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An optimal approach for maximizing the number of adjacencies in multi floor layout problem

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Multi-floor facility layout problem concerns the arrangement of departments on the different floors. In this paper, a new mathematical model is proposed for multi-floor layout with unequal department area. Maximising the number of useful adjacencies among departments is considered as the objective function. The adjacencies are divided into two major categories: horizontal and vertical adjacencies. The horizontal adjacency may be occurred between the departments assigned to same floors while the vertical can be happened between departments assigned to any consecutive floors. A minimum common boundary length (surface area) between any two horizontal (vertical) adjacent departments is specified. The efficiency of the model is demonstrated by six illustrative examples. The proposed model is practical in multi-floor plant where the existence of adjacencies between departments is useful or essential due to possible establishment of conveyor, transferring pipes, lift truck route, etc.

Keywords: adjacency; mathematical model; multi-floor facility layout; unequal department

1. Introduction

Facility layout problem (FLP) concerns with finding the appropriate arrangement of departments. The basic meaning of facility is the space in which a business's activities take place (work station) and the basic objective of facility layout is to ensure a smooth flow of work, material and information through a system. It can be considered as an important component of a business's overall operations, both in terms of maximising the effectiveness of the production process and meeting the needs of employees. The layout and design of the space impact greatly how the work, the flow of work, materials and information through the system is done. The key to good facility layout and design is the integration of the needs of manpower, materials and machinery in such a way that they create a well-designed functioning system.

FLP can be categorised into two distinct classes, namely single floor and multi-floor layout. In real-world situation, it would be worthwhile to design the plant layout on a multi-floor instead of a single floor. This is a common practice in the areas with high land cost or scares of the available land (Drira, Pierreval, and Hajri-Gabouj 2007). Also, sometimes it is more practical to arrange departments into multiple floors rather than single floor due to the nature of material or the process specification. In addition, a compact building shape may allow for more efficient environmental control.

In multi-floor facility layout problem (MFFLP), facilities can be located in the limited numbers of floors so that the factory space can be utilised efficiently. In MFFLP, there are two general types of flow, horizontal and vertical, between facilities. Horizontal flow occurs between facilities located on the same floor. In contrast, the vertical flow can be seen between facilities on different floors. Generally transferring the materials between two departments, located on different floors are done using elevators, conveyors, transferring pipes, etc.

It is impossible to develop a layout model which can capture all the relevant aspects of the problem. This partly explains why a huge number of papers consider only a particular aspect of this problem. One of the interesting and applicable approaches for the assessment of different layouts is the number (or value) of created useful adjacencies among departments or facilities. It, therefore, stands to reason that maximising the number of useful adjacencies between facilities is a desirable objective function considered in layout problems. Typically, most of the papers in the related literature utilise graph theory for maximising the number of adjacencies (Foulds and Robinson 1978; Boswell 1994; John and Hammond 2000; Osman 2006). Graph theory is used to maximise the number of adjacencies between departments with total disregard for the physical shape of departments; therefore, in a layout drawing based on the graph theory, departments can be T- or L-shaped, and it is difficult to actualise such a layout in the real world. This constraint seems to be reason enough for us to present a new mathematical model for maximising the number of useful adjacencies

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among rectangular departments or facilities. Therefore, this study proposes a new model for maximising the number of horizontal and vertical adjacencies among different rectangular departments in multi-floor layouts. Proposition of the vertical and horizontal adjacencies is useful especially in design of the multi-floor process plant layout, while materials are mostly in the fluid type and have to be handled through the transferring pipes.

2. Literature review

The theoretical beauty and practical applications of the FLP have focused the attentions of the researchers to this combinatorial optimisation problem. Unfortunately, the FLP has been reported as a NP-complete problem (Kusiak and Heragu 1987). Therefore, only heuristics or meta-heuristics solution approach can be employed to obtain the solution of the large-sized problems. The single-floor layout problem has been studied extensively in the past decades (Jain, Khare, and Mishra 2013; Drira, Pierreval and Hajri-Gabouj 2007). Meller and Gau (1996) categorised the layout planning problems, as discrete and continues problems. In the discrete version of the problem, the departments are usually the collections of equal-sized unit squares. In the continuous version, the departments are polygons with orthogonal sides and vertices that can be located anywhere inside the building floor. There are several optimal solutions algorithms that have been developed by Heragu and Kusiak (1991), Meller, Narayanan, and Vance (1999), Barbosa-PÓvoa, Mateus, and Novais (2001), Sherali, Fraticelli, and Meller (2003).

In contrast, the research on the multi-floor FLP is limited. MFFLP is categorised as a particular case of FLP and all solutions approaching MFFLP can be classified into two main groups: one-stage algorithms and two-stage algorithms. In one-stage approaches, all departments, except the fixed ones, are assigned to the floors during progress of the solution approach. In two-stage approaches, first each department is assigned to a specific floor, and then, the solution procedure arranges the layout of a floor according to the assigned departments.

The early research in which the MFFLP was introduced was the work of Johnson (1982). Later, Meller and Bozer (1996) proposed a simulated annealing (SA) algorithm for solving MFFLP, their attempt was focused on the minimisation of the total vertical and horizontal material flow costs. A considerable number of well-known researchers have endeavoured to solve MFFLP by applying different meta-heuristics algorithms such as genetic algorithm (Berntsson and Tang 2004; Krishna and Jaffari 2011; Kia et al. 2014), tabu search (TS) (Abdinnour-Helm and Hadley 2000), SA (Tam 1992; Kevani et al. 2010). In many algorithms, minimising the total material handling costs is considered as the objective function (Kohara, Yamamoto, and Suzuki 2008). In some others, a multi-objective function approach is followed considering other interests such as maximising the total adjacencies value (Lee, Roh, and Jeong 2005), improving the safety factor (Jung et al. 2011; Park et al. 2011), and minimising the total cost of installing the elevators (Matsuzaki, Irohara, and Yoshimoto 1999) and facility building cost (Hathhorn, Sisikoglu, and Sir 2013). Moreover, there are several exact approaches for solving the small size MFFLP problems (Patsiatzis and Papageorgiou 2002; Patsiatzis and Papageorgiou 2003; Hahn, MacGregor Smith, and Zhu 2010).

Considering two-stage approaches, Meller and Bozer (1997) were the first researchers who used a mathematical model to assign the departments to the floors and then utilised a heuristic algorithm to arrange the assigned departments in each floor. Abdinnour-Helm and Hadley (2000) compared two different models. In first model, a heuristic algorithm was used to assign the departments to a specific floor while in the second a deterministic mathematical model was utilised for this purpose. Moreover, in both models they planned the layout of the departments within each floors, by using the TS algorithm. They concluded the solutions obtained by mathematical model are better than the heuristic one. Bernadi and Anjos (2012) have developed a deterministic mathematical model which assigns departments to the floors. They compared two solution approaches of MFFLP. They concluded that simultaneous arrangement of departments in all floors leads to better solution than independent layout of each floor.

Hosseini, Mirzapour, and Wong (2013) explored the effects of applying the systematic layout planning (SLP) method on the MFFL of a card and packet production company. They employed simulation as an evaluation tool for comparing three alternative layouts obtained by SLP. Ghadikolaei and Shahanaghi (2013) proposed a dynamic MFFL model in which the material flow data was fluctuating over time periods. A mathematical model and SA-based solution method was developed for the proposed problem and the results were reported.

3. Statement of the problem

Generally two facilities are considered to be adjacent if they share a common wall or divider with some minimal tolerance length between them. In this study, the shape of all departments are considered to be rectangular norm and adjacencies can occur along with X-, Y- or Z-axis. We want to determine the centre point coordination and assign each facility/department to the appropriate floor in order to maximise the horizontal and vertical adjacencies. In a single-floor layout, each department can be adjacent with other departments in four directions (see Figure 1(a)) but in multi-floor layout there are six possible directions as it is illustrated in Figure 1(b) in which top and bottom adjacencies can be added.

Our purpose from horizontal adjacencies is all possible adjacencies that can be created among the departments located on a specified floor. For creating the horizontal adjacencies, it is sufficient that two departments having a common boundary length along X- or Y-axis with each other.

Consideration of the vertical adjacencies with common surface area between departments becomes more attractive when material is in the form of fluid and has to be handled through the transferring pipes. Usually in these situations, in order to transfer material between two non-adjacent departments which located on two different floors, at least one (or more) pipe bend(s) is (are) needed. Unfortunately using pipe bends in transferring rout can create numerous problems such as loosing heat and changing pressure in the outer wall of the bend; therefore it is more desirable to design a layout scheme with minimum using of pipe bends. This aim can be satisfied by maximising the number of adjacencies among departments; because two vertically (or horizontally) adjacent departments can be connected via a straight pipe.

Another important issue is the case of using elevators for transporting material between departments. In these cases, it is vigorous to have the vertical straight line for material handling, in order to transfer materials vertically without a major restriction. With this type of vertical adjacency we intend any two departments which are located on two consecutive floors, with at least some common surface area between them. In other words, two departments are considered vertically adjacent when department i lies on floor k, and department j lies on floor $k-1$ or $k+1$, and two departments i and j have a common surface along Z-axis (see Figure 2).

4. Formulation

In this study, all departments can be located anywhere on each floor in view of the non-overlapping constraints. One further assumption is that all parameters are predetermined and certain. The objective function and constraints are developed as follows:

4.1 *Objective function*

As mentioned earlier, maximising the number of useful adjacencies is taken as the objective function of the proposed model; hence, considering the f_{ij}(a positive number) as the adjacent value, the objective function can be written as:

$$\max Z = \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij}\left(\frac{1}{2}N_{ij} + NV_{ij}\right) \tag{1}$$

where n indicates the number of unequal facilities that should be located in the given floors. N_{ij} and NV_{ij} are two binary variables that imply horizontal and vertical adjacencies, respectively, as follows:

$$N_{ij} = \begin{cases} 1 & \text{when two department } i \text{ and } j \text{ are horizentally adjacent} \\ 0 & \text{otherwise} \end{cases}$$

$$NV_{ij} = \begin{cases} 1 & \text{when two department } i \text{ and } j \text{ are vertically adjacent} \\ 0 & \text{otherwise} \end{cases}$$

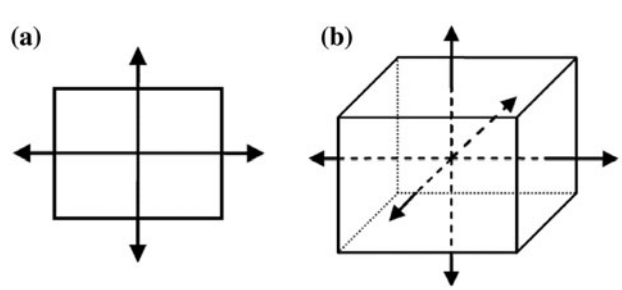

Figure 1. Difference between all possible directions for creating adjacencies in single floor (Figure 1(a)) and Multi-floor (Figure 1(b)).

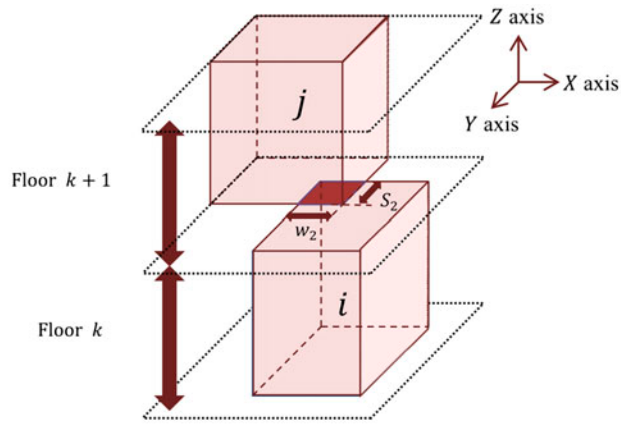

Figure 2. Two vertically adjacent departments.

Noticeably, $1/2$ is a correction factor that prevents to double enumeration of the horizontal adjacencies. In general, the contribution (importance) of the horizontal and the vertical adjacencies in the objective function may be weighted up or down. However, in this study we assumed to be in same order. Moreover it is assumed when two departments i and j are vertically or horizontally adjacent, material can easily be transferred from i to j, and vice versa.

4.2 *Floor constraints*

Generally, the number of floors and departments is specified and each department should be assigned to exactly one floor. To accomplish this, a new binary variable is introduced as follows:

$$V_{ij} = \begin{cases} 1 & \text{if two departments } i \text{ is assigned to floor } k \\ 0 & \text{otherwise} \end{cases}$$

In view of the mentioned variable, the floor constraint can be written as:

$$\sum_{k=1}^{K} V_{ik} = 1 \qquad \forall i \in 1, 2, \ldots, n \tag{2}$$

in which K indicates the number of available floors. Noticeably, in order to assign a specific department such as i to identifying floor k, it suffices to add $V_{ik} = 1$ in to constraints of model. For incorporating some constraints such as non-overlapping, minimum common boundary length and identifying horizontal adjacencies, it is important to determine that two departments i and j are assigned to the same floor or not. Hence, a new binary variable Z_{ij} is introduced, with the value of one when two departments are located on the specific floor and zero otherwise. The value of Z_{ij} can be obtained through Equations (3)–(5) (Patsiatzis and Papageorgiou 2002)

$$Z_{ij} \geq V_{ik} + V_{jk} - 1 \qquad \forall i,j = 1, 2, \ldots, n, \quad i \neq j, \quad k = 1, 2, \ldots, K \tag{3}$$

$$Z_{ij} \leq 1 - V_{ik} + V_{jk} \qquad \forall i,j = 1, 2, \ldots, n, \quad i \neq j, \quad k = 1, 2, \ldots, K \tag{4}$$

$$Z_{ij} \leq 1 + V_{ik} - V_{jk} \qquad \forall i,j = 1, 2, \ldots, n, \quad i \neq j, \quad k = 1, 2, \ldots, K \tag{5}$$

Equation (3) enforce the Z_{ij} takes one when two departments i and j are located on the same floor (i.e. $V_{ik} = V_{jk} = 1$), otherwise if at least one of two variables V_{ik} or V_{jk} take zero, then Z_{ij} will be zero too.

4.3 *Non-overlapping constraints*

The constraints (6)–(9) are added to the model in order to ensure that all departments are assigned to the same floor, have no overlap with each other. Now two new binary variables p_{ij} and q_{ij} are introduced and four disjunctive constraints are included in the model. Non-overlapping constraint is satisfied when at least one of them is active.

$$x_i - x_j + M\left(1 - Z_{ij} + p_{ij} + q_{ij}\right) \geq \frac{1}{2}\left(l_i + l_j\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{6}$$

$$-x_i + x_j + M\left(2 - Z_{ij} - p_{ij} + q_{ij}\right) \geq \frac{1}{2}\left(l_i + l_j\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{7}$$

$$y_i - y_j + M\left(2 - Z_{ij} + p_{ij} - q_{ij}\right) \geq \frac{1}{2}\left(d_i + d_j\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{8}$$

$$-y_i + y_j + M\left(3 - Z_{ij} - p_{ij} - q_{ij}\right) \geq \frac{1}{2}\left(d_i + d_j\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{9}$$

(x_i, y_i) is the centre point coordination of department i, and l_i/d_i represent the length/width of department i (unequal size), and M is a huge positive number. Note that, the $Z_{ij} = 0$, states that two departments i and j are located on different floors and all constraints (6)–(9) are inactive.

4.4 *Distance between two departments*

Distance between any pair of departments is used in other constraints; distance along with X- and Y-axis can be obtained by defining four new variables. These variables are:

$$x_{ij}^{+} = \begin{cases} x_i - x_j & \text{distance along } X - \text{axis, if department } i \text{ is to right of } j \\ 0 & \text{other wise} \end{cases}$$

$$x_{ij}^{-} = \begin{cases} x_j - x_i & \text{distance along } X - \text{axis, if department } i \text{ is to left of } j \\ 0 & \text{other wise} \end{cases}$$

$$y_{ij}^{+} = \begin{cases} y_i - y_j & \text{distance along } Y - \text{axis, if department } i \text{ is to above of } j \\ 0 & \text{other wise} \end{cases}$$

$$y_{ij}^{-} = \begin{cases} y_j - y_i & \text{distance along } Y - \text{axis, if department } i \text{ is to below of } j \\ 0 & \text{other wise} \end{cases}$$

Value of these variables can be obtained by:

$$x_i - x_j = x_{ij}^{+} - x_{ij}^{-} \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{10}$$

$$y_i - y_j = y_{ij}^{+} - y_{ij}^{-} \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{11}$$

Regarding the definition of the new variables, the distance along with X- and Y-axis is calculated as $(x_{ij}^{+} + x_{ij}^{-})$ and $(y_{ij}^{+} + y_{ij}^{-})$, respectively.

4.5 *Horizontal adjacencies constraints:*

Equations (12) and (13) are used for determining horizontal adjacencies. Clearly, when the distance along the X/Y-axis between centre points of two departments i and j, allocated to the same floor, is less than half of the sum of its lengths/width, the possibility of the existence of an adjacency along the X/Y-axis arises. Hence, Equations (12) and (13) are added to the model to identify the horizontal adjacencies using two new types of binary variables NX_{ij} and NY_{ij}.

$$x_{ij}^{+} + x_{ij}^{-} \leq \frac{1}{2}\left(l_i + l_j\right) + M\left(1 + Z_{ij} - 2NX_{ij}\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{12}$$

$$y_{ij}^{+} + y_{ij}^{-} \leq \frac{1}{2}\left(d_i + d_j\right) + M\left(1 + Z_{ij} - 2NY_{ij}\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{13}$$

For satisfying the adjacent condition, both binary variables NX_{ij} and NY_{ij} have to be equal to 1 simultaneously. Note that in both Figure 3(a) and (b), the value of NY_{ij} is equal to 1, but only in Figure 3(a), the value of NX_{ij} is 1 too, and therefore two departments i and j are horizontally adjacent. In other words, the two binary variables NX_{ij} and NY_{ij} should be equalled to 1, simultaneously, in order to affirm two departments i and j are horizontally adjacent with each other.

In view of the above explanation, the following constraints are included in the model:

$$N_{ij} - NX_{ij} - NY_{ij} + 1.5 \geq 0 \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{14}$$

$$1.5N_{ij} - NX_{ij} - NY_{ij} \leq 0 \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{15}$$

As stated, N_{ij} is a binary variable that indicate two departments i and j are horizontally adjacent or not. Therefore, its value is 1 when both binary variables NX_{ij} and NY_{ij} are equal to 1 and N_{ij} is equal to zero when at least one of the two variables NX_{ij} and NY_{ij} are zero.

4.6 *Minimum common boundary length constraints*

Based on the adjacency definition, it is assumed that two departments i and j are adjacent when they have a minimum common boundary length with each other. This common boundary can be used for handling equipment and pipes that transfer material between two departments. Now two new parameters W_1 and $S_1$are introduced which represents the minimum common boundary length along X- and Y-axis, respectively (see Figure 3(a)). For modelling the relevant constraints, two new types of binary variables NTX_{ij} and NTY_{ij} are introduced. Therefore, the following Equations (16)–(19) should be added to the model.

$$x_i - x_j \leq \frac{1}{2}\left(l_i + l_j\right) - W_1 + M\left(1 + Z_{ij} - 2NTX_{ij}\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{16}$$

$$x_i - x_j \geq M\left(2NTX_{ij} - Z_{ij} - 1\right) + W_1 - \frac{1}{2}\left(l_i + l_j\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{17}$$

$$y_i - y_j \leq \frac{1}{2}\left(d_i + d_j\right) - S_1 + M\left(1 + Z_{ij} - 2NTY_{ij}\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{18}$$

$$y_i - y_j \geq M\left(2NTY_{ij} - Z_{ij} - 1\right) + S_1 - \frac{1}{2}\left(d_i + d_j\right) \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{19}$$

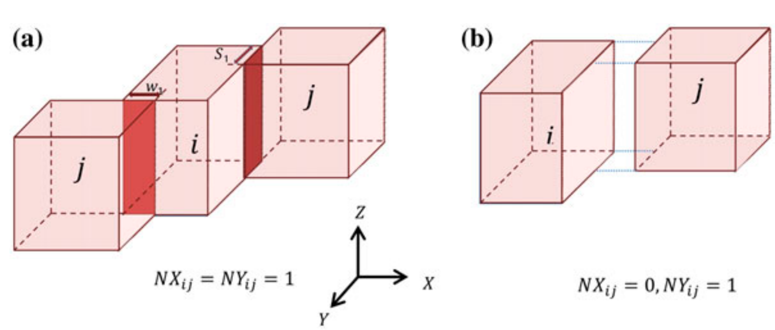

Figure 3. One of the necessary condition for horizontal adjacencies of two departments.

In constraints (16) and (17), the binary variable NTX_{ij} is equal to 1, when any two departments i and j have common boundary length along X-axis and is zero otherwise. This even holds without consideration of the distance between i and j along Y-axis. The same explanation can be applied for NTY_{ij}.

Considering the non-overlapping constraints (6)–(9) vs. minimum common boundary length constraints (16)–(19), it is concluded that the two variables NTX_{ij} and NTY_{ij} cannot be equal to 1, simultaneously. Therefore, if one of the two variables becomes equal to 1, it means that two departments have a common boundary with each other. Hence, the following constraint is added to model for covering the achievement of the minimum common boundary length.

$$NT_{ij} = NTX_{ij} + NTY_{ij} \qquad \forall i, j = 1, 2, \ldots, n, \quad i \neq j \tag{20}$$

where NT_{ij} is a binary variable indicating the achievement of the common boundary along X- or Y-axis.

4.7 *Floor space constraints*

In this study, a rectangular shape of land area is assumed to be used with the length H and width L. Therefore, the following constraints have to be included.

$$x_i - \frac{1}{2} l_i \geq 0 \qquad \forall i = 1, 2, \ldots, n \tag{21}$$

$$x_i + \frac{1}{2} l_i \leq H \qquad \forall i = 1, 2, \ldots, n \tag{22}$$

$$y_i - \frac{1}{2} d_i \geq 0 \qquad \forall i = 1, 2, \ldots, n \tag{23}$$

$$y_i + \frac{1}{2} d_i \leq L \qquad \forall i = 1, 2, \ldots, n \tag{24}$$

4.8 *Vertical adjacencies constraints*

When two departments i and j are located on two sequential floors, it is possible to create vertical adjacencies with each other. For this purpose, it is sufficient that two departments i and j having a common surface with each other along the Z-axis (see Figurer 2). These constraints can be modelled by defining two new types of binary variables NVX_{ijk} and NVY_{ijk}. NVX_{ijk} ($NVY_{ijk,}$) is a binary variable, and equal to 1 when any two departments i and j, belong to any two sequential floors k and $(k+1)$, overlap along the $X(Y)$-axis and zero otherwise. In order to define common surface specification, two new parameters W_2 and S_2 are introduced, which indicate, respectively, the minimum length and width of the common surface along the X- and Y-axis (see Figure 2). The model includes the following constraints (25)–(28) as:

$$x_i - x_j \leq \frac{1}{2}(l_i + l_j) + H(1 + V_{ik} + V_{j,(k+1)} - 3NVX_{ijk}) - W_2 \qquad \forall i, j = 1, \ldots, n; \quad i \neq j; \quad k = 1, \ldots, K-1 \tag{25}$$

$$x_j - x_i \leq \frac{1}{2}(l_i + l_j) + H(1 + V_{ik} + V_{j,(k+1)} - 3NVX_{ijk}) - W_2 \qquad \forall i, j = 1, \ldots, n; \quad i \neq j; \quad k = 1, \ldots, K-1 \tag{26}$$

$$y_i - y_j \leq \frac{1}{2}(d_i + d_j) + L(1 + V_{ik} + V_{j,(k+1)} - 3NVY_{ijk}) - S_2 \qquad \forall i, j = 1, \ldots, n; \quad i \neq j; \quad k = 1, \ldots, K-1 \tag{27}$$

$$y_j - y_i \leq \frac{1}{2}\left(d_i + d_j\right) + L\left(1 + V_{ik} + V_{j,(k+1)} - 3NVY_{ijk}\right) - S_2 \qquad \forall i,j = 1,\ldots,n; \quad i \neq j; \quad k = 1,\ldots,K-1 \tag{28}$$

In constraints (25)–(28), the two parameters L and H play the same role of big-M or a huge number. Considering this fact, where the vertical adjacencies may be happened between two sequential floors, two consecutive binary variables V_{ik} and $V_{j,(k+1)}$ have been employed. Also these constraints become active when two departments i and j, which are located in two sequential floors k and $k+1$, have some overlap with each other along X- or Y-axis.

Obviously, NVX_{ijk} and NVY_{ijk} can be equal to 1 at only one floor. Therefore, the following constraints (29) and (30) are added as:

$$NSX_{ij} = \sum_{k=1}^{K-1} NVX_{ijk} \qquad \forall i,j = 1,2,\ldots,n \tag{29}$$

$$NSY_{ij} = \sum_{k=1}^{K-1} NVY_{ijk} \qquad \forall i,j = 1,2,\ldots,n \tag{30}$$

If the value of NSX_{ij} (NSY_{ij}) is equal to 1, it means that two departments i and j probably overlap each other between two assuming sequential floors such as k and $k+1$; along the $X(Y)$-axis. However, when the two variables NSX_{ij} and NSY_{ij} are simultaneously equal to 1 then two departments i and j are vertically adjacent and common surface area between them is greater than $W_2 \times S_2$. As a result, the following Equations (31) and (32) are added to model.

$$2NV_{ij} - NSX_{ij} - NSY_{ij} \leq 0 \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{31}$$

$$NSX_{ij} + NSY_{ij} - NV_{ij} \leq 1 \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{32}$$

where NV_{ij} indicates vertically adjacent variables and is equal to 1 when two departments i and j have vertically adjacent conditions.

4.9 *Logical constraints*

Regarding the definition of variables and the structure of mathematical model, the following constraints (33)–(35) are inevitable.

$$NX_{ij} \leq NT_{ij} \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{33}$$

$$NY_{ij} \leq NT_{ij} \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{34}$$

$$N_{ij} = N_{ji} \qquad \forall i,j = 1,2,\ldots,n, \quad i \neq j \tag{35}$$

5. Numerical examples

The performance and efficiency of proposed model have been demonstrated through six illustrative examples from the relevant literature. All examples have been coded by ILOG.OPL.CPLEX.6.3 software, on a portable computer (2 GB RAM and 2 GHz CPU Intel Core 2 Duo). To reveal the quality of resultant layout, the first example is more elaborated and is completely described.

Table 1. Adjacent value for example 1.

(i,j)	f_{ij}	(i,j)	f_{ij}
(1, 2)	342	(5, 8)	259
(1, 5)	322	(5, 9)	532
(2, 3)	838	(5, 10)	680
(2, 5)	220	(5, 11)	438
(2, 11)	461	(6, 7)	474
(3, 4)	829	(8, 9)	27
(4, 5)	461	(9, 10)	231
(5, 6)	439	(10, 11)	658

Table 2. Departments dimension and optimal solution for example 1.

	Dimension		Optimal Location		
Departments	l_i	d_i	x_i	y_i	Assigned floor
1	1.8	1.7	1.1	0.85	1
2	1.9	1.8	2.95	1.4	1
3	2.0	1.6	1.2	3.1	1
4	1.7	1.5	0.95	1.75	2
5	1.9	1.8	2.75	1.6	2
6	1.9	1.5	1.05	3.25	2
7	1.9	1.8	1.05	3.1	3
8	2.0	1.9	1.0	1.25	3
9	1.8	1.6	2.9	2.8	3
10	2.0	1.5	3.0	3.25	2
11	1.8	1.7	3.1	3.15	1

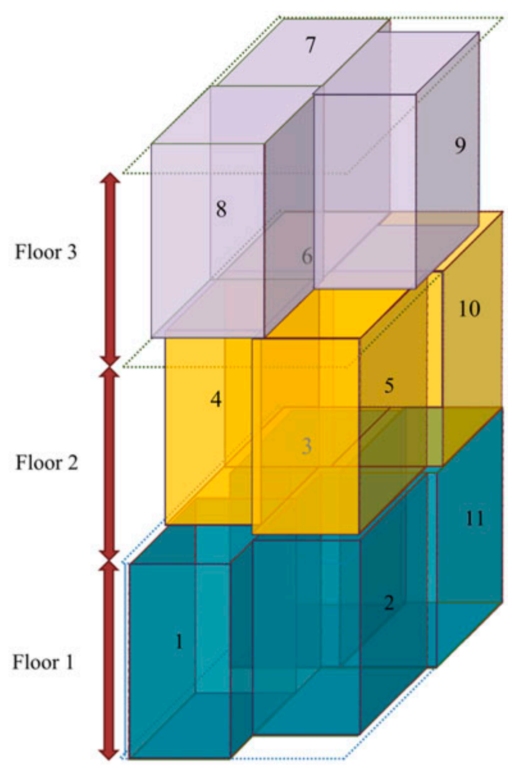

Figure 4. Schematic view of optimal layout for example 1.

5.1 *Example 1*

This example was first introduced by Goetschalckx and Irohara 2007. In this example, there are 11 departments to be arranged in a multi-floor plant with three potential floors. Each floor has a rectangular shape with equal length and width of four units ($H = L = 4$). To adapt the data of this example, the material flow among these 11 departments are considered as adjacent benefit between them and is illustrated in Table 1. This adaptation is realistic as any increase/decrease in flow between two departments corresponds to increase/decrease in the adjacent value between them. Each department has a rectangular shape with the length l_i and width d_i and are depicted in Table 2. In addition, the minimum common boundary length along the X- and Y-axis are the same and are considered to be equal to $S_1 = W_1 = 0.2$. The minimum common surface dimension is also taken to be $S_2 = W_2 = 0.2$.

The optimal centre point coordination and the optimal assignment of floor for each department are summarised in Table 2.

Table 3. Adjacent value for examples 2 through 6.

Example 2		Example 3		Example 4		Example 5		Example 6	
(i,j)	f_{ij}	(i,j)	f_{ij}	(i,j)	f_{ij}	(i,j)	f_{ij}	(i,j)	f_{ij}
(1, 2)	200	(1, 3)	950	(1, 2)	120.0	(1, 3)	150	(1, 2)	175
(1, 5)	200	(2, 4)	380	(1, 4)	12.0	(2, 3)	150	(2, 3)	160
(2, 3)	200	(2, 6)	570	(1, 5)	42.0	(3, 4)	50	(3, 4)	205
(3, 4)	200	(3, 5)	570	(1, 12)	12.0	(3, 5)	300	(3, 5)	15
(4, 5)	200	(3, 7)	190	(2, 3)	195.0	(5, 6)	300	(4, 5)	205
(5, 6)	200	(4, 5)	285	(2, 4)	135.0	(6, 7)	250	(5, 6)	190
(5, 7)	200	(5, 8)	456	(4, 5)	45.0	(6, 13)	50	(6, 7)	190
(6, 7)	200	(5, 9)	304	(4, 6)	135.0	(7, 8)	280	(7, 8)	190
		(6, 7)	456	(4, 12)	27.0	(7, 10)	30	(8, 9)	45
		(7, 10)	285	(6, 7)	135.0	(8, 9)	250	(9, 8)	15
		(7, 11)	285	(7, 8)	165.0	(8, 12)	30	(9, 10)	30
				(7, 11)	13.5	(8, 14)	80	(10, 11)	35
				(8, 9)	90.0	(9, 10)	250	(11, 3)	25
				(8, 11)	24.0	(10, 11)	150	(8, 12)	140
				(9, 10)	90.0	(10, 12)	100	(12, 13)	140
				(10, 11)	24.0	(11, 12)	150	(13, 14)	140
				(11, 12)	36.0	(13, 14)	50	(14, 15)	140
						(14, 8)	20	(15, 16)	140

Table 4. Departments dimension for examples 2 through 6.

	Example 2		Example 3		Example 4		Example 5		Example 6	
Item	l_i	d_i	l_i	d_i	l_i	d_i	l_i	d_i	l_i	d_i
1	5.22	5.22	5.0	3.0	6.0	4.8	3.0	4.5	2.5	2.0
2	11.42	11.42	6.0	5.0	7.2	6.0	2.5	3.5	2.5	3.0
3	7.68	7.68	4.0	6.0	6.0	7.2	4.0	3.0	3.0	4.0
4	8.48	8.48	6.5	5.0	4.8	6.0	3.0	4.5	2.5	2.0
5	7.68	7.68	6.0	3.0	4.8	6.0	2.5	4.0	2.5	2.5
6	2.60	2.60	4.0	5.5	6.0	4.8	4.0	4.0	3.0	3.0
7	2.40	2.40	4.0	5.0	4.8	6.0	4.5	3.5	3.5	3.5
8			5.0	3.0	7.2	4.8	3.5	3.0	3.0	2.0
9			4.0	6.0	4.8	6.0	3.0	4.5	3.5	2.5
10			2.0	1.0	7.2	6.0	3.0	3.0	3.0	3.0
11			3.0	2.0	6.0	4.8	2.0	3.5	2.0	2.0
12					7.2	4.8	4.0	2.0	3.5	1.5
13							3.5	3.0	4.0	1.5
14							2.5	3.5	3.5	3.0
15									3.5	2.5
16									2.0	1.5

Figure 4 shows a schematic view of optimal solution obtained for example 1. Since, the optimal value is 7211, implying that in the final solution all useful adjacencies have been created, due to the fact that the solution obtained is an upper bound for creating the useful adjacencies.

5.2 *Other examples*

Unfortunately a limited number of multi-floor plant layout test problems have been reported in the relevant literature. We selected five more process layout test problems, from the literature, for evaluating the efficiency of the proposed mathematical model. Table 3 through Table 5 summarises extend and the data of these examples. In Table 3, the adjacency values among the departments for each example are presented.

Table 4 contains the length and width of the departments for each example.

Furthermore, the Table 5 is devoted for the further information such as the corresponding reference, scientific name of process, number of departments and etc., related with these examples.

Table 5. Brief explanation of examples 2 through 6.

	Example 2	Example 3	Example 4	Example 5	Example 6
Reference	Papageorgiou and Rotstein (1998)	Georgiadis, Rotstein, and Macchietto (1997)	Jayakumar and Reklaitis (1996)	Meyers (1986)	Meyers (1986)
Process name	Ethylene Oxide	A bath production system	Cosmetic-grade isopropyl alcohol.	Maleic anhydride process	Cis-polybutadiene process
Number of departments	7	11	12	14	16
Number of floors	2	3	3	3	3
$W_1 = S_1$	0.2	0.5	0.1	0.1	0.1
$W_2 = S_2$	0.2	0.5	0.1	0.1	0.1
H	20.0	10.0	30.0	12.0	15.0
L	20.0	10.0	20.0	12.0	10.0

Table 6. Optimal result of each department in examples 2 through 6.

	Example 2			Example 3			Example 4			Example 5			Example 6		
Time (s)		2			153			172			745			846	
Z^*		1600			4731			1300.5			2590			2820	
Item	Optimal			Optimal			Optimal			Optimal			Optimal		
	x_i	y_i	Floor	x_i	y_i	Floor	x_i	y_i	Floor	x_i	y_i	Floor	x_i	y_i	Floor
1	5.27	2.61	1	3.5	8.5	1	15.2	12.8	2	10.50	9.65	1	7.35	3.50	2
2	13.39	8.31	2	3.0	7.5	3	9.7	11.0	3	9.25	6.25	2	5.65	5.70	3
3	3.84	16.16	2	6.0	7.0	2	3.1	16.4	3	7.00	6.00	1	7.50	8.00	2
4	11.72	14.12	1	3.25	2.5	3	15.7	16.9	3	4.90	2.25	1	10.15	5.60	1
5	3.84	6.24	2	7.0	2.5	2	20.5	17.0	3	7.65	2.50	1	10.25	5.65	2
6	8.98	1.30	2	2.0	7.25	2	15.0	17.6	2	4.50	2.50	2	12.80	3.10	1
7	6.48	1.20	2	2.5	2.5	1	20.3	17.0	1	5.75	6.25	2	13.25	6.25	2
8				7.0	1.5	1	26.3	11.7	1	4.85	9.40	1	10.40	3.80	3
9				8.0	6.0	1	27.6	6.4	2	2.00	9.75	2	13.25	1.55	3
10				3.0	4.0	2	21.6	7.4	2	5.00	9.50	2	10.10	2.90	2
11				2.5	1.0	2	21.2	12.8	2	5.60	10.25	3	7.90	5.30	3
12							21.6	17.6	2	8.50	11.0	2	7.25	5.25	2
13										1.75	6.00	2	6.90	3.95	1
14										2.15	6.15	1	4.35	3.00	2
15													5.25	1.55	3
16													7.90	3.55	3

The optimal solution obtained for these examples are shown in Table 6. More specifically, the optimal coordinate of the departments, the optimal assignment of departments to each floor, solution time and optimal value are reported in this table.

As it can be realised, the result reveals the efficiency of the proposed model. Furthermore, the proposed model contemplates computational speed in obtaining the optimal solution. For instance, the optimal solution of Example 6, by the proposed approach, is reached in 14.1 min, while solution of the same example with the objective function of material handling cost, using CPLEX software, obtained in more than 120 min. Clearly, this difference is related with the objective function.

6. Conclusion

In this paper, a new mathematical model for multi-floor FLP has been proposed. For further compatibility with the real world, the departments are considered quadrangle which may be vertically or horizontally adjacent with each other. Consideration of two types of the adjacencies, namely horizontal type and vertical type of the adjacencies, provides a better flexibility in designing of the multi-floor layout especially when one attempts to arrange the departments in a process layout problem. In the proposed model, minimum common boundary length (surface area) between any two horizontal (vertical) adjacent departments is also considered. The objective function of the optimisation problem is set as maximising the number of useful adjacencies among departments. The efficiency of the model is evaluated and demonstrated by six illustrative examples. The results of the computational experience reveal the efficiency of the proposed model.

The proposed model can be helpful for optimal arrangement of departments in multi-floor process plants where the existence of adjacencies between departments is useful or essential due to possible establishment of transferring pipes.

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