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Multiple Access Relay Channel With Relay-Sources Feedback

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ABSTRACT Feedback analysis on the rate region of multi-user channels is a problem of great importance in information and communication theory. In this paper, the Slepian-Wolf multiple access relay channel (SW-MARC) with relay-sources feedback is studied as a more general model of MARC where feedback outputs are taken into consideration. First, an inner bound on the capacity region of the discret alphabet and memoryless (DM) model of this channel (DM-SW-MARC) is obtained using partial decode and forward strategy. Second, an outer bound on the capacity region is derived by applying Fano's inequality. Third, the DM results are extended to the continuous alphabet version, where an inner bound on the capacity region of the Gaussian MARC (GMARC) with relay-sources feedback is derived. Fourth, it is shown that our results include the important previous works on multiple-access channel with feedback (Cover-Leung rate region) and without feedback (Slepian-Wolf (SW) rate region, etc.) as special cases. Finally, we investigate the effect of feedback and show that the presence of feedback enables the sources to understand each other's messages, which in turn allows the sources to cooperate to resolve the residual uncertainty at the receiver in an efficient way and at the same time, independent fresh information from the sources is superimposed upon the resolution information.

INDEX TERMS Achievable rate region, partial feedback, regular encoding/ backward decoding, Slepian-Wolf multiple access relay channel.

I. INTRODUCTION

Multiple access relay channel (MARC) is the combination of the MAC and relay channel (RC) which was first introduced and studied by Kramer and Wijngaarder [1]. In this channel, the relay helps the sources to communicate simultaneously with one destination. An example of such a channel model is cooperative uplink communications in the cellular network in which mobile users send their own data to the base station with the help of the relay station. Sankar [2] has proposed different strategies such as compress and forward (CF), amplify and forward (AF) and decode and forward (DF) for discrete memoryless (DM) MARC. An achievable rate region for DM-MARC with a common message was derived in [3] using a regular block Markov encoding/backward decoding scheme which was then extended to the continuous alphabet version. Moreover, a general inner bound on the capacity region for Slepian-Wolf MARC (SW-MARC) was analyzed in [4]. Murin *et al.* [5] obtained an achievable source-channel rate for arbitrarily correlated sources over MARC with correlated side information at both relay and the destination. Sattar *et al.* [6] have proposed two relay cooperation schemes to be applied in MARC with interference cancellation and compute and forward strategy. Performance analysis for transmission of correlated sources over orthogonal and non-orthogonal MARC was carried out in [7] and [8] respectively. A quantize and forward strategy for interleave division multiple access relay channel was investigated in [9]. Also, energy efficiency in the MARC with optimal power allocations minimizing the energy consumption at the desired communication rates has been studied in [10] and [11].

The relay channel has been widely studied since its initiation by Van der Meulan [12] in 1971. Cover and El. Gamal [13] have studied the relay channel, where a general inner bound on the capacity is obtained using a combination of partial decode and forward (PDF) and compress and forward (CF) strategies. Subsequently, an extensive study on some special classes of the relay channel, such as the relay channel with full feedback, degraded and reversely degraded relay channel [13], orthogonal relay channel [14] and semi-deterministic relay channel [15], have proven that strategies of Cover and El. Gamal is optimal and achieves capacity. A unified capacity theorem for the relay channel has been obtained in [16].

Feedback, as one of the most important research areas in network information theory, can increase achievable rate region of several multiuser channels although it does not change the achievable rate of discrete alphabet and memoryless point to point channels. It is interesting to investigate the impact of feedback on the achievable rate region of the MARC. The relay channel with full feedback (receiver to source, receiver to relay and relay to source feedbacks) was proved to be a physically degraded channel in [13] based on which DF has worked as a capacity achieving strategy in this case. Two other partial feedback configurations for the relay channel with relay-source feedback and receiver-source feedback via CF strategy were obtained in [17] and [18]. Moreover, two new generalized noisy feedback configurations were investigated in [19]. Hou *et al.* [20] have derived the achievable rate region through applying regular coding in a general MARC with relay-source feedback.

A. OUR MOTIVATION AND WORK

The problem of sending correlated sources over a channel is an important issue in multiuser information theory. Dependency between the sources can be taken into account by splitting each source message into two parts: private message and common message, as introduced by Slepian-Wolf for the MAC with common message [21]. On the other hand, it was shown that relay can facilitate communications between the sources and the destination by increasing the transmission rate and coverage area. Moreover, feedback can cause to resolve residual uncertainty at the receiver.

The main objective of this paper is to derive a general inner bound on the capacity region for DM-MARC with relay-sources feedback by applying regular encoding/backward decoding strategy where there is a common message between the sources that might be considered as a generalization of SW-MAC capacity theorem to the MARC with relay-sources feedback. Furthermore, by applying Fano's inequality, an outer bound for DM-MARC with relay-sources feedback is obtained. In addition, the SW-GMARC which is a full duplex Gaussian model of the SW-MARC is introduced and the results of the DM model are extended to a discrete time continuous alphabet model. Through relay-sources feedback, cooperation among the sources and the relay is set up to resolve residual uncertainty at the receiver. All related previous works are special cases of our results.

B. PAPER ORGANIZATION

The rest of the paper is organized as follows: In the next section, the system model under consideration and definitions are presented. In section III, we present our main theorems on inner and outer bounds for multiple access relay channel with relay-sources feedback. Proof of the achievability rate region and outer bound region is described in section IV and the conclusion is drawn in section V.

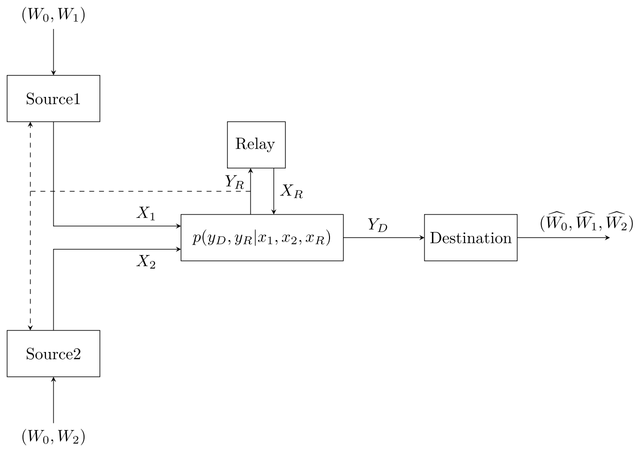

FIGURE 1. Multiple access relay channel with relay-sources feedback.

II. SYSTEM MODEL AND NOTATIONS

A. NOTATION

Random variables are denoted by capital letters, while their realizations are shown by respective lower case letters. Whenever the dimension of a random vector is clear from the context, the random vector is denoted by a bold-face letter, that is, X denotes the random vector $(X_1, X_2, \ldots, X_n)$, and $x = (x_1, x_2, \ldots, x_n)$ shows a specific sample value of it. However, in those cases where it is important to emphasize explicitly the dimension of a random vector X^i denotes the random vector $(X_1, X_2, \ldots, X_i)$, and $x^i = (x_1, x_2, \ldots, x_i)$ shows a specific sample value of X^i. The alphabet of a scalar random variable X is designated by a calligraphic letter $\mathcal{X}$. The n-fold Cartesian power of a generic alphabet $\mathcal{V}$, that is, the set of all n-vectors over $\mathcal{V}$, are denoted by $\mathcal{V}^n$. $\mathcal{A}_\varepsilon^n(X)$ denotes the set of all ε-typical n-sequences according to $p(x)$. Furthermore, $h(X)$ and $h(X|Y)$ denote differential entropy of X and differential entropy of X given Y, respectively; $I(X; Y)$ denotes the mutual information between X and Y. For $X \sim N(0, \sigma_X^2)$, $h(X) = \frac{1}{2}\log(2\pi e \sigma_X^2)$. For simplicity in notation, let $C(x) = \frac{1}{2}\log(1 + x)$, for $x \geq 0$ and $\bar{\alpha} = 1 - \alpha$.

B. SYSTEM MODEL

As shown in Fig. 1, a discrete memoryless multiple access relay channel with relay-sources feedback consists of source 1, source 2, a relay and a receiver. The channel is defined by tuple $(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_R, p(y_D, y_R|x_1, x_2, x_R), \mathcal{Y}_D \times \mathcal{Y}_R)$ where $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_R)$ are finite sets corresponding to the input alphabets of source1, source 2 and the relay, respectively; the finite sets $(\mathcal{Y}_D, \mathcal{Y}_R)$ are the output alphabets of the relay and the receiver, respectively; and $p(y_D, y_R|x_1, x_2, x_R)$ denotes the collections of probabilities of the channel outputs $(y_R, y_D) \in (\mathcal{Y}_R, \mathcal{Y}_D)$ which might be received on the conditions that channel inputs $(x_1, x_2, x_R) \in (\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_R)$ are transmitted.

The channel is assumed to be memoryless like the following

$$p(y_{R,k}, y_{D,k}|x_1^k, x_2^k, x_R^k, y_R^k, y_D^k) = p(y_{R,k}, y_{D,k}|x_{1,k}, x_{2,k}, x_{R,k}) \quad (1)$$

where $x_{1,k}, x_{2,k}, x_{R,k}, y_{R,k}, y_{D,k}$ denote the inputs and outputs of the channel at time instant k, respectively. A $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ code for the multiple access relay channel with casual partial feedback from the relay to the sources consists of three sets of integers $\mathcal{W}_0, \mathcal{W}_1, \mathcal{W}_2$ where $(\mathcal{W}_0, \mathcal{W}_i) = [1 : 2^{nR_0}] \times [1 : 2^{nR_i}], (i = 1, 2)$, is defined as follows:

1)A set of encoding functions at the sources,

$$\begin{cases} f_{1,k} : (\mathcal{W}_0, \mathcal{W}_1) \times \mathcal{Y}_R^{k-1} \to \mathcal{X}_1, & k = 1, 2, \dots, n \\ f_{2,k} : (\mathcal{W}_0, \mathcal{W}_2) \times \mathcal{Y}_R^{k-1} \to \mathcal{X}_2, & k = 1, 2, \dots, n \end{cases} \quad (2)$$

where $f_{i,k}(w_0, w_i, y_R^{k-1}), i = 1, 2$, is the symbol transmitted at time instant k by the sender i when attempting to convey messages (w_0, w_i) to the receiver after obtaining the previous $k-1$ output symbols $y_R^{k-1} = (y_{R,1}, y_{R,2}, \dots, y_{R,k-1})$ which is already set as feedback from the relay.

2) A set of relaying functions at the relay

$$f_{R,k} : \mathcal{Y}_R^{k-1} \to \mathcal{X}_R, \quad k = 1, 2, \dots, n \quad (3)$$

After obtaining the previous $k-1$ output symbols $y_R^{k-1} = (y_{R,1}, y_{R,2}, \dots, y_{R,k-1})$, $f_{R,k}(y_R^{k-1})$ will be prepared to be transmitted at time instant k by the relay.

3) A decoding function at the destination

$$g : \mathcal{Y}_D^n \to (\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2) \quad (4)$$

The probability of error is defined under the uniform distribution of messages over the product set $(\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2)$ as follows:

$$p_e^n = \frac{1}{2^{n(R_0+R_1+R_2)}} \sum_{\substack{(w_0,w_1,w_2)\in \\ (\mathcal{W}_0,\mathcal{W}_1,\mathcal{W}_2)}} Pr\{g(Y_D^n) \neq (w_0, w_1, w_2)|(w_0, w_1, w_2) sent\} \quad (5)$$

A rate triple (R_0, R_1, R_2) is said to be achievable for the DM-MARC with relay-sources feedback, if a sequence of codes $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ exists for which P_e^n approaches 0 as $n \to \infty$.

III. MAIN THEOREMS

In this section, an inner bound on the capacity region of the DM-SW-MARC with relay-sources feedback is proved to be achievable by applying regular encoding/ backward decoding strategy as Theorem 1. Next, the outer bound is derived in Theorem 2. Finally, inner bound of the DM model is extended to the Gaussian case in Theorem 3. Note that in the proposed MARC with relay-sources feedback, the relay is a generic relay performing PDF strategy to aid transmission between the sources and the destination while there is a common message among the sources. Finally, it is shown that all previous works are special cases of our theorems.

Theorem 1: For the DM-SW-MARC with casual noiseless relay-sources feedback, an inner bound on the capacity region using PDF strategy is given by the union set of rate triple (R_0, R_1, R_2) satisfying:

$$\begin{aligned}
&R_1' \leq I(U_1; Y_R|X_R, V', U', U_0, U_2, V_0, V_1, V_2) \\
&R_2' \leq I(U_2; Y_R|X_R, V', U', U_0, U_1, V_0, V_1, V_2) \\
&R_1' + R_2' \leq I(U_1, U_2; Y_R|X_R, V', U', U_0, V_0, V_1, V_2) \\
&R_0' + R_1' + R_2' \leq I(U_0, U_1, U_2; Y_R|X_R, V', U', V_0, V_1, V_2) \\
&R_V + R_0' + R_1' + R_2' \leq I(U', U_0, U_1, U_2; Y_R|X_R, V', V_0, V_1, V_2) \\
&R_1'' \leq I(X_1; Y_D|X_R, X_2, Q, V', V_0, V_1, V_2, U', U_0, U_1, U_2) \\
&R_2'' \leq I(X_2; Y_D|X_R, X_1, Q, V', V_0, V_1, V_2, U', U_0, U_1, U_2) \\
&R_1'' + R_2'' \leq I(X_1, X_2; Y_D|X_R, Q, V', V_0, V_1, V_2, U', U_0, U_1, U_2) \\
&R_1' + R_1'' \leq I(X_1, X_R, V_1, U_1; Y_D|X_2, Q, V', V_0, V_2, U', U_0, U_2) \\
&R_2' + R_2'' \leq I(X_2, X_R, V_2, U_2; Y_D|X_1, Q, V', V_0, V_1, U', U_0, U_1) \\
&R_0'' + R_1'' + R_2'' \leq I(X_1, X_2, Q; Y_D|X_R, V', V_0, V_1, V_2, U', U_0, U_1, U_2) \\
&R_1' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, V_1, U_1; Y_D|Q, V', V_0, V_2, U', U_0, U_2) \\
&R_2' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, V_2, U_2; Y_D|Q, V', V_0, V_1, U', U_0, U_1) \\
&R_1' + R_0'' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, Q, V_1, U_1; Y_D|V', V_0, V_2, U', U_0, U_2) \\
&R_2' + R_0'' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, Q, V_2, U_2; Y_D|V', V_0, V_1, U', U_0, U_1) \\
&R_1' + R_2' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, V_1, U_1, V_2, U_2; Y_D|Q, V', V_0, U', U_0) \\
&R_1' + R_2' + R_0'' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, Q, V_1, U_1, V_2, U_2; Y_D|V', V_0, U', U_0) \\
&R_0' + R_1' + R_2' + R_0'' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, Q, V_0, U_0, V_1, U_1, V_2, U_2; Y_D|V', U') \\
&R_V + R_0' + R_1' + R_2' + R_0'' + R_1'' + R_2'' \leq I(X_1, X_2, X_R, Q, V', U', V_0, U_0, V_1, U_1, V_2, U_2; Y_D)
\end{aligned} \quad (6)$$

where the union is taken over all joint probability distributions of the form:

$$\begin{aligned}
&p(v', v_0, v_1, v_2, u', u_0, u_1, u_2, q, x_1, x_2, x_R, y_R, y_D)\\
&= p(v')p(v_0|v')p(v_1|v', v_0)p(v_2|v', v_0)p(u'|v')\\
&\quad p(u_0|v', v_0, u')p(q|v', v_0, u', u_0)\\
&\quad \times (\prod_{k=1}^{2} p(u_k|v', v_0, v_k, u', u_0)p(x_k|v', v_0, v_k, u', u_0, u_k, q))\\
&\quad \times p(x_R|v', v_0, v_1, v_2)p(y_R, y_D|x_1, x_2, x_R) \qquad (7)
\end{aligned}$$

and R_V is the feedback rate and R'_k, R''_k are the components of the message rate R_k such that $R_k = R'_k + R''_k, k = 0, 1, 2$.

Outline of proof: Each common message (w_0) and private message (w_1, w_2) are splitted into two parts w'_k and $w''_k, k = 0, 1, 2$, with the rates R'_k and R''_k, respectively. Note that V' carries the relay-sources feedback message information for resolving receiver's residual uncertainty, V_0 is the common message information, $V_k, k = 1, 2$, is the private message information, U' is the new feedback message information, U_0 is the new common message information, $U_k, k = 1, 2$, is the private new message information. Also Q represents all new common messages information that is intended to be transmitted along with all new private messages information by each source. The details of the proof are given in the next section.

Corollary 1: By setting $V' = V_0 = V_1 = V_2 = U' = U_0 = U_1 = U_2 = X_R = \emptyset$, the achievable rate region of Slepian-Wolf MAC in [21] is derived.

Corollary 2: As easily seen, the inner bound on the capacity region of MARC with common message in [3] is obtained by setting $V' = U' = \emptyset$.

Corollary 3: By setting $V_0 = V_2 = U_0 = U_2 = X_2 = Q = 0$, and $V_1 = X_R$, $U_1 = X_1$, the achievable rate for relay channel with relay-source feedback through DF strategy (not studied previously) is derived.

Proposition 1: By setting $U_1 = X_1$, $U_2 = X_2$ and $Q = U_0$ *or* $\emptyset$, an achievable rate region for MARC with a common message and relay-source feedback via full DF strategy is given by equation (9), where the union is obtained by taking account of all distributions of the form:

$$\begin{aligned}
&p(v', v_0, v_1, v_2, u', u_0, x_1, x_2, x_R, y_R, y_D)\\
&= p(v')p(v_0|v')p(v_1|v', v_0)p(v_2|v', v_0)p(u'|v')\\
&\quad \times p(u_0|v', v_0, u')(\prod_{k=1}^{2} p(x_k|v', v_0, v_k, u', u_0))\\
&\quad \times p(x_R|v', v_0, v_1, v_2)p(y_R, y_D|x_1, x_2, x_R) \qquad (8)
\end{aligned}$$

$$\begin{aligned}
&R_1 \leq \min\begin{cases} I(X_1; Y_R|X_R, X_2, U', U_0, V', V_0, V_1, V_2),\\ I(X_1, X_R; Y_D|X_2, U', U_0, V', V_0, V_2),\end{cases}\\
&R_2 \leq \min\begin{cases} I(X_2; Y_R|X_R, X_1, U', U_0, V', V_0, V_1, V_2),\\ I(X_2, X_R; Y_D|X_1, U', U_0, V', V_0, V_1),\end{cases}\\
&R_1 + R_2 \leq \min\begin{cases} I(X_1, X_2; Y_R|X_R, U', U_0, V', V_0, V_1, V_2),\\ I(X_1, X_2, X_R; Y_D|U', U_0, V', V_0),\end{cases}\\
&R_0 + R_1 + R_2 \leq \min\begin{cases} I(X_1, X_2; Y_R|X_R, U', V', V_0, V_1, V_2),\\ I(X_1, X_2, X_R; Y_D|U', V'),\end{cases}\\
&R_V + R_0 + R_1 + R_2 \leq \min\begin{cases} I(X_1, X_2; Y_R|X_R, V', V_0, V_1, V_2),\\ I(X_1, X_2, X_R; Y_D),\end{cases} \qquad (9)
\end{aligned}$$

Theorem 2: An outer bound on the capacity region of a two-user MARC with a common message and relay-sources feedback is the union of all rate triple (R_0, R_1, R_2) satisfying:

$$\begin{aligned}
&R_1 \leq \min\begin{cases} I(X_1, X_R; Y_D|X_2, U, V),\\ I(X_1; Y_D, Y_R|X_2, X_R, U, V),\end{cases}\\
&R_2 \leq \min\begin{cases} I(X_2, X_R; Y_D|X_1, U, V),\\ I(X_2; Y_D, Y_R|X_1, X_R, U, V),\end{cases}\\
&R_1 + R_2 \leq \min\begin{cases} I(X_1, X_2, X_R; Y_D|U, V),\\ I(X_1, X_2; Y_D, Y_R|X_R, U, V),\end{cases}\\
&R_0 + R_1 + R_2 \leq \min\begin{cases} I(X_1, X_2, X_R; Y_D|V),\\ I(X_1, X_2; Y_D, Y_R|X_R, V),\end{cases}\\
&R_V + R_0 + R_1 + R_2 \leq \min\begin{cases} I(X_1, X_2, X_R; Y_D),\\ I(X_1, X_2; Y_D, Y_R|X_R),\end{cases} \qquad (10)
\end{aligned}$$

where the union is taken over all joint probability distributions of the form:

$$\begin{aligned}
&p(v, u, x_1, x_2, x_R, y_R, y_D)\\
&= p(v)p(u|v)p(x_1|v, u)\\
&\quad \times p(x_2|v, u)p(x_R|x_1, x_2)p(y_R, y_D|x_1, x_2, x_R) \qquad (11)
\end{aligned}$$

Corollary 4: As easily seen, the outer bound of MARC with a common message and without feedback is obtained by setting $V = \emptyset$.

Corollary 5: By setting $X_R = Y_R = V = \emptyset$, the outer bound of MAC without feedback is derived.

Theorem 3: An inner bound on capacity region of SW-GMARC with relay-sources feedback is given by

$$\begin{aligned}
&R_1 \leq \max\min\{C(\frac{\alpha_1 P_1}{N_R}), C(\frac{\alpha_1 P_1 + (\sqrt{\mu_1 P_1} + \sqrt{\gamma_R P_R})^2}{N_D})\},\\
&R_2 \leq \max\min\{C(\frac{\alpha_2 P_2}{N_R}), C(\frac{\alpha_2 P_2 + (\sqrt{\mu_2 P_2} + \sqrt{\alpha_R P_R})^2}{N_D})\},\\
&R_1 + R_2 \leq \max\min\\
&\quad \left\{\begin{matrix} C\left(\frac{\alpha_1 P_1 + \alpha_2 P_2}{N_R}\right),\\ C\left(\frac{\left(\begin{matrix}\alpha_1 P_1 + \alpha_2 P_2 +\\ (\sqrt{\mu_1 P_1} + \sqrt{\gamma_R P_R})^2 +\\ (\sqrt{\mu_2 P_2} + \sqrt{\alpha_R P_R})^2\end{matrix}\right)}{N_D}\right)\end{matrix}\right\},\\
&R_0 + R_1 + R_2 \leq \max\min
\end{aligned}$$

$$\left\{\begin{array}{c} C\left(\frac{\alpha_1P_1+\alpha_2P_2+(\sqrt{\gamma_1\,P_1}+\sqrt{\gamma_2\,P_2})^2}{N_R}\right), \\ C\left(\frac{\left(\begin{array}{c}\alpha_1P_1+\alpha_2P_2+ \\ (\sqrt{\mu_1\,P_1}+\sqrt{\gamma_RP_R})^2+ \\ (\sqrt{\mu_2\,P_2}+\sqrt{\alpha_RP_R})^2+ \\ +(\sqrt{\gamma_1\,P_1}+\sqrt{\gamma_2\,P_2})^2+ \\ (\sqrt{\theta_1\,P_1}+\sqrt{\theta_2\,P_2}+\sqrt{\beta_RP_R})^2\end{array}\right)}{N_D}\right)\end{array}\right\},$$

$$R_V+R_0+R_1+R_2 \leq \max\min$$

$$\left\{\begin{array}{c} C\left(\frac{\left(\begin{array}{c}\alpha_1P_1+\alpha_2P_2+ \\ (\sqrt{\gamma_1\,P_1}+\sqrt{\gamma_2\,P_2})^2+(\sqrt{\beta_1\,P_1}+\sqrt{\beta_2\,P_2})^2\end{array}\right)}{N_R}\right), \\ C\left(\frac{\left(\begin{array}{c}\alpha_1P_1+\alpha_2P_2+ \\ (\sqrt{\mu_1\,P_1}+\sqrt{\gamma_RP_R})^2+ \\ (\sqrt{\mu_2\,P_2}+\sqrt{\alpha_RP_R})^2+ \\ (\sqrt{\gamma_1\,P_1}+\sqrt{\gamma_2\,P_2})^2+ \\ (\sqrt{\theta_1\,P_1}+\sqrt{\theta_2\,P_2}+\sqrt{\beta_RP_R})^2+ \\ (\sqrt{\beta_1\,P_1}+\sqrt{\beta_2\,P_2})^2+ \\ \left(\begin{array}{c}(\sqrt{\alpha_1+\beta_1+\gamma_1+\mu_1+\theta_1P_1}+ \\ \sqrt{\alpha_2+\beta_2+\gamma_2+\mu_2+\theta_2P_2}+ \\ \sqrt{\alpha_R+\beta_R+\gamma_RP_R})\end{array}\right)^2\end{array}\right)}{N_D}\right)\end{array}\right\} \tag{12}$$

where the union is obtained by taking account of all parameters $\alpha_i, \beta_i, \gamma_i, \mu_i, \theta_i, \alpha_R, \beta_R, \gamma_R \in [0, 1]$, $i = 1, 2$, such that $\alpha_i + \beta_i + \gamma_i + \mu_i + \theta_i \leq 1, \alpha_R, \beta_R, \gamma_R \leq 1$.

Corollary 6: The inner bound on capacity region for SW-GMARC without feedback via the DF strategy [3] is obtained by setting $\alpha_1+\gamma_1+\mu_1+\theta_1 = 1, \alpha_2+\gamma_2+\mu_2+\theta_2 = 1, \alpha_R + \beta_R + \gamma_R = 1, \beta_1 = \beta_2 = 0$.

Corollary 7: By setting $\alpha_R = \beta_R = P_2 = \theta_1 = \gamma_1 = 0$, the achievable rate for the Gaussian relay channel with relay-source via the DF strategy is obtained. The difference of this corollary with [17] is due to the strategies applied for the coding (DF and CF strategies).

Corollary 8: The achievable rate region for Gaussian multiple access channel with feedback [22] is derived by setting $N_D = N_R, P_R = 0, \theta_1 = \gamma_1 = \theta_2 = \gamma_2 = 0$. The difference of this corollary with [23], which achieves a higher sum rate, is due to the coding schemes which are totally different.
Using equation (12), the achievable sum rate for Gaussian MARC with relay-sources feedback has been computed and shown in Fig. 2. As predicted, the achievable sum rate with feedback is larger than the sum rate without feedback and the feedback increases the sum rate.

IV. PROOFS

Proof of Theorem 1: The achievability proof of inner bound consists of three parts: first, the random code book generation is given, next, the encoding scheme is presented, and third, the decoding scheme is provided.

Random code book generation: consider $n > 0$ and fix a choice of

$$p(v', v_0, v_1, v_2, u', u_0, u_1, u_2, q, x_1, x_2, x_R, y_R, y_D)$$

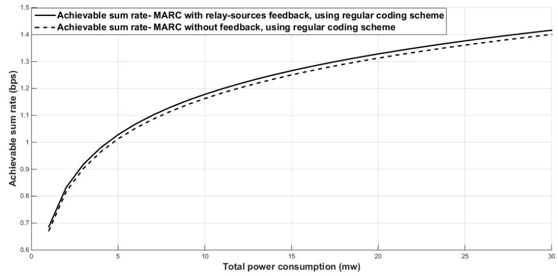

FIGURE 2. Achievable sum rate for Gaussian Multiple access relay channel, with relay-sources feedback and without feedback.

$$\begin{aligned} &= p(v')p(v_0|v')p(v_1|v', v_0)p(v_2|v', v_0)p(u'|v')p(u_0|v', v_0, u') \\ &\quad \times p(q|v', v_0, u', u_0) \\ &\quad \times (\prod_{k=1}^{2} p(u_k|v', v_0, v_k, u', u_0)p(x_k|v', v_0, v_k, u', u_0, u_k, q)) \\ &\quad \times p(x_R|v', v_0, v_1, v_2)p(y_R, y_D|x_1, x_2, x_R) \end{aligned} \tag{13}$$

Note that V' carries the relay-sources feedback information for resolving receiver's residual uncertainty. First, as the center of the clouds, the feedback information message codeword, v'^n,based on $p(v'^n) = \prod_{t=1}^{n} p(v'_t)$ is generated. Next, for each v'^n, the common message codeword v_0^n is generated by superposition coding. Then, for each (v'^n, v_0^n), the private message codeword $v_k^n, k = 1, 2$ is generated. After that, for each v'^n, the new information codeword of feedback, u'^n,is generated. Next, for each (v'^n, v_0^n, u'^n), the common new information codeword u_0^n is generated. For each $(v'^n, v_0^n, v_k^n, u'^n, u_0^n)$, the private new information message codeword u_k^n is generated. q^n represents all new common information messages that is intended to be transmitted along with all new private information messages by each source.

- As the center of the cloud, generate 2^{nR_V} i.i.d n-sequence v'^n,based on $p(v'^n) = \prod_{t=1}^{n} p(v'_t)$ and index them as $v'^n(j'), j' \in [1 : 2^{nR_V}]$.
- For each $v'^n(j')$, by superposition coding, generate $2^{nR'_0}$ i.i.d n-sequence v_0^n,based on $p(v_0^n|v'^n(j')) = \prod_{t=1}^{n} p(v_{0,t}|v'_t(j'))$ and index them as $v_0^n(j_0, j'), j_0 \in [1 : 2^{nR'_0}]$.
- For each $(v'^n(j'), v_0^n(j_0, j'))$, generate $2^{nR'_k}$ i.i.d n-sequence v_k^n, based on $p(v_k^n|v'^n(j'), v_0^n(j_0, j')) = \prod_{t=1}^{n} p(v_{k,t}|v'_t(j'), v_{0,t}(j_0, j'))$ and index them as $v_k^n(j_k, j_0, j'), j_k \in [1 : 2^{nR'_k}, k = 1, 2]$.
- For each $v'^n(j')$, by superposition coding, generate 2^{nR_V} i.i.d n-sequence u'^n,based on $p(u'^n|v'^n(j')) = \prod_{t=1}^{n} p(u'_t|v'_t(j'))$ and index them as $u'^n(l', j'), l' \in [1 : 2^{nR_V}]$.
- For each $(v'^n(j'), u'^n(l', j'), v_0^n(j_0, j'))$, generate $2^{nR'_0}$ i.i.d n-sequence u_0^n,based on $p(u_0^n|v'^n(j'), u'^n(l', j'), v_0^n(j_0, j')) = \prod_{t=1}^{n} p(u_{0,t}|v'_t(j'), u'_t(l', j'), v_{0,t}(j_0, j'))$ and index them as $u_0^n(l_0, j_0, l', j'), l_0 \in [1 : 2^{nR'_0}]$.

- For each $(v'^n(j'), u'^n(l',j'), v_0^n(j_0,j'), u_0^n(l_0,j_0,l',j'), v_k^n(j_k,j_0,j'))$, generate $2^{nR_k'}$ i.i.d n-sequence u_k^n, based on $p(u_k^n|v'^n(j'), u'^n(l',j'), v_0^n(j_0,j'), u_0^n(l_0,j_0,l',j'), v_k^n(j_k,j_0,j')) = \prod_{t=1}^n p(u_{k,t}|v_t'(j'), u_t'(l',j'), v_{0,t}(j_0,j'), u_{0,t}(l_0,j_0,l',j'), v_{k,t}(j_k,j_0,j'))$ and index them as $u_k^n(l_k,j_k,l_0,j_0,l',j')$, $l_k \in [1:2^{nR_k'}]$, $k=1,2$.
- For each $(v'^n(j'), u'^n(l',j'), v_0^n(j_0,j'), u_0^n(l_0,j_0,l',j'))$, generate $2_0^{nR''}$ i.i.d n-sequence q^n, based on $p(q^n|v'^n(j'), u'^n(l',j'), v_0^n(j_0,j'), u_0^n(l_0,j_0,l',j')) = \prod_{t=1}^n p(q_t|v_t'(j'), u_t'(l',j'), v_{0,t}(j_0,j'), u_{0,t}(l_0,j_0,l',j'))$ and index them as $q^n(m_0,l_0,j_0,l',j')$, $m_0 \in [1:2^{nR_0''}]$.
- For each $(v'^n(j'), u'^n(l',j'), v_0^n(j_0,j'), u_0^n(l_0,j_0,l',j'), v_k^n(j_k,j_0,j'), u_k^n(l_k,j_k,l_0,j_0,l',j'), q^n(m_0,l_0,j_0,l',j')$, generate $2^{nR_k''}$ i.i.d n-sequence x_k^n, based on $p(x_k^n|v'^n(j'), u'^n(l',j'), v_0^n(j_0,j'), u_0^n(l_0,j_0,l',j'), v_k^n(j_k,j_0,j'), u_k^n(l_k,j_k,l_0,j_0,l',j'), q^n(m_0,l_0,j_0,l',j') = \prod_{t=1}^n p(x_{k,t}|v_t'(j'), u_t'(l',j'), v_{0,t}(j_0,j'), u_{0,t}(l_0,j_0,l',j'), v_{k,t}(j_k,j_0,j'), u_{k,t}(l_k,j_k,l_0,j_0,l',j'), q_t(m_0,l_0,j_0,l',j')$ and index them as $x_k^n(m_k,m_0,l_k,j_k,l_0,j_0,l',j')$, $m_k \in [1:2^{nR_k''}]$, $k=1,2$.
- For each $(v'^n(j'), v_0^n(j_0,j'), v_1^n(j_1,j_0,j'), v_2^n(j_2,j_0,j'))$, choose an x_R^n with $p(x_R^n|v'^n(j'), v_0^n(j_0,j'), v_1^n(j_1,j_0,j'), v_2^n(j_2,j_0,j')) = \prod_{t=1}^n p(x_{R,t}|v_t'^n(j'), v_{0,t}^n(j_0,j'), v_{1,t}^n(j_1,j_0,j'), v_{2,t}^n(j_2,j_0,j'))$ and index them as $x_R^n(j',j_0,j_1,j_2)$.

Encoding: Encoding is performed in $B+1$ blocks. The messages w_v, $w_{k'}$ and w_k'' are split into B equally sized blocks $w_{v,b}$, $w_{k,b'}$ and $w_{k,b}''$, $k=0,1,2$ and $b=1,2,\ldots,B$.

- **At source terminals:** In block $b=1,2,\ldots,B+1$, the k^{th} encoder sends $x_{k,b}^n(w_{k,b}'', w_{0,b}'', w_{k,b}', w_{0,b}', w_{v,b}, w_{k,b-1}', w_{0,b-1}', w_{v,b-1})$ over the channel, where $w_{v,0}=w_{0,0}'=w_{1,0}'=w_{2,0}'=w_{v,B+1}=w_{0,B+1}'=w_{1,B+1}'=w_{2,B+1}'=w_{0,B+1}''=w_{1,B+1}''=w_{2,B+1}''=1$.
- **At relay terminal:** The relay knows $w_{v,b}, w_{0,b}', w_{1,b}', w_{2,b}'$ from previous decoding step at the relay and transmits $x_{R,b+1}^n(w_{v,b}, w_{0,b}', w_{1,b}', w_{2,b}')$ in block $b+1$.

The rest of the proof, including decoding and eroror analysis, are given in Appendix A.

Proof of Theorem 2: Auxiliary random variables $U_i \triangleq W_0$ and $V_i \triangleq W_v$ are defined.

$$\begin{aligned}
nR_1 &= H(W_1) = I(W_1;Y_D^n) + H(W_1|Y_D^n)\\
&\leq I(W_1;Y_D^n) + n\varepsilon_n\\
&\leq I(W_1;Y_D^n,Y_R^n) + n\varepsilon_n \overset{(a)}{\rightarrow}\\
nR_1 &\leq I(W_1;Y_D^n,Y_R^n|W_0,W_2,W_v) + n\varepsilon_n\\
&= \sum_{i=1}^n I(W_1;Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_2,W_v) + n\varepsilon_n\\
&= \sum_{i=1}^n H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_2,W_v)\\
&\quad - H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_1,W_2,W_v)\\
&\quad + n\varepsilon_n \overset{(b)}{\rightarrow}\\
nR_1 &\leq \sum_{i=1}^n H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_2,W_v,X_{2,i},X_{R,i})\\
&\quad - H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_1,W_2,W_v,X_{1,i},\\
&\qquad X_{2,i},X_{R,i}) + n\varepsilon_n \overset{(c)}{\rightarrow}\\
nR_1 &\leq \sum_{i=1}^n H(Y_{D,i},Y_{R,i}|X_{2,i},X_{R,i},U_i,V_i)\\
&\quad - H(Y_{D,i},Y_{R,i}|X_{1,i},X_{2,i},X_{R,i},U_i,V_i) + n\varepsilon_n\\
&= \sum_{i=1}^n I(X_{1,i};Y_{D,i},Y_{R,i}|X_{2,i},X_{R,i},U_i,V_i) + n\varepsilon_n
\end{aligned} \tag{14}$$

where (a) follows from the fact that (W_1) is independent from (W_0,W_2,W_v), (b) is due to $X_{1,i}$, $X_{2,i}$ are deterministic functions of (Y_R^{i-1},W_v,W_0,W_1), (Y_R^{i-1},W_v,W_0,W_2), respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (c) follows from removing conditioning, and the Markovity of $(Y_D^{i-1},Y_R^{i-1},W_v,W_0,W_1,W_2) \Leftrightarrow (X_{1,i},X_{2,i},X_{R,i}) \Leftrightarrow (Y_{D,i},Y_{R,i})$. The rest of the proof, including the other bounds, are given in Appendix B

Proof of Theorem 3: In this sub-section, we focus on the Gaussian multiple access relay channel with relay-sources feedback which is described as follows: In this channel, the input symbols sequences $\{X_1\}$, $\{X_2\}$ and $\{X_R\}$ are subjected to the following average power constraints:

$$\begin{aligned}
&\frac{1}{n}\sum_{k=1}^n \mathbf{E}(|X_{i,k}|^2) \leq P_i, \quad i=1,2\\
&\frac{1}{n}\sum_{k=1}^n \mathbf{E}(|X_{R,k}|^2) \leq P_R,
\end{aligned} \tag{15}$$

where $\mathbf{E}$ denotes as an expectation operator. For each slot k, X_1, X_2, X_R are sent and channel outputs are:

$$\begin{aligned}
Y_{R,k} &= X_{1,k} + X_{2,k} + Z_{R,k},\\
Y_{D,k} &= X_{1,k} + X_{2,k} + X_{R,k} + Z_{D,k},
\end{aligned} \tag{16}$$

where $Z_R = (Z_{R,1},\ldots,Z_{R,n})$ and $Z_D = (Z_{D,1},\ldots,Z_{D,n})$ are sequences of independent identically distributed (i.i.d) normal random variables with zero mean and variances N_R, N_D which model the noise on the relay and the receiver respectively.

The discrete alphabet achievable rate results can be extended to memoryless channels with discrete time and continuous alphabet [24]. Achievability is established following the proof of Theorem 1 by substituting $U_1 = X_1, U_2 = X_2$ and $Q = \emptyset$ *or* U_0. The resulting information rates suggest that the rate region is achievable if it satisfies (9). We set:

$$\begin{aligned}
V_0 &= \sqrt{\eta_0}L + \sqrt{\eta_0}V'\\
V_1 &= \sqrt{\eta_{1,1}}T_1 + \sqrt{\eta_{1,2}}L + \sqrt{(\eta_{1,1}+\eta_{1,2})}V'\\
V_2 &= \sqrt{\eta_{2,1}}T_2 + \sqrt{\eta_{2,2}}L + \sqrt{(\eta_{2,1}+\eta_{2,2})}V'\\
U' &= \sqrt{\eta_3}P + \sqrt{\eta_3}V'\\
U_0 &= \sqrt{\lambda_1}M + \sqrt{\lambda_2}P + \sqrt{\lambda_3}L + \sqrt{\lambda_1+\lambda_2+\lambda_3}V'
\end{aligned}$$

$$
\begin{aligned}
X_1 &= \sqrt{P_1}(\sqrt{\alpha_1}N_1 + \sqrt{\beta_1}P + \sqrt{\gamma_1}M + \sqrt{\mu_1}T_1 + \sqrt{\theta_1}L \\
&\qquad + \sqrt{(\alpha_1 + \beta_1 + \gamma_1 + \mu_1 + \theta_1)}V') \\
X_2 &= \sqrt{P_2}(\sqrt{\alpha_2}N_2 + \sqrt{\beta_2}P + \sqrt{\gamma_2}M + \sqrt{\mu_2}T_2 + \sqrt{\theta_2}L \\
&\qquad + \sqrt{(\alpha_2 + \beta_2 + \gamma_2 + \mu_2 + \theta_2)}V') \\
X_R &= \sqrt{P_R}(\sqrt{\beta_R}L + \sqrt{\gamma_R}T_1 + \sqrt{\alpha_R}T_2 \\
&\qquad + \sqrt{(\alpha_R + \beta_R + \gamma_R)}V')
\end{aligned} \tag{17}
$$

where $V', L, T_1, T_2, P, M, N_1, N_2$ are all Gaussian random variables with zero mean and unit variance and $\eta_0, \eta_{1,1}, \eta_{1,2}, \eta_{2,1}, \eta_{2,2}, \eta_3, \lambda_1, \lambda_2, \lambda_3, \alpha_i, \beta_i, \gamma_i, \mu_i, \theta_i, \alpha_R, \beta_R, \gamma_R \in [0, 1]$, $i = 1, 2$, such that $\eta_{1,1} + \eta_{1,2} \leq 1$, $\eta_{2,1} + \eta_{2,2} \leq 1$, $\lambda_1+\lambda_2+\lambda_3 \leq 1$, $\alpha_i+\beta_i+\gamma_i+\mu_i+\theta_i \leq 1$, $\alpha_R+\beta_R+\gamma_R \leq 1$.

Considering the relation the channel inputs- channel outputs relation in equation (16), the rates are bounded as follows:

- **At the relay:**

$$
\begin{aligned}
R_1 &\leq I(X_1; Y_R|X_R, X_2, U', U_0, V', V_0, V_1, V_2) \\
&= h(Y_R|X_R, X_2, U', U_0, V', V_0, V_1, V_2) \\
&\quad - h(Y_R|X_R, X_1, X_2, U', U_0, V', V_0, V_1, V_2) \\
&= h(X_1+X_2+Z_R|X_R, X_2, U', U_0, V', V_0, V_1, V_2) - h(Z_R) \\
&= h(\sqrt{P_1}(\sqrt{\alpha_1}N_1 + \sqrt{\beta_1}P + \sqrt{\gamma_1}M + \sqrt{\mu_1}T_1 + \sqrt{\theta_1}L \\
&\quad + \sqrt{(\alpha_1 + \beta_1 + \gamma_1 + \mu_1 + \theta_1)}V') + Z_R|X_R, X_2, \\
&\quad U', U_0, V', V_0, V_1, V_2) - h(Z_R) \overset{(a)}{\Rightarrow} \\
&= h(\sqrt{\alpha_1 P_1}N_1 + Z_R) - h(Z_R) \\
&= \frac{1}{2}\log(2\pi e(\alpha_1 P_1 + N_R)) - \frac{1}{2}\log(2\pi e N_R) \\
&= \frac{1}{2}\log(1 + \frac{\alpha_1 P_1}{N_R}) = C(\frac{\alpha_1 P_1}{N_R})
\end{aligned} \tag{18}
$$

Where (a) follows since knowing (U', U_0, V', V_0, V_1), reveals L, T_1, P, M. Similarly we obtain:

$$R_2 \leq C(\frac{\alpha_2 P_2}{N_R}) \tag{19}$$

$$R_1 + R_2 \leq C(\frac{\alpha_1 P_1 + \alpha_2 P_2}{N_R}) \tag{20}$$

$$R_0 + R_1 + R_2 \leq C(\frac{\alpha_1 P_1 + \alpha_2 P_2 + (\sqrt{\gamma_1 P_1} + \sqrt{\gamma_2 P_2})^2}{N_R}) \tag{21}$$

$$R_V + R_0 + R_1 + R_2 \leq C\left(\frac{\begin{pmatrix} \alpha_1 P_1 + \alpha_2 P_2 + \\ (\sqrt{\gamma_1 P_1} + \sqrt{\gamma_2 P_2})^2 + \\ (\sqrt{\beta_1 P_1} + \sqrt{\beta_2 P_2})^2 \end{pmatrix}}{N_R}\right) \tag{22}$$

the rest of the proof, including the other Gaussian bounds, are given in Appendix C.

V. CONCLUSION

In this paper, inner and outer bounds for the capacity of the DM-SW-MARC with relay-sources feedback configuration were investigated. Moreover, by extending discrete alphabet results to the continuous alphabet version, an achievable rate region for the Gaussian multiple access relay channel with relay-sources feedback was derived. The derived achievable rate region was shown to resolve the residual uncertainty of the receiver in an efficient way due to the impact of the feedback, and our results, while including the previous works as special cases, show that the proposed coding strategy enhances the correlation among the sources and the relay encoder and therefore improves the best known one-way achievable rate.

APPENDIX A
THE ACHIEVABILITY PROOF OF INNER BOUND

- **Decoding and error analysis at the relay terminal:** At the end of block b, after determining x_R^n from y_R^n, the relay looks for $(\widetilde{w_{v,b}}, \widetilde{w'_{0,b}}, \widetilde{w'_{1,b}}, \widetilde{w'_{2,b}})$ in a way that

$$
\begin{aligned}
&(v_b'^n(w_{v,b-1}), v_{0,b}^n(w'_{0,b-1}, w_{v,b-1}), \\
&\ v_{1,b}^n(w'_{1,b-1}, w'_{0,b-1}, w_{v,b-1}), \\
&\ v_{2,b}^n(w'_{2,b-1}, w'_{0,b-1}, w_{v,b-1}), u_b'^n(\widetilde{w_{v,b}}, w_{v,b-1}), \\
&\ u_{0,b}^n(\widetilde{w'_{0,b}}, w'_{0,b-1}, \widetilde{w_{v,b}}, w_{v,b-1}), \\
&\ u_{1,b}^n(\widetilde{w'_{1,b}}, w'_{1,b-1}, \widetilde{w'_{0,b}}, w'_{0,b-1}, \widetilde{w_{v,b}}, w_{v,b-1}), \\
&\ u_{2,b}^n(\widetilde{w'_{2,b}}, w'_{2,b-1}, \widetilde{w'_{0,b}}, w'_{0,b-1}, \widetilde{w_{v,b}}, w_{v,b-1}), \\
&\ x_{R,b}^n(w_{v,b-1}, w'_{0,b-1}, w'_{1,b-1}, w'_{2,b-1}), y_{R,b}^n) \\
&\ \in A_\varepsilon^n(V', V_0, V_1, V_2, U', U_0, U_1, U_2, X_R, Y_R)
\end{aligned} \tag{23}
$$

The relay uses joint decoding strategy to find $(w_{v,b}, w'_{0,b}, w'_{1,b}, w'_{2,b})$ by using typicality decoding of equation (23). This can be done if:

$$
\begin{aligned}
R'_1 &\leq I(U_1; Y_R|X_R, V', U', U_0, U_2, V_0, V_1, V_2) \\
R'_2 &\leq I(U_2; Y_R|X_R, V', U', U_0, U_1, V_0, V_1, V_2) \\
R'_1 + R'_2 &\leq I(U_1, U_2; Y_R|X_R, V', U', U_0, V_0, V_1, V_2) \\
R'_0 + R'_1 + R'_2 &\leq I(U_0, U_1, U_2; Y_R|X_R, V', U', V_0, V_1, V_2) \\
R_V + R'_0 + R'_1 + R'_2 &\leq I(U', U_0, U_1, U_2; Y_R|X_R, V', V_0, V_1, V_2)
\end{aligned} \tag{24}
$$

- **Decoding and error analysis at the destination terminal:** The receiver uses backward decoding by waiting until the last block's transmission is received. It then jointly decodes $w_{v,b-1}, w'_{0,b-1}, w'_{1,b-1}, w'_{2,b-1}, w''_{0,b}, w''_{1,b}, w''_{2,b}$ for $b = B + 1, B, \ldots, 2$ by using $y_{D,b}$ and considering that its previously decoded messages are correct. The receiver tries to find $\widetilde{w_{v,b-1}}, \widetilde{w'_{0,b-1}}, \widetilde{w'_{1,b-1}}, \widetilde{w'_{2,b-1}}, \widetilde{w''_{0,b}}, \widetilde{w''_{1,b}}, \widetilde{w''_{2,b}}$ in a way that

$$
\begin{aligned}
&(v_b'^n(\widetilde{w_{v,b-1}}), v_{0,b}^n(\widetilde{w'_{0,b-1}}, \widetilde{w_{v,b-1}}), \\
&\ v_{1,b}^n(\widetilde{w'_{1,b-1}}, \widetilde{w'_{0,b-1}}, \widetilde{w_{v,b-1}}), v_{2,b}^n(\widetilde{w'_{2,b-1}}, \widetilde{w'_{0,b-1}}, \widetilde{w_{v,b-1}}), \\
&\ u_b'^n(w_{v,b}, \widetilde{w_{v,b-1}}), u_{0,b}^n(w'_{0,b}, \widetilde{w'_{0,b-1}}, w_{v,b}, \widetilde{w_{v,b-1}}), \\
&\ u_{1,b}^n(w'_{1,b}, \widetilde{w'_{1,b-1}}, w'_{0,b}, \widetilde{w'_{0,b-1}}, w_{v,b}, \widetilde{w_{v,b-1}}),
\end{aligned}
$$

$$
\begin{aligned}
&u^n_{2,b}(w'_{2,b}, \widetilde{w'_{2,b-1}}, w'_{0,b}, \widetilde{w'_{0,b-1}}, w_{v,b}, \widetilde{w_{v,b-1}}),\\
&q^n_b(\widetilde{w''_{0,b}}, w'_{0,b}, \widetilde{w'_{0,b-1}}, w_{v,b}, \widetilde{w_{v,b-1}}),\\
&x^n_{R,b}(\widetilde{w_{v,b-1}}, \widetilde{w'_{0,b-1}}, \widetilde{w'_{1,b-1}}, \widetilde{w'_{2,b-1}}),\\
&x^n_{1,b}(\widetilde{w''_{1,b}}, \widetilde{w''_{0,b}}, w'_{1,b}, \widetilde{w'_{1,b-1}}, w'_{0,b}, \widetilde{w'_{0,b-1}}, w_{v,b}, \widetilde{w_{v,b-1}}),\\
&x^n_{2,b}(\widetilde{w''_{2,b}}, \widetilde{w''_{0,b}}, w'_{2,b}, \widetilde{w'_{2,b-1}}, w'_{0,b},\\
&\qquad \widetilde{w'_{0,b-1}}, w_{v,b}, \widetilde{w_{v,b-1}}), y^n_{D,b})\\
&\in A^n_\varepsilon(V', V_0, V_1, V_2, U', U_0, U_1, U_2, Q, X_R, X_1, X_2, Y_D)
\end{aligned}
\tag{25}
$$

The relay uses joint decoding strategy to find $\widetilde{w_{v,b-1}}, \widetilde{w'_{0,b-1}}, \widetilde{w'_{1,b-1}}, \widetilde{w'_{2,b-1}}, \widetilde{w''_{0,b}}, \widetilde{w''_{1,b}}, \widetilde{w''_{2,b}}$ by using typicality decoding of equation (25). This can be done if:

$$
\begin{aligned}
&R''_1\\
&\quad \le I(X_1; Y_D|X_R, X_2, Q, V', V_0, V_1, V_2, U', U_0, U_1, U_2)\\
&R''_2\\
&\quad \le I(X_2; Y_D|X_R, X_1, Q, V', V_0, V_1, V_2, U', U_0, U_1, U_2)\\
&R''_1 + R''_2\\
&\quad \le I(X_1, X_2; Y_D|X_R, Q, V', V_0, V_1, V_2, U', U_0, U_1, U_2)\\
&R'_1 + R''_1\\
&\quad \le I(X_1, X_R, V_1, U_1; Y_D|X_2, Q, V', V_0, V_2, U', U_0, U_2)\\
&R'_2 + R''_2\\
&\quad \le I(X_2, X_R, V_2, U_2; Y_D|X_1, Q, V', V_0, V_1, U', U_0, U_1)\\
&R''_0 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, Q; Y_D|X_R, V', V_0, V_1, V_2, U', U_0, U_1, U_2)\\
&R'_1 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, V_1, U_1; Y_D|Q, V', V_0, V_2, U', U_0, U_2)\\
&R'_2 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, V_2, U_2; Y_D|Q, V', V_0, V_1, U', U_0, U_1)\\
&R'_1 + R''_0 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, Q, V_1, U_1; Y_D|V', V_0, V_2, U', U_0, U_2)\\
&R'_2 + R''_0 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, Q, V_2, U_2; Y_D|V', V_0, V_1, U', U_0, U_1)\\
&R'_1 + R'_2 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, V_1, U_1, V_2, U_2; Y_D|Q, V', V_0, U', U_0)\\
&R'_1 + R'_2 + R''_0 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, Q, V_1, U_1, V_2, U_2; Y_D|V', V_0, U', U_0)\\
&R'_0 + R'_1 + R'_2 + R''_0 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, Q, V_0, U_0, V_1, U_1, V_2, U_2; Y_D|V', U')\\
&R_V + R'_0 + R'_1 + R'_2 + R''_0 + R''_1 + R''_2\\
&\quad \le I(X_1, X_2, X_R, Q, V', U', V_0, U_0, V_1, U_1, V_2, U_2; Y_D)
\end{aligned}
\tag{26}
$$

Due to the distribution equation (7), the Markov chain $(V', V_0, V_1, V_2, U', U_0, U_1, U_2, Q) \Leftrightarrow (X_1, X_2, X_R) \Leftrightarrow (Y_D, Y_R)$ is used to simplify the aforementioned inequalities.

APPENDIX B
THE OUTER BOUND PROOF

$$
\begin{aligned}
nR_1 &= H(W_1) = I(W_1; Y^n_D) + H(W_1|Y^n_D)\\
&\le I(W_1; Y^n_D) + n\varepsilon_n \overset{(a)}{\to}\\
nR_1 &\le I(W_1; Y^n_D|W_0, W_2, W_v) + n\varepsilon_n\\
&= \sum_{i=1}^n I(W_1; Y_{D,i}|Y^{i-1}_D, W_0, W_2, W_v) + n\varepsilon_n\\
\overset{(a)}{\to} &= \sum_{i=1}^n I(W_1; Y_{D,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_2, W_v) + n\varepsilon_n\\
&= \sum_{i=1}^n H(Y_{D,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_2, W_v)\\
&\quad - H(Y_{D,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_1, W_2, W_v) + n\varepsilon_n\\
\overset{(b)}{\to} nR_1 &\le \sum_{i=1}^n H(Y_{D,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_2, W_v, X_{2,i})\\
&\quad - H(Y_{D,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_1, W_2, W_v, X_{1,i},\\
&\qquad X_{2,i}, X_{R,i}) + n\varepsilon_n\\
\overset{(c)}{\to} nR_1 &\le \sum_{i=1}^n H(Y_{D,i}|X_{2,i}, U_i, V_i)\\
&\quad - H(Y_{D,i}|X_{1,i}, X_{2,i}, X_{R,i}, U_i, V_i) + n\varepsilon_n\\
&= \sum_{i=1}^n I(X_{1,i}, X_{R,i}; Y_{D,i}|X_{2,i}, U_i, V_i) + n\varepsilon_n
\end{aligned}
\tag{27}
$$

where (a) follows from the fact that (W_1) is independent from $(W_0, W_2, W_v, Y^{i-1}_R)$, (b) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y^{i-1}_R, W_v, W_0, W_1)$, $(Y^{i-1}_R, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y^{i-1}_R, (c) follows from removing conditioning, and the Markovity of $(Y^{i-1}_D, Y^{i-1}_R, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

$$
\begin{aligned}
n(R_1 + R_2) &= H(W_1, W_2)\\
&= I(W_1, W_2; Y^n_D) + H(W_1, W_2|Y^n_D)\\
&\le I(W_1, W_2; Y^n_D) + n\varepsilon_n\\
&\le I(W_1, W_2; Y^n_D, Y^n_R) + n\varepsilon_n \overset{(a)}{\to}\\
n(R_1 + R_2) &\le I(W_1, W_2; Y^n_D, Y^n_R|W_0, W_v) + n\varepsilon_n\\
&= \sum_{i=1}^n I(W_1, W_2; Y_{D,i}, Y_{R,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_v)\\
&\quad + n\varepsilon_n\\
&= \sum_{i=1}^n H(Y_{D,i}, Y_{R,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_v)\\
&\quad - H(Y_{D,i}, Y_{R,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_1, W_2, W_v)\\
&\quad + n\varepsilon_n \overset{(b)}{\to}\\
n(R_1 + R_2) &\le \sum_{i=1}^n H(Y_{D,i}, Y_{R,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_v, X_{R,i})\\
&\quad - H(Y_{D,i}, Y_{R,i}|Y^{i-1}_D, Y^{i-1}_R, W_0, W_1, W_2,
\end{aligned}
$$

$$W_v, X_{1,i}, X_{2,i}, X_{R,i}) + n\varepsilon_n \overset{(c)}{\rightarrow}$$

$$\begin{aligned} n(R_1+R_2) &\leq \sum_{i=1}^{n} H(Y_{D,i}, Y_{R,i}|X_{R,i}, U_i, V_i) \\ &\quad - H(Y_{D,i}, Y_{R,i}|X_{1,i}, X_{2,i}, X_{R,i}, U_i, V_i) + n\varepsilon_n \\ &= \sum_{i=1}^{n} I(X_{1,i}, X_{2,i}; Y_{D,i}, Y_{R,i}|X_{R,i}, U_i, V_i) + n\varepsilon_n \end{aligned} \tag{28}$$

where (a) follows from the fact that (W_1, W_2) is independent from (W_0, W_v), (b) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y_R^{i-1}, W_v, W_0, W_1), (Y_R^{i-1}, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (c) follows from removing conditioning, and the Markovity of $(Y_D^{i-1}, Y_R^{i-1}, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

$$\begin{aligned} n(R_1+R_2) &= H(W_1, W_2) \\ &= I(W_1, W_2; Y_D^n) + H(W_1, W_2|Y_D^n) \\ &\leq I(W_1, W_2; Y_D^n) + n\varepsilon_n \overset{(a)}{\rightarrow} \\ n(R_1+R_2) &\leq I(W_1, W_2; Y_D^n|W_0, W_v) + n\varepsilon_n \\ &= \sum_{i=1}^{n} I(W_1, W_2; Y_{D,i}|Y_D^{i-1}, W_0, W_v) + n\varepsilon_n \\ \overset{(a)}{\rightarrow} &= \sum_{i=1}^{n} I(W_1, W_2; Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_v) \\ &\quad + n\varepsilon_n \\ &= \sum_{i=1}^{n} H(Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_v) \\ &\quad - H(Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_1, W_2, W_v) \\ &\quad + n\varepsilon_n \overset{(b)}{\rightarrow} \\ n(R_1+R_2) &\leq \sum_{i=1}^{n} H(Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_v) \\ &\quad - H(Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_1, W_2, W_v, \\ &\qquad X_{1,i}, X_{2,i}, X_{R,i}) + n\varepsilon_n \overset{(c)}{\rightarrow} \\ n(R_1+R_2) &\leq \sum_{i=1}^{n} H(Y_{D,i}|U_i, V_i) - \\ &\quad H(Y_{D,i}|X_{1,i}, X_{2,i}, X_{R,i}, U_i, V_i) + n\varepsilon_n \\ &= \sum_{i=1}^{n} I(X_{1,i}, X_{2,i}, X_{R,i}; Y_{D,i}|U_i, V_i) + n\varepsilon_n \end{aligned} \tag{29}$$

where (a) follows from the fact that (W_1, W_2) is independent from (W_0, W_v, Y_R^{i-1}), (b) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y_R^{i-1}, W_v, W_0, W_1), (Y_R^{i-1}, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (c) follows from removing conditioning, and the Markovity of $(Y_D^{i-1}, Y_R^{i-1}, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

$$\begin{aligned} &n(R_0+R_1+R_2) \\ &= H(W_0, W_1, W_2) \\ &= I(W_0, W_1, W_2; Y_D^n) + H(W_0, W_1, W_2|Y_D^n) \\ &\leq I(W_0, W_1, W_2; Y_D^n) + n\varepsilon_n \\ &\leq I(W_0, W_1, W_2; Y_D^n, Y_R^n) + n\varepsilon_n \overset{(a)}{\rightarrow} \\ &n(R_0+R_1+R_2) \\ &\leq I(W_0, W_1, W_2; Y_D^n, Y_R^n|W_v) + n\varepsilon_n \\ &= \sum_{i=1}^{n} I(W_0, W_1, W_2; Y_{D,i}, Y_{R,i}|Y_D^{i-1}, Y_R^{i-1}, W_v) + n\varepsilon_n \\ &= \sum_{i=1}^{n} H(Y_{D,i}, Y_{R,i}|Y_D^{i-1}, Y_R^{i-1}, W_v) \\ &\quad - H(Y_{D,i}, Y_{R,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_1, W_2, W_v) + n\varepsilon_n \overset{(b)}{\rightarrow} \\ &n(R_0+R_1+R_2) \\ &\leq \sum_{i=1}^{n} H(Y_{D,i}, Y_{R,i}|Y_D^{i-1}, Y_R^{i-1}, W_v, X_{R,i}) \\ &\quad - H(Y_{D,i}, Y_{R,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_1, W_2, \\ &\qquad W_v, X_{1,i}, X_{2,i}, X_{R,i}) + n\varepsilon_n \overset{(c)}{\rightarrow} \\ &n(R_0+R_1+R_2) \\ &\leq \sum_{i=1}^{n} H(Y_{D,i}, Y_{R,i}|X_{R,i}, V_i) \\ &\quad - H(Y_{D,i}, Y_{R,i}|X_{1,i}, X_{2,i}, X_{R,i}, U_i, V_i) + n\varepsilon_n \\ &= \sum_{i=1}^{n} I(X_{1,i}, X_{2,i}; Y_{D,i}, Y_{R,i}|X_{R,i}, V_i) + n\varepsilon_n \end{aligned} \tag{30}$$

where (a) follows from the fact that (W_0, W_1, W_2) is independent from (W_v), (b) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y_R^{i-1}, W_v, W_0, W_1), (Y_R^{i-1}, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (c) follows from removing conditioning, and the Markovity of $(Y_D^{i-1}, Y_R^{i-1}, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

$$\begin{aligned} &n(R_0+R_1+R_2) \\ &= H(W_0, W_1, W_2) \\ &= I(W_0, W_1, W_2; Y_D^n) + H(W_0, W_1, W_2|Y_D^n) \\ &\leq I(W_0, W_1, W_2; Y_D^n) + n\varepsilon_n \overset{(a)}{\rightarrow} \\ &n(R_0+R_1+R_2) \\ &\leq I(W_0, W_1, W_2; Y_D^n|W_v) + n\varepsilon_n \\ &= \sum_{i=1}^{n} I(W_0, W_1, W_2; Y_{D,i}|Y_D^{i-1}, W_v) + n\varepsilon_n \overset{(a)}{\rightarrow} \\ &= \sum_{i=1}^{n} I(W_0, W_1, W_2; Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_v) + n\varepsilon_n \\ &= \sum_{i=1}^{n} H(Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_v) \\ &\quad - H(Y_{D,i}|Y_D^{i-1}, Y_R^{i-1}, W_0, W_1, W_2, W_v) \\ &\quad + n\varepsilon_n \overset{(b)}{\rightarrow} \end{aligned}$$

$$\begin{aligned}
&n(R_0+R_1+R_2)\\
&\quad\le \sum_{i=1}^{n} H(Y_{D,i}|Y_D^{i-1},Y_R^{i-1},W_v)\\
&\qquad - H(Y_{D,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_1,W_2,\\
&\qquad\qquad W_v,X_{1,i},X_{2,i},X_{R,i})+n\varepsilon_n \overset{(c)}{\rightarrow}\\
&n(R_0+R_1+R_2)\\
&\quad\le \sum_{i=1}^{n} H(Y_{D,i}|V_i)\\
&\qquad - H(Y_{D,i}|X_{1,i},X_{2,i},X_{R,i},U_i,V_i)+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} I(X_{1,i},X_{2,i},X_{R,i};Y_{D,i}|V_i)+n\varepsilon_n
\end{aligned}\tag{31}$$

where (a) follows from the fact that (W_0, W_1, W_2) is independent from (W_v, Y_R^{i-1}), (b) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y_R^{i-1}, W_v, W_0, W_1), (Y_R^{i-1}, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (c) follows from removing conditioning, and the Markovity of $(Y_D^{i-1}, Y_R^{i-1}, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

$$\begin{aligned}
&n(R_V+R_0+R_1+R_2)\\
&\quad= H(W_v,W_0,W_1,W_2)\\
&\quad= I(W_v,W_0,W_1,W_2;Y_D^n)+H(W_v,W_0,W_1,W_2|Y_D^n)\\
&\quad\le I(W_v,W_0,W_1,W_2;Y_D^n)+n\varepsilon_n\\
&\quad\le I(W_v,W_0,W_1,W_2;Y_D^n,Y_R^n)+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} I(W_v,W_0,W_1,W_2;Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1})+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1})\\
&\qquad - H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_1,W_2,W_v)+n\varepsilon_n \overset{(a)}{\rightarrow}\\
&n(R_V+R_0+R_1+R_2)\\
&\quad\le \sum_{i=1}^{n} H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},X_{R,i})\\
&\qquad - H(Y_{D,i},Y_{R,i}|Y_D^{i-1},Y_R^{i-1},\\
&\qquad\qquad W_0,W_1,W_2,W_v,X_{1,i},X_{2,i},X_{R,i})+n\varepsilon_n \overset{(b)}{\rightarrow}\\
&n(R_V+R_0+R_1+R_2)\\
&\quad\le \sum_{i=1}^{n} H(Y_{D,i},Y_{R,i}|X_{R,i})\\
&\qquad - H(Y_{D,i},Y_{R,i}|X_{1,i},X_{2,i},X_{R,i},U_i,V_i)+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} I(X_{1,i},X_{2,i};Y_{D,i},Y_{R,i}|X_{R,i})+n\varepsilon_n
\end{aligned}\tag{32}$$

where (a) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y_R^{i-1}, W_v, W_0, W_1), (Y_R^{i-1}, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (b) follows from removing conditioning, and the Markovity of $(Y_D^{i-1}, Y_R^{i-1}, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

$$\begin{aligned}
&n(R_V+R_0+R_1+R_2)\\
&\quad= H(W_v,W_0,W_1,W_2)\\
&\quad= I(W_v,W_0,W_1,W_2;Y_D^n)+H(W_v,W_0,W_1,W_2|Y_D^n)\\
&\quad\le I(W_v,W_0,W_1,W_2;Y_D^n)+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} I(W_v,W_0,W_1,W_2;Y_{D,i}|Y_D^{i-1})+n\varepsilon_n \overset{(a)}{\rightarrow}\\
&\quad\le \sum_{i=1}^{n} I(W_v,W_0,W_1,W_2;Y_{D,i}|Y_D^{i-1},Y_R^{i-1})+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} H(Y_{D,i}|Y_D^{i-1},Y_R^{i-1})\\
&\qquad - H(Y_{D,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_1,W_2,W_v)+n\varepsilon_n \overset{(b)}{\rightarrow}\\
&n(R_V+R_0+R_1+R_2)\\
&\quad\le \sum_{i=1}^{n} H(Y_{D,i}|Y_D^{i-1},Y_R^{i-1})\\
&\qquad - H(Y_{D,i}|Y_D^{i-1},Y_R^{i-1},W_0,W_1,W_2,W_v,X_{1,i},X_{2,i},X_{R,i})\\
&\qquad +n\varepsilon_n \overset{(c)}{\rightarrow}\\
&n(R_V+R_0+R_1+R_2)\\
&\quad\le \sum_{i=1}^{n} H(Y_{D,i})\\
&\qquad - H(Y_{D,i}|X_{1,i},X_{2,i},X_{R,i},U_i,V_i)+n\varepsilon_n\\
&\quad= \sum_{i=1}^{n} I(X_{1,i},X_{2,i},X_{R,i};Y_{D,i})+n\varepsilon_n
\end{aligned}\tag{33}$$

where (a) follows from the fact that (W_v, W_0, W_1, W_2) is independent from (Y_R^{i-1}), (b) is due to $X_{1,i}, X_{2,i}$ are deterministic functions of $(Y_R^{i-1}, W_v, W_0, W_1), (Y_R^{i-1}, W_v, W_0, W_2)$, respectively, and deterministic relation between $X_{R,i}$ and Y_R^{i-1}, (c) follows from removing conditioning, and the Markovity of $(Y_D^{i-1}, Y_R^{i-1}, W_v, W_0, W_1, W_2) \Leftrightarrow (X_{1,i}, X_{2,i}, X_{R,i}) \Leftrightarrow (Y_{D,i}, Y_{R,i})$.

APPENDIX C
GAUSSIAN INNER BOUND

- **At the destination:**

$$\begin{aligned}
R_1 &\le I(X_1,X_R;Y_D|X_2,U',U_0,V',V_0,V_2)\\
&= h(Y_D|X_2,U',U_0,V',V_0,V_2)\\
&\quad - h(Y_D|X_R,X_1,X_2,U',U_0,V',V_0,V_2)\\
&= h(X_1+X_2+X_R+Z_D|X_2,U',U_0,V',V_0,V_2)-h(Z_D)\\
&= h(\sqrt{P_1}(\sqrt{\alpha_1}N_1+\sqrt{\beta_1}P+\sqrt{\gamma_1}M+\sqrt{\mu_1}T_1+\sqrt{\theta_1}L\\
&\quad+\sqrt{(\alpha_1+\beta_1+\gamma_1+\mu_1+\theta_1)}V')\\
&\quad+\sqrt{P_R}(\sqrt{\beta_R}L+\sqrt{\gamma_R}T_1+\sqrt{\alpha_R}T_2\\
&\quad+\sqrt{(\alpha_R+\beta_R+\gamma_R)}V')+Z_D|X_2,U',U_0,V',V_0,V_2)\\
&\quad - h(Z_D) \overset{(b)}{\Rightarrow}
\end{aligned}$$

$$
\begin{aligned}
&= h(\sqrt{\alpha_1 P_1}N_1 + (\sqrt{\mu_1 P_1} + \sqrt{\gamma_R P_R})T_1 + Z_D) - h(Z_D)\\
&= \frac{1}{2}\log(2\pi e(\alpha_1 P_1 + (\sqrt{\mu_1 P_1} + \sqrt{\gamma_R P_R})^2 + N_D))\\
&\quad - \frac{1}{2}\log(2\pi e N_D)\\
&= \frac{1}{2}\log(1 + \frac{\alpha_1 P_1 + (\sqrt{\mu_1 P_1} + \sqrt{\gamma_R P_R})^2}{N_D})\\
&= C(\frac{\alpha_1 P_1 + (\sqrt{\mu_1 P_1} + \sqrt{\gamma_R P_R})^2}{N_D}) \qquad (34)
\end{aligned}
$$

Where (b) follows since knowing (U', U_0, V', V_0, V_2), reveals L, T_2, P, M. Similarly we obtain:

$$
\begin{aligned}
&R_2\\
&\quad \leq C(\frac{\alpha_2 P_2 + (\sqrt{\mu_2 P_2} + \sqrt{\alpha_R P_R})^2}{N_D}) \qquad (35)\\
&R_1 + R_2\\
&\quad \leq C\left(\frac{\begin{pmatrix}\alpha_1 P_1 + \alpha_2 P_2 + \\ (\sqrt{\mu_1\, P_1} + \sqrt{\gamma_R P_R})^2 + \\ (\sqrt{\mu_2\, P_2} + \sqrt{\alpha_R P_R})^2\end{pmatrix}}{N_D}\right) \qquad (36)\\
&R_0 + R_1 + R_2\\
&\quad \leq C\left(\frac{\begin{pmatrix}\alpha_1 P_1 + \alpha_2 P_2 + \\ (\sqrt{\mu_1\, P_1} + \sqrt{\gamma_R P_R})^2 + \\ (\sqrt{\mu_2\, P_2} + \sqrt{\alpha_R P_R})^2 + \\ +(\sqrt{\gamma_1\, P_1} + \sqrt{\gamma_2\, P_2})^2 + \\ (\sqrt{\theta_1\, P_1} + \sqrt{\theta_2\, P_2} + \sqrt{\beta_R P_R})^2\end{pmatrix}}{N_D}\right)\\
&\qquad (37)\\
&R_V + R_0 + R_1 + R_2\\
&\quad \leq C\left(\frac{\begin{pmatrix}\alpha_1 P_1 + \alpha_2 P_2 + \\ (\sqrt{\mu_1\, P_1} + \sqrt{\gamma_R P_R})^2 + \\ (\sqrt{\mu_2\, P_2} + \sqrt{\alpha_R P_R})^2 + \\ (\sqrt{\gamma_1\, P_1} + \sqrt{\gamma_2\, P_2})^2 + \\ (\sqrt{\theta_1\, P_1} + \sqrt{\theta_2\, P_2} + \sqrt{\beta_R P_R})^2 + \\ (\sqrt{\beta_1\, P_1} + \sqrt{\beta_2\, P_2})^2 + \\ \begin{pmatrix}(\sqrt{\alpha_1 + \beta_1 + \gamma_1 + \mu_1 + \theta_1}P_1 + \\ \sqrt{\alpha_2 + \beta_2 + \gamma_2 + \mu_2 + \theta_2}P_2 + \\ \sqrt{\alpha_R + \beta_R + \gamma_R}P_R)\end{pmatrix}^2\end{pmatrix}}{N_D}\right) \qquad (38)
\end{aligned}
$$

REFERENCES

[1] G. Kramer and A. J. van Wijngaarden, "On the white Gaussian multiple-access relay channel," in *Proc. IEEE Int. Symp. Inf. Theory*, Jun. 2000, p. 40.

[2] L. Sankar, "Relay cooperation in multiaccess networks," Ph.D. dissertation, Graduate School—New Brunswick Rutgers, New Brunswick, NJ, USA, 2007.

[3] M. Osmani-Bojd and G. A. Hodtani, "On multiple-access relay channel with common message," *Trans. Emerg. Telecommun. Technol.*, vol. 27, no. 8, pp. 1030–1043, 2016.

[4] A. Sahebalam and G. A. Hodtani, "General and new inner bound for multiple-access relay channel and two certain capacity theorems," *IET Commun.*, vol. 7, no. 13, pp. 1348–1359, 2013.

[5] Y. Murin, R. Dabora, and D. Gündüz, "Source-channel coding theorems for the multiple-access relay channel," *IEEE Trans. Inf. Theory*, vol. 59, no. 9, pp. 5446–5465, Sep. 2013.

[6] Z. Sattar, Y. Jiang, and F. C. M. Lau, "Relay cooperation schemes for the multiple access relay channel: Compute-and-forward and successive interference cancellation," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2014, pp. 1070–1075.

[7] X. Zhou, P.-S. Lu, K. Anwar, and T. Matsumoto, "Correlated sources transmission in orthogonal multiple access relay channel: Theoretical analysis and performance evaluation," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1424–1435, Mar. 2014.

[8] J. He, I. Hussain, V. Tervo, M. Juntti, and T. Matsumoto, "Performance analysis for transmission of correlated sources over non-orthogonal MARCs," in *Proc. 9th Int. Symp. Turbo Codes Iterative Inf. Process. (ISTC)*, Sep. 2016, pp. 66–70.

[9] L. Liu, Y. Li, Y. Su, and Y. Sun, "Quantize-and-forward strategy for interleave-division multiple-access relay channel," *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1808–1814, Mar. 2016.

[10] E. M. Taghavi, M. Monemizadeh, S. Bahrami, and G. A. Hodtani, "An energy efficient coding scheme for the multiple access relay channel," in *Proc. Int. Symp. Inf. Theory Appl. (ISITA)*, Oct./Nov. 2016, pp. 196–200.

[11] E. M. Taghavi and G. A. Hodtani, "Energy-efficient communications over multiple access relay channels," *Int. J. Commun. Syst.*, vol. 30, no. 18, p. e3368, 2017.

[12] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Probab.*, vol. 3, no. 1, pp. 120–154, 1971.

[13] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-25, no. 5, pp. 572–584, Sep. 1979.

[14] A. El Gamal and S. Zahedi, "Capacity of a class of relay channels with orthogonal components," *IEEE Trans. Inf. Theory*, vol. 51, no. 5, pp. 1815–1817, May 2005.

[15] A. El Gamal and M. Aref, "The capacity of the semideterministic relay channel (Corresp.)," *IEEE Trans. Inf. Theory*, vol. IT-28, no. 3, p. 536, May 1982.

[16] G. A. Hodtani and M. R. Aref, "Unified approach to the capacity evaluation of the relay channel," *IET Commun.*, vol. 3, no. 7, pp. 1208–1215, 2009.

[17] Y. Gabbai and S. I. Bross, "Achievable rates for the discrete memoryless relay channel with partial feedback configurations," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 4989–5007, Nov. 2006.

[18] S. I. Bross and M. A. Wigger, "On the relay channel with receiver–transmitter feedback," *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 275–291, Jan. 2009.

[19] J. Jiang, Y. Xin, and H. V. Poor, "Achievable rates for discrete memoryless relay channels with generalised feedback," *Trans. Emerg. Telecommun. Technol.*, vol. 24, no. 2, pp. 212–231, 2013.

[20] J. Hou, R. Koetter, and G. Kramer, "Rate regions for multiple access relay channels with relay-source feedback," in *Proc. IEEE Inf. Theory Workshop (ITW)*, Oct. 2009, pp. 288–292.

[21] D. Slepian and J. K. Wolf, "A coding theorem for multiple access channels with correlated sources," *Bell Syst. Tech. J.*, vol. 52, no. 7, pp. 1037–1076, 1973.

[22] T. Cover and C. Leung, "An achievable rate region for the multiple-access channel with feedback," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 3, pp. 292–298, May 1981.

[23] L. H. Ozarow, "The capacity of the white Gaussian multiple access channel with feedback," *IEEE Trans. Inf. Theory*, vol. IT-30, no. 4, pp. 623–629, Jul. 1984.

[24] R. G. Gallager, *Information Theory and Reliable Communication*, vol. 2. Vienna, Austria: Springer, 1968.

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