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On Bayesian Shrinkage Estimator of Parameter of Exponential Distribution with Outliers

P. Nasiri
Department of Statistics,
University of Payam Noor,
19395-4697 Tehran, I.R. Iran.
Email: pnasiri@pnu.ac.ir, pnasiri45@yahoo.com

M. Jabbari Nooghabi*
Department of Statistics,
Ferdowsi University of Mashhad,
Mashhad-Iran.
E-mail: jabbarinm@um.ac.ir, jabbarinm@yahoo.com
*Corresponding Author

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Abstract. In this article, we have estimated the scale parameter of exponential distribution with a prior information. A shrinkage estimator is derived for parameter of exponential distribution contaminated with outliers and in the presence of LINEX loss function. An admissible estimator based on the LINEX loss function are compared with different methods of estimations. Numerical study are used to compare the estimators.

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Key Words: Exponential distribution, LINEX loss function, Prior information, Outlier, Shrinkage estimation.

1. Introduction

The following probability density function (pdf) which is exponential distribution is often used in life-testing research.

$$f(x,\theta)=\frac{1}{\theta}e^{\frac{-x}{\theta}},\ x>0,\ \theta>0.$$

Let $(X_1,X_2,\cdots,X_n)$ is a random sample of size n which is derived from the exponential distribution. In this distribution, θ is known as the scale parameter and is the mean life. θ is the average time to failure and $\bar{X}$ is an unbiased estimator of it. By using the squared error loss function (SELF) which is a symmetric, it is not appropriate to estimate mean of

life reliability function. [20] and [18] introduced a new version of loss function which is asymmetric and known as the LINEX loss function (LLF). This approach was modified by [6] and [8]. Also, a modified version of LINEX loss function is exist which is general entropy loss function and proposed by [7].
For any parameter θ, a LLF which is invariant is

$$f(\Delta) = e^{a\Delta} - a\Delta - 1,\ a \neq 0,\ \Delta = \Big(\frac{\hat{\theta}}{\theta} - 1\Big),$$

based on [6]. In this loss function, the grade of asymmetry and orientation are depend on sign and magnitude of a, respectively. The positive value of a is usually considered when overestimation is more benefit than the underestimation and the negative value is taken in the reverse situations. When a is around zero, the loss function is almost squared error and it is a symmetric. Details are found in [13] and [2].
A minimum mean square error (MMSE) estimator of parameter θ in the exponential distribution is obtained by [15] and defined as $\frac{n}{n+1}\bar{X}$ under the class of $k\bar{X}$. [13] have used the Searls's estimator and explained that it was inadmissible under the LLF.
[19] derived optimal shrinkage estimations for the parameters of exponential distribution based on record values. [14] presented shrinkage estimation of the parameter of exponential type-II censored data under LLF. [4] have estimated $P(X < Y)$ in the exponential distribution with common location parameter by using shrinkage method. [12] discussed several methods of shrinkage estimation of $P(Y < X)$ in the exponential distribution mixing with exponential distribution. [1] have used beta priors and new Bayes estimators of population proportion of respondents possessing stigmatized attribute to extend Mangat Randomized Response Technique. Also, [16] have considered a new methodology for Bayesian analysis of mixture models under doubly censored samples. They evaluated the Bayesian estimation of parameters of the two-component mixture of Rayleigh distribution under square root gamma, Maxwell and half normal priors using two loss functions. Further, [3] have presented a modified factor-type estimator under two-phase sampling. This method is found by incorporating information like coefficient of variation, kurtosis, skewness and correlation coefficient.
Based on [17], when θ_0 indicates a conjecture of θ a shrinkage estimator is

$$\widehat{\theta}_{sh} = c(\hat{\theta} - \theta_0) + \theta_0.$$

One can evaluate the shrinkage factor c depend on the guessed value θ_0. This method of estimation is now used in different subjects.
If we assume source distributed which may be a small plot of plants. When the weather is normal, the plants distribute the pollen and it intersperse such as an exponential distribution far from the origin. Also, in some situation, for example in fog or light rain, the herbage reduced their diffusion of pollen, but still exponentially distributed with other scale parameter. [9] have assumed viral spores (BYMDV) and estimated the parameters of its pdf.
So, in this paper we constructed the structure as: Section 2 is to present density of $(X_1, X_2, \cdots, X_n)$ contaminated with outliers. In sections 3 and 4, shrinkage estimators of the scale parameter of an exponential distribution contaminated with outliers under squared error and LLF are discussed. In section 5, the minimum risk of the two loss functions are derived.

2. Joint Density of $(X_1, X_2, \cdots, X_n)$ with Outliers

Let a random sample of size n shows the interval of a sampled plant from a plant from a plot of plants which is infected by a virus. Here, most of data are coming from the airborne dispersal of the spores and follow the exponential distribution. A small number of data from n random variables (denote by k) are remained and to be transport barley yellow mosaic dwarf virus (BYMDV) have moved the virus into the plants while the aphids cuisines on the sap. [10] have estimated the parameters of the exponential distribution contaminated with outliers which is coming from uniform distribution.
Then, consider $(X_1, X_2, \cdots, X_n)$ are distributed such that k of them are coming from pdf $g(x, \theta, \beta)$

$$g(x, \theta, \beta) = \frac{\beta}{\theta} e^{\frac{-\beta x}{\theta}}, \ x > 0, \ \beta > 0, \ \theta > 0, \tag{2. 1}$$

and the other $(n-k)$ are generated from pdf $f(x, \theta)$ as

$$f(x, \theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, \ x > 0, \ \theta > 0. \tag{2. 2}$$

So, the joint pdf of $(X_1, X_2, \cdots, X_n)$ is

$$f(x_1, x_2, \cdots, x_n) = \frac{k!(n-k)!}{n!} \prod_{i=1}^{n} f(x_i, \theta) \sum^{*} \prod_{j=1}^{k} \frac{g(x_{A_{i_j}}, \theta, \beta)}{f(x_{A_{i_j}}, \theta)}, \tag{2. 3}$$

with $\sum^{*} = \sum_{A_1=1}^{n-k+1} \sum_{A_2=A_1+1}^{n-k+2} \cdots \sum_{A_k=A_{k-1}+1}^{n}$. Then, for $g(x, \theta, \beta)$ and $f(x, \theta)$ which are given in (2.1) and (2.2), respectively, $f(x_1, x_2, \cdots, x_n)$ is

$$\begin{aligned} f(x_1, x_2, \cdots, x_n) &= \frac{k!(n-k)!}{n!} \frac{e^{\frac{-\sum x_i}{\theta}}}{\theta^n} \sum^{*} \prod_{j=1}^{k} \frac{\beta e^{\frac{-\beta x_{A_j}}{\theta}}}{e^{\frac{-x_{A_j}}{\theta}}} \\ &= \frac{k!(n-k)!\beta^k}{n!\theta^n} e^{\frac{-\sum x_i}{\theta}} \sum^{*} \prod_{j=1}^{k} \frac{e^{\frac{-\beta x_{A_j}}{\theta}}}{e^{\frac{-x_{A_j}}{\theta}}} \\ &= \frac{k!(n-k)!\beta^k}{n!\theta^n} e^{\frac{-\sum x_i}{\theta}} \sum^{*} \prod_{j=1}^{k} e^{-(\beta-1)\frac{x_{A_j}}{\theta}}. \end{aligned} \tag{2. 4}$$

Therefore, by using equation (2.4), we obtain the marginal density of X as follows.

$$\begin{aligned} f(x, \theta, \beta) &= \frac{k}{n} g(x, \theta, \beta) + \frac{n-k}{n} f(x, \theta) \\ &= \frac{k\beta}{n\theta} e^{\frac{-\beta x}{\theta}} + \frac{n-k}{n\theta} e^{\frac{-x}{\theta}}, \qquad x > 0. \end{aligned}$$

3. Shrinkage Estimation of θ with SELF

Here, the shrinkage estimator of parameter θ when θ_0 is a guess value of it, is given by

$$\widehat{\theta}_{sh} = c(\hat{\theta} - \theta_0) + \theta_0, \ c \in [0, 1].$$

Let us suppose that $\widehat{\theta}_{shsel} = c(\bar{X} - \theta_0) + \theta_0$ be the estimator in the SELF. Assume that this risk denotes by R_s. Hence

$$\begin{aligned} R_s &= E[\theta - \widehat{\theta}_{shsel}]^2 = E[\theta - c(\bar{X} - \theta_0) - \theta_0]^2 \\ &= E[\theta + (c-1)\theta_0 - c\bar{X}]^2 \\ &= [\theta + (c-1)\theta_0]^2 + c^2 E(\bar{X}^2) - 2c[\theta + (c-1)\theta_0]E(\bar{X}). \end{aligned}$$

By using the marginal distribution of X, $E(\bar{X})$ and $V(\bar{X})$ are obtained respectively as

$$E(\bar{X}) = \frac{k\theta}{n\beta} + \frac{(n-k)\theta}{n},$$

and

$$V(\bar{X}) = \frac{\theta^2}{n^3}\left[\frac{k^2}{\beta^2} + (n-k)^2\right].$$

Hence

$$\begin{aligned} R_s &= [\theta + (c-1)\theta_0]^2 + \frac{c^2\theta^2}{n^2}\left[\frac{k^2}{n\beta^2} + \frac{(n-k)^2}{n} + \frac{k^2}{\beta^2} + (n-k)^2 + \frac{2k(n-k)}{\beta}\right] \\ &- \frac{2c\theta}{n}[\theta + (c-1)\theta_0]\left[\frac{k}{\beta} + (n-k)\right]. \end{aligned}$$

Let

$$A = \frac{k^2}{n\beta^2} + \frac{(n-k)^2}{n} + \frac{k^2}{\beta^2} + (n-k)^2 + \frac{2k(n-k)}{\beta},$$

and

$$B = \frac{k}{\beta} + (n-k),$$

then, R_s will be

$$R_s = [\theta + (c-1)\theta_0]^2 + \frac{c^2\theta^2}{n^2}A - \frac{2c\theta}{n}[\theta + (c-1)\theta_0]B. \quad (3.\ 5)$$

Now, we have to minimized the risk and find c_0 such that

$$\frac{dR_s}{dc} = 0.$$

Therefore

$$\frac{dR_s}{dc} = \theta_0[\theta + (c-1)\theta_0] + \frac{c\theta^2}{n^2}A - \frac{\theta}{n}[\theta + (c-1)\theta_0]B - \frac{c\theta\theta_0}{n}B = 0,$$

and

$$c_0 = \frac{n\theta_0^2 + \theta^2 B - (n+B)\theta\theta_0}{n\theta_0^2 + \frac{\theta A}{n} - 2\theta\theta_0 B}.$$

For θ_0=0 and $c_0 = \frac{nB}{A}$, it is given by [10].
Hence

$$\widehat{\theta}_{shsel} = c_0(\bar{X} - \theta_0) + \theta_0.$$

Substitute c_0 in (3.5), imply that

$$MinR_s = [\theta + (c_0 - 1)\theta_0]^2 + \frac{c_0^2\theta^2}{n^2}A - \frac{2c_0\theta}{n}[\theta + (c_0 - 1)\theta_0]B.$$

4. Shrinkage Estimator with LLF

In this section, an estimator of θ is derived under LLF based on shrinkage method. The following function is LLF

$$f(\Delta) = e^{a\Delta} - a\Delta - 1,\ a \neq 0,\ \Delta = \left(\frac{\widehat{\theta}_{shll}}{\theta} - 1\right),$$

where

$$\widehat{\theta}_{shll} = c(\hat{\theta} - \theta_0) + \theta_0.$$

Let us suppose that

$$\widehat{\theta}_{shll} = c(\bar{X} - \theta_0) + \theta_0,$$

be the estimator under LLF and the risk is denoted by LR_s. Hence

$$LR_s = E(L(\triangle)) = E\left(e^{a\triangle} - a\triangle - 1\right),$$

where

$$\begin{aligned}\Delta &= \frac{\widehat{\theta}_{shll}}{\theta} - 1 = \frac{1}{\theta}[c\hat{\theta} - c\theta_0 + \theta_0] - 1 \\ &= \frac{c\hat{\theta}}{\theta} + (1-c)\frac{\theta_0}{\theta} - 1 = \frac{c\hat{\theta}}{\theta} + s,\end{aligned}$$

and

$$s = (1-c)\frac{\theta_0}{\theta} - 1.$$

So

$$\begin{aligned}LR_s &= E(L(\triangle)) = E\left(e^{a\triangle} - a\triangle - 1\right) \\ &= E\left(e^{a\ \frac{c\hat{\theta}}{\theta}+s} - a\left(\frac{c\hat{\theta}}{\theta} + s\right) - 1\right) \\ &= e^{as}E\left(e^{\frac{ac}{\theta}\hat{\theta}}\right) - \frac{ac}{\theta}E(\hat{\theta}) - as - 1,\end{aligned}$$

where

$$E(\hat{\theta}) = E(\bar{X}) = \frac{k\theta}{n\beta} + \frac{(n-k)\theta}{n},$$

$$
\begin{aligned}
E\left(e^{\frac{ac}{\theta}\hat{\theta}}\right) &= E\left(e^{\frac{ac}{\theta}\bar{X}}\right) = E\left(e^{\frac{ac}{n\theta}\sum X_i}\right) = E\left(e^{\frac{ac}{\theta}X}\right) \\
&= \int_0^\infty \frac{k\beta}{n\theta} e^{\frac{acx}{\theta}} e^{\frac{-\beta x}{\theta}} dx + \frac{(n-k)}{n\theta} \int_0^\infty e^{\frac{acx}{\theta}} e^{\frac{-x}{\theta}} dx \\
&= \frac{k}{n} \int_0^\infty \frac{\beta}{\theta} e^{-\left(1-\frac{ac}{\beta}\right)\frac{\beta x}{\theta}} dx + \frac{n-k}{n} \int_0^\infty \frac{1}{\theta} e^{-(1-ac)\frac{x}{\theta}} dx \\
&= \frac{k}{n}\frac{\beta}{(\beta-ac)} + \frac{(n-k)}{n}\frac{1}{(1-ac)}.
\end{aligned}
$$

Therefore

$$
LR_s = e^{as}\left(\frac{k\beta}{n(\beta-ac)} + \frac{(n-k)}{n(1-ac)}\right) - ac\left(\frac{k}{n\beta} + \frac{(n-k)}{n}\right) - as - 1.
$$

Now, we have to minimize the risk. Hence, c_0 is found such that $\frac{dLR_s}{dc} = 0$, ie.

$$
\begin{aligned}
h(c) &= -\frac{a\theta_0}{\theta} e^{as}\left(\frac{k\beta}{n(\beta-ac)} + \frac{(n-k)}{n(1-ac)}\right) + e^{as}\left(\frac{k\beta a}{n(\beta-ac)^2} + \frac{(n-k)a}{n(1-ac)^2}\right) \\
&- a\left(\frac{k}{n\beta} + \frac{(n-k)}{n}\right) + a\frac{\theta_0}{\theta} = 0. \qquad (4.\ 6)
\end{aligned}
$$

Therefore, $h(c) = 0$ is solved by using Newton-Raphson method as

$$
c_j = c_{j-1} - \frac{h(c_{j-1})}{h'(c_{j-1})},\ j = 1, 2, ...,
$$

where $h'(c) = \frac{dh(c)}{dc}$.
After selection the proper value of c, LR_s will be minimum for $c = c_0$, where c_0 is the solution of the equation (4.6).

5. Numerical experiments and discussions

Here, to see the performance of the estimators ie. R_s and LR_s, sampling experiments by using **R** statistical software are used. The results are given in Tables 1-6 for k=1,2, β=0.5, 1.5, θ=2,3, θ_0=0.75 and a=-0.2. Bias of $\hat{\theta}$ (ie. Bias($\widehat{\theta}$)) is defined as $E(\hat{\theta}) - \theta$.

Table 1. Bias of the estimators, R_s and LR_s for k=1, β=1.5, θ=2, θ_0=0.75 and a=-0.2.

n	Bias($\widehat{\theta}_{shsel}$)	R_s	Bias($\widehat{\theta}_{shll}$)	LR_s
10	-7.370582	6461.77	-0.867558	0.002586
20	-7.622106	5706.11	-0.874153	0.002691
30	-6.558582	1661.77	-0.888976	0.002655
40	-6.317610	862.48	-0.889309	0.002671
50	-6.094307	702.66	-0.896411	0.002644
60	-6.102083	677.75	-0.889351	0.002701

Table 2. Bias of the estimators, R_s and LR_s for k=2, β=1.5, θ=2, θ_0=0.75 and a=-0.2.

n	Bias($\widehat{\theta}_{shsel}$)	R_s	Bias($\widehat{\theta}_{shll}$)	LR_s
10	-6.671847	1735.64	-0.885200	0.002698
20	-6.462133	1665.02	-0.879905	0.002660
30	-6.379222	1380.26	-0.873978	0.002758
40	-6.145077	724.14	-0.885146	0.002711
50	-6.115715	683.22	-0.881738	0.002747
60	-5.970897	611.50	-0.890469	0.002696

Table 3. Bias of the estimators, R_s and LR_s for k=1, β=1.5, θ=3, θ_0=0.75, a=-0.2.

n	Bias($\widehat{\theta}_{shsel}$)	R_s	Bias($\widehat{\theta}_{shll}$)	LR_s
10	-19.151860	63021.03	-1.384436	0.004786
20	-16.937740	19807.44	-1.389674	0.004950
30	-15.755640	13068.62	-1.413402	0.004930
40	-15.565560	11551.14	-1.407027	0.004982
50	-15.369630	10245.82	-1.406485	0.005003
60	-14.905750	8715.08	-1.421279	0.004972

Table 4. Bias of the estimators, R_s and LR_s for k=2, β=1.5, θ=3, θ_0=0.75 and a=-0.2.

n	Bias($\widehat{\theta}_{shsel}$)	R_s	Bias($\widehat{\theta}_{shll}$)	LR_s
10	-17.621820	75474.21	-1.340366	0.004911
20	-15.478530	14127.74	-1.410833	0.004901
30	-15.574820	12204.55	-1.393489	0.005014
40	-15.120370	10115.33	-1.406576	0.005007
50	-14.813120	8989.30	-1.417287	0.004981
60	-15.012150	8839.64	-1.403421	0.005041

Table 5. Bias of the estimators, R_s and LR_s for k=1, β=0.5, θ=3, θ_0=0.75 and a=-0.2.

n	Bias($\widehat{\theta}_{shsel}$)	R_s	Bias($\widehat{\theta}_{shll}$)	LR_s
10	-31.177520	133815.00	-1.512323	0.004325
20	-20.504490	52664.87	-1.474874	0.004652
30	-18.030170	23910.56	-1.457887	0.004785
40	-17.160260	18089.33	-1.442318	0.004867
50	-16.324460	13794.63	-1.446929	0.004867
60	-16.018730	12267.45	-1.440376	0.004900

Table 6. Bias of the estimators, R_s and LR_s for k=2, β=0.5, θ=3, θ_0=0.75 and a=-0.2.

n	Bias($\widehat{\theta}_{shsel}$)	R_s	Bias($\widehat{\theta}_{shll}$)	LR_s
10	-31.341310	122380.00	-1.624322	0.003857
20	-24.419250	105833.60	-1.529319	0.004465
30	-19.762300	39089.02	-1.507678	0.004602
40	-18.353870	24394.75	-1.477247	0.004758
50	-17.301090	18342.36	-1.472677	0.004784
60	-17.016030	15612.78	-1.451695	0.004880

From Tables 1 to 6 LR_s is less than R_s. Also, comparing the bias shows that the absolute value of bias of the estimators of θ is decreasing respect to n. In addition, the bias of shrinkage estimator of θ under LLF is less than the bias of the corresponding estimator under SELF. Further more, the values of LR_s and R_s are decreasing when n increased. Overall conclusion is that we can say that to estimate θ under g and f the LLF has better performance than the SELF.

6. Actual Example

To show the performance of the estimators, we have selected an actual example which is based on a data set discussed by [11] and [5]. When the size of the crushed rock is larger than the rocks crushed, a rock crushing machine is working. Else, it has been reset. The following data shows the sizes of the crushed rocks up to the third reset of the machine.

9.30, 0.60, 24.40, 18.10, 6.60, 9.00, 14.30, 6.60, 13.00, 2.40, 5.60, 33.80

By using Kolmogorov-Smirnov (Statistic=0.20685, critical value at the level of 5%=0.37543 and p=0.61239) and Anderson-Darling (Statistic=0.33653, critical value at the level of 5%=2.5018) tests, data follows exponential distribution with parameter $\hat{\theta}$=11.97461. Here, we assume that β=1.5, θ_0 is the median and a=-0.2. So, the shrinkage estimators of θ, R_s, LR_s and the corresponding likelihood respect to k=1, 2 and 3 are shown in Table 7.

Table 7. Shrinkage estimators of θ, R_s, LR_s and the corresponding likelihood for k=1,2,3, θ_0=0.75 and a=-0.2.

k	$\widehat{\theta}_{shsel}$	R_s	$L(\widehat{\theta}_{shsel})$	$\widehat{\theta}_{shll}$	LR_s	$L(\widehat{\theta}_{shsel})$
1	8.919453	0.1000459	3.401296e-19	9.332078	7.137392e-05	4.115446e-19
2	8.864743	0.1514201	2.779157e-19	9.372426	8.634380e-05	3.613990e-19
3	8.800492	0.2294922	2.204463e-19	9.419562	1.044919e-04	3.149668e-19

Table 7 shows that the likelihood is maximized when k=1 under both loss functions. Therefore, in the example the number of outliers (k) is equal to 1 and the value of the shrinkage estimators of unknown parameter θ, R_s and LR_s could be taken from the first row of Table 7.

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