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Effect of long-range correlated disorder on the transverse localization of light in 1D array of optical waveguides

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ABSTRACT

In this paper, the effects of the long-range correlated diagonal disordered optical waveguide arrays in the presence and absence of the positive Kerr nonlinearity are analyzed numerically. The calculated inverse localization length shows that the long-range correlation in a disordered system causes a decrease in the transverse localization in linear optical waveguide arrays. In the presence of positive Kerr nonlinearity, the inverse localization length is increased by increasing the nonlinear parameters in long-range correlated disordered systems in comparison with the uniform distribution disordered systems. This means that the long range correlation causes an enhancement of transverse localization in nonlinear waveguides in contrast with linear waveguide arrays. The calculated participation ratio and effective beamwidth confirm these results for linear and nonlinear systems.

1. Introduction

The localization phenomenon was introduced for the first time by Anderson for electron's wave function in disordered potential in 1958 [1]. It was shown that all electron wave functions are exponentially localized in any 1D system in the presence of even small disordered potential. After a long time, some of the theoretical (numerical) and experimental studies of 1D disordered systems showed some deviation from this picture [2–15]. Some numerical studies showed that if the disordered potential is tailored in a specific manner, such as those that are chosen from long-range or short-range correlated sequences, or in dimer and generalized M-mer systems, the extended states appear in some electron energy intervals [2–15]. The presence of extended states in dimer systems were confirmed experimentally by measuring the conductance of some polymers such as polyaniline and electronic super lattices [14,15]. Izrailiev et al. showed, both theoretically and experimentally, that the correlation in disorder causes not only suppressed localization, but it may enhance the localization in some correlated systems [7–10]. Some natural systems such as nucleotide in DNA and the trace of particles in Brownian motion also demonstrate the long-range correlated sequence [11–13].

Localization phenomena are due to the wave interference, so it is natural to be expected to introduce them in any wave system such as condensed matter, elastic and optical systems [16–21]. One of the interesting topics in light localization is the transverse localization (*TL*) which was first predicted in 1989 [22], and was experimentally observed in disordered optical lattices in 2007 [23]. Later on, TL was also investigated in other disordered optical lattices [24–33]. Optical lattices can be realized experimentally by several techniques such as the optical induced technique, laser writing and lithographic methods [23,27–33]. One of the most realizable systems is 1D array of optical waveguides [32,33]. Disorder can be introduced by randomly changing the propagation constant of each waveguide (diagonal disorder) or randomly changing the coupling coefficients between them (off-diagonal disorder) [42].

In this paper, we consider a 1D array of optical waveguides with diagonal disorder. In Section 2 the theoretical models and methods for the generation of a long-range correlated sequence and the *TL* of light in a linear and a positive Kerr nonlinearity system (such as in fused silica waveguides) are introduced. Section 3 is devoted to results of numerical simulations and discussion. Finally it is ended by conclusion section.

2. Theoretical models and methods

2.1. Theoretical models for linear and nonlinear coupled waveguides

A 1D array of optical waveguides has been considered. We used the slowly varying envelope approximation (SVEA) to writing the propagation equations for light in linear and nonlinear waveguides.

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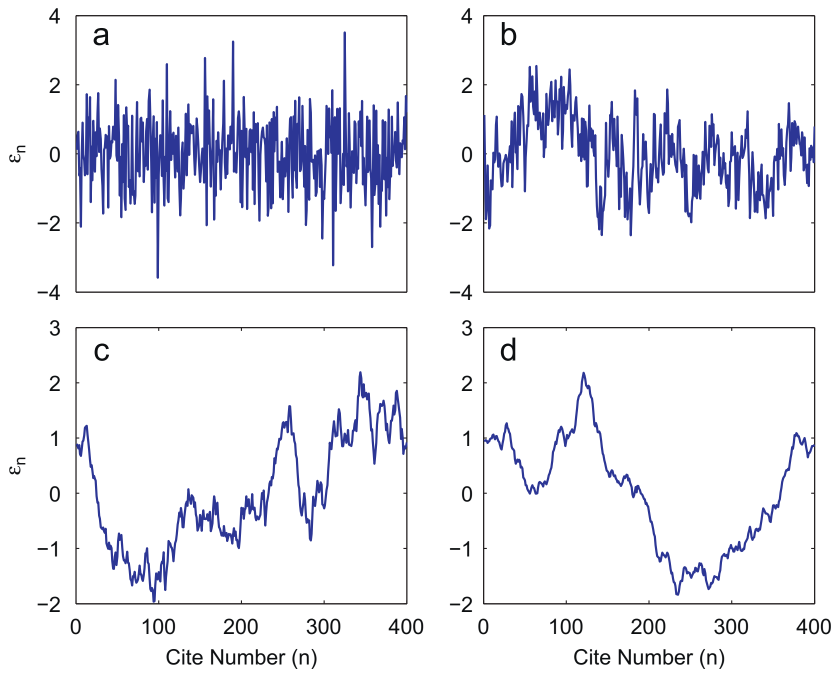

Fig. 1. Long range correlated sequences (a) $\alpha=0.0$, (b) $\alpha=1.0$, (c) $\alpha=2.0$, and (d) $\alpha=2.5$.

The appropriate equations in this approximation can be written in tight-binding model as follows [27–36]:

$$i\frac{dE_n}{dz}+K_nE_n+C_{n,n+1}E_{n+1}+C_{n,n-1}E_{n-1}+\gamma|E_n|^2E_n=0, \quad (1)$$

where n=1, 2,…,N. N is the number of waveguides, E_n and K_n are amplitudes of electric fields and propagation constants of the nth waveguide, respectively. $C_{n,m}$ are coupling coefficients between the nth and mth waveguide. $\gamma=n_2\omega/cA_{eff}$ is the third order Kerr parameter, n_2, A_{eff}, c and ω are nonlinear refractive index, effective area of fundamental modes, speed of light in free space and frequency, respectively. K_n depends to the width and refractive index of the nth waveguide. $C_{n,m}$ depends on the separation distance and refractive index of material between the nth and mth waveguides, that can be calculated by coupled mode theory [37]. In the weak coupling regime, the coupling coefficients can be adjusted by appropriately choosing the distance between waveguides. We assumed that the coupling coefficients are constant, but propagation constants were chosen from long-range correlated sequence. In order to rewrite Eq. (1) in a dimensionless form the following variables are employed:

$$C_{n,n+1}=C,\quad \varepsilon_n=\frac{K_n}{C},\quad s=Cz,\quad U_n=\frac{E_n}{\sqrt{P}},\quad \chi=\frac{\gamma P}{C}$$

where P is the intensity of the injected light. Eq. (1) versus dimensionless variables are as follows [33]:

$$i\frac{dU_n}{ds}+\varepsilon_nU_n+U_{n+1}+U_{n-1}+\chi|U_n|^2U_n=0,\quad n=1,2,\ldots,N \quad (2)$$

The system of Eq. (2), for $\chi=0$ is linear, while in the presence of self-focusing Kerr nonlinearity ($\chi>0$), the system of Eq. (2) is nonlinear.

As already mentioned, we have considered the third order Kerr type nonlinearity and neglected the higher order ones. This assumption is correct when the waveguide arrays are written with Ti: Sapphire laser on fused silica wafer [43]. In this case, the typical value of separation distances and the coupling coefficients between each waveguides are about 48 μm and 13.5 m^{-1}, respectively [43]. The normalized strength of third and fifth order Kerr nonlinearity are $\chi=\chi^{(3)}=\gamma^{(3)}P/C=2\pi n_0n_2P/\lambda CA_{eff}$ and $\chi^{(5)}=\gamma^{(5)}P^2/C=2\pi n_0n_4P^2/\lambda CA_{eff}^2$, respectively. The parameters of fused silica waveguides array are as follows [34,44]:

$$n_0=1.47,\quad \lambda=0.8\ \mu\text{m},\quad A_{eff}=50\ \mu\text{m}^2,\quad n_2=3.2\times10^{-20}\ \frac{m^2}{W}$$

$$n_4=-1.59\times10^{-38}\,\frac{m^4}{W^2}$$

In our simulation, the third order Kerr nonlinearity changes in the range $[0,10]$. For the $\chi^{(3)}_{max}=10$, the relative strength of the fifth to the third order Kerr nonlinearity is about $\chi^{(5)}/\chi^{(3)}_{max}\simeq1.75\times10^{-4}$. Therefore, the higher order terms of Kerr type nonlinearity can be ignored in fused silica waveguide arrays. The effects of higher order Kerr nonlinearity must be considered for other types of waveguide arrays such as those are made from *AlGaAs* , *PTS*polymers and other high nonlinear materials, for them the higher order Kerr nonlinearity introduces the significant effects [34,45].

In this work, we have focused on fused silica waveguides, and considered only third order Kerr type nonlinearity.

2.2. Generation of long-range correlated sequence

In Eq. (2), ε_n is chosen from a long-range correlated sequence in such a way that the spectral density decays as a power law ($S(k)\propto1/k^\alpha$). Spectral density is the Fourier transform of two-point correlation function. α is the long-range correlation exponent. In order to generate the long-range correlated sequence ϵ'_n is chosen by the following relation [11–13]:

$$\epsilon'_n=\sum_{m=1}^{N/2}\left[m^{-\alpha}\left(\frac{2\pi}{N}\right)^{1-\alpha}\right]^{1/2}\cos\left(\frac{2\pi nm}{N}+\phi_m\right), \quad (3)$$

where ϕ_m s are uniformly distributed in the $[0,2\pi]$ interval. The ϵ_n sequence is defined by the ϵ'_n rescaled sequence:

$$\epsilon_n=\frac{\epsilon'_n-\overline{\epsilon'_n}}{\Delta}, \quad (4)$$

Where $\Delta=\sqrt{\langle\epsilon'^2_n\rangle-\langle\epsilon'_n\rangle^2}$ and $\overline{\epsilon'_n}$ are the standard deviation and the mean of ϵ'_n, respectively. Some examples of long-range correlated sequences, which are generated by this method, are presented in Fig. 1 for four different values of correlation exponents.

2.3. Studying the transverse localization

There are two important measures for studying the *TL* in disordered waveguides: the transverse localization length and participation ratio (*PR*). The Rung–Kutta Fehlberg method [38] is employed to solve the systems of governing equations (2) up to the $s=s_0$ point. The solutions $[U_n(s=s_0)]$ are fitted by a least square process in the exponential function [39]:

$$\langle |U_n(s=s_0)|^2 \rangle = |U_{max}(s=s_0)|^2 \exp\left[-\frac{|n-m|}{l}\right], \tag{5}$$

where m $(1 \le m \le N)$ is the coordinate of maximum amplitude waveguide. l is the normalized transverse localization length and $\langle \ldots \rangle$ denotes averaging over large numbers of realizations due to the statistical nature of the Anderson localization.

PR is a qualitative measure of *TL*, and is defined by [24–26,33]:

$$PR(s) = \left\langle \frac{(\sum_{n=1}^{N} |U_n(s)|^2)^2}{\sum_{n=1}^{N} |U_n(s)|^4} \right\rangle. \tag{6}$$

In the case of a completely transverse localized system $(U_n(s)=\delta_{n,m})$ the *PR* of the system is approximately equal to one $(PR \simeq 1)$, while in a completely extended system (delocalized) $(U_n(s)=1/\sqrt{N})$ the participation ratio approaches to the number of waveguides $(PR \simeq N)$. The effective beam width is defined as the square root of the participation ratio $(W_{eff}(s)=\sqrt{PR(s)})$ [24–26].

3. Numerical results and discussion

For numerical simulation a finite number of coupled waveguides has been considered. Light is injected in one of the waveguides $(U_n(s)=\delta_{n,n_0})$ and fixed boundary conditions are exerted $(U_0(s)=U_{N+1}(s)=0)$. The inverse localization length and *PR* for linear and nonlinear systems are calculated.

3.1. Linear waveguides

To consider the effect of long-range correlated disorder on the localization length, in the absence of Kerr nonlinearity, the linear system of Eq. (2) $(\chi=0)$ with $N=200$, $n_0=100$ are solved numerically for different values of correlation exponent (α). The inverse localization length versus correlation exponent is shown in Fig. 2. As it is expected by increasing the long-range correlation exponent (α), the transverse localization decreases. This behavior is similar to destruction of localization in electronic systems in the presence of a long range correlated disorder [11–13]. As shown in Fig. 3 the *PR* increases along the propagation direction and the effective beam width enhances due to the increase in the correlation exponent i.e. light is expanded over a larger number of waveguides for higher correlation exponents.

The light intensity profile for uniform distributed and long-range correlated disordered waveguide arrays are shown in Fig. 4 (a) and (b) respectively. As it is expected light is expanded over large numbers of waveguides in correlated disordered waveguide arrays, by propagation along waveguides.

3.2. Nonlinear waveguides

In the self-focusing arrays χ is positive. All array parameters are chosen as those used in the previous section. The system of Eq. (2) is solved numerically, in the presence of positive Kerr nonlinearity. Variation of the inverse localization length versus nonlinear parameter χ for three different correlation exponents is presented in Fig. 5. As shown in Fig. 5 for low values of nonlinear parameters $(\chi < \chi_c = 4.4)$, the inverse localization length is approximately constant. In the low nonlinear regime, the inverse localization length decreases by increasing the long-range correlation exponent i.e. the transverse localization decreases by increasing the correlation &QJ;exponent for $\chi < \chi_c$. This figure shows that the system behaves similar to the linear system in low nonlinear regime, as it was shown in the previous section. χ_c is the critical nonlinear parameter. &QJ;By increasing the nonlinear parameter $(\chi > \chi_c)$, the correlated systems are localized faster than the uniform distributed disordered

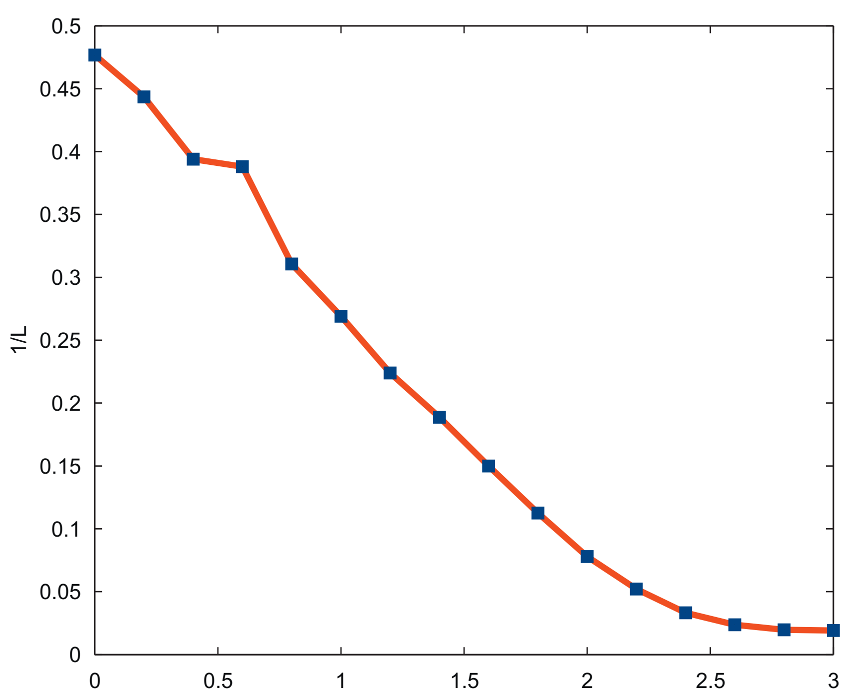

Fig. 2. Inverse localization Length versus long range correlation exponent parameter for linear waveguides, $n_0=100$, $N=200$, $nr=1000$, $s_0=1000$.

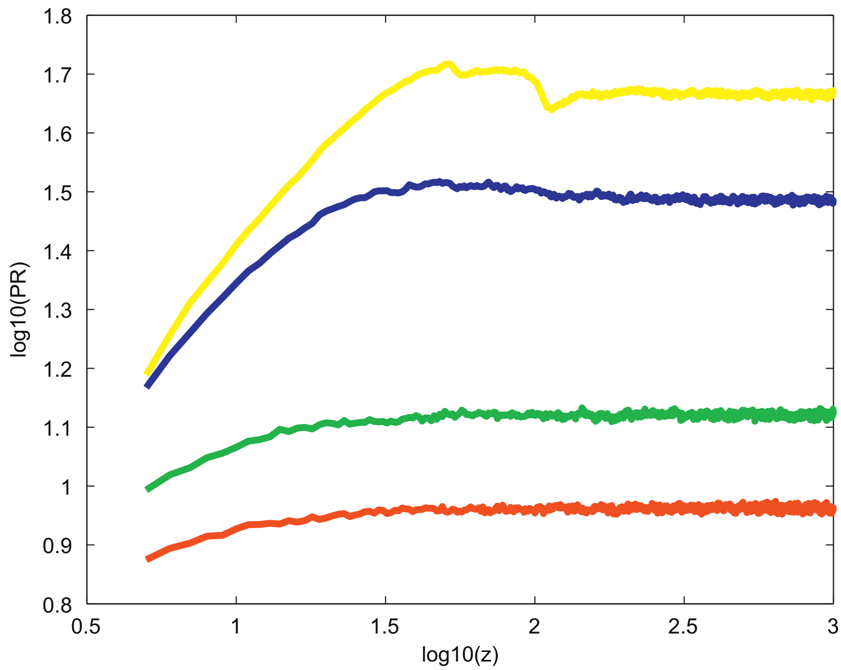

Fig. 3. Participation ratio for linear waveguides, $\alpha=0.0$ (red), $\alpha=0.8$ (green), $\alpha=1.6$ (blue), $\alpha=2.0$ (yellow). $N=200$, $n_0=100$, $nr=1000$, $s_0=1000$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

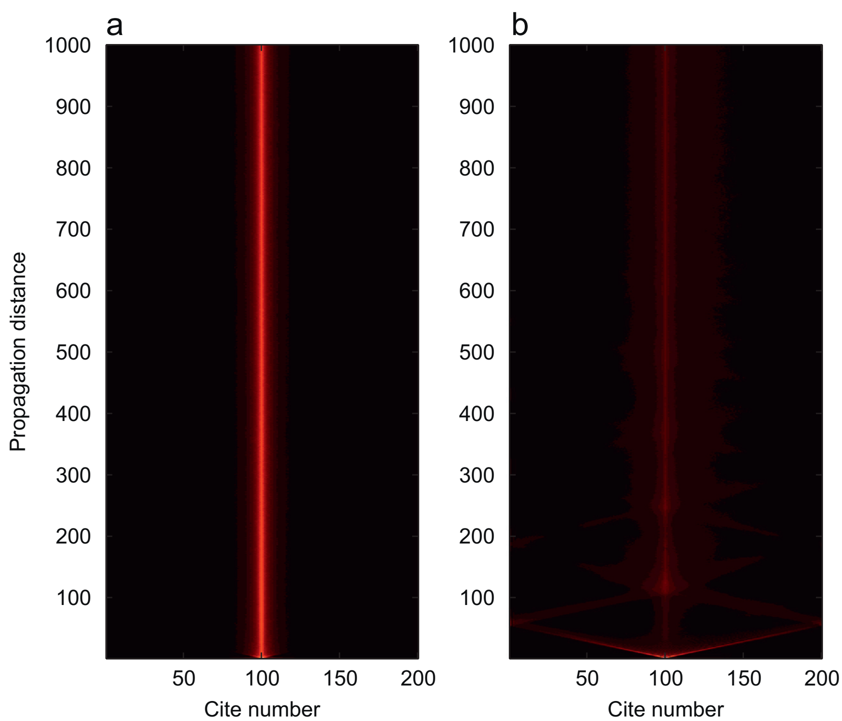

Fig. 4. Light intensity profile for (a) uniform distributed $\alpha=0.0$ and (b) long-range correlated $\alpha=3.0$ disordered waveguide arrays. $N=200$, $n_0=100$, $nr=1000$, $s_0=1000$.

waveguide arrays ($\alpha=0$). As it is expected, by increasing the nonlinear Kerr parameter self-trapping occurs and the probability of the confinement of light near the initial injected waveguide increases, therefore, self trapping causes *TL* of light in high nonlinear parameters [40,41]. Fig. 5 also shows that the long range correlated disorder enhances the self trapping phenomena in high nonlinear parameters.

For positive χ waveguide arrays, there are two important mechanisms for localization of light in the transverse direction: disorder and self-trapping. Both enhance the *TL*, but the *TL* is dominated by the effect of self-trapping and localization in transverse direction is often due to the self-trapping effect at $\chi>\chi_c$. On the other hand, the probability of self-trapping in ordered systems is higher than in disordered systems. Therefore by increasing the disorder strength the probability of self-trapping is decreased. Although increasing the disorder strength enhances the *TL* due to disorder but decreased the *TL* due to self-trapping. Therefore it is natural to expect that when the disorder is chosen

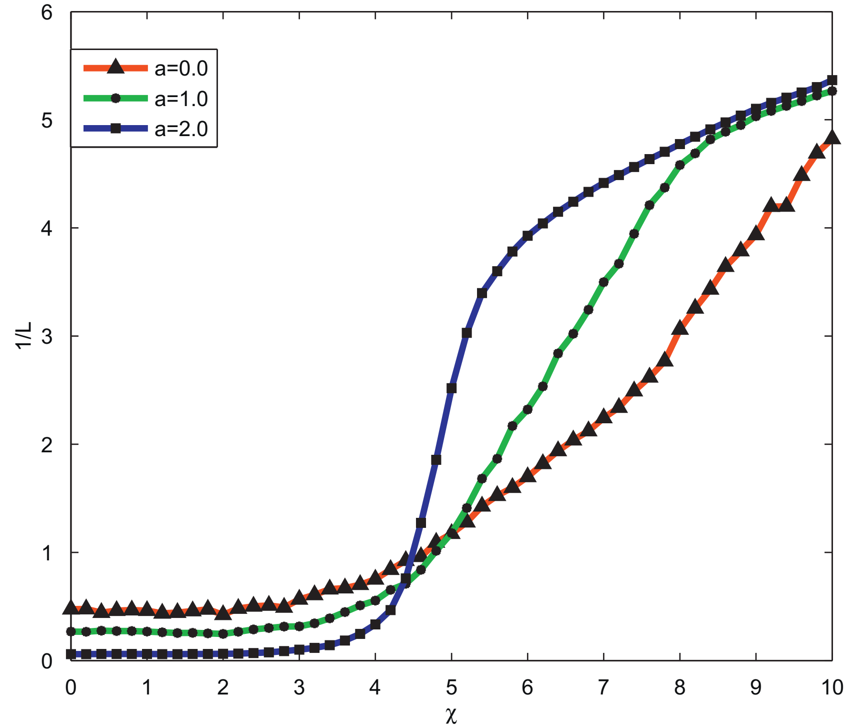

Fig. 5. Inverse localization Length versus nonlinear parameter, for different values of correlation exponent.

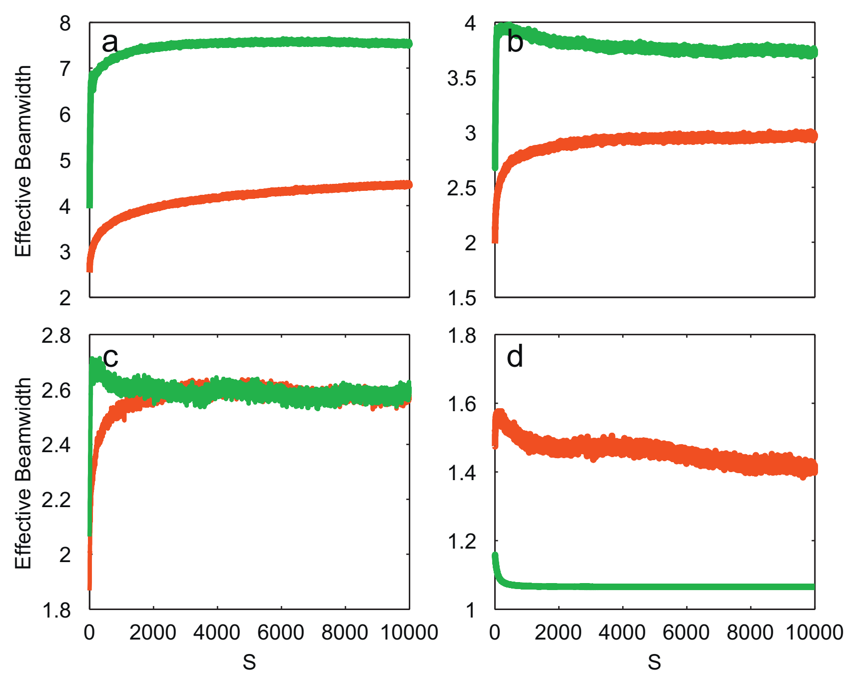

Fig. 6. Effective beam width versus propagation distance for positive Kerr nonlinearity, and $\alpha=0.0$ (red) and $\alpha=2.0$ (green). a: $\chi=2.0$, b: $\chi=4.0$, c: $\chi=4.4$, d: $\chi=6.0$. $N=200$, $n_0=100$, $nr=1000$, $s_0=10{,}000$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

from a long-range correlated sequence for $\chi>\chi_c$, *TL* is enhanced in comparison when a disorder with a uniform distribution is used ($\alpha=0$). So, by increasing the long-range correlation exponent (α) the *TL* is enhanced.

Fig. 6 shows the effective beamwidth versus propagation distance for a long-range correlated ($\alpha=2.0$) and uniformly distributed disordered array ($\alpha=0$), for different values of positive Kerr nonlinear parameters. For $\chi<\chi_c$ the effective beamwidth for the long-range correlated disordered array is larger than the uniformly distributed disordered array and light could be expanded over large number of waveguides, therefore the long-range correlation in disorder causes the destruction of the *TL* at $\chi<\chi_c$. In the case of $\chi>\chi_c$, this situation is reversed and the effective beamwidth for long-range correlated disordered array becomes lower than that of

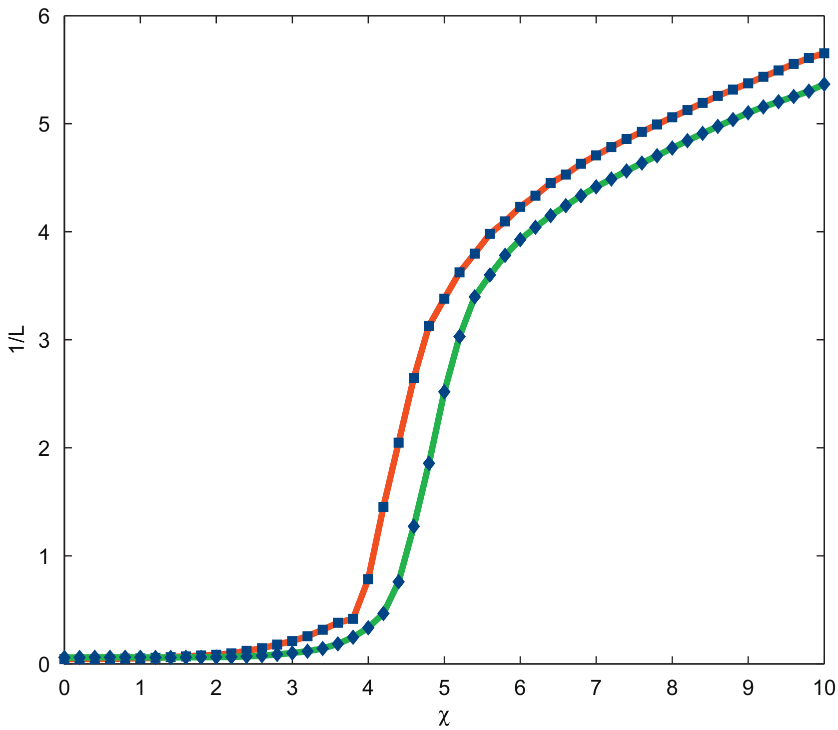

Fig. 7. Inverse localization Length versus nonlinear parameter, $\alpha=2.0$, $n_0=1$ (red), $n_0=100$ (green). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

uniformly distributed disordered array. This figure also shows, the long-range correlation in disorder enhances the self trapping effects. These results confirm those which were obtained from Fig. 5.

In highly nonlinear parameters, self-trapping causes the fast localization of light near the initially injected waveguide. The surface modes are defined as modes propagating in edge waveguides ($n=1,\ N$) and bulk modes are defined as modes propagating in intermediate waveguides ($n\neq1,\ N$). As it is expected, self-trapping has more effect on localization of surface modes than bulk modes, because for surface modes there is only one way (left or right) for coupling light in other waveguides but for bulk modes, light can be coupled to the both side waveguides. The inverse localization length versus the nonlinear parameters for surface and bulk modes for long range correlated disordered array are compared in Fig. 7. As expected the surface modes are localized faster than the bulk modes as χ is increased.

4. Conclusion

In summary, the effects of long-range correlated disorder on transverse localization of light in arrays of linear and nonlinear optical waveguides have been studied numerically. We see that the long-range correlation in diagonal disorder, decreases the *TL* in linear waveguide arrays. But in the presence of a positive Kerr type nonlinearity in disordered waveguide arrays, self-trapping occurs and light confines near the initial injected waveguide when $\chi>\chi_c$. The interplay between disorder and self-trapping decreases the probability of self-trapping at $\chi>\chi_c$. Although an increase in the disorder strength enhances the *TL* due to disorder but decreases the *TL* due to self-trapping. Therefore, it is reasonable to expect an enhancement in the *TL* when the diagonal disorder is chosen from a long-range correlated sequence at $\chi>\chi_c$, in comparison with the *TL* when a uniformly distributed disordered system in chosen ($\alpha=0$). We also see that, in the presence of the long-range correlated disorder, the surface modes are localized faster than bulk modes in high positive nonlinear Kerr effects, similar to the uniform distributed arrays.

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