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Solving fuzzy number quadratic programming problems using variable neighborhood search algorithm

Khatere Ghorbani-Moghadam[1,*], Reza Ghanbari [2] and Nezam Mahdavi-Amiri [3]

[1] *Faculty of Mathematical Sciences, Sharif University of Technology, Tehran, Iran*

[*] ***Lecturer.*** *Email: kh_ghorbani@mehr.sharif.ir*

[2] *Faculty of Mathematical Sciences, Department of Applied Mathematics,*

Ferdowsi University of Mashhad, Mashhad, Iran

Email: rghanbari@um.ac.ir

[3] *Faculty of Mathematical Sciences, Sharif University of Technology, Tehran, Iran*

Email: nezamm@sharif.edu

Abstract

Recently, we proposed modified Kerre's method as a more efficient approach than the original Kerre's approach for comparison of *LR* fuzzy numbers. Using the modified Kerre's method, we proposed a new variable neighborhood search algorithm for solving fuzzy number quadratic programming problems. In our algorithm, the local search is defined based on descent directions. We show the effectiveness of our proposed method in comparison with some available methods by using a non-parametric statistical sign test.

Keywords: Fuzzy quadratic programming problem, Modified Kerre's method, Ranking function.

1. Introduction

One type of fuzzy number quadratic programming problem is defined as follows:

$$(QP)\quad \begin{cases} \min \tilde{f}(x) = \dfrac{1}{2} x^T \tilde{Q} x + \tilde{c}^T x \\ s.t. \\ Ax \le b, \\ x \ge 0, \end{cases} \qquad (1)$$

where $x \in \mathbb{R}^n$, $\tilde{c} = (c^L / c / c^R) \in F(\mathbb{R}^n)$, $\tilde{Q} = (Q^L / Q / Q^R) \in F(\mathbb{R}^{n\times n})$, an $n \times n$ symmetric matrix with fuzzy entries, $b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m\times n}$, with $F(\mathbb{R}), F(\mathbb{R}^n), F(\mathbb{R}^{n\times n})$ respectively denoting the set of all triangular fuzzy numbers, vectors and matrices.

Fuzzy quadratic programming problem with fuzzy coefficients have been treated with different methods. For example, Luozhong et al. [6] analyzed a fuzzy quadratic programming problem with fuzzy parameters. To solve it, they defined a crisp model equivalent to the original fuzzy problem, and then used the optimal solution of the model as the optimal solution of the quadratic programming problem. Nasseri [7] gave a method to solve fuzzy version of the quadratic optimization problem with fuzzy coefficients by using a ranking function. Kheirfam [4], using a fuzzy ranking and arithmetic operations, transformed a quadratic programming problem with fuzzy numbers and fuzzy variables to a crisp model with non-linear objective and linear constraints and then by solving the new problem, he obtained a fuzzy optimal solution. Also, Kheirfam and Verdegay [5] made a strict sensitivity analysis of a fuzzy quadratic programming problem considering simultaneous and independent variations in the right-hand-side of the constraints and the coefficients of the objective function.

Here, we are to solve a type of fuzzy quadratic programming problem defined as (1) directly. We propose a new variable neighborhood search (VNS) algorithm with its local search intended to find a feasible solution by using descent directions. To compare fuzzy objective values, we make use of our modified Kerre's method [2].

In Section 2, we provide basic aspects of modified Kerre's method. In Section 3, we present our VNS algorithm for solving (QP). We conclude in Section 4.

2. Modified Kerre's method and its properties

In the original Kerre's method [1] for comparison of two triangular fuzzy numbers such as $\tilde{M}=(a/b/c)$ and $\tilde{N}=(a',b',c')$, first the fuzzy max $(\tilde{\max})$ of these two fuzzy numbers is computed using the extension principle. Then, according to this method, it is said that $\tilde{M}\le\tilde{N}$, if $d(\tilde{M},\tilde{\max}(\tilde{M},\tilde{N}))-d(\tilde{N},\tilde{\max}(\tilde{M},\tilde{N}))\ge 0$ and $\tilde{M}\ge\tilde{N}$, $d(\tilde{M},\tilde{\max}(\tilde{M},\tilde{N}))-d(\tilde{N},\tilde{\max}(\tilde{M},\tilde{N}))\le 0$, where $d(.,.)$ is the Hamming distance between two fuzzy numbers (see details in [1]). In our modified Kerre's method [2], we devised a technique that lead to a direct and efficient formula, $r(\tilde{M},\tilde{N})$ below, for use in the comparison of two triangular fuzzy numbers, without the need to compute the fuzzy maximum, $\tilde{\max}$. The value of $r(\tilde{M},\tilde{N})$ is defined to be (see [2]),

$$r(\tilde{M},\tilde{N})=\begin{cases}\dfrac{(c'-a')}{2}+\dfrac{(c-a)}{2}, & c\le a'\\ \dfrac{(c'+a')}{2}-\dfrac{(c+a)}{2}, & b=b'\\ \dfrac{(c'-a')}{2}+\dfrac{(c-a)}{2}-\bar{y}(c-a'), & b<b'\end{cases} \qquad (2)$$

where $\bar{y}$ is the common membership value of the intersecting point of the left arm of $\tilde{N}$ and the right arm of $\tilde{M}$. Using (2), we showed that if $r(\tilde{M},\tilde{N})\ge 0$, then $\tilde{M}\le^{K}\tilde{N}$ else $\tilde{M}\ge^{K}\tilde{N}$.

3. VNS algorithm for solving a quadratic programming problem with fuzzy objective coefficients

VNS is a meta-heuristic algorithm first proposed by Hansen and Meladenovic [3]. In each iteration ($k=1,\ldots,\max\_iter$) of the algorithm, a search composed of at most $t_{\max}$ local searches is made to improve upon a given point x_0. A local search starts with a so-called shaking phase to select a randomly generated point x_1. Then, the local search uses x_1 to obtain another solution x_2. Afterwards, VNS compares x_2 to x_0 in terms of their objective function values. If the objective function value at x_2 is less than the one at x_0, then the initial solution will be changed to x_2 and the algorithm proceeds to the next iteration; otherwise, VNS widens the neighborhood around x_0 and a new local search is made in the widened neighborhood. The VNS algorithm is terminated either after finding a new x_0 for the next iteration or after conducting $t_{\max}$ local searches. Finally, the best solution is the x_0 obtained after $\max\_iter$ iterations. Algorithm 1 gives the steps needed for solving the problem (QP).

Algorithm 1 VNS algorithm for solving quadratic programming problems.

1. **Inputs:** neighboring structures $N_t = 1,\ldots,\max$, an initial solution x_0, $\max\_iter$ (maximum number of iterations). **Output**: x^*.
2. **While** $k \le \max\_iter$ **do**

   begin

   2.1 $t = 1$.

   2.1.1 **if** $t < t_{\max}$ **then do**

   begin

   2.1.2 Using some criteria, select a point x_1 around neighborhood of x_0 $(x_1 \in N_t(x_0))$.

   2.1.3 Apply a local search on x_1 and name the new point as x_2.

   2.1.4 **If** $\tilde{f}(x_2) \ge^K \tilde{f}(x_0)$ **then** let $t = t + 1$ and go to 2.1.1 **else** go to 2.2.

   end

   else go to 2.3.

   2.2 $x_0 = x_2$.

   2.3 $k = k + 1$.

   end
3. Return $x^* = x_0$.

Notes on Step 2.1.2:

Neighboring structure: We define the neighboring structure as $\| x' - x_0 \| < t\varepsilon$, for $t = 1,\ldots,t_{\max}$, where x' is in the t-th neighborhood of x_0 and ε is a arbitrary parameter (we set to be $\varepsilon = 10^{-3}$).

- **Selection of a point in the neighborhood of x_0:** A point x' in neighborhood of x_0 is selected if

  (*i*) $(t-1)\varepsilon < \| x - x_0 \| \le t\varepsilon$, (*ii*) $Ax' \le b$, (*iii*) $x' \ge 0$.

We define $x' = x_0 + \alpha d$, where $\alpha \ge 0$. So, we need to find α and d to satisfy conditions (*i*), (*ii*) and (*iii*). Therefore, α and d must satisfy the following conditions:

(1) $$(t-1)\varepsilon < \alpha d \le t\varepsilon,$$

(2)

$$\alpha \le \min\{\min\{\frac{(b_i - a_i^T x)}{a_i^T d},\ a_i^T d > 0\},\ \min\{\frac{-x_j}{d_j},\ d_j < 0\}\},\ i = 1,\ldots,m, \quad j = 1,\ldots,n.$$

Notes on Step 2.1.3:

- **Local search:** Supposing that x_k is a feasible solution at iteration k, we want to find a feasible solution with less value of the objective function as compared to the objective value due to x_k. In other words, we should find s_k such that $\tilde{f}(x_k + s_k) <^{K} \tilde{f}(x_k)$. Since the given results for the modified Kerre's method depend on the mean value of the fuzzy numbers, to find s_k we consider the following three cases:

Case 1: $\frac{1}{2}x_k^T Q x_k + c^T x_k \le \frac{1}{2}(x_k + s_k)^T Q(x_k + s_k) + c^T(x_k + s_k) \to \frac{1}{2}x_k^T Q s_k + \frac{1}{2}s_k^T Q x_k + \frac{1}{2}s_k^T Q s_k + c^T s_k \ge 0$,

Case 2: $\frac{1}{2}(x_k + s_k)^T Q(x_k + s_k) + c^T(x_k + s_k) \le \frac{1}{2}x_k^T Q x_k + c^T x_k \to \frac{1}{2}x_k^T Q s_k + \frac{1}{2}s_k^T Q x_k + \frac{1}{2}s_k^T Q s_k + c^T s_k \le 0$,

Case 3: $\frac{1}{2}(x_k + s_k)^T Q(x_k + s_k) + c^T(x_k + s_k) = \frac{1}{2}x_k^T Q x_k + c^T x_k \to \frac{1}{2}x_k^T Q s_k + \frac{1}{2}s_k^T Q x_k + \frac{1}{2}s_k^T Q s_k + c^T s_k = 0$.

To find the descent direction, we proposed five crisp mathematical programming problems for the above cases by using the sign of $r(\tilde{f}(x_k + s_k), \tilde{f}(x_k))$. We generated and solved some randomly test examples with triangular fuzzy coefficients and made a comparative study to show the effectiveness of the proposed algorithm. We used the non-parametric statistical sign test to analyze the performance of our algorithm. The obtained results showed that the objective function values obtained by our proposed algorithm were less than the ones obtained by the algorithms due to Luozhong et al. [6], Nasseri [7] and Kheirfam [4].

4. Conclusion

Considering a quadratic programming problem with fuzzy numbers, we made use of a modified Kerre’s method for comparison of LR fuzzy numbers to present an algorithm to solve the problem directly. In contrast to some available methods in the literature, here we proposed a variable neighborhood search algorithm using the modified Kerre’s method without the need to change the problem into a crisp program.

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