



Contents lists available at ScienceDirect

Journal of Manufacturing Systems

journal homepage: www.elsevier.com/locate/jmansys



A new linear adjacency approach for facility layout problem with unequal area departments

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ARTICLE INFO

Article history:

Received 12 February 2015

Received in revised form 6 August 2015

Accepted 15 September 2015

Available online xxx

Keywords:

Generalized version of adjacency

Facility planning of unequal department sizes

Mixed integer mathematical programming

Linear adjacency relation

ABSTRACT

In this study a new version of adjacency, which provides a more flexible layout design, is proposed. In the proposed version, departments which are nonadjacent yet close to each other are considered to be adjacent with a smaller adjacency rating. It is shown that the proposed adjacency is a generalized version of the traditional adjacency. A mathematical programming model is developed for the proposed facility layout problem. To show the flexibility and efficacy of the proposed model, a computational study is conducted. The solution of an illustrative example as well as the solutions of several test problems, reveal flexibility and efficacy of the proposed model.

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1. Introduction

The facility layout problem (FLP) is a combinational optimization problem, dealing with the relative placement and orientation of departments, items, workstations, equipment, etc. The output results can affect the overall efficiency and productivity of plants. This is due to the fact that from 20 to 50 percent of the total operating expenses and from 15 to 70 percent of the total manufacturing costs are attributed to materials handling costs [1]. An effective facility layout can reduce these costs by 10 to 30 percent annually [2]. Minimization of material handling costs is the most popular measure of effectiveness in FLP. Other types of measures such as minimization of rearrangement costs [3], maximization of safety factors [4], and maximization of adjacencies have also been applied.

In this context, the number of trips between pairs of departments and the distance between them are actually used for deciding on the desirability of locating a department next to each other. Therefore the measure of effectiveness, for finding a proper layout design, can be defined as a function of the closeness ratings and the distances. A problem that may arise when designer employs the department rating is unrealistic shapes of the departments, such as T-shaped, L-shaped, U-shaped or narrow rectangle shaped,

in the final layout design. Due to unrealistic department shapes, there is an essential need for exploring the problems of maximizing the adjacencies for rectangle departments shape. Furthermore, thorough investigation of the relevant literatures reveals that the consideration of the adjacency issue and the minimum common boundary length, contemplated in the same mathematical model, has often been neglected. Due to high applicability of the adjacency criterion, especially in the process layout, this paper is focused on this type of criterion.

In this article, we propose a new definition for the adjacencies and present a mathematical model for maximizing a revised version of the adjacencies among rectangular departments. It is assumed that all the departments in the model have the same need/demand to be close to one another except their tendency defined by the adjacency ratings. The proposed mathematical model presents a more realistic representation of the problem when compared to the more traditional version. By the proposed adjacency concept, departments which are nonadjacent yet close to each other are considered to be adjacent with a smaller adjacency rating. More precisely, we consider two departments as adjacent to each other with the full adjacency rating (i.e., a rating of 1) if they share a common boundary. Otherwise we consider two departments are adjacent with the smaller rating of less than one, if they are close to each other within a pre-specified distance, r^* . We therefore proposed an adjacency rating which continuously varies between 0 and 1 according to the distance from each other. A more precise definition of the proposed adjacency will be described in the later sections of this paper.

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Apart from the foregoing introductory notes, this paper is structured as follows. Section 2 presents the related works as the literatures review. Section 3 describes the new adjacency concept which is used in the proposed model. Section 4 presents the development of the mathematical model. Section 5 dedicated to the computational experiments. Finally, in Section 6 the conclusion of this research will be presented.

2. Literature review

The facility layout problem (FLP) is an NP-hard combinatorial optimization problem [5], and the exact solution of the problem is complex and highly time consuming. Therefore, exact approaches cannot solve FLPs with more than 20 facilities in reasonable time [6]. Due to this obstacle, many approximate approaches such as genetic algorithm [7], simulated annealing [8], ant colony [9], tabu search [10], particle swarm optimization [11], and other heuristic algorithms [12] have been proposed. Many FLP exact approaches have focused on a single objective [13], either quantitative or qualitative for the design of the facility layout.

A thorough review of the related literatures reveal that most of exact approaches are devoted to formulation of the FLP in the form of linear or nonlinear mixed integer programming models. Among them is the exact mixed integer programming (MIP) formulation of Phuaksaman [14]. The model therein uses a distance based objective but is not based on the discrete representation as in the quadratic assignment problem. Camp et al. [15] proposed a nonlinear programming (NLP) technique for a FLP with unequal-size departments to minimize the material handling cost. They presented a computational study which its results showed their solutions are comparable to, and in many cases better than, those of other algorithms.

Tate and Smith [16] implemented a genetic search for unequal-size department's layout and showed how optimal solutions are affected by constraints on permitted department shapes, as specified by a maximum allowable aspect ratio for each department. Meller et al. [17] proposed a modified mixed integer programming for a FLP in which perimeter constraint were improved. Sheralli et al. [18] presented an improved MIP model of Meller et al. [17] and proposed effective solution strategies for the facility layout problem. They also developed several modeling and algorithmic enhancements that are demonstrated to produce more accurate solutions while also decreasing the solution effort required. Anjos et al. [19] considered one-dimensional facility layout problem, which consists in finding an optimal placement of facilities on a straight line. They suggest a heuristic procedure which extracts a feasible solution to the one-dimensional space allocation problem.

Hale et al. [20] developed a model for the FLP for unequal department sizes. Essentially, their model divided each department into several sub-departments based upon its relative size. However, their model could not ensure department shape or contiguity. Xiao et al. [21] also investigated the FLP for unequal department sizes. Their research involved a two-step approach in which good models are improved upon via simulated annealing.

Anjos and Vannelli [22] presented a new framework for the FLP by combining two new mathematical-programming models for efficiently finding competitive solutions for the facility-layout problem. One of these models was a relaxation of the layout problem and was intended to find good starting points for the iterative algorithm used to solve the other model. The second model was an exact formulation of the facility-layout problem as a non-convex mathematical program with equilibrium constraints. They showed that layouts with relatively little computational effort can be obtained.

Bhowmik [23] presented an iterative heuristic algorithm and branch and bound algorithm for a FLP with locating set of departments, each requiring equal area on a given site. They employed the adjacency matrix as the number of trips between pairs of departments. Through computational study they concluded that, the processing time is very high for large datasets and departments tend to split between levels.

Kaveh et al. [24] proposed a strategy for using harmony search algorithm in facility layout optimization problems. They employed two real-world cases to demonstrate the efficiency of their model and claimed that their method can find an optimal facility arrangement in an existing layout. Deechongkit and Srinon [25] developed a mixed integer programming for solving facility layout problem. Their formulation tested problems from the literature to minimize material handling cost. They employed departments and shop floors and tested problems with fixed rectangular dimensions. They claimed that the classic mixed integer programming may provide solution with 5–10% less area than initial layout design requirements which can be considered as an infeasible solution. Therefore the main purpose for proposing a new MIP was to eliminate such infeasibility and to show the efficiency of this model. Other optimization approaches for solving the FLP includes dynamic programming [26], quadratic assignment [27], nonlinear programming [15], graph theoretic [28], and linear mixed integer programming [29–31]. There are also a large number of studies considering single row facility layout problems [32–34]. For assessing the structural complexity of manufacturing systems layout in the physical domain refer to [35].

3. New definition for adjacencies

Traditionally, two departments i and j are considered as adjacent departments when they simultaneously have two conditions:

- (I) Two departments i and j are attached if they are side by side.
- (II) All attached departments share a common boundary length.

In relevant literature, the concept of adjacencies among departments has been focused as a binary matter. This type of the adjacency is very strict and imposes some inflexibility in construction of layout plans.

In this study, a revised version of adjacency is proposed. In the proposed version, two departments i and j are adjacent if they are located within a predetermined distance from each other. We propose a new continuous variable, called adjacency degree, which measures the adjacency degree between any pairs of departments. The adjacency degree is a bounded variable on the interval of $[0,1]$. The distance between two departments i and j and their associate adjacency degree are inversely correlated. As the distance between two departments i and j increases, the amount of adjacency between them proportionally decreases. The adjacency degree between two departments i and j along X-axis or Y-axis, is zero if distance between two departments i and j along X-axis or Y-axis, is more than a pre-specified value of r^* , where r^* is defined as the maximum allowable distance between i and j to be considered as adjacent. Therefore, the adjacency degree decreases linearly from one to zero, when the Tchebychev distance between the two departments varies from zero to r^* .

Fig. 1 illustrates the greatness of the adjacency degree by the intensity of the shaded area. The darkest part indicates the adjacency degree of one and the lightest shows the adjacency degree of close to zero.

It is noted that adjacency degrees can be defined differently along X-axis and Y-axis. In other words, the value of r^* along X-axis or Y-axis can be different. For simplicity, we consider the same

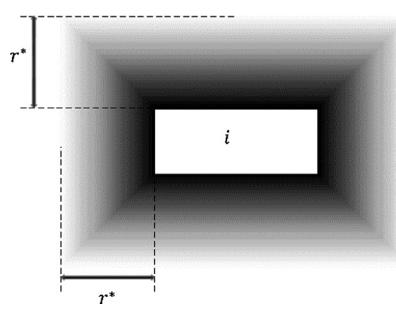


Fig. 1. An illustrative representation of the degree of adjacency.

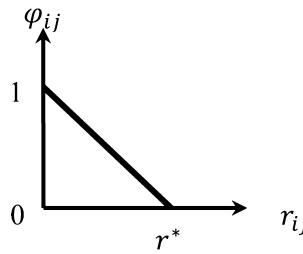


Fig. 2. Curvature relationship of adjacency degree and distance.

value for both the X-axis and the Y-axis. For a better illustration, the graphical functional relation for variation of the proposed adjacency degree versus distances is depicted in Fig. 2. In this figure φ_{ij} indicates the adjacency degree and $r_{ij} = \text{Max}(r_{ij}^X, r_{ij}^Y)$ indicates the Chebychev distance between the two departments.

It should be noted that the distance between two departments is measured by the length of an imaginary straight line connecting the nearest boundary points of two departments in a perpendicular manner. As an illustrative presentation of distances between two departments along X-axis or Y-axis refer to Fig. 3(1) and (2), respectively.

The proposed version of adjacency is a generalized version of the traditional adjacency. This from the fact that by assigning zero to the minimum allowable distance ($r^* = 0$), the traditional adjacency is obtained.

In addition, the proposed adjacency phenomenon is more helpful for the optimal arrangement of departments in the process layout problems where the existence of adjacency between departments is useful/essential due to possible establishment of conveyor, transferring pipe, lift truck route, elevator, etc. This straight transferring route may provide a more efficient material handling system. Consider the case of pipelines for transfer of material. To connect two non-adjacency departments, one (or more) pipe bend is needed. Unfortunately using pipe bends in transferring rout can create serious problems such as loosing heat and pressure changes in the outer wall of the bends. Therefore designers may prefer to have a layout scheme with the minimum use of the pipe bends. This can be satisfied by maximizing the number of the adjacencies among the departments through which the pair of adjacent departments are connected by a straight pipe [30].

4. Development of the mathematical model

In the proposed facility layout model, it is assumed that all departments are rectangular shaped and all the parameters are deterministic. By these assumptions, the nomenclatures of the variables and parameters for the proposed FLP are defined as follows:

Parameters	
n	Number of departments
l_i	Length of department i in the X direction
d_i	Width of department i in the Y direction
H, L	Dimensions of the land, length and width, respectively
f_{ij}	Adjacency value of i and j
s_1	Minimum common boundary length along Y-axis
w_1	Minimum common boundary length along X-axis
r^*	Maximum allowable distance for adjacency
M	A large number
Variables	
Continuous variables	
(x_i, y_i)	Coordinates of the geometrical center of department i
x_{ij}^U	Relative distance between i and j along X-axis, if is placed to the right of j , e.g., $x_{ij}^U = 0$ if i is placed to the left of j
x_{ij}^L	Relative distance between i and j along X-axis, if i is placed to the left of j , e.g., $x_{ij}^L = 0$ if i is placed to the right of j
y_{ij}^U	Relative distance between i and j along Y-axis, if i is placed above j , e.g., $y_{ij}^U = 0$ if i is placed to the below of j
y_{ij}^L	Relative distance between i and j along Y-axis, if i is placed below j , e.g., $y_{ij}^L = 0$ if i is placed to the above of j
r_{ij}^X	Nearest distance between boundaries of two departments i and j along X-axis
r_{ij}^Y	Nearest distance between boundaries of two departments i and j along Y-axis
r_{ij}	Distance between boundaries of two departments i and j
φ_{ij}	Adjacency degree between i and j
$\varphi_{ij}^X, \varphi_{ij}^Y$	Two variables for calculating the adjacency degree
Binary variables	
E_{1ij}, E_{2ij}	Two variables used in non-overlapping constraints
N_{ij}^X, N_{ij}^Y	Variables used for ensuring the adjacency of departments i and j
$\rho_{ij}^X, \rho_{ij}^Y, \rho_{ij}$	Variables used for satisfying the minimum common boundary length between i and j

4.1. Objective function

Maximization of the total adjacency ratings among departments is considered as the objective function. The objective function can be written as the following equation:

$$\text{MaxZ} = \sum_{i=1}^{n-1} \sum_{j=i+1}^n f_{ij} \times \varphi_{ij} \quad (1)$$

Since creating the adjacencies between two departments is a desirable criterion, the maximization of the objective function is selected.

4.2. Non-overlapping constraints

In order to avoid situations where two departments i and j overlap along either the X-axis or the Y-axis, four disjunctive constraints are added to the mathematical model. If any one of the four constraints, shown by Eqs. (2)–(5), is satisfied, we can ensure that no overlapping has occurred.

$$x_i - x_j + M (E_{1ij} + E_{2ij}) \geq \frac{1}{2} (l_i + l_j) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (2)$$

$$x_j - x_i + M (1 - E_{1ij} + E_{2ij}) \geq \frac{1}{2} (l_i + l_j) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (3)$$

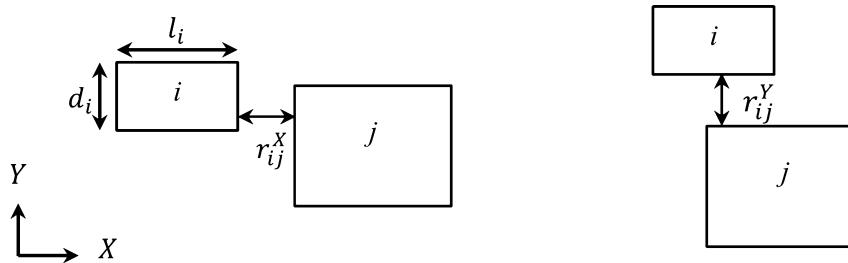


Fig. 3. The allowable distance between two departments along X-axis (1) and Y-axis (2).

$$y_i - y_j + M(1 + E1_{ij} - E2_{ij}) \geq \frac{1}{2}(d_i + d_j) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (4)$$

$$y_j - y_i + M(2 - E1_{ij} - E2_{ij}) \geq \frac{1}{2}(d_i + d_j) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (5)$$

In these relations, $E2_{ij}$, $E1_{ij}$ are binary variables and are acting as follows:

- (1) Relation (2) becomes active when $E1_{ij} = E2_{ij} = 0$, which implies that department i is placed on the right side of department j .
- (2) Relation (3) becomes active when $E1_{ij} = 1$, $E2_{ij} = 0$, which implies that department i is placed on the left side of department j .
- (3) Relation (4) becomes active when $E1_{ij} = 0$, and $E2_{ij} = 1$, which implies that department i is placed above the department j .
- (4) Relation (5) becomes active when $E1_{ij} = E2_{ij} = 1$, which implies that department i is placed below the department j .

In this paper the orientation of the departments are considered to be fixed. However, for a non-fixed orientation design, a new binary variable δ_{ij} can be defined so that l_i and d_i are, respectively, replaced by l_{oi} and d_{oi} in which $l_{oi} = l_i\delta_{ij} + d_i(1 - \delta_{ij})$ and $d_{oi} = l_i(1 - \delta_{ij}) + d_i\delta_{ij}$.

4.3. Land dimension constraints

Constraints (6)–(9) are defined for ensuring that all departments must be located in a rectangular area between the origin and Cartesian coordinate (H, L).

$$x_i - \frac{1}{2}l_i \geq 0 \quad \forall i = 1, 2, \dots, n \quad (6)$$

$$x_i + \frac{1}{2}l_i \leq H \quad \forall i = 1, 2, \dots, n \quad (7)$$

$$y_i - \frac{1}{2}d_i \geq 0 \quad \forall i = 1, 2, \dots, n \quad (8)$$

$$y_i + \frac{1}{2}d_i \leq L \quad \forall i = 1, 2, \dots, n \quad (9)$$

4.4. Distance between two departments

In this work, the distance between two departments is measured by rectilinear distances. Therefore, the distance along with X-axis and Y-axis can be presented by $(x_{ij}^U + x_{ij}^L)$ and $(y_{ij}^U + y_{ij}^L)$, respectively. Therefore the associated constraints are defined by the following equation.

$$|x_i - x_j| = x_{ij}^U + x_{ij}^L \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (10)$$

$$|y_i - y_j| = y_{ij}^U + y_{ij}^L \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (11)$$

The value of x_{ij}^U , x_{ij}^L , y_{ij}^U , y_{ij}^L can be obtained by the following constraints:

$$x_i - x_j = x_{ij}^U - x_{ij}^L \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (12)$$

$$y_i - y_j = y_{ij}^U - y_{ij}^L \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (13)$$

Two variables x_{ij}^U and x_{ij}^L are linearly dependent variables and therefore they cannot take a nonzero value simultaneously. The same logic can be used for y_{ij}^U and y_{ij}^L .

4.5. Maximum adjacent radius area

If two departments i and j are relatively placed in an enclosed area, they can be adjacent with each other. For this situation Eqs. (14) and (15) should be added to model.

$$x_{ij}^U + x_{ij}^L \leq \frac{1}{2}(l_i + l_j) + M(1 - N_{ij}^X) + r^* \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (14)$$

$$y_{ij}^U + y_{ij}^L \leq \frac{1}{2}(d_i + d_j) + M(1 - N_{ij}^Y) + r^* \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (15)$$

In Eq. (15), if $N_{ij}^Y = 1$, then the position of department j in relation to department i can be shown as in Fig. 4. In other words, if N_{ij}^Y equal 1, then the center point of department j can be located anywhere along two parallel horizontal lines of δ' and δ'' . In addition, the distance between two parallel lines is equivalent to $(d_i + d_j + 2r^*)$.

A similar expression can also be applied to N_{ij}^X . So, if $N_{ij}^X = 1$, then the center point of department j lies in the middle of two vertical parallel lines. Referring to Fig. 5 it can be seen that, if $N_{ij}^X = N_{ij}^Y = 1$, then the center points of department j can be located anywhere along two parallel horizontal lines of δ' and δ'' , and two vertical lines of ν' and ν'' . It implies that the distance between two departments i and j is less than the maximum adjacency radius (i.e., r^*).

Consequently, for the adjacencies of two departments, it is necessary to have N_{ij}^X and N_{ij}^Y to be simultaneously equal to 1. This is modeled by constraints (16) and (17).

$$1.5N_{ij} - N_{ij}^X - N_{ij}^Y \leq 0 \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (16)$$

$$N_{ij} - N_{ij}^X - N_{ij}^Y + 1.5 \geq 0 \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (17)$$

From relation (16) and (17) when $N_{ij}^X = N_{ij}^Y = 1$, we will have $1.5N_{ij} - 2 \leq 0$, and $N_{ij} - 2 + 1.5 \geq 0$. Since N_{ij} is a binary variable the

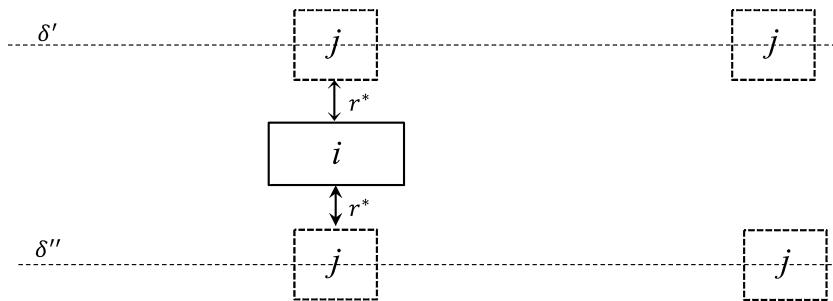


Fig. 4. The allowable allocation of center point of a department along X-axis.

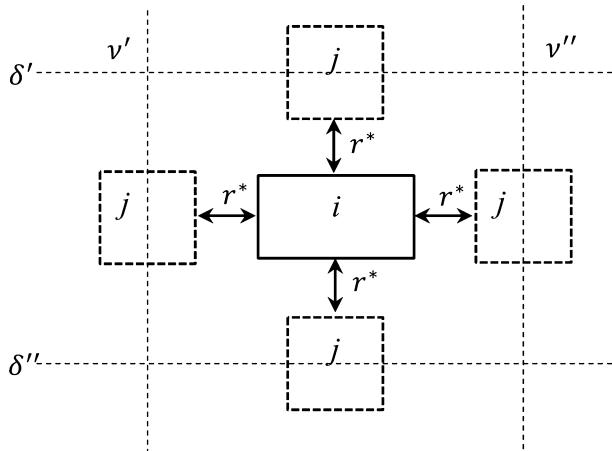


Fig. 5. The allowable allocation of center points of a department along X-axis and Y-axis.

only way for satisfying (17) is $N_{ij} = 1$, while relation (16) is automatically satisfied. In the other hand, we want to force N_{ij} to become zero when one or both of N_{ij}^X and N_{ij}^Y become zero. This will be accomplished by (16). In this relation if one or both of N_{ij}^X and N_{ij}^Y become zero, we will have either $1.5N_{ij} - 1 \leq 0$ or $1.5N_{ij} \leq 0$. The only way for satisfying (16) is $N_{ij} = 0$, while relation (17) is automatically satisfied.

4.6. Common boundary length

We consider, two departments i and j as adjacent, when their common boundary length along X-axis and Y-axis is greater than w_1 and s_1 , respectively. This is formulated by constraint (18) and (19) as follows:

$$|x_i - x_j| \leq \frac{1}{2} (l_i + l_j) - w_1 + M (1 - \rho_{ij}^X) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (18)$$

$$|y_i - y_j| \leq \frac{1}{2} (d_i + d_j) - s_1 + M (1 - \rho_{ij}^Y) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (19)$$

The constraints numbered by (18) and (19) can be converted to relations (20)–(23).

$$x_i - x_j \leq \frac{1}{2} (l_i + l_j) - w_1 + M (1 - \rho_{ij}^X) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (20)$$

$$x_i - x_j \geq M (\rho_{ij}^X - 1) + w_1 - \frac{1}{2} (l_i + l_j) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (21)$$

$$y_i - y_j \leq \frac{1}{2} (d_i + d_j) - s_1 + M (1 - \rho_{ij}^Y) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (22)$$

$$y_i - y_j \leq M (\rho_{ij}^Y - 1) + s_1 - \frac{1}{2} (d_i + d_j) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (23)$$

Comparing the non-overlapping and common boundary length constrains, it can be concluded that the two binary variables ρ_{ij}^X and ρ_{ij}^Y cannot simultaneously be equal to 1. Consequently, one of two binary variables ρ_{ij}^X and ρ_{ij}^Y has to be zero. However, for identifying the adjacency of two departments i and j , Eq. (24) should be added to the model.

$$\rho_{ij} = \rho_{ij}^X + \rho_{ij}^Y \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (24)$$

According to this model, if $\rho_{ij} = 1$, then the common boundary length between two departments i and j is satisfied at least along either X-axis or Y-axis.

4.7. Determination of the adjacency degree value

As stated in Section 2, if the distance between two departments i and j (i.e., r_{ij}) is less than the maximum allowable adjacent radius, then the adjacency degree between them takes on a value on the range [0,1]. Therefore relations (25) and (26) have to be defined.

$$r_{ij} \leq r^* : 0 \leq \varphi_{ij} \leq 1 \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (25)$$

$$r_{ij} > r^* : \varphi_{ij} = 0 \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (26)$$

Now, it should be determined if the distance between any two departments i and j , lies in the limited boundary range $[0, r^*]$. For this purpose, the indicating variable N_{ij} is sufficient. Subsequently the adjacent value is achievable by interpolating. Thus we will have relations (27) and (28).

$$r_{ij}^X \leq r^* : \frac{0 - \varphi_{ij}^X}{r^* - r_{ij}^X} = \frac{\varphi_{ij}^X - 1}{r_{ij}^X} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (27)$$

$$r_{ij}^Y \leq r^* : \frac{0 - \varphi_{ij}^Y}{r^* - r_{ij}^Y} = \frac{\varphi_{ij}^Y - 1}{r_{ij}^Y} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (28)$$

Since the objective function is in the form of maximization, terms (27) and (28) are equivalent to terms (29) and (30). Note that

Table 1

Adjacency value between departments in illustrative example.

(i,j)	f_{ij}	(i,j)	f_{ij}
(1,2)	10	(2,4)	3
(1,3)	8	(2,5)	12
(1,4)	6	(3,4)	5
(1,5)	8	(3,5)	4
(2,3)	7	(4,5)	9

as shown in Fig. 3, r_{ij}^X and r_{ij}^Y are equal to $\left(x_{ij}^U + x_{ij}^L\right) - 1/2(l_i + l_j)$ and $\left(y_{ij}^U + y_{ij}^L\right) - 1/2(d_i + d_j)$, respectively.

$$r^* \times \varphi_{ij}^X \leq \frac{1}{2}(l_i + l_j) + r^* - (x_{ij}^U + x_{ij}^L) + M(1 - N_{ij}^X) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (29)$$

$$r^* \times \varphi_{ij}^Y \leq \frac{1}{2}(d_i + d_j) + r^* - (y_{ij}^U + y_{ij}^L) + M(1 - N_{ij}^Y) \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (30)$$

4.8. Logical constraints

Considering parameter's definition, constraints (31)–(38) are inevitable.

$$\varphi_{ij}^X \leq N_{ij} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (31)$$

$$\varphi_{ij}^Y \leq N_{ij} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (32)$$

$$N_{ij}^X \leq \rho_{ij} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (33)$$

$$N_{ij}^Y \leq \rho_{ij} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (34)$$

$$\varphi_{ij} \leq \varphi_{ij}^X \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (35)$$

$$\varphi_{ij} \leq \varphi_{ij}^Y \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (36)$$

$$\varphi_{ij} = \varphi_{ji} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (37)$$

$$N_{ij} = N_{ji} \quad \forall i, j = 1, 2, \dots, n, \quad i \neq j \quad (38)$$

5. Computational experiment

Computational experiment consists of three sections. Section 5.1 presents an illustrative example and Section 5.2 describes the important aspects of the proposed model. Finally Section 5.3 evaluates the efficiency of the proposed model, by solving eight test problems selected from literatures. All the test problems were modeled using ILOG.OPL.CPLEX.6.3 software, and run on an Intel® Core 2 Duo 2 GHz portable PC with 2GB RAM.

5.1. An illustrative example

This example is selected from the study of Neghab et al. [36] and modified for the proposed model. This example involves 5 departments that should be arranged in the unrestricted land area. In the original example the length and width of departments are not deterministic and are considered as boundary intervals ($l_i \in [l'_i, l''_i]$ and $d_i \in [d'_i, d''_i]$). To adapt this data, it is assumed that, the dimensions of department are equal to the average of the maximum and minimum length and width (i.e., $l_i = (l'_i + l''_i)/2$ and $d_i = (d'_i + d''_i)/2$). Furthermore, the material flow matrix is substituted by the adjacency value matrix and is represented in Table 1. Moreover, the minimum common boundary length along X-axis

Table 2

Department sizes and optimal center points for illustrative example.

Department	Department sizes		Optimal center point	
	l	d_i	x_i	y_i
1	6.0	10	16.00	7.00
2	12.0	8	6.00	15.00
3	9.0	11	16.50	17.50
4	5.5	6	10.25	3.00
5	7.5	5	9.25	8.50

Table 3

Created adjacency matrix of illustrative example.

5	4	3	2	NB _{ij}
1	1	1	0.8	1
1	0	1	–	2
0.8	0	–	–	3
1	–	–	–	4

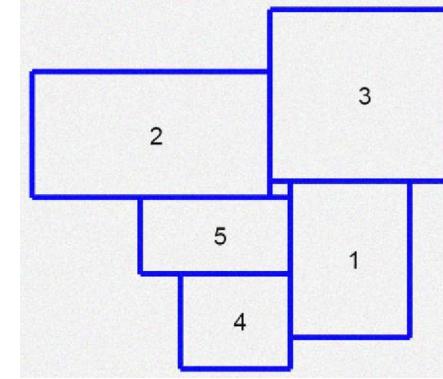


Fig. 6. Optimal solution of illustrative example.

and X-axis are assumed to be the same and equal to $S_1 = W_1 = 1$. The maximum adjacency radius is also set to be 5 (i.e., $r^* = 5$).

After solving this example, the objective function value of 61.2 is obtained. The optimal center point of each department and the created adjacency matrix are tabulated in Tables 2 and 3, respectively.

The optimal layout is shown in Fig. 6.

The enclosed space among four departments 1, 2, 3, and 5 can have very useful application. This and other aspects of the proposed model, with regard of the solution of this example, will be discussed in Section 5.2.

5.2. Important aspects of the proposed model

As stated in Section 2, in this paper a new linear concept for adjacencies has been developed. The new definition often leads to a more flexible layout plan. The appropriate value of maximum adjacency radius (r^*) influences the quality of solutions significantly. By assigning a large value to r^* , the adjacency rating tends to 1 regardless of the distance between two departments. In other words,

$$r^* \rightarrow +\infty \text{ then } \begin{cases} \varphi_{ij} = 1 \\ \text{or} \\ \varphi_{ij} \rightarrow 1 \end{cases} \quad \forall i, j \in 1, 2, \dots, n \quad (39)$$

Although the larger value for r^* provides a more flexibility for creating adjacency, but it may create a problem by which one or more departments to be placed between the two adjacent departments. For example, consider four rectangular departments i, j, k and l ; suppose the value of the adjacencies between any pair of

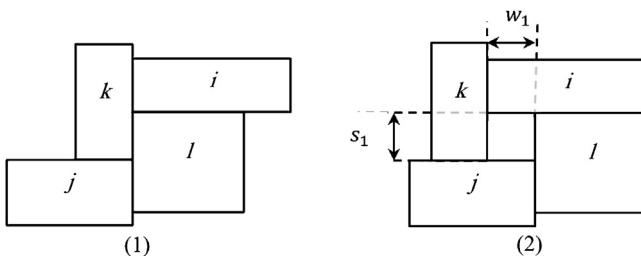


Fig. 7. The adjacency degrees obtained by solution of the proposed model.

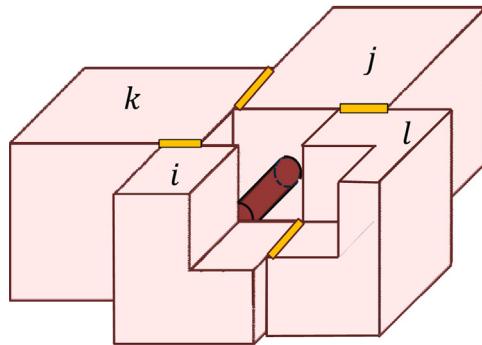


Fig. 8. The possible connection phenomenon according to the proposed adjacency

departments is equal. Using the traditional definition of adjacency, at least one of six possible adjacencies will be overlooked. The determination of an adequate value for r^* can lead to the model which creates more adjacencies with the lower adjacency degree while traditional adjacency ignores some of them. These facts are illustrated in Fig. 7(1) and (2). As it can be realized, in Fig. 7(1), there are five adjacencies with degree 1 while in Fig. 7(2) there are four adjacencies with degree 1 and two adjacencies with a degree less than 1.

Fig. 8 depicts the possible connection phenomenon which can be achieved by the new proposed adjacency. As it is shown, two departments i and j or two departments k and l can be considered as adjacent due to the possibility of being connected by a straight pipe.

Therefore, if designer determines a proper value of the maximum adjacency radius a better total adjacency can be obtained. It would be appropriate to consider the two following points in determination of r^* .

- (1) It is more appropriate if the designers determine the value of (r^*) such that the model preferably create more numbers of adjacencies with low degree versus losing some. Therefore, regarding the equality of all adjacency values, relations (40)–(42) are included.

$$\varphi_{ij} + \varphi_{kl} \geq 1 \quad (40)$$

Suppose $LM = \max\{s_1, w_1\}$

$$\varphi_{ij} + \varphi_{kl} \geq 1 : \varphi_{ij} + \varphi_{kl} \geq \underbrace{\varphi_{ij}}_{\text{LM}} + \underbrace{\varphi_{kl}}_{\text{LM}} \geq 1 \quad (41)$$

$$\underbrace{\varphi_{ij}}_{\text{LM}} + \underbrace{\varphi_{kl}}_{\text{LM}} = 2 \times \underbrace{\varphi_{ij}}_{\text{LNM}} \geq 1 : r^* \geq 2 \text{LM} : r^* \geq 2 \times \max \{s_1, w_1\}$$
(42)

The stated point in relation (42) is an optional matter for designers, and it is not necessary to be satisfied. Even if it is

assumed to be equal to zero, the classical definition for adjacencies will emerge.

- (2) For assigning a value to the maximum adjacency radius, it is logical to consider a value which is less than the size of each department. Its significance becomes more clear when two departments i and j are adjacent and no one lies within the shared space between i and j . In other words, placing any department between the shared spaces of two adjacent departments is not appropriate. This hint can be mathematically presented as follows Eq. (43)

$$r^* \leq \min \{l_i, d_i\} \quad \forall i = 1, 2, \dots, n \quad (43)$$

5.3. Test problems

To evaluate the performance and efficiency of the proposed model, eight test problems are selected from literatures and are solved by the proposed model and the traditional method. Before presenting the results of the computational experiment, we proposed **Theorem 1**. As it will be seen later the computational results are well coherent with this theorem.

Theorem 1. The optimal solution of the traditional models with the objective function of maximizing the total adjacency is a lower bound for the optimal solution of proposed model.

Proof. Let us assume A as the optimal solution of a traditional model with objective function value of Z_A^T . Then any pair of departments in A is considered to be adjacent when they simultaneously satisfy the following two conditions:

- (1) Two departments i and j are attached side by side either along X -axis or Y -axis.
 - (2) All attached departments share a minimum common boundary length of w_1 along X -axis or s_1 along Y -axis.

Since in the proposed model, two departments are also considered to be adjacent with the adjacency degree of 1, when the same two conditions are hold. In other hand, A is a feasible solution for the proposed model. Consequently, these results imply that the objective function of the proposed model is greater than or equal to Z_A^T . \square

To adapt the scope of test problems, the material flow between departments are substituted by adjacency values. These values and the other related data are presented in [Table 4](#). Furthermore, [Table 5](#) contains the length and width of the departments for each test problem.

Table 6 shows the optimal center coordinate points of the departments for the proposed model (P-model) versus the traditional model (T-model).

Finally the solutions obtained by the proposed model are compared with the solutions of the traditional model and their results are presented in [Table 7](#). In this table, the corresponding reference of selected test problem is listed in second column. In third column, the number of departments is presented. In columns 4, the appropriate value of $s_1 = w_1$ is displayed.

The upper bounds of the optimal solutions are calculated through assigning the value of one to the adjacency rating while letting $f_{ij} > 0$. These upper bounds are calculated by summation of all the adjacency values. For example, referring the adjacency values of [Table 1](#), the upper bound of this problem assuming the adjacency rating of 1 for each pairs of departments which $f_{ij} > 0$, can be calculated as follow:

$$\text{Upper bound} = 10 + 8 + 6 + 8 + 7 + 3 + 12 + 5 + 4 + 9 \\ = 72 > \text{optimal value} = 61.2$$

Table 4

Adjacency values of test problems.

Jayakumar and Rekiatis [37]		Penteado and Ceric [38]		Gonzales and Realff [39]		Georgiadis et al. [40]		Ozyurt and Realff [41]		Khare et al. [42]		Meyers [43]		Meyers [43]	
(i, j)	f_{ij}	(i, j)	f_{ij}	(i, j)	f_{ij}	(i, j)	f_{ij}	(i, j)	f_{ij}	(i, j)	f_{ij}	(i, j)	f_{ij}	(i, j)	f_{ij}
(1,2)	2525	(1,2)	400	(1, 4)	24	(1, 3)	950	(1, 2)	1	(1, 2)	1965	(1, 3)	150	(1, 2)	175
(1,3)	3783	(1,5)	100	(1, 6)	6	(2, 4)	380	(2, 3)	10	(1, 4)	100	(2, 3)	150	(2, 3)	160
(2,3)	631	(2,3)	400	(1, 7)	15	(2, 6)	570	(2, 7)	1	(1, 5)	400	(3, 4)	50	(3, 4)	205
(2,4)	1879	(3,4)	300	(2, 3)	25	(3, 5)	570	(3, 4)	10	(1, 12)	100	(3, 5)	300	(3, 5)	15
(3,5)	1420	(4,5)	300	(2, 5)	6	(3, 7)	190	(3, 5)	10	(2, 3)	3930	(5, 6)	300	(4, 5)	205
		(5,6)	200	(2, 7)	15	(4, 5)	285	(3, 6)	20	(2, 4)	1965	(6, 7)	250	(5, 6)	190
		(5,7)	150	(3, 5)	25	(5, 8)	456	(3, 11)	5	(4, 5)	500	(6, 13)	50	(6, 7)	190
		(6,7)	150	(4, 5)	20	(5, 9)	304	(6, 10)	20	(4, 6)	1565	(7, 8)	280	(7, 8)	190
				(4, 6)	24	(6, 7)	456	(7, 8)	1	(4, 12)	200	(7, 10)	30	(8, 9)	45
				(5, 7)	15	(7, 10)	285	(7, 9)	1	(6, 7)	1565	(8, 9)	250	(9, 8)	15
				(6, 8)	15	(7, 11)	285	(7, 11)	5	(7, 8)	1450	(8, 12)	30	(9, 10)	30
						(10, 11)		5	(7, 11)	100	(8, 14)	50	(10, 11)	35	
									(8, 9)	1250	(9, 10)	250	(11, 3)	25	
									(8, 11)	200	(10, 11)	150	(8, 12)	140	
									(9, 10)	1000	(10, 12)	100	(12, 13)	140	
									(10, 11)	200	(11, 12)	150	(13, 14)	140	
									(11, 12)	300	(13, 14)	50	(14, 15)	140	
												(15, 16)	140		

Using the same approach, the upper bounds of test problems are calculated as presented in column 5. In columns 6 through 8, the total adjacency value (Z^*), the material handling cost and the execution time for the traditional model are tabulated, respectively. Similar results obtained by the proposed model are also presented in columns 10 through 12. The proper value of r^* , ($r^* = \min(l_i, w_i) \quad \forall i = 1, 2, \dots, n$) for the proposed model is also presented in column 9.

As it can be realized, the total adjacency values in all the test problems, obtained by the proposed model, are greater than or equal to the traditional model. In this table the material handling cost of the test problems are also compared. Although the material handling cost are not considered as the measure of effectiveness of the proposed model, as it can be recognized however, for all the test problems except test problem II, the costs obtained by the proposed model is lower than those obtained by the traditional model.

Referring to the values of the execution times, it can be realized that the optimal solutions for medium size test problems are obtained in a reasonable time. However, the larger execution times of proposed model for obtaining the solution of some of the test problems, is probably due to the more number of variables defined in the mathematical model. These extra variables are deliberately implemented in the mathematical model to enhance its flexibility and its efficacy by enabling it for enumeration of a more layout design alternatives during its iterative computational efforts.

The proposed FLP is applicable in the process layout such as chemical, petrochemical, cosmetics and food manufacturing plants. As it can be seen in this subsection the majority of the test problems are selected from the real-world problems. For example problems (I), (II), (VI)–(VIII) each, respectively, present a coffee, an ethylene oxide, a cosmetic-grade isopropyl alcohol, a maleic anhydride, a cis-polybutadiene manufacturing process [30].

Table 5

Departments dimensions of test problems.

Problem	Test problem	l & d 's	Departments													
			1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	I	l_i	15.80	3.10	15.80	6.30	9.44									
		d_i	3.10	3.10	3.10	6.30	3.10									
2	II	l_i	5.22	11.42	7.68	8.48	7.68	2.60	2.40							
		d_i	5.22	11.42	7.68	8.48	7.68	2.60	2.40							
3	III	l_i	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
		d_i	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
4	IV	l_i	5.00	6.00	4.00	6.50	6.00	4.00	4.00	5.00	4.00	4.00	2.00	3.00		
		d_i	3.00	5.00	6.00	5.00	3.00	5.50	5.00	3.00	6.00	1.00	2.00			
5	V	l_i	5.00	6.00	6.00	5.00	6.00	5.00	5.00	2.00	3.00	6.00	4.50			
		d_i	3.00	6.00	6.00	3.00	6.00	5.00	5.00	1.00	2.00	6.00	4.50			
6	VI	l_i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		d_i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	VII	l_i	3.00	2.50	4.00	3.00	2.50	4.00	4.50	3.50	3.00	2.00	4.00	3.50	2.50	
		d_i	4.50	3.50	3.00	4.50	4.00	4.00	3.50	3.00	4.50	3.00	3.50	2.00	3.00	3.50
8	VIII	l_i	2.50	2.50	3.00	2.50	2.50	3.00	3.50	3.00	3.50	3.00	2.00	3.50	4.00	3.50
		d_i	2.00	3.00	4.00	2.00	2.50	3.00	3.50	2.00	2.50	3.00	2.00	1.50	3.00	2.50

I. Jayakumar and Rekiatis [37].

II. Penteado and Ceric [38].

III. Gonzales and Realff [39].

IV. Georgiadis et al. [40].

V. Ozyurt and Realff [41].

VI. Khare et al. [42].

VII. Meyers [43].

VIII. Meyers [43].

Table 6

The optimal center points of departments for the proposed model (P-model) versus the traditional model (T-model).

Problem	Test problem		Departments																
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	I	T-model	x_i	7.90	15.25	23.70	10.55	24.70											
			y_i	1.55	4.65	1.55	6.25	4.65											
		P-model	x_i	26.70	17.25	17.30	12.55	4.70											
			y_i	6.85	6.85	9.95	3.15	7.85											
2	II	T-model	x_i	18.77	21.87	12.32	4.24	12.32	9.82	12.32									
			y_i	5.80	14.12	14.12	7.04	6.44	1.30	1.40									
		P-model	x_i	10.29	18.61	20.00	11.92	3.84	3.84	1.34									
			y_i	16.20	16.20	6.65	4.08	11.16	16.30	16.20									
3	III	T-model	x_i	18.00	10.00	6.00	14.00	10.00	18.00	14.00	14.00								
			y_i	6.00	2.00	4.00	8.00	6.00	10.00	4.00	12.00								
		P-model	x_i	6.00	2.00	4.00	8.00	6.00	10.00	4.00	14.00								
			y_i	2.00	10.00	14.00	6.00	10.00	2.00	6.00	2.00								
4	IV	T-model	x_i	20.30	3.00	15.80	9.25	15.50	8.00	12.00	21.00	16.00	15.00	8.70					
			y_i	5.70	12.00	10.00	13.50	14.50	8.25	4.50	14.50	19.00	2.50	1.00					
		P-model	x_i	14.00	9.50	14.00	3.25	9.20	14.50	17.00	3.70	9.50	14.00	20.50					
			y_i	1.50	14.50	6.00	14.50	10.50	16.75	11.50	7.70	6.00	11.50	11.00					
5	V	T-model	x_i	2.50	7.80	14.00	19.30	13.80	19.30	8.30	5.00	4.30	13.80	12.85					
			y_i	23.0	18.50	13.50	18.00	19.50	8.00	13.00	10.00	13.50	3.00	8.25					
		P-model	x_i	13.75	13.75	8.50	8.50	3.00	3.50	17.25	13.75	13.25	9.00	13.75					
			y_i	1.50	6.00	12.00	16.50	6.00	17.50	11.50	11.50	13.00	21.50	16.25					
6	VI	T-model	x_i	5.50	5.40	4.40	4.50	5.40	3.50	2.50	1.50	0.50	1.50	2.50	3.50				
			y_i	2.40	1.40	0.50	2.40	3.40	1.50	1.50	2.40	2.50	3.40	2.50	2.50				
		P-model	x_i	1.50	1.50	0.50	2.50	1.60	3.50	3.60	4.60	5.60	4.60	3.60	2.60				
			y_i	2.40	3.40	3.50	2.50	1.40	3.40	2.40	1.50	1.40	0.50	1.40	1.50				
7	VII	T-model	x_i	11.10	14.50	14.50	18.00	11.25	11.00	7.75	4.75	1.50	4.00	1.50	4.50	8.25	7.75		
			y_i	2.25	2.75	6.00	8.25	8.50	12.50	16.25	13.00	12.75	16.50	19.00	9.00	12.25			
		P-model	x_i	5.50	2.75	5.25	8.75	5.25	2.00	6.25	6.85	10.00	10.10	12.60	10.60	2.25	3.85		
			y_i	2.25	9.25	6.00	6.00	9.50	13.00	13.85	17.10	13.25	17.00	16.75	19.50	16.50	19.75		
8	VIII	T-model	x_i	17.75	15.25	15.00	17.75	15.25	15.00	11.75	9.00	6.95	10.20	12.50	5.75	2.00	1.75	2.95	5.00
			y_i	1.50	1.50	5.00	6.25	8.25	11.00	9.00	6.25	4.00	1.50	4.00	7.50	7.50	9.75	12.50	10.50
		P-model	x_i	5.25	5.25	2.50	2.75	5.25	8.00	11.25	8.00	4.75	1.50	2.00	11.25	15.00	18.75	18.75	16.00
			y_i	7.00	4.50	7.00	10.00	9.25	7.00	4.00	2.00	1.50	1.50	4.00	1.50	1.50	4.25	4.25	

Table 7

The comparison results of the proposed model versus the traditional model.

Test problem	Ref.	Number of departments	$s_1 = w_1$	Upper bound	Traditional model			Proposed model			
					Total adjacency value (Z')	Material handling cost	Time (s)	r^*	Total adjacency value (Z')	Material handling cost	Time (s)
I	[37]	5	1.0	10,238	10,238	111,105	1	3.1	10,238	109,794	1
II	[38]	7	1.0	2000	2000	18,923	2	2.4	2000	19,145	2
III	[39]	8	2.0	190	190	1116	6	4.0	190	1086	8
IV	[40]	11	0.2	4731	4731	33,787	9	1.0	4731	31,307	17
V	[41]	11	0.2	89	89	834.1	6	1.0	89	833.75	10
VI	[42]	12	0.1	16,790	16,690	26,851	540	5.0	16,778 ^a	23,747	1082
VII	[43]	14	0.1	2590	2560	13,280	1124	2.0	2576 ^a	12,310	1937
VIII	[43]	16	0.2	2150	2150	8691.5	1325	1.5	2150	8623.8	1739

^a Computation is suspended due to the "Gap" of less than 1%.

6. Conclusion

Facility layout is an important problem in the early stage of system design and plays a key role in modern manufacturing systems. In effort to maximize plant efficiency, the design of plant layout should facilitate the production process, minimize material handling and operating cost, and promote ease of the related activities.

In this study a new version of adjacency which provides a more flexible layout design is proposed. In the proposed version, two departments are considered adjacent if they are located within a pre-specified distance from each other. To accomplish this, a new continuous variable, called adjacency degree, is defined which measures the adjacency degree between any pairs of departments. The adjacency degree is defined as a bounded continuous variable on the interval of [0,1]. Also the adjacency degree is defined in such way that it is inversely correlated with the Tchebychev distance between two departments. It is shown that the proposed adjacency is a generalized version of the traditional adjacency. The proposed model is applicable not only in design of the process layout, but it is also applicable in the manufacturing plants with continuous production system, as well as the design of the electric boards. In these types of applications, the straight adjacency among departments is more valuable and provides more efficient arrangement of facilities.

For constructing an optimal layout design, we developed a mathematical programming model which is a more realistic representation of the FLP when compared to more traditional versions. An illustrative example and solutions of several test problems, selected from literature, showed the flexibility and efficacy of the proposed model. The computational experiment also reveals that in some of the final layouts, there is an enclosed space among departments, meaning the model sometimes preferred to create more adjacencies with degree of 1 and others with degree less than 1. This is very important aspect of the proposed model due to the indication that the model attempts to create all possible adjacencies without overlooking them.

The safety aspect of the layout design is an important issue which has to be considered in the layout design. As it can be realized, in the proposed model, we only considered the closeness of departments which are beneficial to the layout design. For the case when adjacency has negative effects, especially due to the safety consideration, the proposed model has to be modified. We therefore recommend this type of modifications for further research.

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