Eigenvalue topology optimization of structures by using MMA

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ABSTRACT

This paper presents a new optimization algorithm for topology optimization of freely vibrating continuum structures for optimal simple and repeated natural eigenfrequencies. Finite element method (FEM) is used to formulate the topology optimization. The modified solid isotropic material with penalization (SIMP) model is used to avoid artificial modes in low density areas. During the optimization process a simple frequency may become multiple. However, the sensitivity of repeated eigenfrequencies is not unique. To capture this behavior, sensitivity of them are calculated by the mathematical perturbation analysis. The eigenvalue topology optimization is considered as a max-min formulation. In order to solve the problem, the method of moving asymptotes (MMA) is used.

Two dimensional, plane elasticity problems with different sets of boundary condition and attachment of a concentrated non-structural mass are considered. Numerical results show the validity and supremacy of the proposed approach.

Keywords: Topology optimization, Eigenfrequency design, SIMP, Multiple eigenvalues, MMA

1. INTRODUCTION

The topology optimization of the continuum structure has great impact in the field of the structural optimization and has been extensively developed in the last several decades. For structural topology optimization design several optimization approaches such as the homogenization approach [1, 2], the classical solid isotropic material with penalization (SIMP) approach [3, 4], and the evolutionary structural optimization approach [5, 6], have been developed.

Although optimization procedure is time-consuming, with the development of computer technology, topology optimization is increasingly becoming a powerful tool for solving structural design problems for optimal eigenfrequencies. However, the number of papers that deal with topology optimization of dynamic problem is limited. Problems of passive design again vibration and noise are of a great impact in many engineering fields. Keeping the eigenfrequencies of a structure away from the external excitation frequencies is a frequent goal of the design of the vibrating structure to avoid resonance.

The first attempt at eigenvalue topology optimization was considered by Diaz and Kikuchi [7] dealt with the reinforcement of given 2D structures. Tenek and Hagiwara [8] dealt with maximizing the eigenfrequencies of plates using the homogenization method and mathematical programming. The problem objective function is defined as scalar weighted sum of the first five eigenfrequencies see [9].

Olhoff and Du [10] dealt with topology optimization for minimum dynamic compliance of continuum structure subjected to force vibration. Maximizing the gap between two adjacent frequencies have been considered in the papers [11]. Topology optimization was applied to maximize fundamental frequency of two-dimensional structures with additional non-structural concentrated mass [12, 13]. The

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objective of optimization problem is formulated by bound formulation which make the proper treatment of multiple eigenvalues easier [11, 14]. The topology optimization formulation of couple-stress continuum structures is investigated for maximizing the first natural frequency [15].

The simultaneous optimization of material properties and the topology of functionally graded (FG) structures was proposed in the context of minimum compliance [16, 17].

In this paper, we present topology optimization of two-dimensional functionally graded material for optimal values of fundamental eigenfrequency. Also possibility of multiple eigenfrequency is considered and Sensitivity of them are computed by the results of mathematical perturbation analysis [18]. Spurious modes related to subregions with low values of material density is captured by using the modified SIMP [14]. The topology optimization is formulated by a bound formulation. The problem of eigenvalue topology optimization is solved by the (MMA) [19].

2. FORMULATION OF TOPOLOGY OPTIMIZATION AND MATERIA MODEL

2.1 Maximization of fundamental eigenfrequency

Based on the finite element analysis the dynamic behavior of continuum structure can be depicted by the general eigenvalue problem

$$(\mathbf{K}-\omega_i^2\mathbf{M})\mathbf{u}_i = 0 . \tag{1}$$

where ω_i is the i'th natural eigenfrequency and u_i is the corresponding eigenvector.

The eigenvalue optimization problem can be considered as max-min formulation

$$\max \quad \left\{ \min_{i=1,\dots,N} \left\{ \omega_i^2 \right\} \right\} . \tag{2}$$

$$\text{s.t.} \quad \mathbf{K}\mathbf{u}_i = \omega_i^2 \mathbf{M}\mathbf{u}_i , \quad i=1,\dots,N . \tag{3}$$

$$\mathbf{u}_i^T \mathbf{M}\mathbf{u}_j = \delta_{ij} \quad i \geq j , \quad i, j = 1,\dots,N . \tag{4}$$

$$\sum_{e=1}^{n} x_e - fV_0 \leq 0 . \tag{5}$$

$$\mathbf{0} < \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{1} . \tag{6}$$

where N is total number of degrees of freedom of the admissible design domain, and f and V_0 are respectively the prescribed volume fraction and design domain volume, $\mathbf{x}$ denotes the vector of element material densities, $\mathbf{x}_{\min}$ represent a vector of lower bound for $\mathbf{x}$.

The max-min problem may equivalently be written by a bound variable formulation that has an advantage when we have multiple eigenfrequencies.

2.2 The classical SIMP model

The purpose of the topology optimization process is to find the void-solid distribution of known amount of the given material. By considering the SIMP model, the element elasticity matrix may be considered as

$$\mathbf{E}_e(x_e) = x_e^p \mathbf{E}_e^0 . \tag{7}$$

where x_e and $\mathbf{E}_e^0$ are respectively the element material density and elasticity matrix of homogeneous solid. To attain a solution which be composed of solid and void region the penalization factor p is used. Also the element mass matrix based on the SIMP model may be considered as

$$\mathbf{M}_e(x_e) = x_e^q \mathbf{M}_e^0 . \tag{8}$$

where $\mathbf{M}_e^0$ is the mass matrix corresponding to the element with fully solid material, and usually the $q = 1$.

2.3 The modified SIMP model

If $p = 3$ and $q = 1$, the SIMP model for eigenvalue topology optimization may cause artificial eigenmodes related to very small corresponding eigenfrequencies. Following Du and Olhof [14] we may replace (2) by the modified SIMP model

$$\mathbf{M}_e(x_e) = \begin{cases} x_e \mathbf{M}_e^0 & x_e \geq .1 \\ (c_1 x_e^6 + c_2 x_e^7)\mathbf{M}_e^0 & x_e \leq .1 \end{cases} . \tag{9}$$

In this equation the two coefficients c_1 and c_2 enforce the continuity at the value $x_e = 0.1$ of the element material density.

3. SENSITIVITY CALCULATION

If we assume the eigenfrequency is simple, then the corresponding eigenvector will be unique. Therefore it is differentiable with respect to design variables. To calculate the sensitivity of unimodal eigenvalue we differentiate (1) with respect to x_e, and achieve

$$\frac{\partial \lambda_i}{\partial x_e} = \mathbf{u}_i^T \left(\frac{\partial \mathbf{K}}{\partial x_e} - \lambda_i \frac{\partial \mathbf{M}}{\partial x_e} \right) \mathbf{u}_i \quad e = 1, \ldots, n . \tag{10}$$

By using the equation (7) and (9), the sensitivity of i'th natural eigenfrequency becomes

$$\frac{\partial \lambda_i}{\partial x_e} = \begin{cases} \mathbf{u}_i^T (p x_e^{(p-1)} \mathbf{K}_e^0 - \lambda_i \mathbf{M}_e^0) \mathbf{u}_i & x_e > 0.1 \\ \mathbf{u}_i^T (p x_e^{(p-1)} \mathbf{K}_e^0 - \lambda_i (6 c_1 x_e^5 + 7 c_2 x_e^6) \mathbf{M}_e^0) \mathbf{u}_i & x_e \leq 0.1 \end{cases} \quad e = 1, \ldots, n . \tag{11}$$

A unimodal eigenfrequency may become multiple during optimization. In this paper the sensitivities of repeated eigenfrequencies are calculated based on the result of the mathematical perturbation analysis [18].

4. OPTIMIZATION ALGORITHM

The proposed iterative procedure is illustrated in a flowchart as Figure 1. The bound variable formulation of eigenvalue topology optimization is carried out in the self-programming matlab software. Checherboard and mesh-dependency problem usually arise in topology optimization problem. To prevent those problems, we have used the mesh-independent filter as described in [4] by weighted averaging of sensitivities over the neighbouring elements (see e.g. Hassani and Hinton [20]).

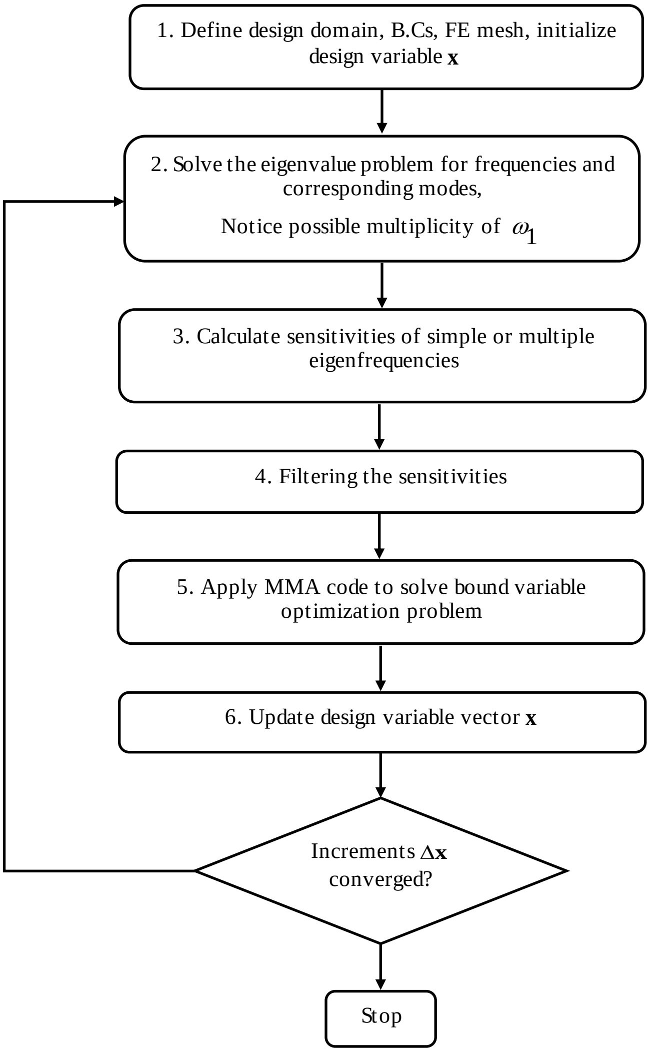

Fig. 1. Flowchart of the proposed optimization algorithm

5. NUMERICAL EXAMPLES

In this section an illustrative examples is presented which show validity of the proposed method. It has been modeled by four-node plane stress elements. In this example we want to maximize the fundamental eigenfrequency of a beam-like 2D structure with simply supported ends shown in Figure 2. The material volume fraction is 50%. The length, width, and thickness of the beam are respectively 8 m, 1 m and 1 m. Young's modulus is $E = 10^7$, poisson's ratio is $\upsilon = 0.3$ and mass density is $\rho = 1$. The existing material is uniformly distributed over the design domain. The first natural frequency of the initial design is ω_1 =68.7 rad/s. The optimal topology is shown Figure 3. The fundamental frequency of the optimal design is ω_1 =166.44 rad/s. The result is agree with the result in Huang et.al. [21] and has relative error less than 3%.

Fig. 2. Beam with simply supported ends

Fig. 3. Topological design of the simply supported beam

6. Conclusions

In this paper the bound variable optimization problem has been solved by using MMA. The modified SIMP model has been employed to handle the localized eigenmodes in low density areas. An algorithm has been developed for eigenvalue topology optimization problem. A numerical example for two-dimensional plane elasticity problem is presented which show validity of the proposed method.

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