Computers & Industrial Engineering 118 (2018) 67–79

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

# Integration of parts transportation without cross docking in a supply chain

Mohammad-Hesam Shaelaie, Mohammad Ranjbar*, Negin Jamili

*Department of Industrial Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran*

A R T I C L E  I N F O

*Keywords:*
Supply chain management
Transportation planning
Heuristic algorithms

A B S T R A C T

In this paper, we consider a supply chain with multiple raw material suppliers, located in close proximity to each other as a single supplier area, who transport the products to an industrial company, as a single customer at the subsequent downstream stage. The goal of this problem is to determine a schedule to integrate the suppliers' products transshipment in order to minimize the total cost which includes transportation and inventory costs, subject to the suppliers' production rate and the customer's daily demands. Following, an integrated transportation system approach, in which all suppliers cooperate with each other by applying a master transportation plan, is designed and evaluated using two linear integer programming models, comparing the integrated transportation model with the non-integrated one. Since solving these models using available commercial solvers is very time consuming, two heuristic algorithms are developed where one of them is combined with two metaheuristic approaches based on genetic and GRASP algorithms. The performance of all developed algorithms are then analysed using randomly generated test instances.

## 1. Introduction

In this study, a specific transportation model is addressed, in which a group of suppliers deliver different products to a large company as a single manufacturer. Considering the suppliers located in close proximity to one another at a distinct location from manufacturing facility, this problem has a wide applicability to real world industrial problems, such as automobile and home appliances manufacturing supply chains, in which components are transferred from suppliers using roadways or railways. As an example, Iran-Khodro car company has many suppliers located around Mashhad city. We assume that the total delivery time from suppliers to the customer is one day and a specific route is taken by all vehicles. In such a supply chain, each supplier usually optimizes its own operational decisions regardless of the other parts. In this case, namely non-integrated system, since orders may have different due dates, suppliers may fail to optimally utilize the vehicles capacity, as they are required to deliver the orders on time and meet the customer's demand. It is also possible for the suppliers to ignore the customer's due dates in order to use the maximum capacity of the vehicles. It can be obviously seen that in both cases, transportation costs are higher, and in terms of customer service level, distribution lead times are longer rather than in an integrated system in which all the suppliers are managed by means of a master strategy that guides the operations by considering the customer's demand rate and its inventory, as well as the suppliers' inventory and production rates. In other words, determining the distribution policy in an integrated system yields more economic savings in transportation costs by simultaneously considering the suppliers plans, as the batching decisions are made for the set of suppliers at the same time, in order to maximize the vehicle's capacity utilization while taking into account the customer's due dates. As a cross dock is a distribution part of the supply chains where multiple smaller shipments are merged into full truck loads, the aforementioned integrated system can be defined as a dummy cross dock accordingly, aiming at improving the performance of the whole supply chain.

The contributions of this articles are threefold: (1) we model both the non-integrated and the integrated transportation system as two linear integer programming models; (2) we develop two heuristic algorithms and combine one of them with two metaheuristic algorithms in order to find good solutions; and (3) we analyze the performance of the developed algorithms and present managerial insights showing when the integrated systems operate better than the non-integrated one.

The remainder of this paper is structured as follows: the next section briefly reviews some related models in the literature. Section 3 describes the problem and the mathematical models. Section 4 is devoted to the methodology developed for the proposed model and Section 5 presents the computational experiments and the obtained comparative results. Finally, Section 6 concludes the paper while providing some future research directions.

## 2. Literature review

The recent interest in supply chain management has led to an

* Corresponding author.
*E-mail addresses:* Hesam_shaelaie@mail.um.ac.ir (M.-H. Shaelaie), m_ranjbar@um.ac.ir (M. Ranjbar), negin.jamili@mail.um.ac.ir (N. Jamili).

https://doi.org/10.1016/j.cie.2018.02.012
Received 28 January 2017; Received in revised form 9 December 2017; Accepted 9 February 2018
Available online 12 February 2018
0360-8352/ 

extensive literature of this topic where tremendous amount of research has been done on various problems. In recent years, some studies have proposed a lot sizing problem incorporated the routing decisions and introduced the production routing problem (PRP) where an integrated production-transportation model is addressed, e.g. Koç, Toptalb, and Sabuncuogluc (2017) and Hajiaghaei-Keshteli, Aminnayeri, and Fatemi Ghomi (2014). In another line of research, the deliveries of products to customers in vender-managed inventory (VMI) systems are integrated into routing problem and an inventory routing problem (IRP) is developed, consequently. Le, Diabat, Richard, and Yih (2013) and Absi, Cattaruzza, Feillet, Ogier, and Semet (2016) focus on such a problem in which inventory and routing decisions are determined simultaneously over a given time horizon. Despite the successful application of IRP and PRP and the wide attention they have received from both practitioners and academic researchers, there is no article studying the exact same characteristics defined in this paper. In the previously mentioned research area, the focus is on the integration of inventory and routing decision, regardless of the items allocation to the vehicles, while the problem we investigate is how to assign multiple products to heterogeneous fleet of vehicles where the products are transferred from suppliers to the customer based on a pre-specified plan. In summary, this problem is regarded as a special case of inventory management (lot sizing problem, LSP; and joint economic lot sizing problem, JELS) combined with bin packing problem (BPP), which highlights the original contribution of this paper. In the following, the problems mentioned above are concisely described and some recently published papers on this topic are provided.

JELS and LSP models are considered to be useful planning tools in case of analyzing the inventory system of a manufacturer where the finished goods are directly transported to the customer. These problems aim at determining production and shipment quantities so that the total cost which includes production inventory setup and transportation costs is minimized subject to satisfying all the customer's orders. The streams of this research emerged from Harris' (1913) seminar paper in which the economic order quantity model was introduced as a simple and efficient tool to avoid excessive inventory costs. A detailed review of the literature in this domain is presented by Glock, Grosse, and Ries (2014). Lee, Han, and Cho (2005) focus on a multi-product dynamic lot sizing problem in which orders are shipped by a single container and is transported from a manufacturer to the customer. Because of the NP-hardness of the proposed problem, a heuristic algorithm based on marginal cost coefficient is developed. A dynamic programming algorithm for solving LSP can be found in Jaruphongsa, ÇEtinkaya, and Lee (2007) where a two-echelon lot sizing model considering less-than-truckload and full-truckload deliveries is proposed. Jaruphongsa and Lee (2008) study the dynamic lot sizing problem with demand time windows and apply two polynomial algorithms to solve the special cases of the problem where split delivery is allowed. Hu and Hu (2016), address a two-echelon stochastic lot sizing problem with sequence-dependent setup cost in order to minimize the total costs under uncertainties.

A review of bin packing problem (BPP) is presented by Lodi, Martello, and Monaci (2002) and Coffman et al. (2013). BPP models deal with an infinite number of capacitated bins and a list of items, where the objective is to pack all the items into a minimum number of bins. As BPP is well-known to be NP-hard (see Garey & Johnson, 1979), several heuristic and metaheuristic methods as well as exact approaches have been developed for solving this problem. A dimension-related classification scheme for the BPP can be found in the literature, differentiating this problem into one-, two- and three-dimensional bin packing. In addition to dimension-related classification, an alternative categorization is suggested in this area, in which a set of constraints such as orientation and guillotine cuts is regarded. A different approach to reviewing BPP models is based on the size of bins which can be constant or variable. The case of variable-sized pin packing problem (VSBPP) was firstly analyzed by Langston (1982), where a number of heterogeneous bins are accessible. Since various types of containers are regarded in this paper, the problem can be almost considered as a generalization of VSBPP. Coming to this type of problem, numerous cases are studied under different assumptions using various solution approaches. Hong, Zhang, Lau, Zeng, and Si (2014) focus on a two-dimensional VSBPP with guillotine constraint and propose a hybrid heuristic algorithm based on simulated annealing and binary search. The VSBPP is also addressed by Alves and De Carvalho (2007). The authors apply the column generation method while analyzing different strategies to stabilize and accelerate it. To tackle the VSBPP, Kang and Park (2003) describe two greedy algorithms in order to optimize the total cost of used bins.

Taking weight and volume of each item into consideration, this study demonstrates a system combining lot sizing, bin packing and scheduling issues, which reflects a principal contribution of this paper.

## 3. Problem statement and modelling

### 3.1. Problem statement

Consider a supply chain consisting of a set of suppliers, $S = \{1,...,|S|\}$, located in close proximity to each other, as well as a customer located at a different site. A variety of items manufactured by $S$ is processed as a batch and delivered to the customer, using an unlimited number of heterogeneous vehicles, within a day via a pre-specified route. The delivery batches might be assigned to different types of vehicles depending on the minimization of total cost, which includes the inventory holding cost and the transportation cost. The objective is to determine the size of batches as well as the vehicles arrangement subjected to some capacity constraints, while minimizing aforementioned performance measure. Other assumptions are listed as follows:

(I) The overall daily production of all the suppliers for each item is a constant parameter equal to the customer's daily demand.
(II) Each type of items is allowed to be manufactured by one or multiple suppliers.
(III) The heterogeneous fleet is considered for the transportation phase, where an infinite supply of each type of them is available.
(IV) Various vehicle types differ in weight and volume capacity as well as delivering costs.
(V) Due to the fixed delivery route, the transportation cost includes no variable routing cost.
(VI) The products are shipped from the suppliers at the beginning of a day (a time period) and delivered to the customer at the beginning of the next day.
(VII) The initial inventory level of suppliers and the customer for each item is a fixed parameter which takes the same value after the planning horizon.
(VIII) The storage space defined for each of the two stages of the supply chain is divided into several sections specified for various items with different capacities.
(IX) The initial inventory level for each facility does not exceed its capacity.
(X) The shortage is not acceptable for the customer.
(XI) Suppliers are not permitted to exchange the items.

### 3.2. Problem modelling

We formulate two models for the addressed problem, where the first one ($IP1$) describes a non-integrated system in which the suppliers make individual decisions to optimize their own transportation system and the second ($IP2$) aims at integrating the coordinated operations of all suppliers in order to keep the whole system profitable. In both cases, the transportation cost is imposed on the suppliers, and decisions are made from their point of view. The parameters and decision variables required for developing the mathematical models are defined in

**Table 1**
Description of required parameters.

| Parameters | Definitions |
|---|---|
| $S = \{1,...,\lvert S\rvert\}$ | Set of suppliers with index $s$ |
| $P = \{1,...,\lvert P\rvert\}$ | Set of item types with index $p$ |
| $V = \{1,...,\lvert V\rvert\}$ | Set of vehicles with index $v$ (for the integrated model) |
| $V^s = \{1,...,\lvert V^s\rvert\}$ | Set of vehicles associated with supplier $s$ with index $v$ (for the non-integrated model) |
| $T = \{1,...,\lvert T\rvert\}$ | Set of time periods with index $t$ |
| $d_p$ | Daily demand of customer for item type $p$ |
| $m_{ps}$ | Daily production of item $p$ associated with supplier $s$ |
| $w_p$ | Weight of item type $p$ |
| $q_p$ | Volume of item type $p$ |
| $wl_v$ | Maximum weight capacity of vehicle $v$ |
| $ql_v$ | Maximum volume capacity of vehicle $v$ |
| $fc_v$ | Fixed acquisition cost of vehicle $v$ |
| $h_p$ | Inventory cost per each item type $p$ imposed on the suppliers |
| $IS_{ps1}$ | Initial inventory level of item type $p$ associated with supplier $s$ at the beginning of the planning horizon ($IS_{ps1} \geqslant m_{ps}$) |
| $IC_{p1}$ | Initial inventory level of item type $p$ associated with the customer at the beginning of the planning horizon ($IC_{p1} \geqslant d_p$) |
| $TQ_{sp}^{max}$ | Storage capacity of supplier $s$ for item type $p$ (specified by the number of items) |
| $TQ_p^{max}$ | Storage capacity of the customer for item type $p$ (specified by the number of items) |

**Table 2**
Description of required decision variables.

| Variables | Definitions |
|---|---|
| $X_{psvt}$ | Amount of item type $p$ transported from supplier $s$ to the customer, on period $t$ and by vehicle $v$ |
| $IS_{pst}$ | Inventory of item type $p$ storage by supplier $s$ at the beginning of period $t$ (before orders distribution) |
| $IC_{pt}$ | Inventory of item type $p$ storage by the customer at the beginning of period $t$ (after receiving the orders) |
| $Y_{vt}$ | A binary decision variable indicating if vehicle $v$ is used on period $t$ |
| $Y_{svt}$ | A binary decision variable indicating if vehicle $v$ is used for supplier $s$ on period $t$ |

Tables 1 and 2, respectively.

It should be noted that the two models are formulated for a single planning horizon in terms of the time units (days). If the same conditions hold at the beginning and end of planning horizon, the obtained schedule can be repeated for the next periods. That is, the finishing of a planning period may overlap the starting of the next period if no changes occur in the input parameters.

### 3.2.1. The non-integrated system model

In the case of non-integrated system of planning, each supplier aims at satisfying the customer's orders while optimizing its own costs, such as transportation and inventory cost, independently. The supplier index $s$ in $Y_{svt}$ is to differentiate between the vehicles applied for each supplier. The set of vehicles is also partitioned into $|S|$ subsets $(V^1,...,V^s,...,V^{|S|})$ so that $V^s$ indicates a given set of vehicle assigned to supplier $s$. The mathematical model of the non-integrated problem is formulated as follows:

***IP*1**:

$$\textbf{\textit{Minimize}} \sum_{s=1}^{|S|}\sum_{v=1}^{|V^S|}\sum_{t=1}^{|T|} fc_v Y_{svt} + \sum_{p=1}^{|P|}\sum_{s=1}^{|S|}\sum_{t=1}^{|T|} h_p\left(IS_{pst} - m_{ps}/2\right) \tag{1}$$

***S. t.***

$$\sum_{p=1}^{|P|} w_p X_{psvt} \leqslant wl_v.\ Y_{svt} \quad \forall\ s \in S\ ;\forall\ v \in V^s\ ;\forall\ t \in T \tag{2}$$

$$\sum_{p=1}^{|P|} q_p X_{psvt} \leqslant ql_v.\ Y_{svt} \quad \forall\ s \in S\ ;\forall\ v \in V^s\ ;\forall\ t \in T \tag{3}$$

$$IS_{pst} = IS_{ps,t-1} + m_{ps} - \sum_{v=1}^{|V^S|} X_{psv,t-1} \quad \forall\ p \in P\ ;\forall\ s \in S\ ;\forall\ t \in T\backslash\{1\} \tag{4}$$

$$IC_{pt} = IC_{p,t-1} - d_p + \sum_{s=1}^{|S|}\sum_{v=1}^{|V^S|} X_{psv,t-1} \quad \forall\ p \in P\ ;\forall\ t \in T\backslash\{1\} \tag{5}$$

$$\sum_{v=1}^{|V^S|}\sum_{t=1}^{|T|} X_{psvt} = |T|\cdot m_{ps} \quad \forall\ p \in P\ ;\forall\ s \in S \tag{6}$$

$$\sum_{v=1}^{|V^S|} X_{psvt} \leqslant IS_{pst} \quad \forall\ p \in P\ ;\forall\ s \in S\ ;\forall\ t \in T \tag{7}$$

$$IC_{pt} \geqslant d_p \quad \forall\ p \in P\ ;\forall\ t \in T \tag{8}$$

$$IS_{pst} \leqslant TQ_{sp}^{max} \quad \forall\ p \in P\ ;\forall\ s \in S\ ;\forall\ t \in T \tag{9}$$

$$IC_{pt} \leqslant TQ_p^{max} \quad \forall\ p \in P\ ;\forall\ t \in T \tag{10}$$

$$IS_{pst}, IC_{pt}, X_{psvt} \in \mathbb{Z}^+ \quad \forall\ p \in P\ ;\forall\ s \in S\ ;\forall\ v \in V^s\ ;\forall\ t \in T \tag{11}$$

$$Y_{svt} \in \{0,1\} \quad \forall\ s \in S\ ;\forall\ v \in V^s\ ;\forall\ t \in T \tag{12}$$

The objective of this model is to minimize the total cost, so that the first term gives the transportation costs and the second term gives the holding costs incurred for the average daily inventory. The holding cost is calculated using $\sum_{p=1}^{|P|}\sum_{s=1}^{|S|}\sum_{t=1}^{|T|} h_p(IS_{pst} - \sum_{v=1}^{|V^S|} X_{psvt} + m_{ps}/2)$, where the amount of items dispatched to the customer is subtracted from the average inventory per day. This equation is then converted to Eq. (1) as shown below.

$$\sum_{p=1}^{|P|}\sum_{s=1}^{|S|}\sum_{t=1}^{|T|} h_p\left(IS_{pst} - \sum_{v=1}^{|V^S|} X_{psvt} + m_{ps}/2\right)$$
$$= \sum_{p=1}^{|P|}\sum_{s=1}^{|S|}\sum_{t=1}^{|T|-1} h_p\left(IS_{pst} - \sum_{v=1}^{|V^S|} X_{psvt} + m_{ps}/2\right)$$
$$+ \sum_{s=1}^{|S|}\sum_{p=1}^{|P|} h_p\left(IS_{ps|T|} - \sum_{v=1}^{|V^S|} X_{psv|T|} + m_{ps}/2\right)$$

Replacing $IS_{pst} - \sum_{v=1}^{|V^S|} X_{psvt} + m_{ps}/2$ by $IS_{ps,t+1} - m_{ps}/2$ and $IS_{ps|T|} - \sum_{v=1}^{|V^S|} X_{psv|T|} + m_{ps}/2$ by $IS_{ps1} - m_{ps}/2$, we then simplify further relations and the second term of the objective is obtained afterwards.

Constraints (2) and (3) describe the relation between $Y_{svt}$ and $X_{psvt}$, as well as the restriction imposed by the capacity of vehicles. Constraint (4) and (5) denote the inventory level of suppliers and the customer, respectively, at the beginning of each time period, before and after the shipments transportation. The balance between the customer's demands and the suppliers' production is guaranteed in constraint (6), where the total amount of production is ensured to be transported in order to maintain the initial inventory level of the suppliers, and the customer as well. Constraint (7) imposes that the shipment quantities of each items on a specific time period do not exceed the initial inventory of that type of item at the beginning of the period. Constraint (8) stipulates the minimum amount of customer's inventory per day, which is certainly not less than the daily demand. Constraint (9) and (10) define the storage capacity of the set of suppliers and the customer, respectively. Finally, the last two constraints are the non-negative ones to determine the type of the decision variables, in which $\mathbb{Z}^+$ indicates the set of non-negative integers.

### 3.2.2. The integrated system model

Considering a joint system, the set of suppliers and the manufacturer cooperate together for the maximum benefit. There is consequently a master policy in which each supplier is able to access the complete set of vehicles and $Y_{vt}$ is the related decision variable. The objective and the constraints of this model, presented below, are conceptually the same as the previously mentioned one, as the only difference is in the accessibility of suppliers to the vehicles.

***IP*2**:

$$\textbf{Minimize} \sum_{v=1}^{|V|}\sum_{t=1}^{|T|} fc_v Y_{vt} + \sum_{p=1}^{|P|}\sum_{s=1}^{|S|}\sum_{t=1}^{|T|} h_p(IS_{pst}-m_{ps}/2) \tag{13}$$

***S. t.***

$$\sum_{p=1}^{|P|}\sum_{s=1}^{|S|} w_p X_{psvt} \leqslant wl_v.\ Y_{vt} \quad \forall\ v\in V\ ;\ \forall\ t\in T \tag{14}$$

$$\sum_{p=1}^{|P|}\sum_{s=1}^{|S|} q_p X_{psvt} \leqslant ql_v.\ Y_{vt} \quad \forall\ v\in V\ ;\ \forall\ t\in T \tag{15}$$

$$IS_{pst} = IS_{ps,t-1} + m_{ps} - \sum_{v=1}^{|V|} X_{psv,t-1} \quad \forall\ p\in P\ ;\ \forall\ s\in S\ ;\ \forall\ t\in T\backslash\{1\} \tag{16}$$

$$IC_{pt} = IC_{p,t-1} - d_p + \sum_{s=1}^{|S|}\sum_{v=1}^{|V|} X_{psv,t-1} \quad \forall\ p\in P\ ;\ \forall\ t\in T\backslash\{1\} \tag{17}$$

$$\sum_{v=1}^{|V|}\sum_{t=1}^{|T|} X_{psvt} = |T| m_{ps} \quad \forall\ p\in P\ ;\ \forall\ s\in S \tag{18}$$

$$\sum_{v=1}^{|V|} X_{psvt} \leqslant IS_{pst} \quad \forall\ p\in P\ ;\ \forall\ s\in S\ ;\ \forall\ t\in T \tag{19}$$

$$IC_{pt} \geqslant d_p \quad \forall\ p\in P\ ;\ \forall\ t\in T \tag{20}$$

$$IS_{pst} \leqslant TQ_{sp}^{max} \quad \forall\ p\in P\ ;\ \forall\ s\in S\ ;\ \forall\ t\in T \tag{21}$$

$$IC_{pt} \leqslant TQ_{p}^{max} \quad \forall\ p\in P\ ;\ \forall\ t\in T \tag{22}$$

$$IS_{pst}, IC_{pt}, X_{psvt} \in \mathbb{Z}^+ \quad \forall\ p\in P\ ;\ \forall\ s\in S\ ;\ \forall\ v\in V\ ;\ \forall\ t\in T \tag{23}$$

$$Y_{vt} \in \{0,1\} \quad \forall\ v\in V\ ;\ \forall\ t\in T \tag{24}$$

## 4. Solution approaches

As mentioned earlier, the bin packing problem, which is a variant of our problem, is known to be a NP-hard optimization problem and thus implies the NP-hardness of the problem considered in this work. As computational tests show, commercial optimization software packages like CPLEX or LINDO are rarely able to find optimal solution of the *IP*1 and *IP*2 models, especially in large-size instances, within a reasonable computation time. Therefore, we focus on developing heuristics, namely rounding algorithm (RA) and single-period algorithm (SPA). As the *IP*2 model shows better performance compared with the *IP*1 (see Section 5), the proposed solution approaches are designed and implemented based on the *IP*2.

It should be noticed that the RA generates only one solution while the SPA has flexible parameters and is able to generate different random solutions. Thus, the SPA is combined with two metaheuristic algorithms based on the genetic algorithm (GA) and the greedy randomized adaptive search algorithm (GRASP) to find better values for its parameters.

### *4.1. The rounding algorithm*

In brief, the basic structure of this method consists of two phases. The quantity of items transported by suppliers in each period is firstly determined and then assigned to capacitated vehicles in the second phase. As the two phases are needed to be solved simultaneously in order to reach an optimized solution, this algorithm is to consider them at the same time. To do so, in the first phase, the RA aims at specifying a distribution plan in each period regardless of some of the constraints imposed in the second phase so that the transportation cost is roughly considered. Thereafter, the assignment problem, in which each supplier's items are dedicated to a specific vehicle to be delivered, is solved. In the following, the framework of this method is summarized in Algorithm 1.

**Algorithm 1:** General framework of the RA

| | |
|---|---|
| Phase 1 | Develop the *IP*3 based on the *IP*2 model in order to determine the total deliveries. |
| | Relax the *IP*3 variables to continuous ones. |
| | Solve the model by CPLEX. |
| | Round the fractional inventory-related variables. |
| | Determine each supplier's deliveries at each period. |
| Phase 2. | Develop *IP*4 in order to solve the assignment problem. |
| | Solve the model using CPLEX. |
| | Obtain the distribution variables to define the type of vehicles transporting each delivery. |

The detailed procedure of the proposed pseudo-code is given below.

#### *4.1.1. Determination of the suppliers' delivery quantity (Phase 1)*

In this phase, we assess the amount of transported items from each supplier's warehouse to the customer at each period, regardless of the type of vehicles they are delivered by, using a relaxed problem and then rounding the obtained solution. To do so, we first reindex the variables and replace $X_{psvt}$ and $Y_{vt}$ with $X_{pskt}$ and $Y_{kt}$, where index $k$ indicates the type of vehicles ($K = \{1,...,|K|\}$, $|K| \ll |V|$) and $X_{pskt}$ denotes the amount of item $p$ transported from supplier $s$ using vehicle type $k$ at period $t$. Since the binary variable $Y_{vt}$ changes to an integer value representing the total number of items carried by vehicle type $k$ ($Y_{kt}$), and the number of variables is decreased by this modification, the formulated model using these variables, defined by *IP*3, is solvable within a less computational time rather than *IP*2. From another point of view, the coordinated decision making in the transportation stage, where each item is assigned to a specific vehicle, is summarized using the proposed reindexing procedure and the supplying plan is more highlighted. It is worth noting that the optimal solution obtained by solving the *IP*3 cannot be considered as the best solution for *IP*2, and since it may exceed the vehicle capacity constraints regarded in *IP*2, the feasibility of the optimal solution of *IP*3 is not guaranteed for *IP*2.

In the next step, by relaxing the integer variables in *IP*3, including $X_{pskt}$, $IS_{pst}$ and $IC_{pt}$, to continuous variables, we obtain LP relaxation, represented as *MIP*, in which the new variables are denoted as $X'_{pskt}$, $IS'_{pst}$ and $IC'_{pt}$. It should be mentioned that since $Y_{kt}$ accounts for the minority of the variables in proportion to the others, there is no need to relax this variable. In other words, modifying $X_{pskt}$, and the inventory-related variables ($IS_{pst}$ and $IC_{pt}$) consequently, is enough to make the problem solvable in a reasonable computational time.

Solving the *MIP* formulation by CPLEX, we then investigate the generated solution. If generated solution is an integer, then it is optimal for both *MIP* and *IP*3 models; otherwise, we then have to change the non-integer values into integers in order to get a feasible solution for *IP*3. In this case, as we are aiming at determining the delivery quantities, the obtained values of $Y_{kt}$ are ignored and $X'_{pskt}$, $IS'_{pst}$ and $IC'_{pt}$ variables are rounded using the two stages provided in the following. The required notations in this procedure are also described in Table 3.

It is worth noting that in order to find the integer values of $X'_{pst}$, we are not allowed to use the floor or ceiling functions since some constraints are violated consequently. Rounding this variable to the largest previous integer means that the delivery from supplier $s$ is less than the customer's demand. Hence, using the floor function may lead to the

**Table 3**
Description of required notations for RA.

| Notations | Definitions |
|---|---|
| $X_{psvt}$, $IS_{pst}$, $IC_{pt}$, $Y_{vt}$ | Decision variables used in the *IP*2 |
| $X_{pskt}$, $IS_{pst}$, $IC_{pt}$, $Y_{kt}$ | Decision variables used in the *IP*3 |
| $X'_{pskt}$, $IS'_{pst}$, $IC'_{pt}$, $Y_{kt}$ | Decision variables used in the *MIP* |
| $X^{new}_{pst}$, $IS^{new}_{pst}$, $IC^{new}_{pt}$ | Rounded integer values of the *MIP* variables (Stage 2) |
| $X'_{pst}$ | The total amount of item $p$ transported from supplier $s$ at period $t$ ($X'_{pst}=\sum_{k=1}^{\vert K\vert} X'pskt$) |
| $X'_{pt}$ | The total amount of item $p$ transported at period $t$ ($X'_{pt}=\sum_{s=1}^{\vert S\vert} X'pst$) |
| $X^{new}_{pt}$ | Rounded integer value of $X'_{pt}$ (Stage 1) |
| $L_{pt}$ | The largest integer value lower than or equal to $X'_{pt}$ ($L_{pt}=\sum_{s=1}^{\vert S\vert}\lfloor X'pst\rfloor$) |

violation of the supplier's warehouse capacity as well as the customer's demand constraint (see constraints (21) and (20), respectively). On the other hand, rounding the aforementioned variable to the smallest following integer by using the ceiling function, may possibly result in exceeding the customer's inventory capacity or a negative inventory level for the supplier, based on constraints (22) and (19) respectively. As a result, the determination of the integer values is considered as a significant problem which is tackled in this paper by a two-stage procedure sketched based on some of the *MIP* model's properties presented as follows.

(I) According to the equation $\sum_{k=1}^{|K|}\sum_{t=1}^{|T|} X'_{pskt}=|T|m_{ps}$, which is one of the *MIP* constraints, the total amount of item $p$ transported from each supplier (regardless of the type of vehicles they are delivered by) in the planning horizon is an integer equal to $|T|m_{ps}$.
(II) If the variable $X'_{pst}$ takes a non-integer value, it can be concluded that there is an item manufactured at a period before $t$, where a part of it is delivered at time $t$ (the fractional part of the non-integer value).
(III) The customer is not able to use an incomplete item. In other words, the fractional parts of the orders are not regarded as the complete items and are not admissible to be used by the customer, since the demands are assumed to be integer.
(IV) As the capacity of warehouses for each item is defined by an integer number, the fractional orders are considered to occupy an integer amount of physical space in the warehouses of the customer as well as the suppliers.

**Stage 1:** *Determining the total deliveries at each period*

Representing the total amount of item $p$ received at period $t$ as $X'_{pt}=\sum_{s=1}^{|S|} X'_{pst}$, the customer is not able to make use of a fractional order. Let us first consider the following information, provided in Table 4, related to one type of item in an instance with 4 suppliers and $|T|=6$.

As an example, the sum of fractional parts of the non-integer deliveries at time periods 1 and 2 is equal to 1.3. In other words, the incomplete parts received at the mentioned periods, which are not admissible to be used by the customer separately, start to accumulate at $t=1$ and turn into a complete order at $t=2$. Since the inventory holding cost is considered for the suppliers, in this stage we aim at rounding the associated numbers, so that the incomplete orders are gathered and delivered at the first period, where is equal to $t=1$ for the aforementioned example. The pseudo-code of this stage is stated for product $p$ at Algorithm 2.

**Table 4**
Information of the given example.

| Suppliers' deliveries | Time periods | | | | | |
|---|---|---|---|---|---|---|
| | $t=1$ | $t=2$ | $t=3$ | $t=4$ | $t=5$ | $t=6$ |
| $X'_{p1t}=$ | 9 | 4.7 | 5.6 | 10.3 | 0 | 3.4 |
| $X'_{p2t}=$ | 5.2 | 8.3 | 3.3 | 4 | 0 | 6.2 |
| $X'_{p3t}=$ | 0 | 4.7 | 4.2 | 5.1 | 9 | 0 |
| $X'_{p4t}=$ | 3.3 | 12.1 | 4.6 | 0 | 17 | 0 |
| $X'_{pt}=$ | 17.5 | 29.8 | 17.7 | 19.4 | 26 | 9.6 |

**Algorithm 2:** Pseudo-code for stage 1 of RA

**Step 1.** set $t=1$, $X'_{pt}=\sum_{s=1}^{|S|} X'_{pst}$ and $dec=0$;
**Step 2.** **if** $X'_{pt}+dec\in\mathbb{Z}^+$ **then**
let $X^{new}_{pt}=X'_{pt}+dec$ and go to Step 4;
**Step 3.** let $X^{new}_{pt}=\lfloor X'_{pt}+dec\rfloor$;
$dec=X'_{pt}+dec-X^{new}_{pt}$;
**Step 4.** **if** $t<|T|$ **then**
let $t=t+1$, $X'_{pt}=\sum_{s=1}^{|S|} X'_{pst}$ and go to Step 2;
**else** Stop;

**Stage 2:** *Determining each supplier's deliveries*

In this stage, the aim is to assign the total deliveries, denoted by $X^{new}_{pt}$ and determined in the previous stage, to the suppliers in order to obtain $X^{new}_{pst}$. To do so, we first define $L_{pt}=\sum_{s=1}^{|S|}\lfloor X'_{pst}\rfloor$ as the minimum amount of item $p$ transported at period $t$. In case $L_{pt}$ is equal to $X^{new}_{pt}$, $X^{new}_{pst}$ is calculated using this equation: $X^{new}_{pst}=\lfloor X'_{pst}\rfloor$. If $L_{pt}<X'_{pt}$, a procedure, summarized in Algorithm 3, is sketched in order to choose a supplier and increase its delivery by 1. The increment of suppliers' deliveries, and the value of $L_{pt}$ consequently, is then repeated till $L_{pt}$ is equal to $X'_{pt}$.

For ease of explanation, in this procedure the associated information with the suppliers, whose deliveries are considered as fractional numbers, is represented using set , $G=\{\delta_i\}$ where $\delta_i$ is an ordered triple $\delta_i=(\delta_i^1,\delta_i^2,\delta_i^3)$. The three mentioned elements are respectively denoting the index of a supplier, the first period in which a fractional order is delivered, and the period when the other parts of the order are also sent and it is regarded as a complete delivery.

Considering the aforementioned instance to better understand this issue, $G$ contains two elements, (1,2,3) and (1,3,6), relating to supplier 1, where (1,2,3) firstly defines the supplier index (supplier 1) and shows that in period 2 a fractional order is transported and then completed at time 3 (the sum of fractional parts of deliveries in period 2 and 3 is equal to 1.3). Checking Table 4, it is easily concluded that 0.3 of the delivery related to period 3 is accumulated and changed to a complete order at period 6. In summary, $G$ is obtained as $G=\{(1,2,3),(1,3,6),(2,1,6),(3,2,4),(4,1,3)\}$. Algorithm 3 depicts this stage for product $p$ as pseudo-code.

**Algorithm 3:** Pseudo-code for stage 2 in RA

**Step 1.** let $t=1$;
**Step 2.** **for** all $s\in S$ let $X^{new}_{pst}=\lfloor X'_{pst}\rfloor$;
let $L_{pt}=\sum_{s=1}^{|S_p|}\lfloor X'_{pst}\rfloor$;
**Step 3.** **if** $L_{pt}=X^{new}_{pst}$ **then**
go to Step 6;
**else** set $G'=\{\delta_i|\delta_i\in G,\delta_i^2\leqslant t\}$;
**Step 4.** set $i^*=arg\{\min_{\delta_i\in G'}\{\delta_i^3\}\}$;
let $X^{new}_{p\delta^1_{i^*}t}=X^{new}_{p\delta^1_{i^*}t}+1$ and $L_{pt}=L_{pt}+1$;

| | |
|---|---|
| **Step 5.** | $G' \leftarrow G' - \{\delta_{i^*}\}$; |
| | $G \leftarrow G - \{\delta_{i^*}\}$; |
| | **if** $L_{pt} < X_{pt}^{new}$ **then** |
| | go to Step 4; |
| **Step 6.** | **if** $t < |T|$ **then** |
| | let $t = t + 1$ and go to Step 2; |
| | **else** Stop; |

Having implemented this algorithm on the given example, we provide the obtained solution in Table 5. Column A and B represents the values calculated in Step 2 and the final values, respectively.

*4.1.2. Assignment of the transport orders to vehicles (Phase 2)*

In this section, the goal is to determine the value of $X_{psvt}$ and $Y_{vt}$, which is achieved using the following assignment model. To do so, the following model, denoted by *IP*4, in which $X_{pst}^{new}$ is an input data, is solved using CPLEX 12.6.

***IP*4**:

$$\textit{Minimize} \sum_{v=1}^{|V|} \sum_{t=1}^{|T|} fc_v Y_{vt} \tag{25}$$

$$\textit{S. t.} \tag{26}$$

$$\sum_{p=1}^{|P|} \sum_{s=1}^{|S|} w_p X_{psvt} \leqslant wl_v.\ Y_{vt} \quad \forall\, v \in V\,;\, \forall\, t \in T \tag{27}$$

$$\sum_{p=1}^{|P|} \sum_{s=1}^{|S|} q_p X_{psvt} \leqslant ql_v.\ Y_{vt} \quad \forall\, v \in V\,;\, \forall\, t \in T \tag{28}$$

$$\sum_{v=1}^{|V|} X_{psvt} = X_{pst}^{new} \quad \forall\, p \in P\,;\, \forall\, s \in S\,;\, \forall\, t \in T \tag{29}$$

$$X_{psvt} \in \mathbb{Z}^+ \quad \forall\, p \in P\,;\, \forall\, s \in S$$
$$Y_{vt} \in \{0,1\}\ \forall\, v \in V\,;\, \forall\, t \in T \tag{30}$$

In the above model, since the inventory constraints are not considered, the values of parameters in various periods are independent of each other. Therefore, running the *IP*4 for each period ($t = 1,\ldots,|T|$) separately, results in much less computing time rather than the provided mode.

## 4.2. The single-period algorithm

The SPA algorithm is designed to adjust the transportation amount of deliveries at each period, based on a three-phase scheme which is summarized in Algorithm 4. Of course, due to the fact that the results of periods are linked with each other, we need to take the dependency among the deliveries of different periods into account to obtain a feasible solution.

**Table 5**
The obtained solution of stage 2 in RA.

| Suppliers' deliveries | Time periods | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $t = 1$ | | $t = 2$ | | $t = 3$ | | $t = 4$ | | $t = 5$ | | $t = 6$ | |
| | *A* | *B* | *A* | *B* | *A* | *B* | *A* | *B* | *A* | *B* | *A* | *B* |
| $X_{p1t}^{new}=$ | 9 | 9 | 4 | 5 | 5 | 5 | 10 | 11 | 0 | 0 | 3 | 3 |
| $X_{p2t}^{new}=$ | 5 | 5 | 8 | 8 | 3 | 3 | 4 | 5 | 0 | 0 | 6 | 6 |
| $X_{p3t}^{new}=$ | 0 | 0 | 4 | 5 | 4 | 4 | 5 | 5 | 9 | 9 | 0 | 0 |
| $X_{p4t}^{new}=$ | 3 | 4 | 12 | 12 | 4 | 5 | 0 | 0 | 17 | 17 | 0 | 0 |
| $L_{pt}=$ | 17 | 18 | 28 | 30 | 16 | 17 | 19 | 20 | 26 | 26 | 9 | 9 |
| $X_{pt}^{new}=$ | 18 | | 30 | | 17 | | 20 | | 26 | | 9 | |

**Algorithm 4:** General framework of the SPA

| | |
|---|---|
| **for** all $t = 1,\ldots,|T|-1$ | |
| ***Phase 1.*** | Calculate $R_{pt}^{min}$ and $R_{pt}^{max}$. |
| | Determine the total amount of deliveries. |
| | Determine each supplier's deliveries at period $t$. |
| ***Phase 2.*** | Assign the transported items to the vehicles by solving the *IP*4 model. |
| ***Phase 3.*** | Determination of each supplier's contribution. |

Regarding the importance of determination of each period deliveries, the SPA starts with the first period ($t = 1$) in order to return (1) the total amount of deliveries, (2) the assignment of deliveries to the vehicles and lastly, (3) each supplier's contribution in the total delivery amount. Thereafter, these steps are repeated for $t = 2,\ldots,|T|$ to complete the partial solution. In the following, the notations required in this algorithm is given in Table 6 and the detailed procedure is described afterwards.

*4.2.1. Determination of the total amount of deliveries (Phase 1)*

The goal of this phase is to determine the total quantity of items transported in a specific period, regardless of the suppliers/vehicles they are delivered from/by. To do so, two parameters denoted by $R_{pt}^{min}$ and $R_{pt}^{max}$, which define the minimum and maximum number of items type $p$ allowed to be delivered at the given period $t$, are calculated by regarding some of the inventory-related constraints. Thereafter, the total delivery is computed using a coefficient, namely $\alpha_{pt}$, which combines $R_{pt}^{min}$ and $R_{pt}^{max}$ and returns the required value. The detailed procedures to obtain the aforementioned parameters are provided in the following.

*4.2.1.1. Calculation of* $\boldsymbol{R_{pt}^{min}}$. Considering $R_{pt}^{min}$ as a parameter illustrating the minimum amount of item $p$ needed in period $t$, it can be computed by regarding the minimum quantity of this item needed for the customer ($\rho_{pt}^{min}$) and the suppliers ($\sum_{s=1}^{|S|} r_{pst}^{min}$). Therefore, $R_{pt}^{min}$ can be obtained using Eq. (31).

$$R_{pt}^{min} = \max(\rho_{pt}^{min}, \sum_{s=1}^{|S|} r_{pst}^{min}) \quad p = 1,\ldots,|P|;\ t = 1,\ldots,|T|-1 \tag{31}$$

From the customer's point of view, the need for item $p$ arises based on its inventory level which cannot be less than a specific value at each period (see constraint (20)). Replacing $X_{psvt}$ with $\chi_{pst}$, as a variable ignoring the vehicles, we then combine constraints (20) and (17) as

**Table 6**
Description of required notations for SPA.

| Notations | Definitions |
|---|---|
| $R_{pt}^{max}$ | Maximum number of items type $p$ allowed to be delivered at period $t$ |
| $R_{pt}^{min}$ | Minimum number of items type $p$ allowed to be delivered at period $t$ |
| $r_{pst}^{max}$ | Maximum number of items type $p$ allowed to be delivered by supplier $s$ at period $t$ |
| $r_{pst}^{min}$ | Minimum number of items type $p$ allowed to be delivered by supplier $s$ at period $t$ |
| $\rho_{pt}^{max}$ | Maximum number of items type $p$ allowed to be received by the customer at period $t + 1$ |
| $\rho_{pt}^{min}$ | Minimum number of items type $p$ allowed to be received by the customer at period $t + 1$ |
| $\chi_{pt}$ | The total amount of item type $p$ transported at period $t$ |
| $\chi_{pvt}$ | The total amount of item type $p$ transported by vehicle $v$ at period $t$ |
| $\chi_{pst}$ | The total amount of item type $p$ transported from supplier $s$ at period $t$ |
| $\alpha_{pt}$ | Percentage of surplus items of type $p$ delivered at time $t$ |
| $pr_{pst}$ | The priority number indicating how to assign the deliveries of item type $p$ to supplier $s$ at period $t$ |

below:

$$IC_{pt}-d_p+\sum_{s=1}^{|S|}\chi_{pst}\geqslant d_p;\Rightarrow\sum_{s=1}^{|S|}\chi_{pst}\geqslant 2d_p-IC_{pt}\quad p=1,...,|P|;\ t=1,...,|T|-1 \tag{32}$$

Hence, we get the following equation for $\rho_{pt}^{min}$, which returns a non-negative value.

$$\rho_{pt}^{min}=max(0,2d_p-IC_{pt})\quad p=1,...,|P|;\ t=1,...,|T|-1 \tag{33}$$

Establishing a recurrence relation which defines $IC_{pt}$ based on $IC_{p,t-1}$ ($\sum_{\tau=1}^{t-1}\chi_{ps\tau}$ for $t=1$ is equal to zero), we can rewrite Eq. (33) as:

$$\rho_{pt}^{min}=max(0,(t+1)d_p-IC_{p1}-\sum_{s=1}^{|S|}\sum_{\tau=1}^{t-1}\chi_{ps\tau})\quad p=1,...,|P|;\ t=1,...,|T|-1 \tag{34}$$

Regarding the suppliers' point of view, $\rho_{pt}^{min}$ can be similarly calculated such that the storage capacity is observed (see constraint (21)). We then obtain the following equation by combining constraints (21) and (16) and using the associated recurrence relation which corresponds to $IS_{pst}$ .

$$r_{pst}^{min}=max\left(0,tm_{ps}+IS_{ps1}-TQ_{sp}^{max}-\sum_{\tau=1}^{t-1}\chi_{ps\tau}\right)\quad \begin{matrix}p=1,...,|P|;\ s=1,...,|S|;\\ t=1,...,|T|-1\end{matrix} \tag{35}$$

It is noteworthy that since the total amount of deliveries is equal to a predefined value (see constraint (18)), the equations presented in this section are defined for $t=1,...,|T|-1$, and the remaining parameters can be calculated as follows:

$$R_{p|T|}^{max}=R_{p|T|}^{min}=\max(0,|T|\cdot d_p-\sum_{s=1}^{|S|}\sum_{\tau=1}^{|T|-1}\chi_{ps\tau}) \tag{36}$$

*4.2.1.2. Calculation of $\boldsymbol{R_{pt}^{max}}$.* As $R_{pt}^{max}$ indicates the maximum number of items type $p$ allowed to be delivered at period $t$, its value results from three constraints ensuring (1) the suppliers' inventory storage capacities, (2) total amount of each item transported from each supplier in the planning horizon which is equal to $|T|.\ m_{ps}$ and (3) the customer's capacity for receiving item $p$ at period $t+1$. Therefore, we first consider the maximum quantity of item $p$ needed for the customer ($\rho_{pt}^{max}$) and the suppliers ($\sum_{s=1}^{|S|}r_{pst}^{max}$), separately, and then obtain $R_{pt}^{max}$ by choosing the maximum of one of the mentioned terms. The value of $\rho_{pt}^{max}$ can be determined due to the Constraints (17) and (22), as below:

$$\sum_{s=1}^{|S|}\chi_{pst}\leqslant TQ_p^{max}+d_p-IC_{pt}\Rightarrow\rho_{pt}^{max}=TQ_p^{max}+d_p-IC_{pt}\quad \begin{matrix}p=1,...,|P|;\\ t=1,...,|T|-1\end{matrix} \tag{37}$$

Lastly, using the recurrence relation resulted from constraint (17), we have Eq. (38) in order to calculate $\rho_{pt}^{max}$.

$$\rho_{pt}^{max}=TQ_p^{max}+td_p-\sum_{s=1}^{|S|}\sum_{\tau=1}^{t-1}\chi_{ps\tau}-IC_{p1}\quad p=1,...,|P|;\ t=1,...,|T|-1 \tag{38}$$

To calculate $r_{pst}^{max}$, we take constraints (16), (18) and (19) into account and reach the following equation, where $|T|.\ m_{ps}-\sum_{\tau=1}^{t-1}\chi_{ps\tau}$ guarantees the balance between the customer's demands and the suppliers' productions and $\sum_{\tau=1}^{t-1}(m_{ps}-\chi_{ps\tau})$ is equal to zero where $t=1$.

$$r_{pst}^{max}=\min(IS_{pst},|T|.\ m_{ps}-\sum_{\tau=1}^{t-1}\chi_{ps\tau})\quad \begin{matrix}p=1,...,|P|;\\ s=1,...,|S|;\end{matrix} \tag{39}$$

$$r_{pst}^{max}=\min(IS_{ps1}+\sum_{\tau=1}^{t-1}(m_{ps}-\chi_{ps\tau}),|T|.\ m_{ps}-\sum_{\tau=1}^{t-1}\chi_{ps\tau})\quad t=1,...,|T|-1 \tag{40}$$

Following the proposed equations, $R_{pt}^{max}$ is then obtained using Eq. (41).

$$R_{pt}^{max}=\min(\sum_{s=1}^{|S|}r_{pst}^{max},\rho_{pt}^{max})\ p=1,...,|P|;\ t=1,...,|T|-1 \tag{41}$$

*4.2.1.3. Determination of the total of deliveries.* In this section, we calculate the quantity of total deliveries at a period, regarding the previously calculated parameters, $R_{pt}^{min}$ and $R_{pt}^{max}$. Before we proceed with presenting the formula, we investigate the feasibility of the solution obtained using this method. The hard constraints of $IP2$, leading to the infeasibility of a solution, are fivefold. By choosing the quantity of deliveries more than $R_{pt}^{min}$, constraints (20) and (21) are satisfied and by choosing it less than $R_{pt}^{max}$, constraints (18), (19) and (21) are satisfied. Thus, choosing an integer value in the interval $[R_{pt}^{min},R_{pt}^{max}]$ leads to the satisfaction of all the hard constraints and feasibility of the solution, consequently. In the following, we prove that the interval $[R_{pt}^{min},R_{pt}^{max}]$ includes at least a feasible value. In other words, we show that $R_{pt}^{min}\leqslant R_{pt}^{max}$.

**Theorem 1.** For all $p=1,...,|P|$ and $t=1,...,|T|$, we have $R_{pt}^{min}\leqslant R_{pt}^{max}$.

**Proof.** See Appendix A.

Obviously, the amount of each item's deliveries in each period is at least $R_{pt}^{min}$. While transporting some extra items for the following periods can result in actual cost savings due to a reduction in distribution cost, the surplus items must not exceed $R_{pt}^{max}-R_{pt}^{min}$. Therefore, we present Eq. (42) to determine $\chi_{pt}$ where $\alpha_{pt}$, denoting the percentage of surplus items of type $p$ delivered at period $t$, is obtained using the two metaheuristic methods developed in Section 4.3.

$$\chi_{pt}=R_{pt}^{min}+\lfloor\alpha_{pt}(R_{pt}^{max}-R_{pt}^{min})\rfloor\quad p=1,...,|P|;\ t=1,...,|T|-1 \tag{42}$$

*4.2.2. Assigning the transported items to the vehicles (Phase 2)*

In order to define the values of $Y_{vt}$, the $IP4$ model introduced in Section 4.1.2 is used. To do so, we first replace $\sum_{s=1}^{|S|}X_{psvt}$ by $\chi_{pvt}$. Thereafter, solving the modified model leads to the assignment of the items to the vehicles.

*4.2.3. Determination of each supplier's contribution (Phase 3)*

As previously mentioned, since some items are produced by various suppliers, determining each supplier's contribution in the total delivery amount becomes an important issue in this problem. In this phase, the items are dedicated to various suppliers to be supplied by, based on a priority number assigned to the suppliers as formalized below:

$$pr_{pst}=\frac{m_{ps}}{TQ_{sp}^{max}-IS_{pst}+1}\quad p=1,...,|P|;\ s=1,...,|S|;\ t=1,...,|T| \tag{43}$$

According to Eq. (43), $pr_{pst}$ is designed to prioritize the suppliers who have smaller storage space rather than the others. Since this measure is dependent on the remaining space of the warehouse as well as the production rate of the suppliers, the priority is always given to the supplier with higher production rate and smaller storage space. It is to be noted that 1 is added to the original value of the denominator in order to have a non-zero integer denominator. Algorithm 5 describes how the orders are assigned to the suppliers for each period $t$ based on this procedure.

**Algorithm 5:** Pseudo-code for phase 3 in SPA

**Step 1.** set $p=1$;
**Step 2.** **if** $\chi_{pt}=0$ **then**
go to Step 4;
**else for** all $s\in S$ calculate $pr_{pst}$;set $s^*=arg\{\max_{s\in S}\{pr_{pst}\}\}$;
**Step 3.** let $\chi_{ps^*t}=\chi_{ps^*t}+1$ and $\chi_{pt}=\chi_{pt}-1$;

go to Step 2;
**Step 4.** **if** $p \leqslant |P|$ **then**
let $p = p + 1$;go to Step 2;
**else** Stop ;

Having calculated $\chi_{pst}$ and $\chi_{pvt}$, we then arbitrarily determine $X_{psvt}$, since this value has no effect on the objective function.

### 4.3. Metaheuristic algorithms for determination of $\alpha_{pt}$

#### 4.3.1. The genetic algorithm

In this section, a genetic algorithm is applied to reach the good values of $\alpha_{pt}$. Introduced by Holland (1992), GA is an evolutionary algorithm inspired by Darwin's theory about evolution to solve optimization problems. For the problem presented in this article, we use a general and simple version of GA described in Man, Tang, and Kwong (2012). In this algorithm, each chromosome is represented by a $|P| \times (|T|-1)$ matrix, where each row denotes one type of item and each column corresponds to a time period. As the decision made for $|T|-1$ periods would automatically lead to the delivery plan of period $|T|$, this period is not considered in the chromosome matrix, depicted in Fig. 1.

The parameters used in this method are now presented in Table 7.

The algorithms starts with an initial population $Pop$, consisting of $|Pop|-1$ randomly generated individuals and one member obtained from implementing the first phase of relaxation procedure presented in Section 4.1.1. Then, the offsprings are generated from the initial population by the one-point crossover operator applied on each row. The mutation operator is then used for the new individuals' genes with the probability of $p_m$ in which $\alpha_{pt}$ is randomly taken from the interval [0,1]. After each iteration, a local search, developed based on the first improvement strategy, is applied on the offspring population with the probability of $p_l$, such that the values of each column, denoted by $\alpha_{pt}$, are modified within the interval $[\alpha_{pt}-\Delta,\alpha_{pt}+\Delta]$ to examine the potential improvements for the solution generated by that chromosome. The algorithm continues to update the next generations until the stopping criterion, defined by a limit on the overall running time, is met.

$$\begin{array}{cc} & T \\ P & \begin{array}{|c|c|c|c|c|} \hline \alpha_{11} & \cdots & \alpha_{1t} & \cdots & \alpha_{1,|T|-1} \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline \cdots & \cdots & \alpha_{pt} & \cdots & \cdots \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline \alpha_{|P|1} & \cdots & \alpha_{|P|t} & \cdots & \alpha_{|P|,|T|-1} \\ \hline \end{array} \end{array}$$

**Fig. 1.** The chromosome representation in GA.

**Table 7**
Description of required notations in GA.

| Notations | Definitions |
|---|---|
| $p_m$ | The mutation probability |
| $p_l$ | The probability of using the local search |
| $\|Pop\|$ | The size of initial population |
| $\Delta$ | The parameter needed for local search |
| $TL$ | The time limit defined as the stopping criterion |

#### 4.3.2. The GRASP algorithm

The GRASP or Greedy Randomized Adaptive Search Procedure, developed by Feo and Resende (1989), is an iterative procedure combining a constructive phase and an improvement phase. In this section, in order to improve the values of $\alpha_{pt}$ by the GRASP, a random initial solution is generated and then improved using the local search introduced in the previous section. The pseudo-code of this algorithm is summarized in Algorithm 6.

**Algorithm 6:** Pseudo-code for GRASP

**While** (time < $TL$) **do**
Generate a random solution;
Apply local search on the solution;
Evaluate fitness;
Compare it with best solution and update the best solution if necessary;
**End**

## 5. Computational results

In this section, computational experiments were conducted to observe the efficiency of the proposed integrated transportation system as well as the developed algorithms. In order to evaluate the performance of the algorithms, they were coded in Visual C++ and run on a portable computer with an Intel Core i5 CPU and 4 GB RAM under Windows 7 operating system. Also, we used the IBM ILOG CPLEX 12.6 to run our models.

### 5.1. Data sets generation

The tests were performed on 135 instances organized into 3 classes of 45 instances each, namely large, medium and small. The data set were randomly generated based upon the following parameters given in Table 8.

For each combination of parameters, five random instances have been generated. Also, the planning horizon is set equal to 6 ($|T| = 6$), equivalent to 6 working days in a week, for all the instances. The remaining parameters are assessed according to the information provided from a car company, Iran-Khodro, as follows. The weight ($w_p$), volume ($q_p$) and the price of each item $p$ are randomly chosen from the intervals [25,30], [0.01,0.1] and [2,600], respectively. For each item type, the yearly inventory cost imposed to the suppliers is 5% of their price and the customer's daily demand for them ($d_p$) is randomly generated from interval [10,800]. Thereafter, we approximate the number of suppliers who produce a specific type of item based on their demand. Table 9 represents the relationship between two parameters $d_p$ and $|S|$, where each supplier's contribution in the total production is defined, i.e. if 600 units of an item is dedicated to one supplier (with the minimum contribution rate of 100%), they are all produced by this supplier, while in the case 3 suppliers (with the minimum contribution rate of 25%), each supplier is responsible for providing at least 150 (600 × 25%) units of the given item.

The initial inventory level of item type $p$ for the customer ($IC_{p1}$) and supplier $s$ ($IS_{ps1}$) is an integer from $[d_p,3d_p]$ and $[m_{ps},3m_{ps}]$, respectively. In order to determine the maximum storage capacity for the customer,

**Table 8**
Parameters of data sets.

| Size of data | Number of suppliers ($\|S\|$) | Number of items types ($\|P\|$) |
|---|---|---|
| Small | {2,3,4} | {2,3,4} |
| Medium | {5,6,7} | {5,10,15} |
| Large | {8,9,10} | {20,25,30} |

**Table 9**
Relationship between parameters $d_p$ and $|S|$.

| $d_p$ | $\|S\|$ | Minimum contribution rate |
|---|---|---|
| $d_p \leqslant 200$ | {1} | 100% |
| $200 < d_p \leqslant 400$ | {1,2} | 40% |
| $400 < d_p \leqslant 600$ | {1,2,3} | 25% |
| $600 < d_p \leqslant 800$ | {1,2,3,4} | 15% |

we generate a random integer in the interval $[minTQ_p,maxTQ_p]$, where how to find min and max is described as follows. The minimum of capacity of the customer's storage needed for item $p$ ($minTQ_p$) is equal to its initial inventory level ($IC_{p1}$) and the maximum ($maxTQ_p$) is calculated by considering a case where all the suppliers transport their initial inventory to the customer. Hence, Eq. (44) is employed to compute $maxTQ_p$ .

$$TQ_p^{max} \leqslant \sum_{s=1}^{|S|} \min(IS_{ps1},Tm_{ps}) + IC_{p1}-d_p \quad p = \{1,...,|P|\} \tag{44}$$

$$\Rightarrow maxTQ_p = \sum_{s=1}^{|S|} \min(IS_{ps1},Tm_{ps}) + IC_{p1}-d_p$$

In a similar manner, to obtain the storage capacity of each supplier, which is randomly taken from the interval $[minTQ_{ps},maxTQ_{ps}]$, two new parameters are introduced and calculated. $minTQ_{ps}$, denoting the minimum capacity needed for supplier $s$ to stock item $p$ is equal to its initial inventory ($IS_{ps1}$), while the maximum value is computed using the Eq. (45).

$$TQ_{ps}^{max} \leqslant IS_{ps1} + t_{ps}^{*} m_{ps}-sl_{ps}^{*} \quad p = \{1,...,|P|\};\ s = \{1,...,|S|\} \tag{45}$$

$$\Rightarrow maxTQ_{ps} = IS_{ps1} + t_{ps}^{*} m_{ps}-sl_{ps}^{*}$$

The maximum inventory level of an item for each supplier is occurred in the case where the customer's demand is satisfied by the other suppliers. For illustration purposes, let us assume item type $p$ which is supplied by two suppliers. For supplier 1, the inventory level of this item reaches it maximum at a period when the customer's demand is totally satisfied by only supplier 2. This period in which the inventory is maximized is determined using Eq. (46), where $t_{ps}$ is a variable representing the time period and takes integer value from the interval $[0,|T|-1]$ and $t_{ps}^{*}$ is the largest value of $t_{ps}$ for which Eq. (46) is true.

$$IC_{p1} + \sum_{s'\neq s} \min(IS_{ps'1} + m_{ps'}(t_{ps}-1),|T|.\ m_{ps'}) \geqslant d_p t_{ps} \quad p = \{1,...,|P|\};\ s = \{1,...,|S|\} \tag{46}$$

Thereafter, we transform the mentioned inequality to an equality by adding a positive slack, namely $sl_{ps}^{*}$. Having found the value of $sl_{ps}^{*}$, we then use Eq. (45) to obtain $maxTQ_{ps}$. Taking the Iran-Khodro Company into account, we have considered 6 types of delivery vehicles for which the related information is given in Table 10.

In order to determine the total number of vehicles as well as those dedicated to each supplier ($|V^s|$), Eqs. (47) and (48) are formalized where $P_s$ denotes the set of items produced by supplier $s$. It is worth mentioning that the total number of vehicles in the $IP2$ ($|V|$) is less than in $IP1$ ($\sum_s |V^s|$) due to the integration of distribution phase.

$$|V^s| = \sum_{k=1}^{|K|} \sum_{p=1}^{|P_s|} \left\lceil \frac{TQ_{ps}^{max}}{min\left(\frac{wl_k}{w_p},\frac{ql_k}{q_p}\right)} \right\rceil \tag{47}$$

$$|V| = \sum_{k=1}^{|K|} \sum_{p=1}^{|P|} \left\lceil \frac{TQ_{p}^{max}}{min\left(\frac{wl_k}{w_p},\frac{ql_k}{q_p}\right)} \right\rceil \tag{48}$$

**Table 10**
Information of various types of vehicles in the test instances.

| Vehicle type | Weight capacity (1000 kg) | Volume capacity ($m^3$) | Fixed acquisition cost (cost unit) |
|---|---|---|---|
| 1 | 2 | 3.84 | 350 |
| 2 | 3 | 16 | 480 |
| 3 | 5.5 | 21.7 | 540 |
| 4 | 10 | 29.3 | 800 |
| 5 | 15 | 35.1 | 950 |
| 6 | 22 | 76.25 | 1200 |

### 5.2. Integration value

In this section, we evaluate the quality of the two approaches proposed in this paper, the solutions obtained from the mathematical models, $IP1$ and $IP2$, were compared using small and medium size instances. We define the percentage of improvement as $\frac{\text{reduction in the objective function value}}{\text{objective function of the non-integrate system}} \times 100$. As a result of integrating the transportation system, we achieve the average of 30% and 25% improvement in the total costs for small-size and medium-size instances, respectively. The detailed results are presented in Table 11.

### 5.3. Performance comparisons of the proposed algorithms

To evaluate the efficiency of algorithms, we have performed some preliminary experiments to determine the values of the parameters. Thus, using the Design Expert v10, we reach the final values for the parameters of GA and GRASP as shown in Table 12.

Since in terms of efficiency, the RA approach runs in much less computing time rather than that required by other methods, we compare this algorithm with the optimal solutions of $IP2$, obtained by running CPLEX. Due to the complexity of large-size instances, the mathematical model is impractical for solving such problems. Therefore, we solve the $MIP$ model, which provides a good lower bound on the optimal value of the problem and compare the obtained solutions of other algorithms with this lower bound.

For each type of problem and procedure, Table 13 shows the average percent of deviation from the lower bound provided by the $MIP$ (%GAP) and the computing time (Time) in seconds.

Regarding the results summarized in the above table, we can derive that the RA procedure would be an appropriate method, since good

**Table 11**
Average of integration improvement for small and large test instances.

| Category | Number | Suppliers ($\|S\|$) | Items types ($\|P\|$) | Avg. of integration improvement (%) |
|---|---|---|---|---|
| Small-size instances | 1 | 2 | 2 | 39 |
| | 2 | 2 | 3 | 40 |
| | 3 | 2 | 4 | 38 |
| | 4 | 3 | 2 | 27 |
| | 5 | 3 | 3 | 33 |
| | 6 | 3 | 4 | 35 |
| | 7 | 4 | 2 | 22 |
| | 8 | 4 | 3 | 12 |
| | 9 | 4 | 4 | 20 |
| Medium-size instance | 10 | 5 | 5 | 10 |
| | 11 | 5 | 10 | 33 |
| | 12 | 5 | 15 | 23 |
| | 13 | 6 | 5 | 32 |
| | 14 | 6 | 10 | 26 |
| | 15 | 6 | 15 | 27 |
| | 16 | 7 | 5 | 18 |
| | 17 | 7 | 10 | 25 |
| | 18 | 7 | 15 | 18 |

**Table 12**
Parameter setting for metaheuristics.

| Parameter | Lower bound | Upper bound | Fitted value |
|---|---|---|---|
| $p_m$ | 0.1 | 0.3 | 0.2 |
| $p_l$ | 0.2 | 0.8 | 0.4 |
| Δ | 0.01 | 0.1 | 0.1 |
| $\|Pop\|$ | 100 | 200 | 100 |

**Table 13**
The average percent of deviation for *IP*2, *MIP* and RA.

| | Small | | Medium | | Large | |
|---|---|---|---|---|---|---|
| | %GAP | Time | %GAP | Time | %GAP | Time |
| *IP*2 | 4.01 | 225.57 | 4.62 | 1888.24 | – | – |
| *MIP* | 0 | 0 | 0 | 0.19 | 0 | 1.69 |
| *RA* | 8.73 | 0.04 | 9.52 | 0.31 | 11.76 | 2.13 |

solutions to *IP*2 are obtained in reasonable time by this algorithm. It should be noticed that obtained solutions from *MIP* are not necessarily feasible since some of the transported items and also inventories of supplies and customer may not be integer.

A similar table is provided for the metaheuristics, in which the SPA is used as a solution generation procedure. The results from the comparisons of the proposed algorithms (Table 14) reveal that the performance of GA-based procedure is better than GRASP within the same time limit (*TL*). As expected, the performance of both algorithms is improved by increasing the time limit. In order to compare GRASP with RA, we imposed CPU run times of RA for small, medium and large size instances (0.04, 0.31 and 2.13 s) to GRASP and we obtained the following percent of deviations: 10.46, 10.23, and 9.83. Based on these results, a fair comparison tells us that GRASP is not always better than RA but it should be noticed that RA cannot be run for different time limits. In other words, if we consider time limit option, GRASP has absolutely better performance than RA. Based upon similar reasoning, we conclude that GA has better performance than RA as well.

### 5.4. Managerial insights

In this section, various managerial insights based on an extensive set of computational tests are derived in order to evaluate the impact of some parameters in both integrated and non-integrated systems of the proposed supply chain. For this purpose, we conducted some experiments in which the results are tested by changing a specific input parameter, while fixing the others to their fitted values. The following sections report the numerical results associated with different parameters, including the production rate, initial inventory level and, etc. It should be noted that all computational results reported in this section are provided by solving the models *IP*1 and *IP*2 using CPLEX.

#### 5.4.1. Impact of the production rate

In order to investigate the impact of production rate in the value of

**Table 14**
The average percent of deviation for metaheuristics.

| | | Small %GAP | Medium %GAP | Large %GAP |
|---|---|---|---|---|
| *TL* = 10 s | GA | 6.07 | 6.92 | 7.2 |
| | GRASP | 6.94 | 8.03 | 11.72 |
| *TL* = 20 s | GA | 5.92 | 6.54 | 6.89 |
| | GRASP | 6.53 | 7.24 | 9.50 |
| *TL* = 30 s | GA | 5.91 | 6.41 | 6.08 |
| | GRASP | 6.10 | 6.74 | 8.11 |

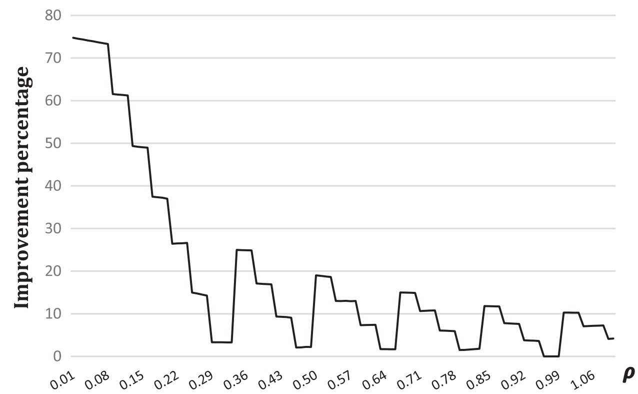


**Fig. 2.** Impact of the production rate.

integration, the model is solved under the following assumptions.

(I) The system includes only 4 suppliers with similar properties, such as the production rate and the initial inventory level.
(II) One type of product is considered in the system, where all the suppliers are capable of producing it.
(III) The suppliers' initial inventory is equal to their production in one period.
(IV) The customer's initial inventory is equivalent to its demands in three periods.
(V) No capacity limitation is specified for the suppliers' and customer's warehouses.
(VI) One type of vehicle is regarded for the transportation stage.

The computational results are summarized and depicted in Fig. 2, where the horizontal axis represents the ratio of daily production rate of all item types to the vehicle capacity for each supplier, i.e., $\rho = \min\left(\frac{\sum_p m_{ps} \times w_p}{wl_v}, \frac{\sum_p m_{ps} \times q_p}{ql_v}\right)$, and the vertical axis displays percentage of improvement brought by the integration.

The results from Fig. 2 reveal that the improvement percentage gradually decreases by increasing the suppliers' production rate. This can be explained as follows. As the production rate increases in the non-integrated system, there are more opportunities for the suppliers to consolidate items to joint deliveries. In the other words, each supplier can separately utilize almost the full capacity of each vehicle and achieve an efficiency of each trip. Also, because the suppliers are able to fully exploit the warehouses, in case of higher production rates, there are more consolidation possibilities. As a result, in the lower production rates, the total efficiency of the integrated system is improved due to the access of the suppliers to a common set of vehicles.

#### 5.4.2. Impact of the customer's initial inventory

Since the initial inventory level is an opportunity for items consolidation, it is identified as a critical factor for the efficiency of the integrated system. To analyze the impact of this parameter, the aforementioned assumptions in the previous section are made, except for the customer's inventory (case 4). Also, we consider $\rho$ = 0.1, 0.4 and 0.8 as the low, medium and high cases respectively. Fig. 3 illustrates the value of integration in special cases, where the customer's inventory is equivalent to its demands needed to be satisfied in 1, 3 and 6 periods. The suppliers' initial inventory is also regarded as the customer's demand in a single period.

Focusing on Fig. 3, the following two interpretations can be pointed out. (1) In terms of the customer's initial inventory, the integration efficiency almost deteriorates as the inventory increases. It is due to the fact that in the non-integrated system, suppliers have enough time to consolidate their items to joint deliveries because of the more inventory kept in stock. (2) The changes in the inventory level are much more

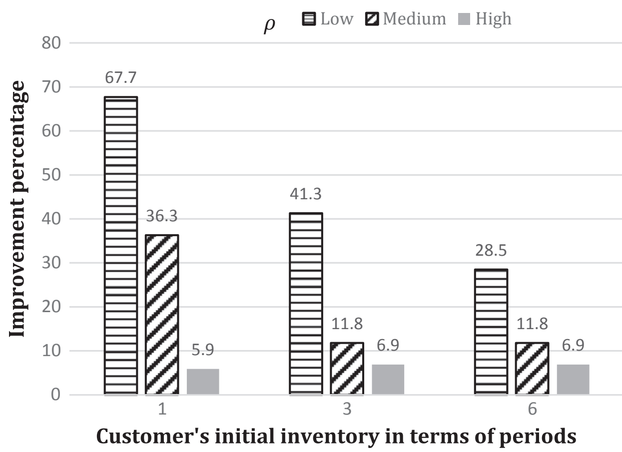


**Fig. 3.** Impact of the customer's initial inventory.

pronounced in the lower production rates. This means that increasing the customer's initial inventory makes a smaller difference if the suppliers' production rate is high, and the relative difference continues to decrease till the impact of inventory on the integration seems to be negligible.

### 5.4.3. Impact of the storage capacity

To analyze the impact of the customer's storage capacity, we changed this parameter from $minTQ_p$ (0% growth) to $maxTQ_p$ (100% growth) and summarized the computational results in Fig. 4.

From the above figure, in which the horizontal axis indicates the percentage of growth in the customer's storage, the efficiency of the integrated system declines with decreasing the customer's storage capacity, especially in low production rates. This can be explained as follows. First, in case of a reduction in the mentioned parameter, since it is not possible for the customer to receive some items, the suppliers have this opportunity to consolidate the items, therefore, the efficiency of non-integrated system improves. Secondly, due to the lack of storage space, the customer's demands are met in more trips and with more transportation costs rather than cases where there is enough capacity in the customer's place (see Fig. 5).

From the supplier's point of view, any reduction in its storage capacity leads to transportation of more items to the customer as well as increasing the transportation costs, so the integration is significantly efficient in this case, as it is verified in Fig. 4.

Analyzing the given figures, it can be interpreted that the impact of supplier's storage on the value of integration is more than the customer's capacity, such that in the low production rates, the improvement percentage varies from 72% to 28%.

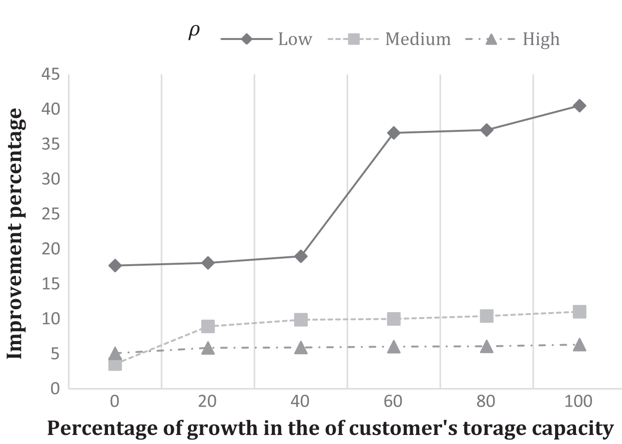


**Fig. 4.** Impact of the customer's storage capacity.


**Fig. 5.** Impact of the supplier's storage capacity.

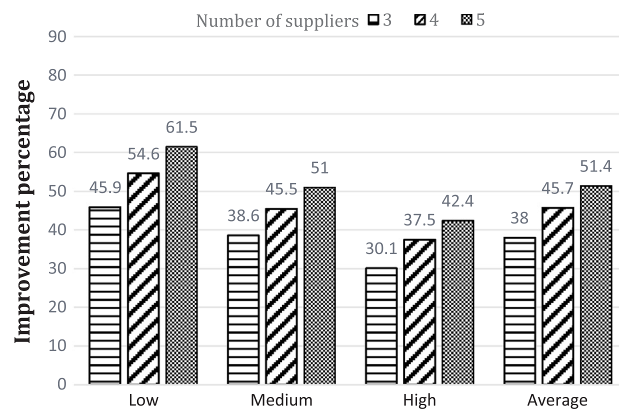


**Fig. 6.** Impact of the number of suppliers.

### 5.4.4. Impact of the number of suppliers

The number of suppliers can also play a crucial role in the integration value. Fig. 6 presents the impact of this parameter on the improvement made by integrating the system in terms of various production rates. This could be interpreted that in case the customer's demand is met by a few number of suppliers, a good performance of an integrated plan is guaranteed.

### 5.4.5. Impact of the variety of vehicles types

To investigate the impact of vehicles diversification on the integrated approach, we conducted an experiment where the 6 types of vehicles (weight and volume capacity of vehicles increase in the order $wl_1 < wl_2 < \ldots < wl_k$ and $ql_1 < ql_2 < \ldots < ql_k$, see Table 10) are employed separately in different instances. The results are displayed in Fig. 7 from which we can make the following observations:

(I) The systems where vehicle 1 is the single type employed in the distribution stage, involve the maximum delivery costs as well as the minimum efficiency in terms of integration value. This is because the small capacity of this vehicle results in the more trips made for delivering the products and the less efficiency of the integrated system, consequently. In addition, the relative efficiency decreases when the production rate increases.

(II) Since by enhancing the capacity of vehicle fleet in test problems where the production rate is low, the transportation costs of the non-integrated system increase, the impact of integration is more pronounced. Therefore, it is not efficient to employ large-capacity vehicles in such problems.

(III) Increasing the vehicles capacity often represents a significant improvement in the performance of the integrated system.

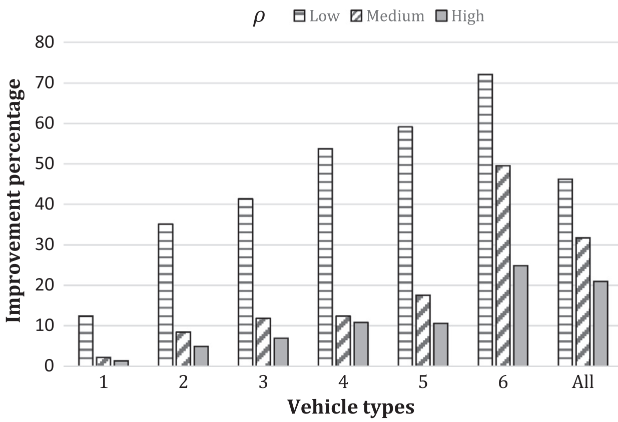


**Fig. 7.** Impact of the variety of vehicle types.

(IV) In case all the six types of vehicle are employed, the costs reduction brought by the integration is nearly 25% and 50% while high and low production rates, respectively, are regarded.

## 6. Conclusions

In this article we have addressed an integration concept for a production and distribution operations in a two-stage supply chain, where multiple suppliers are located in close proximity to each other so that the integration is practical. For the proposed problem, we have presented two integer linear programming models for both integrated and non-integrated approach, as well as some solution methods in order to tackle the NP-hardness of the problem. The quality and efficiency of these methods have been evaluated through computational experiments applied on the randomly generated test problems in various environments. We have also investigated the value of integration by comparing the two approaches, which demonstrates an average of 30% and 25% improvement of costs for small and medium size of instances, respectively.

Due the vast area of research on integration of supply chains, several directions can be considered on the future studies. (1) The suppliers' production rate can be defined as a variable to be determined in planning periods. (2) The number of shipments made by delivery vehicles can be limited. (3) Some suppliers located in various places can be taken into account in order to develop the supply chain.

## Appendix A. Proof of Theorem 1

Since $R_{pt}^{min} = \max(\rho_{pt}^{min}, \sum_{s=1}^{|S|} r_{pst}^{min})$ and $R_{pt}^{max} = \min(\sum_{s=1}^{|S|} r_{pst}^{max}, \rho_{pt}^{max})$, we consider the problem into four cases, and prove that $R_{pt}^{max} - R_{pt}^{min} \geqslant 0$ for all the cases. Note that $\sum_{s=1}^{|S|} m_{ps} = d_p$ in all the cases.

**Case A:** $R_{pt}^{min} = \sum_{s=1}^{|S|} r_{pst}^{min}$ and $R_{pt}^{max} = \sum_{s=1}^{|S|} r_{pst}^{max}$.

Considering Eqs. (35) and (40), the two following cases can be regarded for supplier $s$ and item $p$.

1. $r_{pst}^{max} - r_{pst}^{min} = TQ_{ps}^{max} - m_{ps}$
2. $r_{pst}^{max} - r_{pst}^{min} = (|T| - t)m_{ps} + TQ_{ps}^{max} - IS_{ps1}$

As we have $TQ_{ps}^{max} \geqslant m_{ps}$, $TQ_{ps}^{max} \geqslant IS_{ps1}$ and $(|T| - t)m_{ps} \geqslant 0$ for all $s = 1,\dots,|S|$, $r_{pst}^{max} - r_{pst}^{min} \geqslant 0$ is true, thus we have $\sum_{s=1}^{|s|} r_{pst}^{max} - \sum_{s=1}^{|s|} r_{pst}^{min} \geqslant 0$ and it follows that $R_{pt}^{max} - R_{pt}^{min} \geqslant 0$.

**Case B:** $R_{pt}^{min} = \sum_{s=1}^{|S|} r_{pst}^{min}$ and $R_{pt}^{max} = \rho_{pt}^{max}$.

Considering Eqs. (35) and (40), we have $R_{pt}^{max} - R_{pt}^{min} = (TQ_p^{max} - IC_{p1}) + (TQ_{ps}^{max} - IS_{ps1})$.
Since for all $s = 1,\dots,|S|$ we have $TQ_{ps}^{max} \geqslant IS_{ps1}$ and $TQ_p^{max} \geqslant IC_{p1}$ for the customer, then we have $R_{pt}^{max} - R_{pt}^{min} \geqslant 0$ consequently.

**Case C:** $R_{pt}^{min} = \rho_{pt}^{min}$ and $R_{pt}^{max} = \sum_{s=1}^{|S|} r_{pst}^{max}$.

Considering Eqs. (34) and (40), the two following cases are investigated.

1. $R_{pt}^{max} - R_{pt}^{min} = (\sum_{s=1}^{|S|} IS_{ps1} - d_p) + (IC_{p1} - d_p)$

$$= \left(\sum_{s=1}^{|S|} IS_{ps1} - \sum_{s=1}^{|S|} m_{ps}\right) + (IC_{p1} - d_p) = \sum_{s=1}^{|S|} (IS_{ps1} - m_{ps}) + (IC_{p1} - d_p)$$

2. $R_{pt}^{max} - R_{pt}^{min} = (|T|.\ d_p - td_p) + (IC_{p1} - d_p)$

As we have $IS_{ps1} - m_{ps} \geqslant 0$, $IC_{p1} - d_p \geqslant 0$ and $(|T| - t)d_p \geqslant 0$, hence $R_{pt}^{max} - R_{pt}^{min} \geqslant 0$.

**Case D:** $R_{pt}^{min} = \rho_{pt}^{min}$ and $R_{pt}^{max} = \rho_{pt}^{max}$.Considering Eqs. (34) and (38), we have $R_{pt}^{max} - R_{pt}^{min} = TQ_p^{max} - d_p > 0$.

## References

Absi, N., Cattaruzza, D., Feillet, D., Ogier, M., & Semet, F. (2016). Inventory Routing Problem over the long term: a math-heuristic approach. In *Verolog 2016* [June].

Alves, C., & De Carvalho, J. V. (2007). Accelerating column generation for variable sized bin-packing problems. *European Journal of Operational Research, 183*(3), 1333–1352.

Coffman Jr, E. G., Csirik, J., Galambos, G., Martello, S., & Vigo, D. (2013). Bin packing approximation algorithms: survey and classification. In *Handbook of combinatorial optimization* (pp. 455–531). New York :Springer.

Feo, T., & Resende, M. G. C. (1989). A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters,* 867–871.

Garey, M. R. & Johnson, D. S. (1979). *Computers and intractability: A guide to the theory of NP-completeness*.

Glock, C. H., Grosse, E. H., & Ries, J. M. (2014). The lot sizing problem: A tertiary study. *International Journal of Production Economics, 155*, 39–51.

Hajiaghaei-Keshteli, M., Aminnayeri, M., & Fatemi Ghomi, S. M. T. (2014). Integrated scheduling of production and rail transportation. *Computers and Industrial Engineering, 74*, 240–256.

Harris, F.W. (1913). How many parts to make at once. *The Magazine of Management, 10* (2), 135–136 [152].

Holland, J. H. (1992). *Adaptation in natural and artificial systems.* Ann Arbor, MI: University of Michigan Press.

Hong, S., Zhang, D., Lau, H. C., Zeng, X., & Si, Y. W. (2014). A hybrid heuristic algorithm for the 2D variable-sized bin packing problem. *European Journal of Operational Research, 238*(1), 95–103.

Hu, Z., & Hu, G. (2016). A two-stage stochastic programming model for lot-sizing and scheduling under uncertainty. *International Journal of Production Economics, 180*, 198–207.

Jaruphongsa, W., ÇEtinkaya, S., & Lee, C. Y. (2007). Outbound shipment mode considerations for integrated inventory and delivery lot-sizing decisions. *Operations Research Letters, 35*(6), 813–822.

Jaruphongsa, W., & Lee, C. Y. (2008). Dynamic lot-sizing problem with demand time windows and container-based transportation cost. *Operations Research Letters, 2*(1), 39–51.

Kang, J., & Park, S. (2003). Algorithms for the variable sized bin packing problem. *European Journal of Operational Research, 147*(2), 365–372.

Koç, U., Toptalb, A., & Sabuncuogluc, I. (2017). Coordination of inbound and outbound transportation schedules with the production schedule. *Computers and Industrial Engineering, 103*, 178–192.

Langston, M. A. (1982). Improved 0/1-interchange scheduling. *BIT Numerical Mathematics, 22*(3), 282–290.

Le, T., Diabat, A., Richard, J. P., & Yih, Y. (2013). A column generation-based heuristic algorithm for an inventory routing problem with perishable goods. *Optimization Letters, 7*(7), 1481–1502.

Lee, W. S., Han, J. H., & Cho, S. J. (2005). A heuristic algorithm for a multi-product dynamic lot-sizing and shipping problem. *International Journal of Production Economics, 98*(2), 204–214.

Lodi, A., Martello, S., & Monaci, M. (2002). Two-dimensional packing problems: A survey. *European Journal of Operational Research, 141*(2), 241–252.

Man, K. F., Tang, K. S., & Kwong, S. (2012). *Genetic algorithms: Concepts and designs.* Springer Science & Business Media.