# Comparison Between Two Different Computational Schemes for Stress Constraint in Topology Optimization

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**Abstract**
This paper presents and evaluates an approach for handling stress constraint in topological design. The Finite Element Method (FEM) and the Solid Isotropic Material with Penalization (SIMP) is used to formulate the topology optimization. A new scheme is presented to evaluate the von Mises stress at the element centroids. A single P-norm integrated stress constraint is used to reduce the computational time. The stress constraint sensitivity is calculated by using the adjoint method. The optimization problem is solved by the Method of Moving Asymptotes (MMA). Two dimensional, plane elasticity problem is considered. The resulted topology shows the validity of the proposed approach.

**Keywords:** *Topology optimization, Stress based design, SIMP, Global stress measure, MMA*

## Introduction
Structural optimization is a powerful method to generate light weight structure. Topology optimization of continuum structures has a great impact in the field of the structural optimization [1].

Although optimization procedure is time-consuming, with the development of computer technology, topology optimization is increasingly becoming a powerful tool for solving eigenvalue topology optimization [2] and stress based problem. Two main subjects arise from stress constraint topology optimization. The first is "singularity" problem. The singularity is observed where non-disappearing stresses remain as the design variables tend to zero [3]. Convergence to the optimal solution is practically unattainable. To avoid this situation, we use a stress penalization introduced by [4]. The second is high number of stress constraint. We can ideally enforce one stress constraint per element. This local measure of stress constraint requires to impose a high number of constraint because of the large number of elements involved. Therefore, this approach needs very large computational time. To overcome such an issue, von Mises stresses from several stress evaluation points are replaced with a single stress measure by using a P-norm function [3].

In this paper, we present topological design of two-dimensional stress based problems. Different strategies has been used to keep the aforementioned difficulties under control. The problem of topology optimization is solved by the (MMA) [5].

## Theoretical Base
Based on the finite element analysis the equilibrium equation can be written as

$$\vec{K}\vec{u} = \vec{F} \tag{1}$$

where $\vec{K}$ is the global stiffness matrix, $\vec{u}$ and $\vec{F}$ are the global displacement and force vectors of the structure, respectively. The purpose of the topology optimization process is to find the void-solid distribution of known amount of the given material. By considering the SIMP model, the global stiffness matrix $\vec{K}$ is assembled from the element stiffness matrices $\vec{K}_e$ as

$$\vec{K}\left(\rho(\boldsymbol{x})\right) = \sum_{e=1}^{N}\left(\rho_e(\vec{x})\right)^p \vec{K}_e \tag{2}$$

where $\vec{x}$ is the vector of design variables, $e$ is the element index, $\rho_e$ is the element filtered design variable, $N$ is the total number of elements and $p$ a penalization factor, is introduced with a view to generate black-and-white designs, and is set to 3.

## Stress computation
In the finite element analysis the solid material stress vector is calculated as

$$\vec{\sigma}_e = \vec{E}\vec{B}\vec{u} \tag{3}$$

where $\vec{E}$ and $\vec{B}$ are elastic stiffness and strain-displacement matrixes, respectively.

To prevent singularity phenomenon, the elemental stresses are relaxed as

$$\vec{\sigma}_e = (\rho_e(\vec{x}))^q \vec{E}\vec{B}\vec{u} \tag{4}$$

If we set $q$ to 3, the stress penalization works well [6].

## Optimization problem
We can ideally enforce one constraint per element. This means the number of constraint is large. Hence sensitivity analysis is time consuming. To reduce the computational time, a single global stress constraint is used instead of $N$ local constraints. A P-norm function is used to calculate the global stress measure as

$$\sigma^{PN}(\boldsymbol{x}) = \left(\frac{1}{N}\sum_{e=1}^{N}\left(\sigma_e^{vM}(\boldsymbol{x})\right)^p\right)^{\frac{1}{p}} \tag{5}$$

where $p$ is the P-norm factor. A good selection for $p$ should provide acceptable smoothness for optimization algorithm and acceptable approximation of the maximum stress value. On the basis of [6] we choose $p = 8$ in the numerical examples.

The topology optimization problem can be expressed as

$$\begin{cases} \min\limits_{x} \sum\limits_{e=1}^{N} m_e \rho_e(x) \\ \text{subject to:} \begin{cases} \sigma^{PN}(x) \le \bar{\sigma} \\ 0 < x_{min} \le x_e \le 1, \quad e = 1, \dots, N \end{cases} \end{cases} \tag{6}$$

where $m_e$ is the element mass, $\bar{\sigma}$ is the stress limit and $x_{min}$ is a lower bound for $x_e$ used to avoid singularity.

In the topology optimization problems the number of design variables are large, but the number of constraint can

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be kept limited. It is worth noting, due to using a single global stress constraint in place of $N$ local stress constraints, the adjoint sensitivity analysis is much more suitable and computationally efficient [6].

### Innovation in stress computation

In most of the researches in this area, the stress is already evaluated at the element centroids (elemental von Mises stresses). It turns out, the stress happens to be most accurately interpolated at the Gauss points (integration points) in the isoparametric formulation. In this paper for improving the accuracy, the stresses are calculated at the Gauss points. Then they are being extrapolated from integration points to the nodes. The elemental stress is actually an averaged value of the stresses obtained in the corresponding nodes.

### Numerical example

In this section an illustrative example is presented, see Fig. 1. It is modeled by four-node plane stress elements. The existing material is uniformly distributed over the design domain. A 3000 N load is distributed symmetrically over nine nodes to prevent stress concentration. Geometrical and physical properties of the well-known MBB beam are given in Table 1.

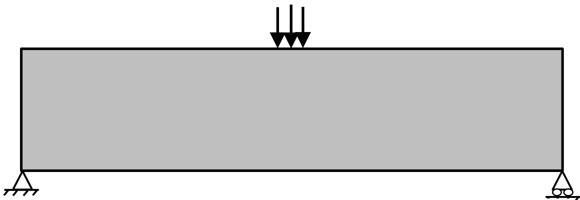

**Fig. 1: An MBB beam**

**Table 1: Geometrical and physical properties of the MBB beam**

| Parameter | Value | Unit |
| --- | --- | --- |
| Young's modulus | 71 | GPa |
| Poisson's ratio | 0.33 | --- |
| density | $2.8 \times 10^{-9}$ | ton/$mm^3$ |
| beam length | 0.6 | m |
| beam width | 0.2 | m |
| beam thickness | 0.001 | m |
| yield limit ($\bar{\sigma}_1$) | 350 | MPa |
| yield limit ($\bar{\sigma}_2$) | 275 | MPa |

### Result and Discussion

Due to the symmetry in the beam, only the right half the beam is presented. The resulted topological designs by the proposed method for different values of yield limit, are shown in Fig. 2 and Fig. 3. The result of previous attempt by [3] is shown in Fig. 3. Another example of stress-based design [7] is presented in Fig. 4. The optimal mass of the resulted structures (Fig. 2 and Fig. 3) by the presented approach have the values $\text{Mass}_{opt1} = 18.95\text{ g}$ and $\text{Mass}_{opt2} = 24.22\text{ g}$, which are more optimized than the result in [3], also the topological design in Fig. 2 is more practical than Fig. 5, which shows the validity and supremacy of the approach used in this paper.

**Fig. 2: Topological design by the proposed approach for $\bar{\sigma}_1$=350 MPa**

**Fig. 3: Present study for $\bar{\sigma}_2$=275 MPa**

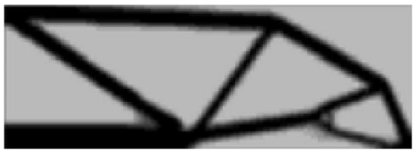

**Fig. 4: MBB beam topological design $\bar{\sigma}_1$=350 MPa [3]**

**Fig. 5: MBB beam Stress-based design [7]**

### Concluding Remarks

In this paper the stress based topology optimization problem has been solved by using MMA. A numerical example of stress constraint topological design for two dimensional plane elasticity problem is presented. The numerical example demonstrate that our approach effectively avoids high stress concentration. The result also shows that the proposed numerical framework generates black-and-white structure and is a promising approach.


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