مرزهای قیمت اختیار قرارداد اختیار فروش بر عملکرد بازارهای اختیار فروش و بازار روز بعد انرژی الکتریکی مطالعه شده است. بدین منظور، ابتدا یک مدل تعادلی جامع مشترک برای بازارهای اختیار فروش و روز بعد ارائه شده است. تأثیر متقابل بازار اختیار فروش و بازار روز بعد، عدم قطعیت در قیمت سوخت، تأثیر مرزهای قیمت اختیار، و کشش تقاضا مصرف‌کنندگان به قیمت اجرا، قیمت اختیار و قیمت بازار روز بعد در این مدل لحاظ گردیده است. سپس یک مدل جدید برای قیمت‌گذاری قرارداد اختیار فروش ارائه گردیده است. در پایان نیز، با اعمال این مدل به یک شبکه آزمون، تأثیر مرزهای قیمت اختیار بر نقطه تعادل مشترک بازارهای اختیار فروش و روز بعد مطالعه شده است.

1. INTRODUCTION

Electricity option markets have an important role in hedging power producers against quantity and price risks [1-2]. Participants of an option market can trade standard option contracts. Each standard option contract has a specific Mega Watt size, a specified strike price, and a specified delivery period [3]. Strike prices and delivery periods of standard option contracts are determined by the related option market operator [4-5]. A trader chooses the desired option contract based on the desired delivery period and the desired strike price, and then offers the required MW size and a suitable premium price to buy or sell it [4-5]. If bids of a seller and a buyer are matched, the deal is done [4-5].

In some electricity financial markets, the related market operator determines upper and lower bounds for premium bids in the option market in order to prevent from rapid changes in premium bids in the option market [4]. These premium bounds may affect the strategies of producers in the option market and consequently in the day-ahead markets. Therefore, they may affect the electricity price of the physical market, profits of participants, and social welfare. These effects should be assessed by option market operator before applying the bounds to premium prices.

Impacts of option contracts on the bidding strategies of physical market participants are studied in [6-12]. To hedge risk-averse producers and consumers against price risks, an optimal strategy for selecting optional forward contracts is presented in [6]. Optimal bidding strategy of a load serving entity for buying forward and option contracts are determined in [7] in order to hedge the load serving entity against quantity risks in a physical competitive market. An option market beside a physical electricity market is considered in [8] and an approach for calculating optimal strike price from the viewpoint of a market maker is proposed. In [9] and [10], a multi stage stochastic model is presented to determine the optimal strategies of a risk-averse producer in forward, option and pool markets considering price and generation availability risks. Reference [11] develops a stochastic optimization model for determining the bidding strategy of a producer in an energy call option auction. In this auction, bidders can offer both premium and strike prices. An integrated risk management framework for strategic trading of a producer in spot, option, and fuel markets is proposed in [12].

The equilibrium of both physical and option markets are studied in [2, 13-18]. In [13], a new forward contract with bilateral options is introduced in order to hedge the price volatility risks of buyers and sellers in the physical market. In [14], a two period equilibrium model for financial and physical electricity markets is presented. In [14] strategic producers compete with their rivals by setting their supply functions in a spot market and by setting their generation power in a financial option market. In [15] effects of put and call option contracts on the strategies of producers in a physical market with Cournot competition is studied.

The influence of call option contracts at the equilibrium of joint spot and option markets is studied in [16]. Reference [16] considers a Cournot model for spot market. Reference [17] evaluates prices of put and call Asian options using interest rate theory and day-ahead market equilibrium. In this approach demand is forecasted and electricity price variability is modeled by calibrating the volatility parameter as an input. The proposed approach in [17] is based on day-ahead market equilibrium, while the presented research work in this paper is based on the equilibrium of the joint day-ahead and option markets. The impact of day-ahead pricing on the equilibrium of the joint option and day-ahead markets is presented in [18]. Supply function model is used to model day-ahead market in [18].

In this paper, first an equilibrium model for modeling the joint option and day-ahead markets is presented. In the presented model day-ahead market is modeled with Cournot model. Then the impacts of premium bounds on the strategies of put option market participants are considered. The difference between this research work and the available researches are as below. This paper considers details of financial derivatives contractssuch as premium price, strike prices, ,premium boundsand interaction between financial and physical electricity markets.

The main contributions of this paper are 1) presenting a more realistic equilibrium model for a joint put option and day-ahead markets, 2) analyzing the impacts of premium bounds on equilibrium of joint put option and day-ahead markets, and 3) presenting a method for put option pricing. Contributions 2 and 3 are very useful in the operation of joint option and day-ahead markets [4].

A few methods for electricity option pricing are presented based on the historical data of electricity spot markets [19-20]. The proposed model in this paper can be used as an option pricing method that can consider the impacts of future changes in the understudy power system such as construction of new power plants and demand growth during delivery period, and the strategic behavior of producers in the option and day-ahead markets.

In this paper, impacts of premium price bounds on the performance of joint option and day-ahead markets under fuel price uncertainty are studied from the viewpoint of market regulators. Bids of producers in the option and day-ahead markets are needed for this study. However, bids of producers are unknown and change in different situations in oligopoly markets. In order to overcome this problem and take into account the interaction of market participants, it is assumed that the understudy put option and day-ahead markets have reached to their Nash equilibrium [21-24].

The paper is organized as follows. In Section II an equilibrium model for a joint put option and day-ahead markets is presented. Option pricing and the methods for determining premium bounds are discussed in Section III. By applying the presented model to a four producer power system, impacts of premium bounds on equilibrium of joint put option and day-ahead markets are studied in Section IV. Concluding remarks are provided in Section V.

2. MODELING JOINT OPTION AND DAY-AHEAD ELECTRICITY MARKETS

It is assumed that the understudy power system consists of a physical day-ahead electricity market and a financial option market. Suppose fuel price changes over the time. The physical market participants can hedge themselves against risks in quantity and price of trading electric energy by concluding derivative contracts in the option market. Put and call option contracts are two different derivative instruments and are traded independently. Here, we focus on European put option contracts as an independent hedging tool.

2.1. Markets Structure and Decision Framework

The understudy physical electricity market is an oligopoly day-ahead market with poolco structure and supply function competition [2, 16, 18, 25]. It is also assumed that transmission network is lossless and has no constraint to avoid the impact of congestion on the simulation results and consequently a uniform electricity pricing is considered for the day-ahead market. Fuel price changes over the time and is an uncertain variable. Load is elastic with constant elasticity. However, consumers are not strategic.

The understudy financial electricity market is a put option market with physical delivery [5, 9, 18]. In European Electricity Exchange (EEX), the underlying of an option contract is a future contract and it has physical delivery. Participants of a physical electricity market can trade standard put option contracts in the associated option market. In delivery period, whenever the day-ahead market price is less than the strike price of the contracted option, buyers of put option contract exercise their right to sell the contracted MW at the contracted strike price [5].

Although put option and day-ahead markets are operated independently, participation of power producers and consumers in both markets connects these markets together, especially if the put option market has physical delivery as EEX market [5]. If the put option market has physical delivery, strategic behavior of power producers in the put option market affects residual load in the day-ahead market and consequently the strategic behavior of power producers in the day-ahead market, and in turn, the day-ahead market price. Change in day-ahead market price may affect the strategic behavior of participants in the put option market.

Financial and physical market operators are independent. However, usually there is a market regulator or a supervisory board that regulates financial and physical electricity markets as EEX [5]. In this paper, the impacts of premium price bounds on the performance of joint option and day-ahead markets under fuel price uncertainty are studied from the viewpoint of this market regulator or supervisory board. Delivery period of an option contract usually consists of

24 hours or specified hours of a specified week, month, season, or year. Without loss of generality, it is assumed that delivery period consists of specified hours of several consecutive days. These hours are referred to as *study hours*. Hours of delivery period are numerated as t_j where $j = 1, 2, \dots, T$.

In order to model uncertainty in fuel price, possible scenarios for fuel prices are identified. Suppose Ω is the set of possible scenarios for fuel price. Load changes during the delivery period. Suppose inverse demand function at study hour t of scenario s of the delivery period is as follows.

$$\lambda_{st} = N_t^{Dh} - \gamma^{Dh} Q_{st}^L \qquad t = t_1, t_2, \dots, t_T \;\; \forall s \in \Omega \tag{1}$$

where λ_{st} and Q_{st}^L are electricity price and total network load at hour t of scenario s, respectively. N_t^{Dh} and γ^{Dh} are coefficients of inverse demand function at hour t in $\$/MWh$ and $\$/MW^2h$, respectively. Generation cost of producer i at the study hour t of scenario s is as below.

$$C_i(Q_{ist}^O + Q_{ist}^{Dh}) = \rho_s \left(a_i (Q_{ist}^O + Q_{ist}^{Dh}) + \frac{1}{2} b_i (Q_{ist}^O + Q_{ist}^{Dh})^2 \right) \tag{2}$$

where ρ_s is the fuel price at scenario s in $\$/Mbtu$, Q_{ist}^O is the exercise volume of option contract of producer i at hour t of scenario s, Q_{ist}^{Dh} is the day-ahead generation power of producer i at hour t of scenario s, and a_i and b_i are coefficients of the cost function of producer i in $Mbtu/MWh$ and $Mbtu/MW^2h$, respectively. It is assumed that each producer offers an affine supply function to independent system operator (ISO) as its bid at day-ahead market.

The slope of bid of each producer is assumed to be equal to the slope of its marginal cost function. Each producer determines the intercept of its bid function by maximizing its profit. Producer i bids as follows for hour t of scenario s at day-ahead market.

$$bid_{ist} = \alpha_{ist} Q_{ist}^{Dh} + \frac{1}{2} \rho_s b_i {Q_{ist}^{Dh}}^2 \tag{3}$$

where bid_{ist} and α_{ist} are the bid of producer i and its intercept at hour t of scenario s in the day-ahead market respectively.

Timeline for producers' decision-making in the option and day-ahead markets is shown in Fig. 1. Consider a delivery period. Producers should make the following decisions optimally to maximize their profits over this delivery period:

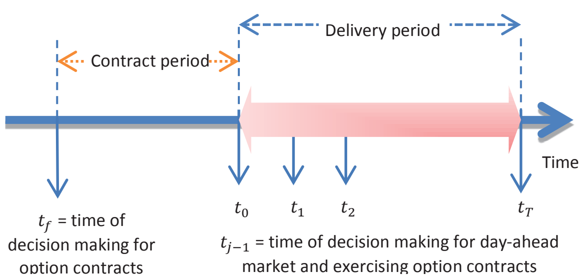

Figure 1. Timeline for decision-making by producers in option and day-ahead markets.

1) Several months before starting the delivery period each producer should decide about the volume of the option contract that should buy from the option market for this delivery period. Suppose producer i buys Q_i^O MW option contract from the option market at contract time t_f as it is shown in Fig. 1.
2) One day before each day of the delivery period fuel price scenario is specified. Suppose scenario s occurs. At this time producer i should decide about its bid, i.e. α_{ist}, for each hour t of scenario s at the day-ahead market.
3) One day before each days of the delivery period, producer i should decide what portion of its option contract must be exercised at each study hour of the next day. It is assumed that producer i exercises Q_{ist}^O MW of its total option contract, i.e. Q_i^O, at hour t of scenario s of the delivery period. Here it is assumed that the exercised volume of option contracts is a continuous variable. In real world it may be a discrete variable with a small step size.

2.2. ISO Optimization Participating in option market is not mandatory. Hence, producers can be categorized in two sets A and B. Set A consists of the producers that attend in both option and day-ahead markets. Set B consists of the producers that only attend in day-ahead market. At each scenario of fuel price, ISO maximizes the social welfare in day-ahead market. The optimization problem of ISO at scenario s is formulated as follows.

$$\max_{Q_{st}^L, Q_{jst}^{Dh}, Q_{lst}^{Dh}} SW_{st} = \int_{\sum_{j\in A} Q_{jst}^O}^{Q_{st}^L} \left(N_t^{Dh} - \gamma^{Dh} Q_d\right) dQ_d - \sum_{m \in A\cup B} \left(\alpha_{mst} Q_{mst}^{Dh} + \frac{1}{2} \rho_s b_m {Q_{mst}^{Dh}}^2 \right) \tag{4}$$

$s.t.$:

$$Q_{st}^L = \sum_{j\in A} \left(Q_{jst}^O + Q_{jst}^{Dh}\right) + \sum_{l \in B} Q_{lst}^{Dh} \tag{5}$$

$$0 \le Q_{jst}^{Dh} \le \overline{Q_j} - Q_{jst}^O \qquad \forall j \in A, \tag{6}$$

$$0 \le Q_{lst}^{Dh} \le \overline{Q_l} \qquad \forall l \in B, \tag{7}$$

where SW_{st} is social welfare of day-ahead market at hour t of scenario s, $\overline{Q_j}$ is the maximum generation capacity of producer j in MW, and $\mathcal{T}$ is the set of study hours in delivery period. Inequalities (6) and (7) enforce generation limits of producer i at every hour of the delivery period. Based on [26], constraints (6) and (7) can be moved into the optimization problem of each producer.

By rearranging the Karush-Kuhn-Tucker (KKT) optimality conditions of optimization (4) to (5), market price and generation of producer i in day-ahead market can be written as functions of bids of all producers as follows.

$$Q_{ist}^{Dh} = \frac{1}{b_i}\left(u_{st} + \boldsymbol{V}_{is}^T \boldsymbol{\alpha}_{st} - w_s \boldsymbol{1}^T \boldsymbol{Q}_{st}^{\boldsymbol{O}}\right) \qquad \forall i \in A \cup B, \forall s \in \Omega, \forall t \in \mathcal{T} \tag{8}$$

$$\lambda_{st} = \rho_s\left(u_{st} + \boldsymbol{Y}_s^T \boldsymbol{\alpha_{st}} - w_s \boldsymbol{1^T Q_{st}^O}\right) \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{9}$$

where $\boldsymbol{\alpha_{st}}$ is a $n(A \cup B) \times 1$ vector which consists of intercepts of bids of all producers, $\boldsymbol{Q_{st}^O}$ is a $n(A) \times 1$ vector which consists of volume of option contracts of producers that attend in the option market. Vectors $\boldsymbol{V}_{is}$ and $\boldsymbol{Y}_s$, and scalars D_s, u_{st} and w_s are defined in the following.

$$H_{is} = 1 + \gamma^{Dh} \sum_{(m \in A \cup B | m \neq i)} \left(\frac{1}{\rho_s b_m}\right) \quad \forall i \in A \cup B, \forall s \in \Omega \tag{10}$$

$$D_s = \rho_s + \gamma^{Dh} \sum_{m \in A \cup B} \left(\frac{1}{b_m}\right) \quad \forall s \in \Omega \tag{11}$$

$$V_{is}(m) = \begin{cases} -H_{is}/D_s & m = i \\ \gamma^{Dh}/(\rho_s b_m D_s) & m \neq i \end{cases} \quad \forall m \& i \in A \cup B, \forall s \in \Omega \tag{12}$$

$$Y_s(m) = \gamma^{Dh}/(\rho_s b_m D_s) \quad \forall m \in A \cup B, \forall s \in \Omega \tag{13}$$

$$u_{st} = N_t^{Dh}/D_s \quad \forall t \in \mathcal{T}, \forall s \in \Omega \tag{14}$$

$$w_s = \gamma^{Dh}/D_s \quad \forall s \in \Omega \tag{15}$$

2.3. Producer's Optimization

In this section, first the optimization problem for each producer in set A and B is modeled. Then, KKT optimality conditions for each set of producers are extracted. Market Nash equilibrium is computed by solving the KKT conditions of all producers' optimization problems.

The optimization problem for producer i of set A, who participates both in the option and day-ahead markets, is formulated as follows.

$$\max_{Q_{ist}^O, \alpha_{ist}, Q_i^O, f_{iK}} \; E(\pi_i) = \sum_{s \epsilon \Omega} \sum_{t=t_0}^{t_T} p_s \Big(Q_{ist}^O K + Q_{ist}^{Dh} \lambda_{st} - \rho_s \Big(a_i \big(Q_{ist}^O + Q_{ist}^{Dh}\big) + \frac{1}{2} b_i \big(Q_{ist}^O + Q_{ist}^{Dh}\big)^2 \Big) \Big) - Q_i^O T f_{iK} e^{rT_C} \tag{16}$$

$s.t.$:

$$Q_{ist}^O - Q_i^O \leq 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} : \omega_{ist} \tag{17}$$

$$K - f_{iK} e^{rT_C} \leq N^O - \gamma^O \sum_{i \in A} Q_i^O \quad : \beta_i \tag{18}$$

$$0 \leq \frac{1}{b_i}\left(u_{st} + V_{is}^T \alpha_{st} - w_s 1^T Q_{st}^O\right) \quad \forall s \in \Omega, \forall t \in \mathcal{T} : \underline{\mu}_{ist} \tag{19}$$

$$\frac{1}{b_i}\left(u_{st} + V_{is}^T \alpha_{st} - w_s 1^T Q_{st}^O\right) \leq \overline{Q}_i - Q_{ist}^O \quad \forall s \in \Omega, \forall t \in \mathcal{T} : \overline{\mu}_{ist} \tag{20}$$

$$f_{iK} \leq f_{up}^K \quad : \overline{\varphi}_{iK} \tag{21}$$

$$f_{down}^K \leq f_{iK} \quad : \underline{\varphi}_{iK} \tag{22}$$

Eqs. (8) and (9)

$$Q_{ist}^O \geq 0, Q_i^O \geq 0, f_{iK} \geq 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{23}$$

where K is strike price of the option contract in \$/MWh, f_{iK} is the premium bid of producer i at strike price K in the option market in \$/MWh, f_{down}^K and f_{up}^K are the lower and upper bounds of premium bid at strike price K in \$/MWh, p_s is the probability of scenario s, r is interest rate, T_C is trading period in year or duration time between contract time and start of delivery period, N^O and γ^O are the intercept and slope of invers demand function in the option market respectively, ω_{ist} is the dual variable of upper capacity limit for exercising option contract of producer i at study hour t of scenario s, β_i is the dual variable of consumer elasticity constraint in the option market, $\underline{\mu}_{ist}$ and $\overline{\mu}_{ist}$ are the dual variables of lower and upper capacity limits of producer i at study hour t of scenario s, respectively, and $\underline{\varphi}_{iK}$ and $\overline{\varphi}_{iK}$ are the dual variables of lower and upper bounds of premium bid of producer i at strike price K, respectively.

The first term of objective function (16) denotes the income of producer i from the exercising option contracts at different hours of the delivery period. The second term of (16) denotes income of producer i from the physical day-ahead market over the delivery period. The sum of third to sixth terms of (16) which are located inside parenthesis indicates the total generation cost of producer i over the delivery period. The last term of (16) denotes the cost of buying put option contract.

Decision making about option exercising by producer i at hour t of scenario s is modeled by maximizing $\left(Q_{ist}^O K + Q_{ist}^{Dh} \lambda_{st}\right)$ in the objective function, considering the fact that demand function is constant at hour t of scenario s. If strike price K is greater than day-ahead market price λ_{st}, the profit of scenario s is maximized if $Q_{ist}^O K$ is maximized, i.e., if Q_{ist}^O is equal to Q_i^O, or if producer i exercises its option contract. If strike price K is smaller than λ_{st}, the profit of producer i is maximized if $Q_{ist}^{Dh} \lambda_{st}$ is maximized, i.e., if Q_{ist}^O is equal to zero or if the producer i does not exercise its option contract.

Inequalities (17) impose the upper limit of producer i for exercising of option contract at every hour of the delivery period. Constraints (18) model the sensitivity of consumers versus strike and premium prices in the option market. Using equations (8), inequalities (6) can be rewritten as inequalities (19) and (20). For each strike price K, inequalities (21) and (22) impose the upper and lower bounds of premium bids in the option market, respectively. Since producers of set A attend both in the option and day-ahead markets, decision variables of the optimization problem of producer i of this set are f_{iK}, Q_i^O, $Q_{ist}^O \; \forall s \in \Omega \, \& \, \forall t \in \mathcal{T}$, and $\alpha_{ist} \forall s \in \Omega \, \& \, \forall t \in \mathcal{T}$. The KKT conditions of each producer i who participate both in the option and day-ahead markets are as below.

$$\frac{\partial \mathcal{L}_i}{\partial \alpha_{ist}} = -p_s \rho_s \Bigg(\left(\frac{\gamma^{Dh} + b_i H_{is}}{{b_i}^2 D_s}\right) \left(u_{st} + \boldsymbol{V}_{is}^T \boldsymbol{\alpha_{st}} - w_s \boldsymbol{1^T Q_{st}^O}\right) - \left(\frac{\rho_s H_{is}}{b_i D_s}\right) \left(u_{st} + \boldsymbol{Y}_s^T \boldsymbol{\alpha_{st}} - w_s \boldsymbol{1^T Q_{st}^O}\right) + \left(\frac{H_{is}}{b_i D_s}\right) \left(a_i + b_i Q_{ist}^O\right) \Bigg) + \underline{\mu}_{ist} - \overline{\mu}_{ist} = 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{24}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial Q_{ist}^O} = -p_s\left(K - \rho_s\left(\frac{\gamma^{Dh}\rho_s}{b_i D_s}\right)\left(2u_{st} + (\boldsymbol{V}_{is}^T + \boldsymbol{Y}_s^T)\boldsymbol{\alpha_{st}} - 2w_s \mathbf{1}^T\boldsymbol{Q_{st}^O}\right) - \rho_s\left(1 - \frac{\gamma^{Dh}}{b_i D_s}\right)\left(a_i + b_i Q_{ist}^O + \left(u_{st} + \boldsymbol{V}_{is}^T\boldsymbol{\alpha_{st}} - w_s\mathbf{1}^T\boldsymbol{Q_{st}^O}\right)\right)\right) + \omega_{ist} + \underline{\mu}_{ist}\left(\frac{\gamma^{Dh}}{H_{is}}\right) + \overline{\mu}_{ist} b_i\rho_s\right\} \geq 0 \ \perp Q_{ist}^O \geq 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{25}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial Q_i^O} = f_{iK} T e^{rT_C} - \sum_{s\in\Omega}\sum_{t=t_0}^{t_T}\omega_{ist} + \gamma^O\beta_i\sum_{j\in A} Q_j^O + \beta_i\left(K - f_{iK}e^{rT_C} - N^O + \gamma^O\sum_{j\in A} Q_j^O\right)\right\} \geq 0 \ \perp Q_i^O \geq 0 \tag{26}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial f_{iK}} = Q_i^O T e^{rT_C} - e^{rT_C}\beta_i\sum_{j\in A} Q_j^O + \overline{\varphi}_{iK} - \underline{\varphi}_{iK}\right\} \geq 0 \perp f_{iK} \geq 0 \tag{27}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \omega_{ist}} = Q_i^O - Q_{ist}^O\right\} \geq 0 \perp \omega_{ist} \geq 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{28}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \beta_i} = \left(N^O - \gamma^O\sum_{j\in A} Q_j^O - K + f_{iK}e^{rT_C}\right)\sum_{j\in A} Q_j^O\right\} \geq 0 \perp \beta_i \geq 0 \tag{29}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \underline{\mu}_{ist}} = \frac{1}{b_i}\left(u_{st} + \boldsymbol{V}_{is}^T\boldsymbol{\alpha_{st}} - w_s\mathbf{1}^T\boldsymbol{Q_{st}^O}\right)\right\} \geq 0 \perp \underline{\mu}_{ist} \geq 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{30}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \overline{\mu}_{ist}} = \overline{Q_i} - \frac{1}{b_i}\left(u_{st} + \boldsymbol{V}_{is}^T\boldsymbol{\alpha_{st}} - w_s\mathbf{1}^T\boldsymbol{Q_{st}^O}\right) - Q_{ist}^O\right\} \geq 0 \ \perp \overline{\mu}_{ist} \geq 0 \quad \forall s \in \Omega, \forall t \in \mathcal{T} \tag{31}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \overline{\varphi}_{iK}} = f_{up}^K - f_{iK}\right\} \geq 0 \ \perp \overline{\varphi}_{iK} \geq 0 \tag{32}$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \underline{\varphi}_{iK}} = f_{iK} - f_{down}^K\right\} \geq 0 \ \perp \underline{\varphi}_{iK} \geq 0 \tag{33}$$

where $\mathcal{L}_i$ is the Lagrangian of the optimization problem of producer i of set A.

Every producer k of set B, participates only in the day-ahead market. Therefore, by ignoring (17), (18), (21), (22), and (23) and setting Q_{ist}^O and Q_i^O equal to zero in (16) and (20), the optimization problem of producer k of set B is obtained. In the same way, the KKT conditions of each producer k of set B can be extracted by omitting (25), (26), (27), (28), (29), (32), and (33) and by setting Q_{ist}^O and Q_i^O equal to zero in (24) and (31).

By substituting λ_{st} and Q_{ist}^{Dh} from (8) and (9) into the objective function (16), the optimization problem (16)-(23) is converted into a quadratic programming as follows:

$$\max_X E\{\pi_{is}\} = -\frac{1}{2}X'A\,X + B^T X \qquad s.t.: \quad CX \leq D \tag{34}$$

Vectors X, B, and D, and matrices A and C are given in Appendix A. In appendix A, It is proved that KKT conditions are sufficient optimality conditions for this problem. The equilibrium of the joint option and day-ahead markets can be calculated by solving the set of KKT conditions of optimization problems of all producers.

3. Option Pricing and Premium Bounds

In some exchange markets option price is estimated and announced to option market participants during trading period. An example of put option pricing at Australian electricity exchange on 28 Feb. 2016 for *Calendar Year 2017 NSW's Base Load Strip Options* is illustrated in Fig. 2 [27]. As Fig. 2 shows, a premium price is estimated for each strike price. Different models for option pricing in financial markets have been presented [19-20]. All of these models depend upon the historical data of physical market [19-20, 25].

Black-Scholes model is one of the famous models for option pricing in financial markets, however it is not suitable for option pricing of electricity markets [28]. None of the presented option pricing models such as Black-Scholes and Binomial tree models can consider system changes in trading period like construction of new power plants and demand growth [19]. In this paper, premium price of the equilibrium of the joint option and day-ahead markets is considered as estimation for option price. Hence, the proposed model in Section II can be used for option pricing. The proposed model is able to consider the predictable changes of the power system during trading period.

In order to prevent from contracts with very high or very low premium prices, financial market operators may exert upper and lower bounds to premium bids [4-5]. Option market participants are restricted to bid within the predetermined bounds for option premium. These bounds may affect the strategies of producers in option markets and consequently the strategies of producers in day-ahead markets. The impacts of the imposed premium bounds can be identified by comparing equilibrium points of joint option and day-ahead markets with and without considering premium bound. In the next Section the impacts of imposed premium bounds on the operation of joint option and day-ahead markets are studied by applying the proposed model to a test system.

4. Case Study

In this section the proposed model is applied to a four-producer power system. Generators of producers 1 to 4 are the same as generators of areas 1 to 4 of IEEE 300-bus test system. The marginal cost function of each producer is computed by aggregating the marginal cost functions of his or her generators and fitting an affine function to it. Capacities of the producers and coefficients of their marginal cost functions are given in Table I. Suppose producers 1 and 2 hedge themselves against price volatility in the day-ahead market by buying European put option one year before starting of delivery period, i.e. trading period is one year or $T_C = 1$. Producers 3 and 4 only participate in day-head market, i.e. producers 1 and 2 are in set A and producers 3 and 4 are in set B as it is shown in Table I. Suppose at contract time t_f, twenty scenarios for fuel price over delivery period are identified.

TABLE 1. Characteristics of all producers.

	Number of producer	Coefficients of marginal cost functions		Generation capacity (GW)
		$a_i(Mbtu/MWh)$	$b_i(Mbtu/MW^2h)$	
Set A	1	7.3137	0.003739	11.40
	2	18.108	0.001483	12.00
Set B	3	19.066	0.001776	8.721
	4	12.943	0.153700	0.558

TABLE 2. Expected value of intercept of inverse demand function at different hours of the delivery period.

hour t	1	2	3	4	5	6	7	8	9	10
N_t^{Dh} (\$/MWh)	44	39	49	48	48.5	47	48	43	40	46

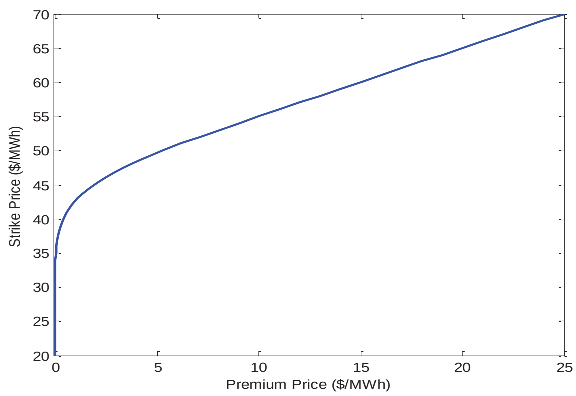

Figure 2. Option pricing for *NSWs Base Load Strip Options, Calendar Year 2017*, in Australia electricity exchange at Feb. 28, 2016 [27].

Fuel prices and their probabilities for different scenarios calculated using distribution of fuel price. It is assumed that distribution of fuel price is $\mathcal{N}(15,1.5)$ \$/Mbtu. The intercept and the slope of invers demand function in the option market at contract time t_f are equal to $\$45/MWh$ and $\$-0.0003/MW^2h$, respectively.

Suppose delivery period of the understudy option contracts consists of a single hour of ten consecutive days. It is assumed that although demand changes during the delivery period, the slope of demand function in day-ahead market remains constant and equal to $\gamma^{Dh} = \$-0.0003/MW^2h$. Demand in different days of delivery period is specified with N_t^{Dh} as it is given in Table II.

Simulation results show that by setting K equal to zero, no option contracts are concluded in the equilibrium of joint option and day-ahead markets.

In this situation, the minimum and maximum prices of day-ahead market over all scenarios and all hours of delivery period, λ^{min} and λ^{max}, are \$29.23/MWh and \$45.91/MWh, respectively. In order to consider strike prices greater than, equal to, and less than day-ahead market price in the study, which in the finance parlance are named *in the money*, *at the money*, and *out the money* strike prices respectively, it is assumed that the strike price varies from \$25/MWh to \$50/MWh with step size \$1/MWh.

Equilibrium of the joint option and day-ahead markets is calculated for each strike price considering fuel price uncertainty, and load change in the delivery period. At each equilibrium point, premium price of option contracts, expected value of day-ahead market price, expected profit of each producer, and the expected value of total social welfare of the joint option and day-ahead markets are computed and discussed. In order to study the impacts of the imposed premium bounds on the operation of the joint option and day-ahead markets, a restricted and a non-restricted case are analyzed in the following.

4.1. Non-restricted Case In the non-restricted case, there are no premium bounds or the bounds are chosen such that optimal premium bids of producers are not restricted by the premium bounds at the equilibrium of the joint option and day-ahead markets. The optimal premium bids in the non-restricted case for the first and second producers and for different strike prices are shown in Fig. 3. As it is shown in Fig. 3, the optimal premium bids of the first and second producers are equal at the equilibrium of the joint option and day-ahead markets. If bid of a producer is a little smaller than the bid of another one, maximum possible contracts are concluded with this producer and the profit of the other producer decreases noticeably.

In some option markets, a settlement premium price is computed for each day of trading period by the financial market operator in order to use in mark-to-marketing process [3]. In financial markets, usually, settlement premium price of a day is equal to weighted average of premium prices of option contracts that are traded in that day or in part of that day [4]. The settlement premium price of the understudy put option market is shown in Fig. 3 for different strike prices by a solid line. Since the optimal premium bids of the first and second producers are equal, the settlement premium price is equal to optimal premium bids of producers, as shown in Fig. 3.

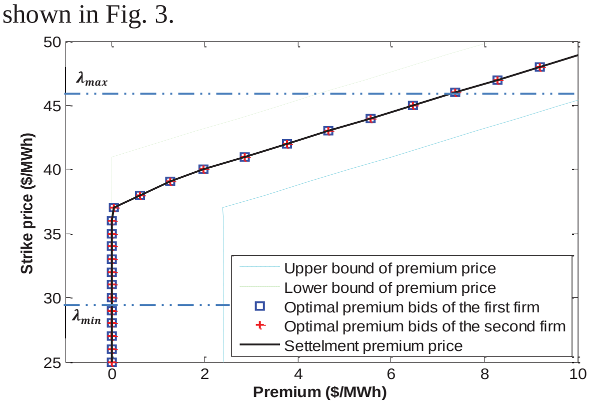

Figure 3. Optimal premium bids of the first and second producers and the related settlement premium price in the non-restricted case.

Comparison of Fig. 3 and Fig. 2 shows that the settlement premium price curve that is obtained from the proposed method is very similar to the actual one that is obtained from the historical data of AEX.

Total volume of concluded option contracts of all producers for the non-restricted case is shown in Fig. 4. If strike prices is less than λ^{min}, concluding option contract is not profitable for producers and no option contract is concluded. As strike price exceeds λ^{min}, concluding option contract gets profitable for producers. Moreover, when strike price exceeds λ^{min}, the price of buying electricity from option market, i.e. $K - f_{iK}e^{rT_C}$, is less than maximum acceptable price of consumers, which is determined by demand function. Hence, total volume of concluded option contracts reach to maximum capacities of producers in set A as strike price increases from λ^{min} to \$31.8/MWh, as it is shown in Fig. 4. For strike prices between \$31.8/MWh and λ^{max}, as strike price increases, consumer price increases and consequently total volume of concluded option contracts decreases due to price elasticity of load. For strike prices greater than λ^{max}, increase in premium price and strike price is so that consumer price remains constant and consequently total volume of concluded option contracts remains constant as strike price increases.

Expected volume of total exercised option contracts of all producers over delivery period for the non-restricted case is shown in Fig. 5. The expected is computed over all scenarios of delivery period and total indicates summation over all hours of delivery period and all producers. As it is shown in Fig. 5, for strike prices between λ^{min} and \$32.6/MWh, as strike price increases both total volume of concluded option contracts and the probability that strike price being greater than day-head market price increase. Hence, expected volume of total exercised option contracts increases noticeably, as strike price increases. For strike prices between \$32.6/MWh and \$34.4/MWh, as strike price increases total volume of concluded option contracts decreases as it is shown in Fig. 4, but the probability that strike price being greater than day-head market price increases with higher rate than decreasing of total volume of concluded option contracts. Hence, expected volume of total exercised option contracts increases as shown in Fig. 5.

As strike price increases from \$34.4/MWh to \$37/MWh, decreasing rate of total volume of concluded option contracts gets greater than increasing rate of probability of exercising option contracts. Thus, total expected volume of exercised option contracts decreases as illustrated in Fig. 5. For strike prices between \$37/MWh and \$40/MWh, the rate of decreasing of total volume of concluded option contracts decreases as strike prices increases, as shown in Fig. 4. Therefore, expected volume of total exercised option contracts increases since decreasing rate of total volume of concluded option contracts gets smaller than increasing rate of probability of exercising option contracts, as shown in Fig. 5.

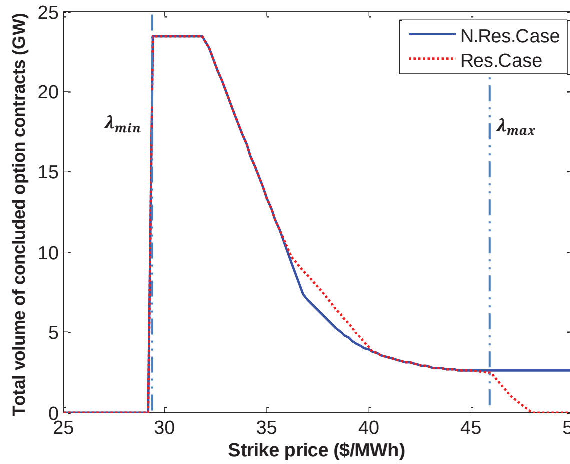

Figure 4. Total volume of concluded option contracts of all producers in non-restricted case (N.Res.Case) and restricted case (Res.Case).

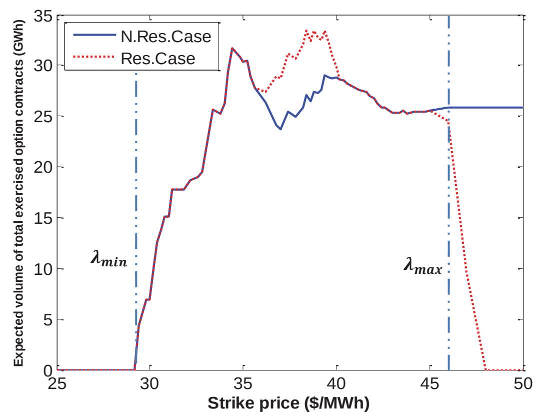

Figure 5. Expected volume of total exercised option contracts over delivery period in non-restricted case (N.Res.Case) and restricted case (Res.Case).

As strike price exceeds λ^{max}, both total volume of concluded option contracts and the probability that strike price being greater than day-head market price remain constant and hence expected volume of total exercised option contracts remain constant at equilibrium.

As strike price increases from λ^{min} to λ^{max}, different constraints may be activated or inactivated in different scenarios. Hence, small fluctuations appears in the expected volume of total exercised option contracts and also expected value of other variables in strike prices between λ^{min} and λ^{max} as it is seen in Fig. 5, and other figures that are discussed in the rest of the paper. The small fluctuations will not appear if the variables are drawn for a specific hour of a specific scenario.

The expected value of day-ahead market price over all hours and scenarios for the non-restricted case is shown in Fig. 6. If total exercised option contracts for hour t of scenario s increases (decreases), residual demand for day-ahead market decreases (increases) and consequently, day-ahead market price decreases (increases).

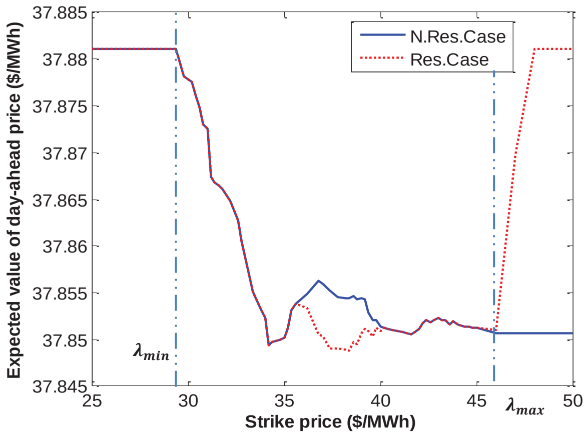

Figure 6. Expected value of day-ahead market price in non-restricted case (N.Res.Case) and restricted case (Res.Case).

Therefore, variations in expected value of day-ahead market price are in opposite direction of variations in expected volume of total exercised option contracts, as it is observed in Fig. 5 and Fig. 6.

Expected value of total social welfare of the joint option and day-ahead markets over delivery period is calculated as (35).

$$E(TSW_s) = \sum_s \sum_{t=t_0}^{t_T} p_s \left(\left(N_t^{Dh} Q_{st}^L - \frac{1}{2} \gamma^{Dh} {Q_{st}^L}^2 \right) - \left(\sum_{j \in A} \rho_s \left(a_j \left(Q_{jst}^{Dh} + Q_{jst}^O \right) + \frac{1}{2} b_j \left(Q_{jst}^{Dh} + Q_{jst}^O \right)^2 \right) + \sum_{l \in B} \rho_s \left(a_l Q_{lst}^{Dh} + \frac{1}{2} b_l {Q_{lst}^{Dh}}^2 \right) \right) \right) \tag{35}$$

where TSW_s is the total social welfare of both option and day-ahead markets in scenario s. Expected value of total social welfare at the equilibrium of the joint option and day-ahead markets is illustrated in Fig. 7 for the non-restricted case. If expected volume of total exercised option contracts for hour t of scenario s increases (decreases), day-ahead market price decreases (increases), total consumption increases (decreases) due to price elasticity of load, and consequently total social welfare at hour t of scenario s increases (decreases). Therefore, variations in expected value of total social welfare are in the same direction of variations in expected volume of total exercised option contracts, as it is seen in Fig. 5 and Fig. 7. Expected value of total profit of the first and second producers from both option and day-ahead markets are illustrated in Fig. 8 for the non-restricted case.

As it is shown in Fig. 8, expected value of total profit of the first and second producers increase as strike price exceeds λ^{min}, where strike price is enough high to encourage producers to buy put option contract and it is less than maximum acceptable price of consumers. Maximum expected value of total profit of each producer occurs between λ^{min} and λ^{max} at the highest strike price at which its optimal premium bid is still zero. Based on (16), as optimal premium bids of producers increase from zero, total cost of each producer increases and consequently expected value of total profit of each producer decreases as shown in Fig. 8.

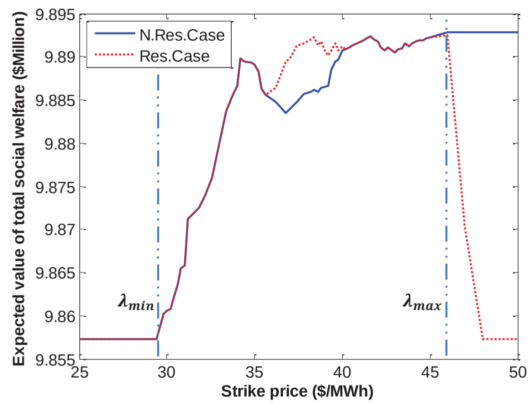

Figure 7. Expected value of total social welfare of the joint option and day-ahead markets in non-restricted case (N.Res.Case) and restricted case (Res.Case).

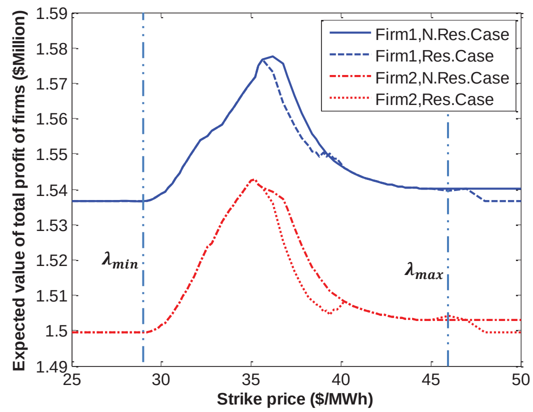

Figure 8. Expected value of total profit of the first and second producers in non-restricted case (N.Res.Case) and restricted case (Res.Case).

In the non-restricted case, when strike price exceeds λ^{max}, all concluded option contracts are exercised at all hours of delivery period. In addition when strike price exceeds λ^{max}, variation of premium price are so that $K - f_{iK} e^{rT_c}$ remains constant at the equilibrium. Therefore profits of producers in set A remain constant.

4.2. Restricted Case

In the restricted case, premium bounds are chosen such that optimal premium bids of producers are restricted by the premium bounds at some strike prices at the equilibrium of the joint option and day-ahead markets. The optimal premium bids of the first and second producers in the restricted case for different strike prices are shown in Fig. 9. The lower and upper bounds of premium bids of the restricted case are specified by dashed and dotted lines in Fig. 9 respectively. As it is shown in Fig. 9, the optimal premium bids of the first and second producers reach to their lower bound at strike prices between \$36/MWh and \$40/MWh and reach to their upper bound at strike prices greater than \$46/MWh in the restricted case. Total volume of concluded option contracts, expected volume of total exercised option contracts, expected value of day-ahead market price, expected value of total social welfare, and total expected value of profit of the first and second producers for the restricted case are shown in Figs. 4 to 8 beside the non-restricted curves.

When premium bids of producers reach to the lower bound at the equilibrium, optimal premium bids of producers increase in restricted case in comparison to non-restricted case, loads are encouraged to sell more put option contract, total volume of concluded option contracts increases as it is shown in Fig. 4, and consequently expected volume of total exercised option contracts increases as illustrated in Fig. 5. In this situation, in restricted case in comparison to non-restricted case, expected value of day-ahead market price decreases, expected volume of total consumption increases, and hence expected value of total social welfare increases as shown in Fig. 6 and Fig. 7, respectively. When premium bids of producers reach to the lower bound, optimal premium bids and concluded volume of option contract of each producer in set A increase in comparison to non-restricted case, hence expected value of total profit of each producer decreases as illustrated in Fig. 8.

On the other side, when the premium bids of producers reach to the upper bound at the equilibrium, optimal premium bids of producers decrease in comparison to non-restricted case, based on (18) producers decrease their concluded volume of option contracts, and consequently expected volume of total exercised option contracts decreases as shown in Fig. 5. By decreasing of expected volume of total exercised option contracts, expected volume of total consumption decreases, day-ahead market price increases and consequently expected value of total social welfare decreases in comparison to non-restricted case as it is illustrated in Fig. 6 and Fig. 7, respectively.

In comparison to non-restricted case, total volume of concluded option contracts decreases when optimal premium bids of producers reach to their upper bound. Thus, each producer must decrease its concluded option contracts. Hence, the expected total profits of the first and second producers decrease as shown in Fig. 8.

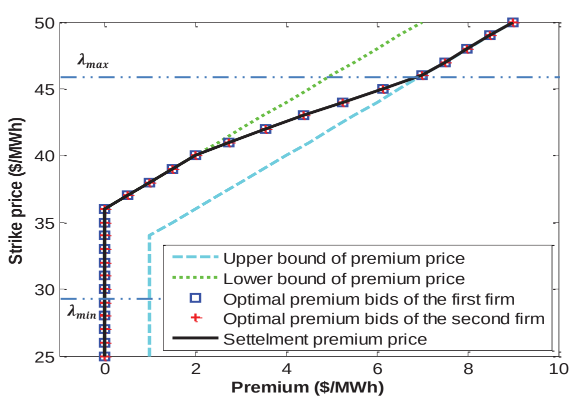

Figure 9. Optimal premium bids of the first and second producers and the related settlement premium price in the restricted case.

5. CONCLUSION

In this paper, the impacts of premium bounds of put option contracts on the operation of joint option and day-ahead markets are studied. In general, decision making about price bounds of an option are finalized before putting it in market for trading. Therefore, the selected bounds for premium prices can affect the strategies of producers over contract period. Consequently it may affect contracted and exercised volumes of option contracts, generation in day-ahead markets, and day-ahead market price. Since the premium bounds are selected by electricity market regulator, who regulates physical and financial electricity markets, he/she should analyze the impacts of premium bounds on the operation of the joint option and day-ahead markets. The presented model provides a tool for this analysis.

Despite of available option pricing methods, which are based on historical data, the presented option pricing method is based on the equilibrium of the joint option and day-ahead markets at the delivery period. This model is able to consider the predictable changes in the power system during trading period. Comparison of simulation results of the proposed option pricing method with the actual result of AEX demonstrates the accuracy of the proposed method.

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Appendix A: Quadratic Form of the Problem

For the sake of simplicity and without loss of generality, here we focus on a single scenario single hour delivery period problem. Matrices X, A, B, C, D for a single scenario single hour delivery period problem are as follows:

$$X = [\alpha_{ist} \quad Q_{ist}^{O} \quad Q_i^{O} \quad f_{iK}]' \tag{A.1}$$

$$A_{11} = p_s \rho_s \left(\frac{2\rho_s H_{is}\gamma^{Dh} + b_i {\rho_s}^2 {H_{is}}^2}{{b_i}^2 {D_s}^2}\right) \tag{A.2}$$

$$A_{12} = p_s \rho_s \left(\frac{{\gamma^{Dh}}^2 + b_i \rho_s H_{is}\gamma^{Dh} - {b_i}^2 {\rho_s}^2 {H_{is}}^2}{{b_i}^2 {D_s}^2}\right) \tag{A.3}$$

$$A_{21} = p_s \rho_s \left(\frac{{\gamma^{Dh}}^2 + b_i \rho_s H_{is}\gamma^{Dh} - {b_i}^2 {\rho_s}^2 {H_{is}}^2}{{b_i}^2 {D_s}^2}\right) \tag{A.4}$$

$$A_{22} = p_s b_i \rho_s \left(\frac{2{\gamma^{Dh}}^2 + {b_i}^2 {\rho_s}^2 {H_{is}}^2}{{b_i}^2 {D_s}^2}\right) \tag{A.5}$$

$$A_{34} = A_{43} = Te^{rT_C} \tag{A.6}$$

$$A_{13} = A_{14} = A_{23} = A_{24} = A_{31} = 0 \tag{A.7}$$

$$A_{32} = A_{33} = A_{41} = A_{42} = A_{44} = 0 \tag{A.8}$$

$$B = \begin{bmatrix} p_s \rho_s \left(a_i \frac{H_{is}}{b_i D_s} + \frac{\gamma^{Dh} N_t^{Dh} + {\gamma^{Dh}}^2 M_{is}}{{b_i}^2 {D_s}^2}\right) \\ p_s \left(K - \rho_s \left(a_i \frac{H_{is}}{D_s} + \frac{N_t^{Dh} H_{is} + \gamma^{Dh} M_{is} H_{is}}{{D_s}^2}\right)\right) \\ 0 \\ 0 \end{bmatrix}' \tag{A.9}$$

$$C = \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & \gamma^{O} & -e^{rT_C} \\ 1 & \frac{\gamma^{Dh}}{H_{is}} & 0 & 0 \\ -1 & \frac{b_i D_s - \gamma^{Dh}}{H_{is}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{A.10}$$

$$D = \begin{bmatrix} 0 \\ N^{O} - K + \gamma^{O} \sum_{j \neq i} Q_j^{O} \\ \frac{N_t^{Dh} + \gamma^{Dh} M_{is}}{H_{is}} \\ \frac{\overline{Q_i} b_i D_s - N_t^{Dh} - \gamma^{Dh} M_{is}}{H_{is}} \\ f_{up} \\ -f_{down} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A.11}$$

Where M_{is} is as follows:

$$M_{is} = \sum_{j \neq i} \left(\frac{\alpha_{jst}}{b_j}\right) + \sum_l \left(\frac{\alpha_{lst}}{b_l}\right) - \sum_{j \neq i} Q_{jst}^{O} \tag{A.12}$$

As it is seen, matrix A is a block diagonal and nonsingular matrix. Moreover, as it is seen in (A.10) matrix C is a full rank matrix and its null space is empty. Therefore, based on [29] and [30], KKT conditions are sufficient optimality conditions for this problem.