Analytical Approaches for Determining Optimal Tilt Angle and Orientation of PV Modules Considering Regional Climate Conditions

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***Abstract*—The increasing growth of solar power plants installation necessitates performing accurate investigations on new methods to find optimum tilt angel and orientation of photovoltaic (PV) panels. Using optimum methods increases collected solar irradiance, and consequently, maximize the expected annual harvested energy, which means better financial performance of PV power plants. In this paper, the optimum tilt angle and orientation of PV panels are calculated by introducing two accurate analytic methods, which take into account the climate conditions of the installation region by considering wind speed, irradiance and ambient temperature. The results of this paper, which are supported by Monte Carlo simulation, indicate that compared with the traditional method, an increase in expected annual produced energy of solar panels is achievable using the optimum tilt angel and orientation calculated by the presented methods.**

***Keywords*-photovoltaic; tilt angle; orientation; analytical method; Monte Carlo.**

I. Introduction

The use of photovoltaic (PV) panels for electricity production is increasing rapidly. According to [1], the global installed capacity of solar PV reached up to 303 gigawatts in 2016. It is estimated that this trend will continue. Produced power of a PV panel depends highly on the received solar irradiance that varies with it's orientation and tilt angle. Maximum irradiance can be absorbed using a dual-axis sun tracker which follows the sun trajectory. However, using a sun tracking system will increase the total cost of the PV system significantly. Therefore, there is a trade-off between the amount of power produced by PV panels and the cost of using sun tracker systems. In many cases, considering the global price drop of PV panels, new PV systems are designed with a fixed position, i.e. without any active part to follow the sun. In these cases, determining the optimum tilt angle and orientation of the PV panels in order to get maximum energy is a crucial issue.

So far, various models have been proposed to formulate the output power of solar panels under different conditions. An accurate I-V model of solar cells is inherently implicit and nonlinear, which calls for iterative calculations to obtain an analytical expression of electric current as a function of voltage. For example, in [2] three physically-based models describing the PV cell (corresponding to three, four and five parameter equivalent electric circuit) and two thermal models for cell temperature estimation are compared. In models presented in [3], various parameters are involved, such as ambient temperature, irradiance, and wind speed. The model derives the operating panel temperature taking into account the power conversion performed by the panel. There are also studies that have been able to drive an explicit model for the panel like the work in [4] that proposed an exponential function of I-V characteristic equation using the padé approximants and an explicit analytical description of current was obtained. In another work by this author in [5], a new method based on Taylor series expansion was proposed which yields an explicit description of current. Furthermore, two different modified five-parameter models based on Taylor series expansion were constructed.

The environmental conditions are one of the factors that can greatly influence the performance of solar panels, as discussed in [6]. In this work, the effects of module temperature, ambient temperature, wind speed, wind direction and relative humidity on the PV panel is examined based on the field monitored data. Finally, authors in [7] focus on modeling solar panel characteristics by curve fitting the nonlinear I-V equation using three main points: open circuit, maximum power, and short circuit.

In [8], solar radiation is predicted using two different approaches. After that, the optimal tilt angle is calculated using both short-term and long-term analysis techniques and finally, daily and monthly optimal tilt angles are achieved. Hourly solar radiation on fifteen PV modules with different tilt angles is estimated in [9] using four mathematical models and annual weather data. Besides, optimal tilt angle in a specific area (Wuhan, China) for yearly and semi-yearly adjustment is determined. Obtained results are verified using practical experiments and the most accurate model is determined. Statistical distribution of measured radiance is integrated in [10] to calculate the solar radiation on tilted panels using a numerical approach. The annual and monthly radiation

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absorbed by different sloped surfaces and different orientations in a specific region in Hong Kong is determined and output energy of PV panels is simulated using a computer program called TRNSYS. Then, an optimum tilt angle and orientation for maximum annual absorbed radiation is presented. Karafil *et al.* [11] mathematically analyzed the monthly, seasonal and annual optimum fixed tilt angles of PV panels based on solar angles. They presented optimum tilt angles for Bilecik city in Turkey and evaluated their results by practical experiments.

The harmony search (HS) meta-heuristic algorithm is employed in [12] to calculate the optimum tilt and orientation angles by maximizing the extraterrestrial radiation on the panel surface for a specific period of time. In [13], researchers took into account the hourly power demand of New York City to accommodate the tilt and orientation such that the solar energy reaching the panel best match the energy consumption pattern. Genetic algorithm (GA) and the simulated-annealing (SA) method is utilized in [14] to calculate the optimum installation angle for fixed solar-cell panels in different areas around Taiwan. The results from GA and SA are compared with hardware experimental results.

The aim of this paper is to propose two analytical methods which can handle the calculation of optimal tilt angle and orientation of a PV panel as a stochastic optimization problem. After calculating the results, Monte Carlo simulation is used for evaluation. The proposed methods are applied on an explicit model for PV generation considering panel's temperature as a function of ambient temperature, wind speed and solar radiation. One-minute historical data from the National Renewable Energy Laboratory (NREL) [15], located in the United States, is used to obtain results and assess the methods' efficiency.

The remainder of the paper is organized as follows. In section II the proposed methods are introduced. Simulation results are presented in section III and finally, section IV draws relevant conclusions.

II. Proposed Methods

A. Mathematical Principles

Regardless of installing a PV panel on a rooftop or on the ground as a power plant, one of the key decisions is how to orientate the panel. Since using a sun tracker will force further investment and maintenance costs, many decide to install the panel at a fixed position. The question which arises here is at what angle should the panel be installed to get the most out of the sun beam.

There are many models for expressing different aspects of PV panel characteristics. In this study, the model must explicitly relate the output power produced by the panel, $P_{pv}(t)$, to the input variables such as solar radiation, ambient temperature, wind speed, sun's position in the sky and panel orientation. For this purpose, the model brought in [16] was adopted:

$$P_{pv}(t) = P_n Y_d \frac{S_{module}(t)}{I_s}[1 - \frac{K_p}{100}(T_c(t) - T_{STC})] \quad (1)$$

TABLE I
The Constant Parameters

Parameter	Unit	Value
P_n	Watt	310
Y_d	–	0.93
I_s	W/m^2	1000
K_p	$\%/^\circ C$	0.41
T_{STC}	$^\circ C$	25
T_{NOCT}	$^\circ C$	45
S_0	W/m^2	800
h	W/m^2 $^\circ C$	6.62
T_0	$^\circ C$	20
ν_0	m/s	1

where P_n is the nominal output power of the panel (W), I_s is standard radiation (W/m^2), K_p is power temperature coefficient ($\%/^\circ C$), T_{STC} is temperature in standard test conditions ($^\circ C$), $T_c(t)$ is the panel temperature ($^\circ C$), Y_d is the derating factor due to air pollution and dust concentration. $S_{module}(t)$ is the perpendicular component of irradiance. According to [3] the panel temperature $T_c(t)$ in (1) is a function of ambient temperature, wind speed and irradiance, which is shown in (2).

$$T_c(t) = T_{amb}(t) + \frac{S(t)(T_{NOCT} - T_0)}{S_0 + h(\nu(t) - \nu_0)(T_{NOCT} - T_0)} \quad (2)$$

In (2), $T_{amb}(t)$ is ambient temperature ($^\circ C$), T_{NOCT} is the nominal operating cell temperature ($^\circ C$) which depends on cell characteristics, $\nu(t)$ is wind speed velocity (m/s), ν_0 is nominal wind speed (m/s), $S(t)$ is global irradiance (W/m^2) and h is a convection parameter and its suggested value in literature [17] is 6.62 ($W/m^{2\circ}C$). The parameters T_0=20$^\circ C$ and S_0=800 W/m^2 are constants.

To determine the power produced by a PV panel, one of the key parameters is the perpendicular component of the sun's beam on the panel's surface. In order to calculate this component on an arbitrary oriented surface, the method proposed in [18] was adapted. In this method, two angels are utilized to describe the sun's position in the sky and two other angels for determining the panel tilt and orientation. Using these four angels, the inner product of the unit vector pointing at the sun's position and the unit normal vector of the panel can be obtained, which results in finding the coefficient needed to express the perpendicular component of the sun's beam intensity. The direct irradiance data must be converted to vertical radiation using the following equation:

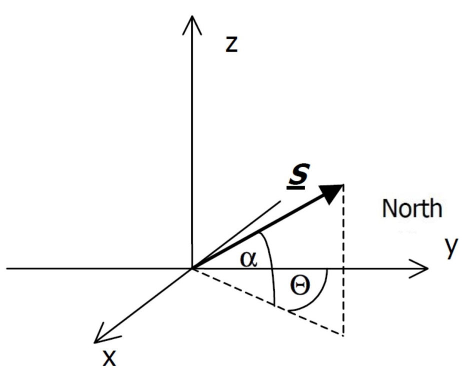
$$S_{module} = S_{in}[\cos(\alpha)\sin(\beta)\cos(\psi - \theta) + \sin(\alpha)\cos(\beta)] \quad (3)$$

where α is the sun elevation angle which is the angle the sun is above the horizontal plane, θ is the azimuth angle of the sun with respect to the north, β is the module's tilt angle with respect to the horizontal plane and ψ is the panel's azimuth angle. S_{in} is the direct beam intensity on the panel measured by the sensors and S_{module} is it's perpendicular component on the panel surface. A panel facing the south will have $\psi = 180^\circ$ and a panel facing the north will have $\psi = 0^\circ$. In (3) the angels β and ψ are the controlling parameters in this paper and α,

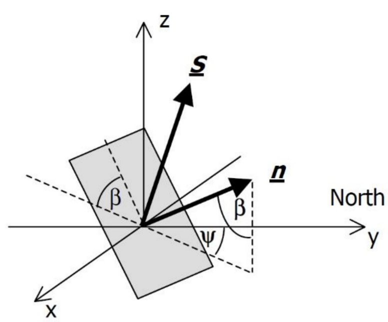
θ and S_{in} have to be extracted from weather reports. Fig. 1 shows the angels mentioned above. Note the vector $\underline{\mathbf{S}}$ is the unit vector pointing at the sun at any moment and $\underline{\mathbf{n}}$ is the unit normal vector of the module.

Panel parameters were extracted from the JA Solar datasheet provided for model JAP6(K). The technical characteristics of panel along with other constants are presented in Table I. The climate data needed for this paper is obtained from NREL available in [15]. From the many stations available in the database, the main station named NREL Solar Radiation Research Laboratory was chosen. This base is located at latitude 39.742° north, longitude 105.18° west and elevation 1828.8 meters AMSL. From this base a set of one-minute daytime (sunrise to sunset) data record was extracted from 1/1/2007 to 1/1/2017. The data Consists of direct solar irradiance (W/m^2), global irradiance (W/m^2), azimuth and elevation angle (degree), temperature ($°C$) and wind speed (m/s). This ten-year data adds up to 2,659,118 individual measuring for each parameter.

This long term monitoring of the region's atmospheric parameters leads to a good understanding of it's climate conditions. Thus, it can be assumed that the weather conditions will approximately stay the same for the near future. Hence, the results obtained from this historical data are valid for forthcoming years.



(a)



(b)

Fig. 1. Sun position unit vector and panel's normal vector [18]: (a) $\underline{\mathbf{S}}$ is the unit vector pointing the sun. The sun's azimuth and altitude angles are shown, (b) $\underline{\mathbf{n}}$ is the normal vector of the module. The panel's tilt and orientation angles are shown.

B. Statistic Properties Calculation (SPC) Method

Moments of a random variable are introduced as statistical properties which can be used for different purposes. For instance, the probability density function (PDF) of a random variable can be formed using it's moments. Moments of a random variable are calculated as follows [19]:

$$\mu_{X,n} = E\{(X-c)^n\} = \int_{-\infty}^{+\infty} (X-c)^n f_X(X)\, dX \quad (4)$$

where X is a random variable, $f_X(X)$ is the PDF of variable X and the moments are calculated around point c. These moments can also be calculated using a moment generation function:

$$M_X(t) = E\{e^{tX}\}, \qquad t \in R \quad (5)$$

the n^{th} raw moment, i.e. the n^{th} moment around $c = 0$, is obtained by calculation of n^{th} derivative of (5) with respect to variable t and evaluating it at $t = 0$. In this method, the Taylor series expansion of the objective function will be used. Therefore, equations which calculate statistical properties of expressions including powers and multiplication of several random variables are needed. Considering independence of random variables, these equations are as follows:

- multiplication of two independent random variable X and Y:
$$\mu'_{XY,n} = \mu'_{X,n} . \mu'_{Y,n} \quad (6)$$
- moment of n^{th} power of a random variable:
$$\mu'_{X^n,m} = \mu'_{X,nm} \quad (7)$$

The general form of the SPC method which calculates the statistical properties of a function including random variables is shown in (8):

$$y = f(x_1, x_2, ..., x_n) \quad (8)$$

where x_i's are random variables which their samples are available. Fig. 2 shows this procedure. First, random variables must be uncorrelated and expressed in terms of z_i's, which are zero-mean unit variance uncorrelated variables:

$$\bar{X} = A\bar{Z} \quad (9)$$

where A is the transformation matrix which is calculated according to [20]. Then, multivariate Taylor series expansion of function f is produced to form an equivalent polynomial in terms of variables z_i's. Finally, moments of this polynomial are calculated using (4), (6) and (7). It should be noted that to use (6) and (7) in an accurate way, the random variables must be statistically independent, but in the proposed method, they have been only uncorrelated, and generally, these two concepts are not the same. This difference was considered as an error in modeling. The algorithm of this method is as follows:

- Data preprocessing: As for any real data analysis method, the data must first be prepared before any process takes place. In this stage, the data is scanned for any abnormal

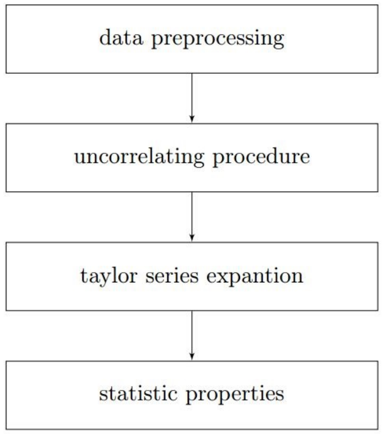


Fig. 2. The flowchart of SPC method.

value and those found are filtered or if possible, replaced by another data from a similar sensor.

- Uncorrelating: Moments of output power are calculated using (6) and (7). As mentioned above, in this method random variables must be transformed to uncorrelated variables and then the moments of output power will be calculated with (6) and (7). In (1), T_c, S_{in}, α and θ are random variables which their samples are available in data records. In other words, there are four corresponding vectors as the inputs to the uncorrelating procedure.
- Taylor series expansion and calculation of statistical properties of output power: Taylor series expansion is used to form an equivalent polynomial expression with variables x_i's. Now the moments can be obtained using (4), (6) and (7).

C. Control Variables Extraction (CVE) Method

In this section, the control variables extraction method was utilized with the same weather record and PV model introduced in Section II-A. To have a better insight on (1), it can be re-written as follows:

$$\begin{aligned} Y =& a\sin(\beta)\cos(\psi) + b\sin(\beta)sin(\psi) \\ &+ c\cos(\beta) \end{aligned} \tag{10}$$

where coefficients a, b and c are defined as:

$$a(t) = \cos(\alpha(t))\cos(\theta(t))\eta(t) \tag{11}$$

$$b(t) = \cos(\alpha(t))\sin(\theta(t))\eta(t) \tag{12}$$

$$c(t) = \sin(\alpha(t))\eta(t) \tag{13}$$

where $\eta(t)$ is:

$$\eta(t) = S_{in}(t)\frac{Y_d}{I_s}(1 - \frac{K_p}{100}(T_c(t) - T_{STC}))P_n \tag{14}$$

Equations (11)-(13) can be written in the form of matrices as follows:

$$\mathbf{A} = \{a(t)| \quad t = 1 : N\} \tag{15}$$

$$\mathbf{B} = \{b(t)| \quad t = 1 : N\} \tag{16}$$

$$\mathbf{C} = \{c(t)| \quad t = 1 : N\} \tag{17}$$

where, N is the total number of measurements. Having the ten year, minute by minute daytime data, one can create (10) for each minute.

Here, the first moment or expected value of the PV panel production is used to find the maximum expected power output of the PV panel.

$$\begin{aligned} E[\mathbf{Y}] =& E[\mathbf{A}\sin(\beta)\cos(\psi)] \\ &+ E[\mathbf{B}\sin(\beta)\sin(\psi)] + E[\mathbf{C}\cos(\beta)] \end{aligned} \tag{18}$$

Equation (18) shows the expected value of (10). As ψ and β are control parameters and can be taken out from the expectation operator, (18) can be expressed as:

$$\begin{aligned} E[\mathbf{Y}] =& \sin(\beta)\cos(\psi)E[\mathbf{A}] \\ &+ \sin(\beta)\sin(\psi)E[\mathbf{B}] + \cos(\beta)E[\mathbf{C}] \end{aligned} \tag{19}$$

Equation (19) expresses that, for calculating the expected value of PV output power, the expected value of coefficients must be substituted. Considering this, the following equation can be obtained:

$$\begin{aligned} E[\mathbf{Y}] =& A_{avg}\sin(\beta)\cos(\psi) \\ &+ B_{avg}\sin(\beta)\sin(\psi) + C_{avg}\cos(\beta) \end{aligned} \tag{20}$$

D. Monte Carlo Method

Monte Carlo simulation is used as a stochastic optimization method [21]. In this work, Monte Carlo simulation has been employed to calculate the optimum orientation and tilt of PV panel. In other words, the tilt and azimuth angles of panel have been changed by definite steps and according to (1) and (2), average output power is calculated for different orientations using a large number of time samples and their corresponding sun angles, irradiance and panel temperature.

III. Simulation Results

Table II summarizes the optimal PV panel orientation angles calculated by the three mentioned methods. Because of various weather conditions during the day, i.e. different radiation and temperature values, the expected value of output power is lower than nominal power which is expressed in standard test conditions (STC). According to [22], the traditional way of determining panel's tilt angle and orientation is to set the tilt angle slightly less than region's latitude and in the northern hemisphere, the panel should be facing the south.

As mentioned in Section II-A the latitude of the study site was 39.742° North. The results in the three methods show the optimal tilt angle (β) is close to the site latitude. The panel azimuth angle (ψ) which means that the direction the panel should be facing towards the south-east (roughly 170°). The reason might be that the panel could produce more power in the morning because of the cooling process which takes place over the night hours.

Fixing the panel in the suggested positions of Table II leads to more power production compared to the traditional method. For instance in the CVE method the expected PV power production rises by 1.13%. This increase in production

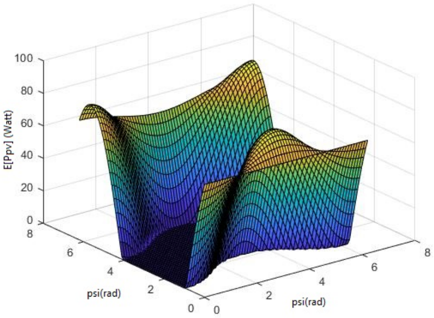


Fig. 3. The expected value of produced power for SPC method.

comes at no cost in the design stage for the utility or panel owner and directly increases the profit gained.

It should be noted these results are valid for this location only, finding the optimal tilt angle for other places requires weather records of the region.

A. SPC Method

As mentioned in Section II-B, this method can produce PV output power moments. For example the first moment or the expected value of PV output power is according to (21):

$$\begin{aligned} P_{pv} =& -47.1587\sin(\beta)\cos(\psi) + 6.3855\sin(\beta)sin(\psi) \\ &+ 69.6145\cos(\beta) \end{aligned} \quad (21)$$

As expected, (21) is a function of control parameters (ψ and β) and it's functional form is shown in Fig. 3. The feasible range for ψ and β was assumed according to (22).

$$\begin{aligned} 90 \leq \psi \leq 270 \\ 0 \leq \beta \leq 90 \end{aligned} \quad (22)$$

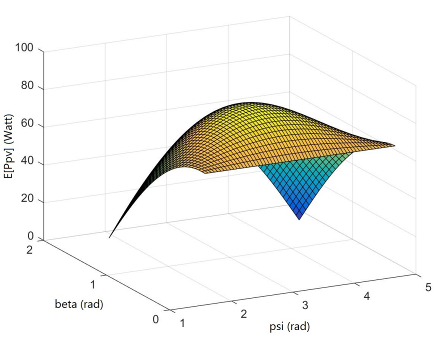


Fig. 4. The expected value of produced power in the feasible range calculated by SPC method.

TABLE II
CALCULATED OPTIMAL PV PANEL ORIENTATION ANGLES

Method	ψ (degree)	β (degree)	P_{pv} $(Watt)$
SPC	172.30	34.38	84.326
CVE	166.95	36.33	94.598
Monte Carlo	169	34	89.396

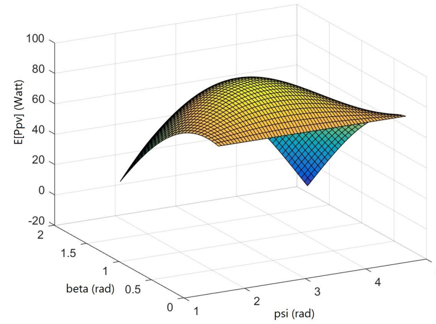


Fig. 5. The expected value of produced power in the feasible range calculated by CVE method.

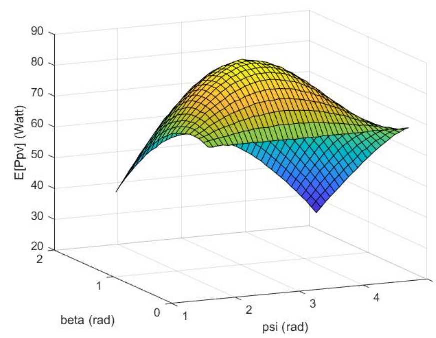


Fig. 6. The expected value of produced power in the feasible range calculated by Monte Carlo method.

Fig. 4 shows the expected value in the feasible set. There is one maximum point corresponding to the optimal angles presented in Table II.

B. CVE Method

Equation (23) shows the expression obtained for the PV output power expectation from this method. Fig. 5 illustrates the functional form of (23).

$$\begin{aligned} P_{pv} =& -54.6176\sin(\beta)\cos(\psi) + 12.6480\sin(\beta)sin(\psi) \\ &+ 76.1953\cos(\beta) \end{aligned} \quad (23)$$

C. Monte Carlo Simulation

The Monte Carlo method is a stochastic means of calculation, therefore it does not generate an explicit equation for the expected value of PV output power. Simulation results are presented in Fig. 6.

IV. CONCLUSION

In this work, two analytical methods have been proposed to precisely calculate the optimal orientation and tilt angle of PV panels to harvest maximum energy. These methods are general and can be utilized for more complex optimization problems with many stochastic variables. Also, a Monte Carlo simulation was performed to verify the results. The methods take into account regional climate parameters, such as ambient temperature, wind speed and irradiance. Using real ten-year one-minute weather records, an optimal fixed position for solar panels was obtained from each method.

The results indicate fixing the solar panels in the optimal position will increase the annual expected produced energy by 1.13% compared with the traditional method in which PV panels are facing south (in northern hemisphere) with tilt angle approximately equivalent to the location latitude. This is a cost free change in the design stage and directly increases the profit of the utility and the internal rate of return (IRR) of the investment. The results will vary from place to place depending on regional climate conditions, emphasizing the importance of data gathering and recording.

Determining an optimal orientation for different seasons can be obtained by modifying the proposed equations. In addition, the interaction between wind and solar production with consideration of electricity spot market uncertainties can be studied using the proposed methods and can be the focus of future studies.

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