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Size-dependent analysis of functionally graded nanoplates using refined plate theory and isogeometric approach

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Abstract

The aim of this paper is the static analysis of functionally graded nanoplates by using the isogeometric analysis (IGA) approach and nonlocal elasticity theory. To consider small scale effects for nanoscale structures, the nonlocal elasticity theory is used. The refined plate theory which is suitable for both thin and thick plates, uses only four independent unknowns and satisfies the free transverse shear stress conditions on the top and bottom surfaces of plate and so a shear correction factor is not needed. The displacement field of model is derived based on physical neutral surface position. The IGA approach can easily formulate C^1 continuous elements by using Non-Uniform Rational B-Spline (NURBS) functions. Finally, numerical results are compared with other available solutions.

Keywords: Functionally graded materials, Isogeometric approach, Neutral surface position, Nonlocal elasticity theory, Refined plate theory.

1. INTRODUCTION

In recent years, researchers have widely studied the behavior of nanostructures. The continuum mechanics approach providing more simplicity than molecular dynamics approach, is widely used to study the mechanical behavior of nanostructures. The local continuum theories do not model the behavior of nanoscale structures properly. In order to consider small scale effects in nanoscale structures, different size-dependent continuum mechanics models have been developed such as the couple stress theory [1,2], gradient theory [3], nonlocal elasticity theory [4-6], strain gradient theory [7], modified couple stress theory [8], modified strain gradient theory [9] and surface energy theory [10]. Many publications show that the nonlocal elasticity theory considering small scale effects can well predict the behavior of nanostructures. A review of continuum mechanics models for size-dependent analysis of beams and plates can be studied in reference [11]. Most of the studies in the literature for functionally graded (FG) nanoplates are based on the classical and first order shear deformation theories and a few studies are available using other shear deformation theories and numerical methods [11].

To analyze plate structures, many theories have been presented. The classical plate theory (CPT) gives acceptable results for thin plates. The first order shear deformation theory (FSDT) which accounts for transverse shear deformation effects, requires a shear correction factor to satisfy the free transverse shear stress conditions on the top and bottom surfaces of plate. To avoid the use of shear correction factor, many higher order shear deformation theories (HSDT) have been proposed such as third order shear deformation theory (TSDT), sinusoidal shear deformation theory (SSDT), hyperbolic shear deformation theory (HSDT), etc. For more details about higher order shear deformation theory, study references [12, 13]. In order to reduce the number of unknowns in HSDT, the refined plate theory (RPT) was proposed by Senthilnathan et al. [14].

There are several numerical methods to solve problems. Hughes et al. [15] introduced isogeometric analysis (IGA) approach which represents the exact geometry of problem by the use of Non-Uniform Rational B-Spline (NURBS). The IGA approach can easily form C^1 continuous elements by using B-splines or NURBS approximations. However the IGA approach has been used to analyze various problems, there are only a few studies which analyze nanostructures. Natarajan et al. [16] presented the free vibration analysis of FG nanoplates based on FSDT. Nguyena et al. [17] utilized the IGA approach to analyze FG nanoplates.

In this paper refined hyperbolic shear deformation theory is used to analyze FG nanoplates by using the IGA approach and nonlocal elasticity theory.The following section presents the equations of nonlocal elasticity theory. In section 3 refined hyperbolic shear deformation theory is introduced for FG nanoplates. In section 4, the equations of nanoplate theory is provided based on NURBS basis functions. In section 5, numerical results and discussions are provided. Finally, this paper is closed by conclusions.

2. Nonlocal elasticity theory

According to the nonlocal elasticity theory, the stress at a reference point x is a function of strain field at every point in the body. The stress is defined as [4,5]

$$(1-\mu\nabla^2)\sigma_{ij}^{nl}=\sigma_{ij}^{l} \tag{1a}$$

$$\mu=(e_0 a)^2 \tag{1b}$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{1c}$$

Here, σ_{ij}^{nl} and σ_{ij}^{l} are nonlocal and local stress respectively. μ is nonlocal parameter which represents the small scale effect. e_0 is a constant determined for each material type and a is an internal characteristics length.

3. Refined plate theory for FG nanoplates

3.1. Physical neutral surface

The neutral surface position of FG plates may not coincide with its middle surface due to the lack of symmetry. If the origin of the coordinate system is located on the neutral surface position, FG plates can be easily analyzed with the isotropic plate theories. z_{ms} and z_{ns} planes are considered to determine the neutral surface position of FG plate as shown in Fig. 1.

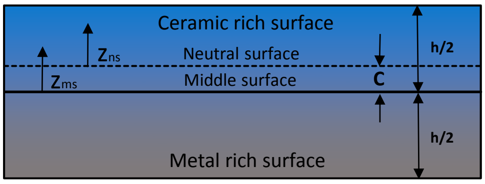


Figure 1. Geometry of functionally graded plate.

Consider a FG rectangular plate with length a, width b and constant thickness h. The position of the neutral surface (C) can be calculated by the use of the equilibrium equation as

$$C=\frac{\int_{-h/2}^{h/2}E(z_{ms})z_{ms}dz_{ms}}{\int_{-h/2}^{h/2}E(z_{ms})dz_{ms}} \tag{2}$$

The nonhomogeneous properties of materials may be obtained by means of the rule of mixture. The volume fraction of ceramic V_C in the new coordinate system can be expressed as

$$V_C=\left(\frac{z_{ms}}{h}+\frac{1}{2}\right)^k=\left(\frac{z_{ns}+C}{h}+\frac{1}{2}\right)^k \tag{3}$$

where the power k is greater than or equal to zero. The Young's modulus of FG plate is defined as:

$$E(z_{ns}) = E_m + (E_c - E_m)\left(\frac{z_{ns}+C}{h} + \frac{1}{2}\right)^k \tag{4}$$

For Mori–Tanaka scheme, the Young's modulus is given as:

$$E(z_{ns}) = E_m + (E_c - E_m)\frac{V_C}{1 + V_m(\frac{E_c}{E_m} - 1)(\frac{1+\nu}{3-3\nu})}, \quad V_m = V_C - 1 \tag{5}$$

3.2. RPT BASED ON PHYSICAL NEUTRAL SURFACE

Based on the RPT, the displacements of a material point located at (x, y, z) in a plate may be written as

$$\begin{aligned} u(x,y,z_{ns}) &= u_0 - z_{ns}\frac{\partial w_b}{\partial x} + g(z_{ns})\frac{\partial w_s}{\partial x} \\ v(x,y,z_{ns}) &= v_0 - z_{ns}\frac{\partial w_b}{\partial y} + g(z_{ns})\frac{\partial w_s}{\partial y} \\ w(x,y,z_{ns}) &= w_b + w_s \end{aligned} \tag{6}$$

where u, v, w are displacements in the x, y, z directions, u_0, v_0, w_b and w_s are mid-plane, bending and shear deflections and β is the rotation of the xy plane due to shear. In this paper $\Psi(z_{ns})$ function is assumed as [13]

$$\Psi(z_{ns}) = ((z_{ns} + C)/h)(cosh(0.5) + 0.5sinh(0.5) - cosh((z_{ns} + C)/h)) \tag{7}$$

The relationships between strains and displacements are described as

$$\begin{aligned} \{\varepsilon\} &= \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z_{ns}\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + g(z_{ns})\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \\ \{\gamma\} &= \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = (1 + g'(z_{ns}))\begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} \end{aligned} \tag{8}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = -\begin{Bmatrix} \frac{\partial^2 w_b}{\partial x^2} \\ \frac{\partial^2 w_b}{\partial y^2} \\ 2\frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial^2 w_s}{\partial x^2} \\ \frac{\partial^2 w_s}{\partial y^2} \\ 2\frac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{bmatrix} \frac{\partial w_s}{\partial x} \\ \frac{\partial w_s}{\partial y} \end{bmatrix} \tag{9}$$

Based on the Hooke's law the stresses are written as

$$(1 - \mu\nabla^2)\{\sigma\} = [Q]\{\varepsilon\}, (1 - \mu\nabla^2)\{\tau\} = [G]\{\gamma\} \tag{10}$$

where the material matrices are given as

$$[Q] = \frac{E(z)}{1 - \nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{10a}$$

$$[G] = \frac{E(z)}{2(1 + \nu)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{10b}$$

The total potential energy can be given as

$$\Pi = U - W - T \tag{11}$$

where U, W and T are strain energy, work done and kinetic energy. The strain energy is defined as

$$U = \frac{1}{2}\iint_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \sigma\,\varepsilon dV = \frac{1}{2}\iint_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (\sigma_x^{nl}\varepsilon_x + \sigma_y^{nl}\varepsilon_y + \tau_{xy}^{nl}\gamma_{xy} + \tau_{xz}^{nl}\gamma_{xz} + \tau_{zy}^{nl}\gamma_{zy})\, dz_{ns} dA \tag{12}$$

By substituting Eq. (8) into Eq. (12), the potential energy of the plate is rewritten as

$$U = \frac{1}{2}\int (N_x\varepsilon_x^0 + N_y\varepsilon_y^0 + N_{xy}\gamma_{xy}^0 + M_x^b k_x^b + M_y^b k_y^b + M_{xy}^b k_{xy}^b + M_x^s k_x^s + M_y^s k_y^s + M_{xy}^s k_{xy}^s + S_{xz}^s\gamma_{xz}^s + S_{yz}^s\gamma_{yz}^s)\, dA \tag{13}$$

where the stress resultants N, M and S are defined as

$$(N_x, N_y, N_{xy}) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (\sigma_x, \sigma_y, \tau_{xy})^{(nl)} dz_{ns} \tag{14a}$$

$$(M_x^b, M_y^b, M_{xy}^b) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (\sigma_x, \sigma_y, \tau_{xy})^{(nl)} z_{ns} dz_{ns} \tag{14b}$$

$$(M_x^s, M_y^s, M_{xy}^s) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (\sigma_x, \sigma_y, \tau_{xy})^{(nl)} g(z_{ns}) dz_{ns} \tag{14c}$$

$$(S_{xz}^s, S_{yz}^s) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (\tau_{xz,}\tau_{yz,})^{(nl)} \Psi'(z_{ns}) dz_{ns} \tag{14d}$$

By substituting Eq. (9) into Eq. (10) and the results into Eqs. (14a-e), the stress resultants are obtained as

$$(1-\mu\nabla^2)\begin{Bmatrix} \{N\} \\ \{M^b\} \\ \{M^s\} \end{Bmatrix} = [D^b]\begin{Bmatrix} \{\varepsilon^0\} \\ \{k^b\} \\ \{k^s\} \end{Bmatrix} \tag{15a}$$

$$(1-\mu\nabla^2)\{S^s\} = [D^s]\{\gamma^s\} \tag{15b}$$

where the material matrices are given as

$$[D^b] = \begin{bmatrix} [A] & [B] & [D] \\ [B] & [C] & [E] \\ [D] & [E] & [F] \end{bmatrix} \tag{16}$$

$$([A],[B],[C],[D],[E],[F]) = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (1, z_{ns}, {z_{ns}}^2, g(z_{ns}), z_{ns}g(z_{ns}), g^2(z_{ns}))\, \frac{E(z_{ns})}{1-\nu^2}\, dz_{ns}$$

$$[D^s] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \frac{E(z_{ns})}{2(1+\nu)} (\Psi'(z_{ns}))^2 dz_{ns}$$

4. FG NANOPLATE FORMULATION BASED ON NURBS BASIS FUNCTIONS

4.1. NURBS FUNCTIONS

A non-decreasing knot vector in the parametric space [15] is define as

$$U = \{u_0, u_1, u_2, \ldots, u_m\},\; u_i \le u_{i+1},\; i = 0,1,2,3,\ldots,m-1 \tag{17}$$
$$m = n + p + 1$$

Here u_i are the i-th knot, p is the polynomial degree and $n+1$ is the number of basis functions. The knots equally spaced in the parametric space are said to be uniform knots. A knot vector the first and the last knots are repeated $p+1$ times, is said to be open knot vector and is defined as

$$U = \left\{\underbrace{a, \cdots, a}_{p+1}, u_{m-p-1}, \underbrace{b, \cdots, b}_{p+1}\right\} \tag{18}$$

The B-spline basis functions of degree p, are defined as

$$N_{i,0}(u) = \begin{cases} 1 & if\ u_i \le u \le u_{i+1} \\ 0 & otherwise \end{cases} \tag{19}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1-u}}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

Piecewise-polynomial B-spline curve and surface ara defined as

$$C(u) = \sum_{i=0}^{n} N_{i,p}(u) \times P_i \ , S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) P_{i,j} \quad \mathrm{a \le u \le b} \tag{20}$$

where $\{P_i\}$ are the control points, $\{P_{i,j}\}$ form a bidirectional control net, $\{N_{i,p}(u)\}$ and $\{N_{j,q}(v)\}$ are the B-spline basis functions of degree p and defined on the knot vectors as

$$U = \left\{\underbrace{0, \dots, 0}_{p+1}, u_{p+1}, \dots, u_{r-p-1}, \underbrace{1, \dots, 1}_{p+1}\right\} \tag{21}$$
$$V = \left\{\underbrace{0, \dots, 0}_{q+1}, v_{q+1}, \dots, v_{s-q-1}, \underbrace{1, \dots, 1}_{q+1}\right\}$$

NURBS curve and surface of degree p are defined as below where $\{w_i\}$ are the weights.

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i}\ , \quad S_{i,j}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j} P_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j}} \tag{22}$$

4.2. RPT FORMULATION BASED ON NURBS APPROXIMATIONS

The weak form of Eq. (11) for the static analysis can be written as

$$\delta\Pi = 0 \ \Rightarrow \ \int \delta\, \{\varepsilon\}^T [\bar{D}] \{\varepsilon\} dV = \int q(1 - \mu\nabla^2)\delta\{u\} dA \tag{23}$$

The formulation of static problems can be simplified as

$$\{F\} = [K]\{D\} \tag{24}$$

where the global stiffness matrix K is computed as

$$[K] = \int ([B^m]^T [D^b][B^m] + [B^s]^T [D^s][B^s]) d\Omega \tag{25}$$

where

$$[B_i^{em}] = \begin{bmatrix} R_{i,x} & 0 & R_{i,y} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{i,y} & R_{i,x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -R_{i,xx} & -R_{i,yy} & -2R_{i,yy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_{i,xx} & R_{i,yy} & 2R_{i,xy} \end{bmatrix}^T \tag{26}$$

$$[B_i^s] = \begin{bmatrix} 0 & 0 & R_{i,x} & R_{i,x} \\ 0 & 0 & R_{i,y} & R_{i,y} \end{bmatrix}$$

and the load vector is expressed as

$$\{F\} = \int q(1-\mu\nabla^2)Rd\Omega, \quad R = [0 \quad 0 \quad R_i \quad R_i\,]^T \tag{27}$$

5. Results and Discussions

In this section the results of the static analysis of FG nanoplates are presented. The efficiency of the present model is shown by comparing the obtained results with other solutions. To obtain results, (p+1)×(q+1) Gauss points are utilized. The material properties of FG nanoplates are listed in Table 1. Simply supported boundary condition is considered as:

$$v_0 = w_b = w_s = 0 \text{ at } x = 0, a \;\&\; u_0 = w_b = w_s = 0 \text{ at } y = 0, b \tag{28}$$

Table 1- Material properties of FG nanoplates.

Material	E (GPa)	ρ (kg/m^3)	ν
Al	70	2702	0.3
Al_2O_3	380	3800	0.3

5.1. Convergence Study

Consider a homogeneous square plate. The convergence of non-dimensional deflection is shown in Table 2. The plate is subjected to sinusoidal load for obtaining deflection results. As observed, the IGA gains very fast convergence for both quartic and cubic NURBS elements. But if the quartic degree is chosen, less meshes will be required to solve problems in comparison with cubic degree. So in this paper a mesh of 7×7 quartic NURBS elements is sufficient and chosen to solve problems.

Table 2- Convergence of deflection $\bar{w} = 10wE_ch^3/qa^4$ homogeneous plates (μ = 0).

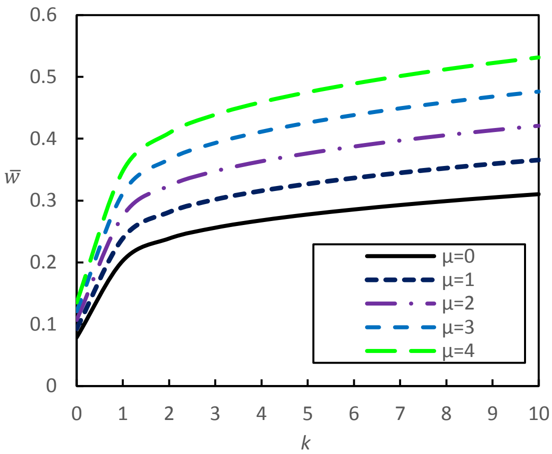
Theory	Meshes	*a/h* = 5	*a/h* = 10	*a/h* = 20
IGA p = 3	3×3	0.3411	0.2941	0.2823
	5×5	0.3432	0.2959	0.2841
	7×7	0.3433	0.2960	0.2842
	9×9	0.3433	0.2960	0.2842
	11×11	0.3433	0.2961	0.2842
IGA p = 4	3×3	0.3434	0.2961	0.2843
	5×5	0.3433	0.2961	0.2842
	7×7	0.3433	0.2961	0.2842
	9×9	0.3433	0.2961	0.2842
	11×11	0.3433	0.2961	0.2842
TSDT [18]		0.3433	0.2961	0.2842

5.2. Example

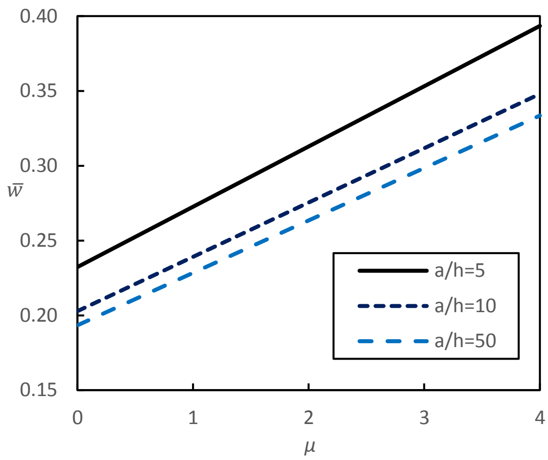
Table 3 shows the non-dimensional deflection for Al/Al_2O_3 square nanoplates with the length a = 10 subjected to a distributed uniform load for different nonlocal parameter μ, a/h and k based on Mori-Tanaka scheme. It is seen that the results are close to the solutions of Nguyena et al. [17], however, the difference increases with the increase of k. In Fig. 3 the effect of the parameters a/h, μ, and k on the deflection of Al/Al_2O_3 square nanoplates subjected to a distributed uniform load are depicted. As observed in Fig. 3, by increasing the nonlocal parameter μ and length–thickness ratio a/h, the deflection increases and decreases respectively.

Table 3- Deflection $\bar{w} = \frac{100h^3D_m}{a^4q_0}w(\frac{a}{2},\frac{b}{2})$ of Al/Al_2O_3 nanoplates under uniform load.

k	Model	a/h = 5			a/h = 10			a/h = 50		
		μ = 0	μ = 1	μ = 4	μ = 0	μ = 1	μ = 4	μ = 0	μ = 1	μ = 4
0	Ref. [17]	0.0902	0.1059	0.1529	0.0787	0.0928	0.1351	0.0750	0.0886	0.1294
	Present	0.0903	0.1060	0.1532	0.0787	0.0928	0.1352	0.0750	0.0886	0.1294
1	Ref. [17]	0.2254	0.2646	0.3821	0.1977	0.2331	0.3393	0.1888	0.2230	0.3256
	Present	0.2323	0.2726	0.3934	0.2028	0.2391	0.3480	0.1933	0.2284	0.3335
2	Ref. [17]	0.2723	0.3194	0.4606	0.2344	0.2763	0.4021	0.2222	0.2625	0.3834
	Present	0.2783	0.3262	0.4701	0.2386	0.2813	0.4092	0.2259	0.2669	0.3897
5	Ref. [17]	0.3247	0.3804	0.5477	0.2730	0.3217	0.4678	0.2564	0.3028	0.4422
	Present	0.3302	0.3870	0.5575	0.2775	0.3270	0.4753	0.2606	0.3079	0.4496
10	Ref. [17]	0.3620	0.4243	0.6109	0.3052	0.3597	0.5231	0.2869	0.3390	0.4950
	Present	0.3679	0.4316	0.6228	0.3104	0.3657	0.5316	0.2919	0.3448	0.5035



(a) a/h = 10



(b) k = 1

Figure 3. Effect of the parameters *a/h*, μ, and *k* on the deflection of Al/Al_2O_3 square nanoplates.

6. Conclusions

In this paper, hyperbolic shear deformation theory has been used to analyze FG nanoplates by using the IGA approach and nonlocal elasticity theory based on physical neutral surface position. The theory uses four independent unknowns and satisfies the free transverse shear stress conditions on the top and bottom surfaces of the plate without a need for shear correction factor. It is observed that by increasing the nonlocal parameter μ and length–thickness ratio a/h, the deflection increases and decreases respectively. The example shows that the hyperbolic model gives good results in comparison with other solutions.

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