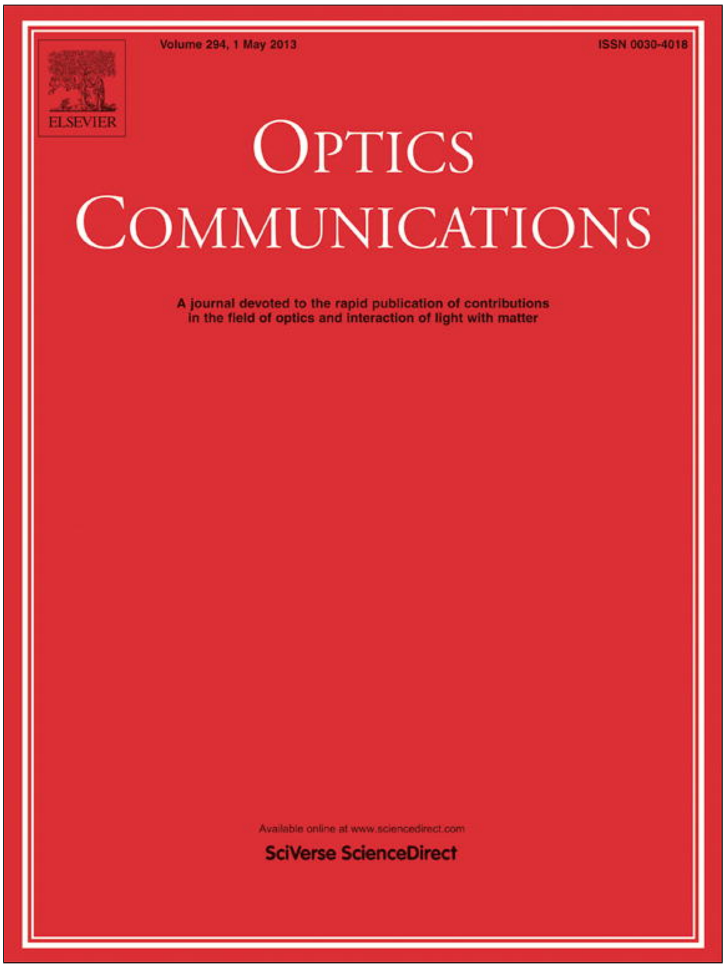
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Effect of Kerr nonlinearity on the transverse localization of light in 1D array of optical waveguides with off-diagonal disorder

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ABSTRACT

In this paper a simulation of the transverse localization of light in 1D array of optical waveguides in the presence of off-diagonal disorder is presented. Effects of self-focusing and self-defocusing Kerr nonlinearity on the transverse localization of surface and bulk modes of the disordered waveguides array are taken into consideration. The simulation shows that in the off-diagonal disordered array at low nonlinear parameters, the transverse localization of light becomes more than that of the corresponding diagonal disordered array. However by increasing the nonlinear parameters the diagonal disordered array is localized more than the associated off-diagonal disordered array for both surface and bulk modes. It is also found that the surface modes become more localized than the bulk modes by increasing the nonlinear parameter. The calculated effective beam width versus propagation distance for off-diagonal disordered arrays confirms these results.

1. Introduction

Since the first paper on the localization of the electron wave function, published by Anderson about half century ago [1], this area has still survived for theoreticians and experimentalists. This phenomenon is due to wave interference in disordered systems, so it is natural to expect that it could be applied in any wave system such as condensed matter systems, elastic and optical systems [2–8]. The Anderson localization of photons can be visualized easily; hence much theoretical and experimental work has been done on the optical Anderson localization [7–11]. One of the interesting topics in light localization is the transverse localization of light which was predicted in 1989 for the first time [12]. About two decades later, this phenomenon was observed experimentally in disordered photonic lattice systems [13]. After this experiment, disordered photonic lattice systems have attracted increasing attention as a ground to study the transverse localization of light [14–23]. One of the most experimentally realizable systems for studying transverse localization is a 1D array of optical waveguides [14,23]. This array can be built by methods such as optical induced techniques in photo refractive materials, laser writing methods or common lithographic methods [13,18–22,14].

In the presence of Kerr effects, Maxwell equations can be reduced to nonlinear Schrödinger equations [13–17,21–24]. To study the effect of Kerr nonlinearity on the transverse localization of light in disordered photonic lattices, the system of nonlinear Schrödinger equations is solved numerically and results are verified experimentally [13–17,21–23]. The impact of the nonlinear Kerr effect on the transverse localization in the diagonal disordered array of waveguides has been investigated theoretically and experimentally [14,23]. To study the effect of positive Kerr effect on the bulk and surface modes in the diagonal disordered waveguides array, nonlinear Schrödinger equations have also been solved numerically [23].

In this work, the 1D array of optical waveguides is considered. Disorder is introduced by changing the coupling coefficient randomly between each of the waveguides. The effect of self-focusing and self-defocusing Kerr nonlinearity on the transverse localization of light in this off-diagonal disordered waveguides array is studied. The results are compared to those of diagonal disordered array in the presence of positive and negative Kerr nonlinear effects.

The paper is organized in four sections. Section 2 contains the theoretical models for diagonal and off-diagonal disordered arrays. Discussion on the numerical simulation results is presented in Section 3 and Section 4 is devoted to conclusions.

2. Theoretical models

Fig. 1 shows the structure of a disordered one dimensional array of single mode-optical waveguides. For the single-mode

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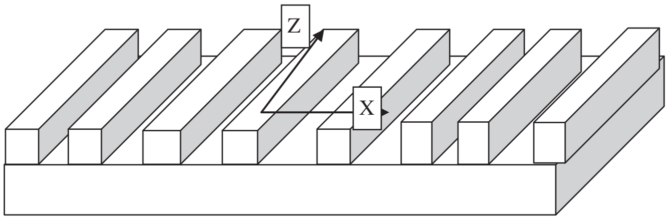

Fig. 1. Schematic of an array of 1D optical waveguides.

operation of optical waveguides, the waveguide width is determined by the light wavelength, while the coupling coefficients between waveguides are determined by the separation distance between adjacent waveguides [25]. At the entrance plane, light is injected in one of the waveguides of the disordered array and can be coupled to the neighboring waveguides by the tunneling effect. The slowly varying envelope approximation (SVEA) method is employed to write the propagation equation in the presence of nonlinear Kerr effect in an array of 1D waveguides [26]. In the tight-binding approximation governing equations have the simple form [14,23,26,27]

$$i\frac{dE_n}{dz}+K_nE_n+C_{n,n+1}E_{n+1}+C_{n,n-1}E_{n-1}+\gamma|E_n|^2E_n=0\,,\quad n=1,2,\ldots,N \tag{1}$$

Here E_n is the amplitude of electric field in waveguide n. N is the total number of waveguides. K_n the propagation constant of light in site n, depends on the refractive index and the width of the corresponding waveguide. $C_{n,m}$ is the coupling coefficient between waveguides n and m. The coupling coefficients can be calculated by coupled mode theory, which depends on the separation distance and refractive index of material between waveguides [25]. The physical dimension of coupling coefficient is the inverse of length. $\gamma=n_2\omega/cA_{eff}$ is the Kerr coefficient [1/Wm], where A_{eff} is the effective area of the fundamental modes. c is the speed of light in free space and n_2 is the nonlinear refractive index [m^2/V]. The nonlinear Kerr coefficient can be positive or negative corresponding to the self-focusing and self-defocusing behavior respectively [23].

Diagonal disorder can be introduced in this system using randomized propagation constants of each waveguide. It is possible by randomized refractive index or the width of each waveguide. The off-diagonal disorder can be introduced by randomizing the coupling coefficient between the nearest neighbor waveguides. Once refractive index is randomized, the diagonal disorder will unavoidably be introduced. The regular part of propagation constants and coupling coefficients are defined by K_0 and C respectively.

In the presence of diagonal and off-diagonal disorders, the following normalized variables are employed [23]:

$$K_n=K_0+C\varepsilon_n, C_{n,n\pm1}=C(1+\varepsilon_{n,n\pm1});\quad s = Cz;\quad U_n=\frac{E_ne^{iK_0s/C}}{\sqrt{P}};$$

$$\chi=\frac{\gamma P}{C}, \tag{2}$$

where P is the power of incident light, and ε_n and $\varepsilon_{n,n\pm1}$ are the normalized random parts of the propagation constants and coupling coefficients respectively. The propagation equation (1) versus the normalized variables can be rewritten as follows:

$$i\frac{dU_n}{ds}+\varepsilon_nU_n+(1+\varepsilon_{n,n-1})U_{n-1}+(1+\varepsilon_{n,n+1})U_{n+1}+\chi|U_n|^2U_n=0, \tag{3}$$

where $n=1,2,\ldots,N$. In diagonal disordered arrays $\varepsilon_{n,n\pm1}$ are zero and ε_n is a random variable distributed on the $[-\Delta_d,\Delta_d]$ interval uniformly, while for off-diagonal disordered arrays $\varepsilon_{n,n\pm1}$ s are distributed uniformly on the interval $[-\Delta_o,\Delta_o]$ and ε_n s are zero. Δ_d and Δ_o are called the strength of diagonal and off-diagonal disordered arrays respectively.

To compare the diagonal and off-diagonal disorder effects on the transverse localization, it is assumed that both of the disorder strength for diagonal and odd-diagonal disordered arrays have the same value ($\Delta_d=\Delta_o=\Delta$). 1 and N waveguides are defined as the surface or edge of the array of optical waveguides and modes of propagation in the one and N waveguides are called the surface modes.

In order to define the initial value for the governing equations it is assumed that light is injected to one of the waveguides i.e. $U_n(s=0)=\delta_{n,n_0}$. When n_0 is set to 1 or N, the surface modes are excited, while for other values ($n_0\neq1,N$) the bulk modes can be excited [23].

The transverse localization length and participation rate are defined as transverse localization measures. If light is localized in the transverse direction, we expect that the light intensity decays exponentially in the transverse direction, with the decay constant being equal to the inverse localization length.

The participation rate (PR) is defined as follows [15–17,23]:

$$PR(s)=\left\langle\frac{\left(\sum_{n=1}^{N}|U_n(s)|^2\right)^2}{\sum_{n=1}^{N}|U_n(s)|^4}\right\rangle \tag{4}$$

Due to the statistical nature of disordered systems the average is taken over realizations of disordered waveguide arrays. In a completely transverse localized system ($U_n(s)\simeq\delta_{n,m}$) the PR of the system is approximately equal to one ($PR\simeq1$), while in a completely extended system the energy is uniformly distributed on the N waveguides, that is the intensity in each waveguide is proportional to the inverse of the number of waveguides and the normalized field intensity is equal to the inverse of the square root of the number of waveguides (delocalized,$U_n(s)\simeq1/\sqrt{N}$). In this case, the participation rate approaches the number of waveguides ($PR\simeq N$). The effective beam width is defined as the square root of the participation rate $\left(w_{eff}=\sqrt{PR}\right)$ [15–17].

To study the evolution of surface and bulk modes of diagonal or off-diagonal disordered waveguide arrays in the presence of focusing or defocusing nonlinearities, the transverse localization length and the effective beam width for different physical parameters are calculated.

3. Results and discussion

For numerical simulation, a 1D array of $N=200$ coupled waveguides with boundary conditions $U_0(s)=U_{N+1}(s)=0$ is taken into consideration. To study the effects of the off-diagonal disorder on the waveguide arrays' behavior, the coupling coefficients random variables are uniformly chosen on the interval $[-\Delta,\ \Delta]$. To obtain the normalized field amplitudes, U_n, the system of governing Eq. (3) is solved by the Runge–Kutta–Fehlberg method [28]. To investigate the statistical behavior of the disordered systems, a number of realizations (nr) of disordered arrays are generated and the field intensity distribution for each of the disordered systems is calculated. Then the statistical calculations are done on the number of different realizations.

3.1. Inverse localization length

For calculating the inverse localization length, after solving Eq. (3) and calculating the evolution of U_n up to the output position s_0, the output data are fitted with the exponential function by the least square method [29]:

$$\left\langle \left|U_n(s=s_0)\right|^2 \right\rangle = \left|U_{max}(s=s_0)\right|^2 \exp\left[-\frac{|n-m|}{l}\right] \tag{5}$$

where m is the number of waveguide in which $|U(s=s_0)|$ takes its maximum value and $\langle \ldots \rangle$ denotes the statistical average over realizations of disordered systems. The parameter l is the transverse localization length.

The inverse localization length versus dimensionless positive and negative nonlinear parameters for three different values of disorder strengths and for surface ($n_0=1$) and bulk ($n_0=10$) modes are calculated and results are presented in Fig. 2. In low nonlinear parameters, by increasing the disorder strength, the transverse localization is enhanced. But as it is expected the self-trapping effect [30,31] causes the high transverse localization at high nonlinearity coefficients both for positive and negative nonlinear parameters, which is obvious from Fig. 2. Fig. 2 shows a transition to high localized states by increasing the nonlinear parameter for surface and bulk modes in low disorder strength.

Similar behavior was reported in diagonal disorder array [23]. Due to the existence of critical nonlinearity coefficients in self-trapping effects [30,31], for low strength of disorders, the variation of localization length versus the nonlinear coefficient has a break point, where the corresponding nonlinear parameter is called the critical nonlinear parameter [23]. The break point is the intersection of the first two linear parts of the piecewise linear approximation of the curves corresponding to the low strength of disorder as shown ($\Delta o=0.4$) in Fig. 2. The values of critical nonlinear parameters are about $\chi_c \approx \pm 3.8$ for surface mode and $\chi_c \approx \pm 3.6$ for bulk mode as can be found in Fig. 2(a) and (b) respectively. Fig. 2 shows that as the disorder level is increased, the inverse localization length curves tend to smooth ones and no critical behavior is observed in figures for high disorder strengths.

For two different disorder strengths ($\Delta o=0.4$, $\Delta o=1.0$) in off-diagonal disordered arrays, the inverse localization length versus nonlinear parameters of the surface and bulk modes are compared in Fig. 3. As shown in Fig. 3a, due to the repulsive effects of boundaries in low nonlinear parameters the surface modes are more extended relative to the bulk modes in low disorder strength. For the diagonal disordered arrays this effect is reported previously [18,23,32]. The boundary repulsive effect is compensated by the localization effect due to high disorder strength (Fig. 3b). In high absolute value of both the positive and negative nonlinear parameters, the self-trapping effect can be found in Fig. 3(a) and (b).

As it is expected self-trapping is saturated for high values of nonlinear parameters and for the intermediate values of nonlinear parameters, the surface modes are more localized than the bulk modes. As it is expected due to the boundary repulsive, for small nonlinear parameters, the surface modes push toward the bulk localized modes, while for the intermediate nonlinear parameter the surface modes are highly localized relative to the bulk modes which is in agreement with those obtained in Fig. 3. In the diagonal disordered array, this effect is reported previously [23].

In order to compare the effects of physical parameters on the localization in diagonal and off-diagonal disordered arrays, the transverse localization in diagonal and off-diagonal disordered arrays for surface and bulk modes and for two different disorder strengths are obtained and results are shown in Fig. 4. The disordered parameters are chosen from the same interval uniformly. In Fig. 4(a–d) in low nonlinear parameters, the off-diagonal disordered arrays are more localized relative to the diagonal disordered arrays, but by increasing the positive and negative nonlinear parameters the diagonal disordered arrays are more localized than off-diagonal disordered arrays. This behavior is similar to that reported in continuous disordered systems [33,34]. As it is expected, due to the self-trapping effect, transition to high localized states is obtained in both diagonal and off-diagonal disordered systems.

The mode profiles for surface and bulk modes as a function of the waveguide number n at s =10,000 for different nonlinearity parameters are shown in Fig. 5. This figure shows that at large

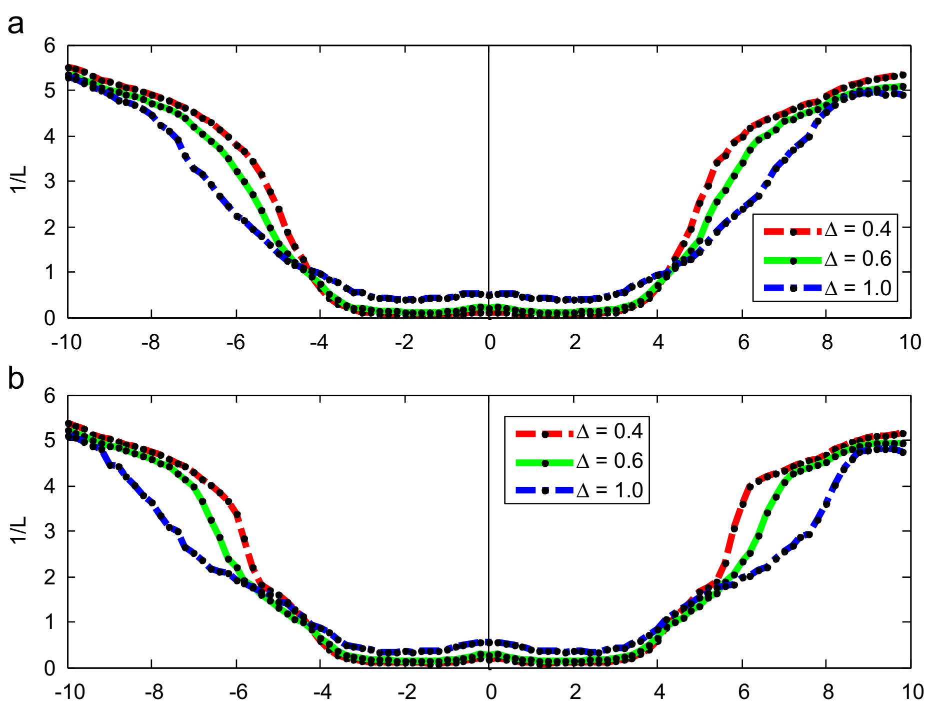

Fig. 2. Inverse localization length of (a): surface mode ($n_0=1$) and (b): bulk mode ($n_0=10$) versus nonlinear parameter for off-diagonal disordered systems, for different values of disorder strength Δ, $\Delta=0.4$ (red), $\Delta=0.6$ (green), $\Delta=1.0$ (blue), nr$=1000$, $s_0=100$. (For interpretation of the reference to color in this figure legend, the reader is referred to the web version of this article.)

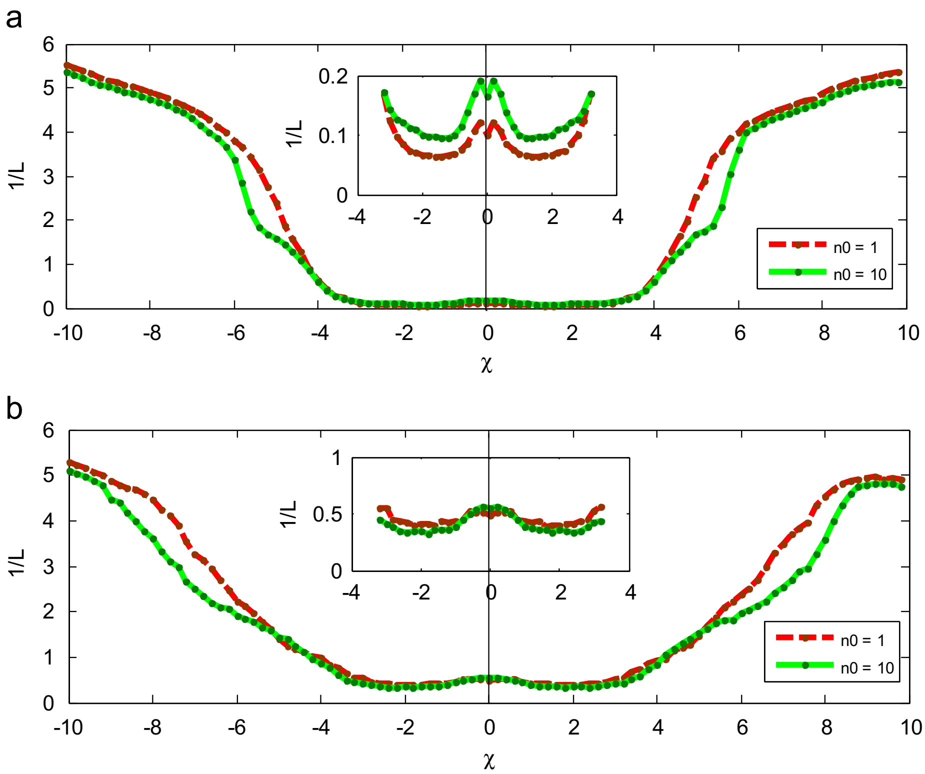

Fig. 3. Inverse localization length of surface mode ($n_0=1$) (red) and bulk mode ($n_0=10$) (green) versus nonlinear parameter for off-diagonal disordered systems, for (a) $\Delta=0.4$, and (b) $\Delta=1.0$, $nr=1000$, $s_0=100$. (Inset: the same figure for low nonlinearity.) (For interpretation of the reference to color in this figure legend, the reader is referred to the web version of this article.)

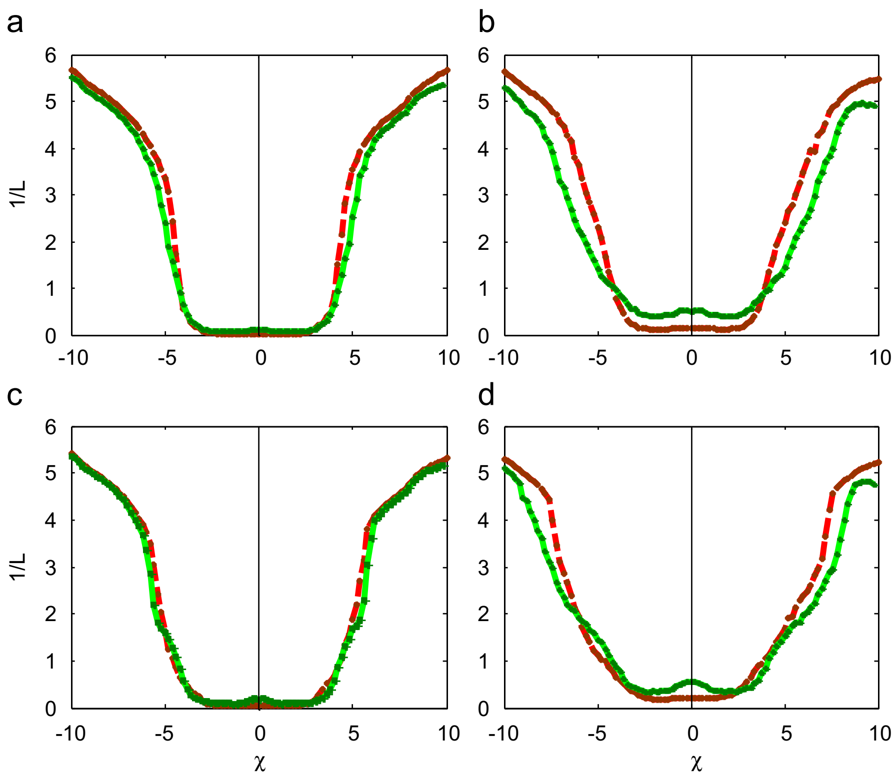

Fig. 4. Inverse localization length versus nonlinear parameter for diagonal (red) and off-diagonal (green) disorder. (a) $n_0=1$, $\Delta=0.4$, (b) $n_0=1$, $\Delta=1.0$, (c) $n_0=10$, $\Delta=0.4$, and (d) $n_0=10$, $\Delta=1.0$. $nr=1000$, $s_0=100$. (For interpretation of the reference to color in this figure legend, the reader is referred to the web version of this article.)

propagation distance the focusing nonlinear array localized slightly more than the defocusing nonlinear waveguides array. The same results are reported in continuous arrays [13].

Two selected propagation images for small ($\chi=1.4$) and high ($\chi=7.0$) nonlinear parameters are shown in Fig. 6. This figure shows high localized light wave in high nonlinear parameters.

3.2. Effective beam width

The effective beam width versus propagation distance, in off-diagonal disordered arrays for various positive nonlinear parameters, is plotted in Figs. 7 and 8 for surface ($n_0=1$) and bulk ($n_0=100$) modes, respectively. It is shown that for the same value of nonlinear parameters, the effective beam width for the bulk modes ($n_0=100$) are higher than that of the surface modes

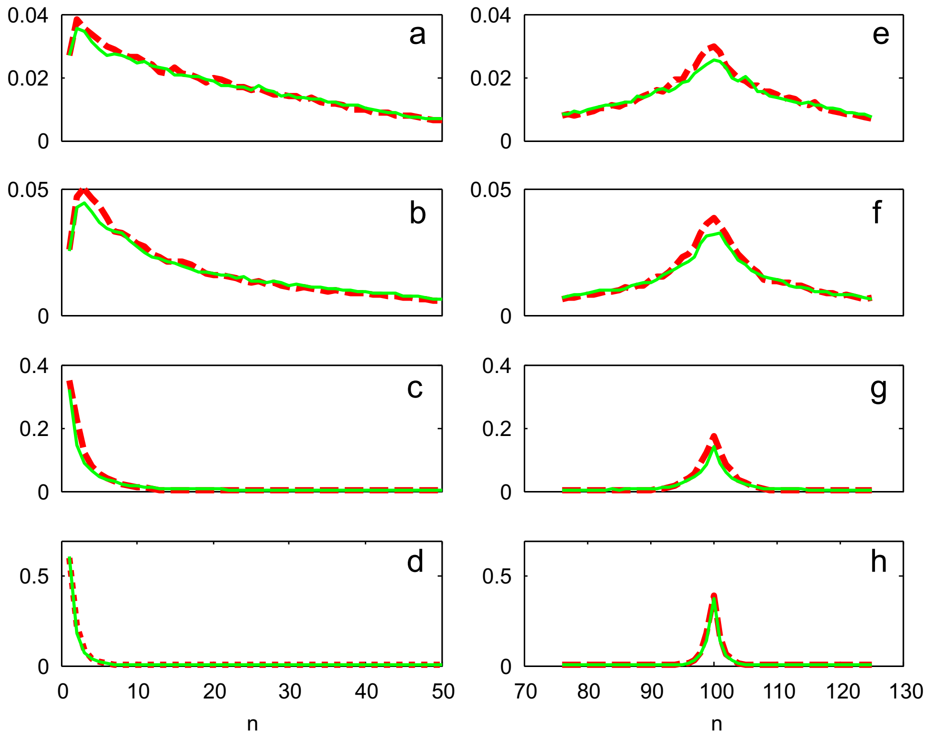

Fig. 5. Mode profiles $\langle |U_n|^2 \rangle$ as a function of the site number n for focusing (red) and defocusing (green) for $N=200$, $\Delta=0.6$, $s=10{,}000$, $nr=1000$, and $\chi=1.2$ [(a), (e)], $\chi=2$ [(b), (f)], $\chi=3.6$ [(c), (g)] and $\chi=4.4$ [(d), (h)]. Left panels denote the surface mode case ($n_0=1$) while right panels refer to the bulk mode case ($n_0=100$). Only part of the array sites are shown for clarity. (For interpretation of the reference to color in this figure legend, the reader is referred to the web version of this article.)

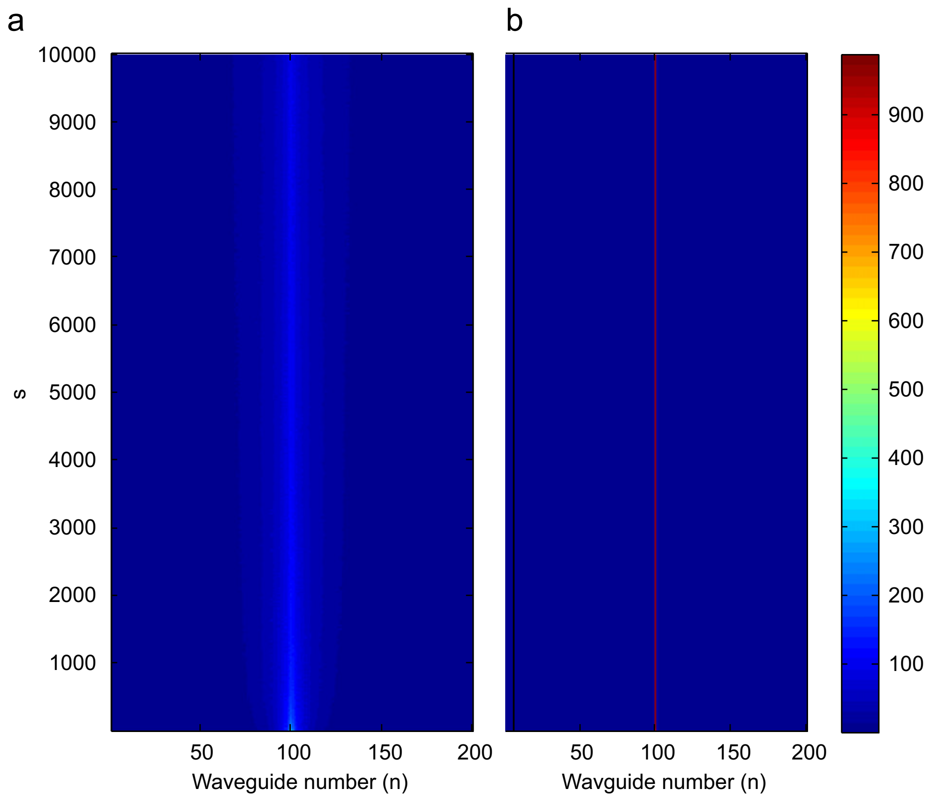

Fig. 6. Propagation images (a) $\chi=1.4$ and (b) $\chi=7.0$. $\Delta=0.6$, $n_0=100$, $nr=1000$.

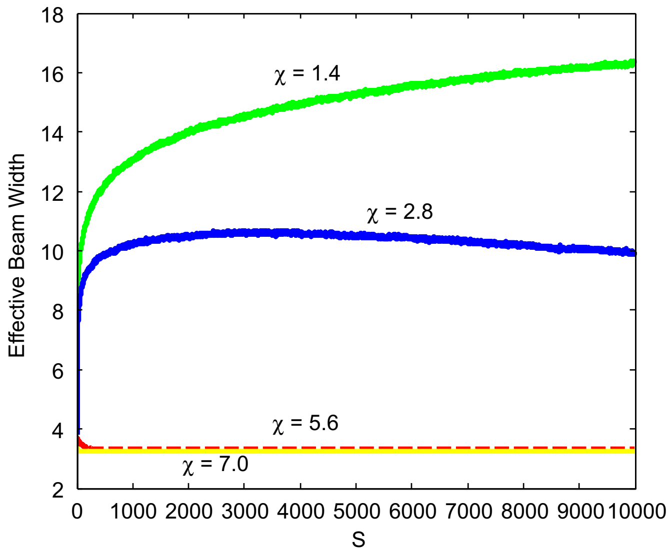

Fig. 7. Effective beam width for off-diagonal disorder, focusing nonlinearity. $\Delta=0.6$, $n_0=1$, nr=1000, $s_0=10{,}000$.

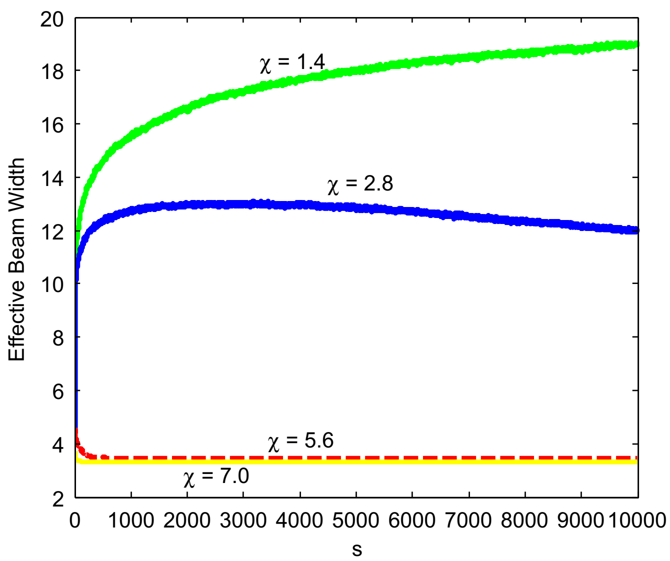

Fig. 8. Participation rate for off-diagonal disorder, focusing nonlinearity. $\Delta=0.6$, $n_0=100$, nr=1000, $s_0=10{,}000$.

($n_0=1$). This result is in agreement with those shown in Figs. 3 and 5. Thus, the incident light tends to expand over a large number of waveguides for bulk modes relative to the surface modes. As shown in these figures, for small values of nonlinear parameter, initially, the effective beam width is increased and then approached a saturated value around the entrance waveguide for bulk modes and shifted toward the bulk mode due to the surface repulsion for surface modes. For nonlinear parameter sufficiently larger than its critical value, self-trapping effect prohibits the expansion of light and light is localized in the entrance waveguide.

4. Conclusion

In summary, the effect of nonlinearity on the transverse localization of light in a 1D array of optical waveguides with off-diagonal disorder was studied numerically. The transverse localization length and the effective beam width are calculated as two important measures to study transverse localization. The transition to high localized states in the off-diagonal disordered array occurs faster than that in the diagonal disordered array in low nonlinear parameters. However by increasing the positive and negative nonlinear parameters the diagonal disordered array is more localized than the off-diagonal disordered arrays. Variation of the participation rate versus propagation distance confirms these results. The curves of inverse localization length versus nonlinear parameters tend to be smoother, by enhancement of the disorder strength. The surface modes are more sensitive to the nonlinear parameter relative to bulk modes.

The large values of nonlinearity parameters cause faster transverse localization for focusing parameters compared to defocusing ones in the diagonal and off-diagonal disordered arrays.

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