$O(D,D)$ constraint on D-dimensional effective actions

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Double field theory is a manifestly T-duality invariant formulation of string theory in which the effective theory at any order of α' is invariant under global $O(D,D)$ transformations and ought to be invariant under gauge transformations which receive α'-corrections. On the other hand, the effective theory in the usual D-dimensional formulation of string theory is manifestly gauge invariant and ought to be invariant under T-duality transformations which receive α'-corrections. We speculate that the combination of these two constraints may fix both the $2D$-dimensional and the D-dimensional effective actions without knowledge of the α'-corrections of the gauge and the T-duality transformations. In this paper, using generalized fluxes, we construct arbitrary $O(D,D)$-invariant actions at orders α'^0 and α', and then dimensionally reduce them to the D-dimensional spacetime. On the other hand, at these orders, we construct arbitrary covariant D-dimensional actions. Constraining the two D-dimensional actions to be equal up to noncovariant field redefinitions, we find that both actions are fixed up to overall factors and up to field redefinitions.

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I. INTRODUCTION

One of the most exciting discoveries in string theory is T-duality [1,2]. This duality may be used to construct the effective field theory at low energy. One approach for constructing this effective action is the double field theory (DFT) approach [3–7]. The DFT is a constraint field theory which doubles the spacetime coordinates, i.e., adds to the usual D-dimensional spacetime coordinates which correspond to the momentum excitations, another D-dimensional coordinates which correspond to the winding excitations. However, a solution to the constraint in its strong form [7] is that the $2D$-dimensional dynamical fields to be independent of the winding coordinates. The T-duality is manifested in this approach as the effective action is $O(D,D)$-invariant by constructions. The effective action is also constrained to satisfy some gauge transformations. The appropriate gauge transformations at the leading order of α' are the generalized diffeomorphisms and double-Lorentz transformations [7,8], however, one of them receive α'-corrections at the higher orders of α' [9,10]. The form of these corrections at order α' have been found in [9,10], however, it is hard to find them at the higher orders. Using the $2D$-dimensional field redefinitions freedom, the effective action may appear in different schemes. The DFT effective action at order α' in one particular scheme has been constructed in [10,11].

Another T-duality based approach for constructing the effective action at higher orders of α', is to use the constraint that the dimensional reduction of the effective action on a circle must be invariant under the T-duality transformations [12]. In this approach, one begins with the most general gauge invariant action in the D-dimensional spacetime. The dimensional reduction of this action on the circle must be invariant under the T-duality transformations. The gauge transformations in this approach are the standard coordinate transformations, the B-field gauge transformations and the nonstandard Lorentz transformation of the B-field which is required for anomaly cancellations. The T-duality transformations at the leading order of α' are the Buscher rules [13,14], however, they receive α'-corrections at the higher orders of α' [15,16]. The form of these corrections at order α' have been found in [15–17],[1] however, it is hard to find them at higher orders of α'. Using the T-duality approach, the standard gravity and dilaton couplings in the effective actions at orders α', α'^2, α'^3 have been reproduced in [19,20].

Since the higher derivative corrections to the gauge transformations in the DFT approach and the α'-corrections

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[1]It has been observed in [18] that the renormalization group flows is covariant under the T-duality transformations at order α'.

to the Buscher rules in the T-duality approach are hard to find in general, it is desirable to find constraints which do not receive α'-corrections. Merging the above two approaches, one may finds such constraints as follows: Using the strong constraint, one can reduce the $2D$-dimensional effective action in the DFT approach to the D-dimensional effective action. This action should then be the same as the D-dimensional effective action in the T-duality approach. The field variables in the two approaches, however, are not the same. Noncovariant field redefinitions are required to relate the two field variables [21]. The T-duality transformations in the DFT approach are the standard Buscher rules [7] whereas the gauge transformations are not the standard gauge transformations. On the other hand, the gauge transformations in the T-duality approach are the standard gauge transformations whereas the T-duality transformations are not the standard Buscher rules. As a result, the two D-dimensional actions must be the same up to noncovariant field redefinitions. Therefore, the effective actions should satisfy the following constraints:

(1) The $2D$-dimensional action is constrained to be invariant under the $O(D,D)$ transformations and under the generalized diffeomorphisms which do not receive α'-corrections. However, it is not constrained to be invariant under the double-Lorentz transformations which receive α'-corrections.
(2) After reducing it to the D-dimensional spacetime and using noncovariant field redefinitions, the action is constrained to be the same as a D-dimensional action which is invariant under the standard coordinate transformations, the B-field gauge transformations and the nonstandard Lorentz transformation of the B-field, however, it is not invariant under the T-duality transformations which receive α'-corrections. We speculate that the above two constraints can fix both the $2D$-dimensional and D-dimensional effective actions. We confirm this idea in this paper by explicit calculations at orders α'^0 and α'.

The outline of the paper is as follows: In Sec. II, we perform the calculations at order α'^0. In particular, in subsection II A, we use generalized metric and dilaton as dynamical fields which are invariant under the double-Lorentz transformations, to construct the most general $O(D,D)$-invariant action at order α'^0. Using the strong constraint, we then reduce it to the D-dimensional action. Then, using the D-dimensional metric, B-field and dilaton, we construct the most general covariant action at order α'^0. Constraining the two actions to be identical, we fix both effective actions. Up to an overall factor, they are exactly the known effective actions in the literature. In Sec. II B, we use the generalized frame and dilaton as the dynamical fields. Using the generalized fluxes, which are invariant under the generalized diffeomorphisms, we construct the most general $O(D,D)$-invariant action at order α'^0, and then reduce it to the D-dimensional action. Comparing it with the covariant D-dimensional action, we fix both the effective actions. The $2D$-dimensional effective action is the same as the action in the literature.

In Sec. III, we extend the calculations to the order α'. In particular, using the generalized fluxes, we first construct the most general $O(D,D)$-invariant action at order α' without fixing its field redefinitions freedom, and then reduce it to the D-dimensional action. To convert the noncovariant field variables in the resulting action to the covariant variables, we use the most general noncovariant field redefinitions. We then compare it with the most general covariant action at order α' up to covariant field redefinitions. The constraint that the two D-dimensional actions must be identical, fixes both actions. Up to an overall factor, the D-dimensional action is exactly the same as the action in the literature. Since the field redefinitions freedom is not fixed in the $2D$-dimensional action, we have found the $2D$-dimensional action with some arbitrary parameters. In one particular scheme in which dilaton appears as an overall factor, we write the effective action.

II. EFFECTIVE ACTION AT ORDER α'^0

Using the strong constraint in the DFT formalism, the effective action of string theory at order α'^0 can be written as $O(D,D)$-invariant and invariant under $2D$-dimensional gauge transformations which are generalized diffeomorphisms and local double-Lorentz transformations. If one uses the generalized metric and dilaton as dynamical fields which are invariant under the double-Lorentz transformations, then the gauge transformations are the generalized diffeomorphisms [8]. On the other hand, if one uses the generalized frame and dilaton as the dynamical fields, then the action can be written in terms of generalized fluxes which are invariant under the generalized diffeomorphisms [7]. Hence, the nontrivial gauge transformations in this case is the double-Lorentz transformations. Using these gauge transformations, the $2D$-dimensional effective actions have been found in [8,22].

In this section we are going to find these actions by comparing the most general $O(D,D)$-invariant action with the most general D-dimensional covariant action.

A. Generalized metric formulation

We begin with the case that the generalized metric $\mathcal{H}_{\mu\nu}$ and dilaton d are the dynamical fields. They are invariant under the double-Lorentz transformations as they carry no index in this space, however, the generalized metric

$\mathcal{H}_{\mu\nu}$ is a matrix that transforms under the $O(D,D)$ transformations as[2]

$$\mathcal{H} \to O\mathcal{H}O^T \tag{1}$$

The D-dimensional coordinate x^a conjugated to the momentum excitations and the D-dimensional coordinate $\tilde{x}_a$ conjugated to the winding excitations, transforms as vector, i.e.,

$$x^\mu \equiv \begin{pmatrix} \tilde{x}_a \\ x^a \end{pmatrix} \to O\begin{pmatrix} \tilde{x}_a \\ x^a \end{pmatrix}; \qquad \partial_\mu \equiv \begin{pmatrix} \tilde{\partial}^a \\ \partial_a \end{pmatrix} \to O\begin{pmatrix} \tilde{\partial}^a \\ \partial_a \end{pmatrix} \tag{2}$$

and the $2D$-dimensional dilaton d is invariant under the $O(D,D)$ transformations [8]. Using these $O(D,D)$ tensors, one can write the most general $O(D,D)$-invariant action at two-derivative level as[3]

$$\begin{aligned}\mathbf{S}_0 = -\frac{2}{\kappa^2\tilde{V}}\int dx d\tilde{x} e^{-2d}(&c_9\mathcal{H}^{\alpha\beta}\partial_\alpha d\partial_\beta d + c_3\mathcal{H}^{\alpha\beta}\partial_\beta\partial_\alpha d + c_1\mathcal{H}^{\alpha\gamma}\mathcal{H}^{\beta\delta}\partial_\gamma\partial_\alpha\mathcal{H}_{\delta\beta} + c_2\mathcal{H}^{\alpha\gamma}\mathcal{H}^{\beta\delta}\partial_\gamma\partial_\delta\mathcal{H}_{\alpha\beta} \\ &+ c_5\mathcal{H}^{\alpha\delta}\mathcal{H}^{\beta\epsilon}\mathcal{H}^{\gamma\varepsilon}\partial_\alpha\mathcal{H}_{\beta\gamma}\partial_\delta\mathcal{H}_{\epsilon\varepsilon} + c_6\mathcal{H}^{\alpha\delta}\mathcal{H}^{\beta\epsilon}\mathcal{H}^{\gamma\varepsilon}\partial_\beta\mathcal{H}_{\alpha\gamma}\partial_\delta\mathcal{H}_{\epsilon\varepsilon} + c_{11}\mathcal{H}^{\alpha\delta}\mathcal{H}^{\beta\gamma}\partial_\gamma\mathcal{H}_{\alpha\beta}\partial_\delta d + c_8\mathcal{H}^{\alpha\delta}\mathcal{H}^{\beta\epsilon}\mathcal{H}^{\gamma\varepsilon}\partial_\delta\mathcal{H}_{\alpha\epsilon}\partial_\varepsilon\mathcal{H}_{\beta\gamma} \\ &+ c_{10}\mathcal{H}^{\alpha\delta}\mathcal{H}^{\beta\gamma}\partial_\alpha\mathcal{H}_{\gamma\beta}\partial_\delta d + c_7\mathcal{H}^{\alpha\delta}\mathcal{H}^{\beta\epsilon}\mathcal{H}^{\gamma\varepsilon}\partial_\beta\mathcal{H}_{\varepsilon\gamma}\partial_\delta\mathcal{H}_{\alpha\epsilon} + c_4\mathcal{H}^{\alpha\epsilon}\mathcal{H}^{\beta\varepsilon}\mathcal{H}^{\gamma\delta}\partial_\alpha\mathcal{H}_{\delta\gamma}\partial_\epsilon\mathcal{H}_{\varepsilon\beta})\end{aligned} \tag{3}$$

where $\mathcal{H}^{\mu\nu}$ is inverse of the generalized metric and $\tilde{V} = \int d\tilde{x}$. Since there is no double-Lorentz index in the couplings (3), this action is invariant under the local double-Lorentz transformations as well.

There is also a $O(D,D)$-invariant metric

$$\eta_{\mu\nu} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \eta^{\mu\nu} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{4}$$

which raises and lowers the $2D$-indicies, i.e.,

$$\mathcal{H}^\mu{}_\nu = \eta^{\mu\alpha}\mathcal{H}_{\alpha\nu}; \quad \mathcal{H}^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta}\mathcal{H}_{\alpha\beta}; \quad \partial^\alpha = \eta^{\alpha\beta}\partial_\beta \tag{5}$$

The symmetry of the effective action under $B_{ab} \to -B_{ab}$, requires the couplings to have even number of constant metric η [8]. The couplings involving odd number of η are antisymmetric under $B_{ab} \to -B_{ab}$. At two derivative level, one can convince oneself that any term which is independent of the winding coordinates and contains two constant metrics, is identical to the couplings in (3). For example, using the constant metric to raise and lower the indices, the term $\partial_\alpha\mathcal{H}^\alpha{}_\beta\partial_\gamma\mathcal{H}^\gamma{}_\mu\mathcal{H}^{\beta\mu}$ can be written as $\partial_\alpha\mathcal{H}^{\alpha\beta}\partial_\gamma\mathcal{H}^{\gamma\mu}\mathcal{H}_{\beta\mu}$. The latter coupling can be written as the couplings in (3) using the identity

$$\mathcal{H}^{\alpha\beta}\mathcal{H}_{\beta\mu} = \delta^\alpha_\mu \tag{6}$$

At higher derivative level, however, there are couplings involving the metric η which can not be written in terms of only generalized metric, e.g., $\partial_\alpha\mathcal{H}^\alpha{}_\beta\partial_\gamma\mathcal{H}^\gamma{}_\lambda\partial_\mu\mathcal{H}^{\mu\lambda}\partial_\nu\mathcal{H}^{\nu\beta}$.

The couplings in (3) with coefficients c_4, c_7, c_{10} become zero using the above identity. Using this identity and total derivative terms, one can relate the coefficients of some of the above terms. One may either use these relations to write (3) in terms of independent couplings and then compare them with the D-dimensional gauge invariant action, or one may fix them after comparing the nonindependent couplings with the D-dimensional gauge invariant action. In the latter case that we are going to do in this paper, they appear as free parameters which can be chosen arbitrarily.

The reduction of the generalized metric and its inverse in terms of the D-dimensional metric and the B-field are [8]

$$\begin{aligned}\mathcal{H}_{\mu\nu} &= \begin{pmatrix} G^{ab} & -G^{ac}B_{cb} \\ B_{ac}G^{cb} & G_{ab} - B_{ac}G^{cd}B_{db} \end{pmatrix}; \\ \mathcal{H}^{\mu\nu} &= \begin{pmatrix} G_{ab} - B_{ac}G^{cd}B_{db} & B_{ac}G^{cb} \\ -G^{ac}B_{cb} & G^{ab} \end{pmatrix}\end{aligned} \tag{7}$$

The reduction of the $2D$-dimensional dilaton to the D-dimensional dilaton and metric is $e^{-2d} = e^{-2\Phi}\sqrt{-G}$. Using the strong constraint that fields do not depend on the coordinate $\tilde{x}$, one can reduce (3) to the following D-dimensional action:

[2]Our index conversion is that the Greek letters $(\mu,\nu,\ldots)$ are the indices of the curved $2D$-dimensional space, the Latin letters $(a,d,c,\ldots)$ are the indices of the curved D-dimensional spacetime, the letters $(A,B,C,\ldots)$ are the indices of flat $2D$-dimensional tangent space, and the letters $(i,j,k,\ldots)$ are the flat D-dimensional tangent space.

[3]We use the *Mathematica* package xAct [23] for performing the calculations in this paper.

$$\mathbf{S}_0 = -\frac{2}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}\Big(c_9 G^{ab}\partial_a\Phi\partial_b\Phi + c_3 G^{ab}\partial_b\partial_a\Phi - \frac{1}{4}c_3 G^{ac}G^{bd}\partial_c\partial_a G_{db} + 2(c_1+c_5)G^{ad}G^{be}G^{cf}\partial_a B_{bc}\partial_d B_{ef}$$
$$+ (c_2+c_6)G^{ad}G^{be}G^{cf}\partial_b B_{ac}\partial_d B_{ef} - \frac{1}{4}c_{11}G^{ad}G^{be}G^{cf}\partial_b G_{fc}\partial_d G_{ae} + \frac{1}{4}(8c_1+c_3+8c_5)G^{ad}G^{be}G^{cf}\partial_a G_{bc}\partial_d G_{ef}$$
$$+ c_6 G^{ad}G^{be}G^{cf}\partial_b G_{ac}\partial_d G_{ef} - \frac{1}{2}c_9 G^{ad}G^{bc}\partial_a G_{cb}\partial_d\Phi + c_{11}G^{ad}G^{bc}\partial_c G_{ab}\partial_d\Phi + \frac{1}{16}c_9 G^{ae}G^{bf}G^{cd}\partial_a G_{dc}\partial_e G_{fb}$$
$$+ (-c_2-c_8)G^{ad}G^{be}G^{cf}\partial_d B_{ae}\partial_f B_{bc} + c_8 G^{ad}G^{be}G^{cf}\partial_d G_{ae}\partial_f G_{bc} + c_2 G^{ac}G^{bd}\partial_c\partial_d G_{ab}\Big) \tag{8}$$

Note that the coefficients c_4, c_7, c_{10} do not appear in above D-dimensional action, so they appear in the final action as arbitrary parameters which can be set to zero.[4] It is interesting to note that even though the generalized metric contains no derivative on the B-field, the above D-dimensional action contains only terms which have derivative on the B-field. It turns out that the dimensional reduction of any $O(D,D)$-invariant coupling produces no B-field without derivative on it. The above $O(D,D)$ and double-Lorentz invariant action is not invariant under the generalized diffeomorphism for arbitrary parameters. We do not impose this constraint for finding the unknown coefficients in this paper.

We now construct the most general D-dimensional action at two-derivative level which is invariant under the coordinate transformations and under the B-field gauge transformations, i.e.,

$$\mathbf{S}_0^c = -\frac{2}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}\Big(a_1 R + a_2\nabla_a\Phi\nabla^a\Phi - \frac{a_3}{12}H^2\Big), \tag{9}$$

where $H_{abc} = \partial_a B_{bc} + \partial_c B_{ab} + \partial_b B_{ca}$ and a_1, a_2, a_3 are three constants. Since the action (8) is in terms of metric and B-field, we rewrite the above covariant action in terms of metric and B-field, i.e.,

$$\mathbf{S}_0^c = -\frac{2}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}\Big(a_2 G^{ab}\partial_a\Phi\partial_b\Phi - a_1 G^{ac}G^{bd}\partial_c\partial_a G_{db} + a_1 G^{ac}G^{bd}\partial_c\partial_d G_{ab} - \frac{1}{4}a_3 G^{ad}G^{be}G^{cf}\partial_a B_{bc}\partial_d B_{ef}$$
$$+ \frac{1}{2}a_3 G^{ad}G^{be}G^{cf}\partial_b B_{ac}\partial_d B_{ef} + a_1 G^{ad}G^{be}G^{cf}\partial_b G_{fc}\partial_d G_{ae} + \frac{3}{4}a_1 G^{ad}G^{be}G^{cf}\partial_a G_{bc}\partial_d G_{ef}$$
$$- \frac{1}{2}a_1 G^{ad}G^{be}G^{cf}\partial_b G_{ac}\partial_d G_{ef} - \frac{1}{4}a_1 G^{ae}G^{bf}G^{cd}\partial_a G_{dc}\partial_e G_{fb} - a_1 G^{ad}G^{be}G^{cf}\partial_d G_{ae}\partial_f G_{bc}\Big). \tag{10}$$

The two D-dimensional Lagrangians (8) and (10) are not equal for any nonzero parameters. However, to compare the two actions, one should take into account noncovariant total derivative terms as well. There are three total derivative terms, i.e.,

$$\mathbf{J} = -\frac{2}{\kappa^2}\int dx\partial_a(e^{-2\Phi}\sqrt{-G}[f_3 G^{ab}\partial_b\Phi + f_2 G^{cb}G^{da}\partial_c G_{db} + f_1 G^{ca}G^{db}\partial_c G_{db}]) \tag{11}$$

where f_1, f_2, f_3 are three arbitrary parameters. Now adding these total derivative terms to (10), the two actions can be equated, i.e., $\mathbf{S}_0 = \mathbf{S}_0^c + \mathbf{J}$, for the following constraints on the parameters:

$$a_2 = 4a_1, \qquad a_3 = a_1,$$
$$c_3 = 2a_1 - c_{11} - 2c_2, \qquad c_5 = -a_1/8 - c_1,$$
$$c_6 = a_1/2 - c_2, \qquad c_8 = -c_2, \qquad c_9 = 2c_{11} + 4c_2,$$
$$f_1 = a_1/2 + c_{11}/4 + c_2/2, \qquad f_2 = -a_1 + c_2,$$
$$f_3 = 2a_1 - c_{11} - 2c_2 \tag{12}$$

The equations in the last line gives the coefficients of the total derivative terms that are needed to equate the two actions.

The equations in the first line constrain the D-dimensional action (9) to be

$$\mathbf{S}_0^c = -\frac{2a_1}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}\Big(R + 4\nabla_a\Phi\nabla^a\Phi - \frac{1}{12}H^2\Big), \tag{13}$$

which is the standard effective action at order α'^0, up to an overall factor. The overall factor must be $a_1 = 1$ to be the

[4]One can check that the above couplings are invariant under the Buscher rules, i.e., if one compactifies the theory on a circle and assumes fields are independent of that directions, then the above couplings would be invariant under the Buscher rules.

effective action of string theory. The equations in the second line constrain the $O(D,D)$-invariant action (3) to be

$$\mathbf{S}_0 = -\frac{2a_1}{\kappa^2\tilde{V}}\int dxd\tilde{x}e^{-2d}\Big(2\mathcal{H}^{\alpha\beta}\partial_\beta\partial_\alpha d+\frac{1}{8}\mathcal{H}^{\alpha\delta}\partial_\alpha\mathcal{H}_{\beta\gamma}\partial_\delta\mathcal{H}^{\beta\gamma} -\frac{1}{2}\mathcal{H}^{\alpha\delta}\partial_\beta\mathcal{H}_{\alpha\gamma}\partial_\delta\mathcal{H}^{\beta\gamma}\Big) \tag{14}$$

where we have also used the identity $\partial_\alpha\mathcal{H}^{\mu\nu} = -\mathcal{H}^{\mu\beta}\mathcal{H}^{\nu\gamma}\partial_\alpha\mathcal{H}_{\beta\gamma}$. The terms with coefficients c_2, c_{11} are total derivative terms, and terms with coefficient c_1, i.e.,

$$c_1\mathcal{H}^{\gamma\alpha}\mathcal{H}^{\delta\beta}\partial_\gamma\partial_\alpha\mathcal{H}_{\delta\beta} - c_1\mathcal{H}^{\delta\alpha}\mathcal{H}^{\epsilon\beta}\mathcal{H}^{\varepsilon\gamma}\partial_\alpha\mathcal{H}_{\beta\gamma}\partial_\delta\mathcal{H}_{\epsilon\varepsilon} \tag{15}$$

become zero using the identity (6), so we have discarded them. The $O(D,D)$-invariant action (14) is the one has been found in [8]. Therefore, the requirement that the $O(D,D)$-invariant action and the covariant D-dimensional action to be identical, fixes both actions, up to an overall factor.

The action (14) has been found in [8] by requiring the $O(D,D)$-invariant couplings (3) to be invariant under the generalized diffeomorphisms which are

$$\delta(e^{-2d}) = \partial_\mu(\zeta^\mu e^{-2d})$$
$$\delta\mathcal{H}_{\mu\nu} = \zeta^\rho\partial_\rho\mathcal{H}_{\mu\nu} + 2(\partial_{(\mu}\zeta^\rho - \partial^\rho\zeta_{(\mu})\mathcal{H}_{\nu)\rho} \tag{16}$$

Unlike the terms in (9) which are invariant under the D-dimensional diffeomorphisms, none of the terms in (3) is invariant under the above $2D$-dimensional generalized diffeomorphisms. Only the combination of terms in (14) is invariant under these transformations [8]. This combination may be defined as the definition of $2D$-dimensional scalar curvature [8]. It is hard to extend the couplings (14) to the higher order of α' because the conventional $2D$-dimensional Riemann curvature does not transform covariantly under the generalized diffeomorphisms [4,24–27].

B. Generalized frame formulation

A convenient frame work for constructing the higher derivative couplings in DFT is the generalized frame construction of the DFT [3,7]. The generalized frame $E_\mu{}^A$ is defined to relate the generalized metric $\mathcal{H}_{\mu\nu}$ to the flat generalized metric $\mathcal{H}_{AB}$ and the $O(D,D)$-metric $\eta_{\mu\nu}$ to flat metric η_{AB}, i.e.,

$$\mathcal{H}_{\mu\nu} = E_\mu{}^A\mathcal{H}_{AB}E_\nu{}^B \qquad \eta_{\mu\nu} = E_\mu{}^A\eta_{AB}E_\nu{}^B \tag{17}$$

In terms of the generalized dilaton d, the generalized frame $E_\mu{}^A$ and its transverse $E^\mu{}_A$, one can construct flat space tensors which transform as scalar under the generalized diffeomorphisms. They are

$$\mathcal{F}_A = 2\partial_A d - E^{\mu B}\partial_B E_{\mu A}$$
$$\mathcal{F}_{ABC} = 3\partial_{[A}E^\mu{}_B E^\nu{}_{C]}\eta_{\mu\nu} \tag{18}$$

where the flat space derivative is $\partial_A = E^\mu{}_A\partial_\mu$. These tensors transform as scalar under the generalized diffeomorphisms [7], i.e.,

$$\delta\mathcal{F}_A = \zeta^\mu\partial_\mu\mathcal{F}_A \qquad \delta\mathcal{F}_{ABC} = \zeta^\mu\partial_\mu\mathcal{F}_{ABC} \tag{19}$$

The flat space derivatives of these tensors transform as scalar under the generalized diffeomophisms as well [7]. However, these tensors do not transform covariantly under local double-Lorentz transformations. It has been shown in [10,11] that these transformations receive α' corrections as well.[5]

One may construct $O(D,D)$-invariant and the generalized diffeomorphism invariant effective actions by considering all contractions of these tensors with constant metric η^{AB} and $\mathcal{H}^{AB}$, i.e., at two-derivative level they are

$$\begin{aligned}\mathbf{S}_0^f = -\frac{2}{\kappa^2\tilde{V}}\int dxd\tilde{x}e^{-2d}(&c_3\mathcal{F}_A\mathcal{F}^A + c_4\partial^A\mathcal{F}_A \\ &+ c_1\mathcal{F}^A\mathcal{F}^B\mathcal{H}_{AB} + c_2\partial^A\mathcal{F}^B\mathcal{H}_{AB} + c_7\mathcal{F}_{ABC}\mathcal{F}^{ABC} \\ &+ c_6\mathcal{H}^{AB}\mathcal{F}_A{}^{CD}\mathcal{F}_{BCD} + c_5\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{F}_{AC}{}^E\mathcal{F}_{BDE} \\ &+ c_8\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{ACE}\mathcal{F}_{BDF})\end{aligned} \tag{20}$$

The flat indices are raised by the flat metric η^{AB}, i.e., $\mathcal{F}^A = \eta^{AB}\mathcal{F}_B$. This action is invariant under the generalized diffeomophisms for arbitrary parameters $c_1,\ldots,c_8$, however, it is not invariant under the local double-Lorentz transformations. Imposing the invariance under the double-Lorentz transformations, one can fix these parameters [11,22]. However, we are not going to fix the parameters in this way.

To fix the parameters $c_1,\ldots,c_8$, instead, we constrain the reduction of this action to be identical with the D-dimensional covariant action (9). The reduction of the metric η^{AB}, $\mathcal{H}^{AB}$ and the generalized frame $E_\mu{}^A$ are [11]:

$$\eta^{AB} = \begin{pmatrix}0 & \delta_i^j\\ \delta_j^i & 0\end{pmatrix}; \qquad \mathcal{H}^{AB} = \begin{pmatrix}\eta_{ij} & 0\\ 0 & \eta^{ij}\end{pmatrix};$$
$$E_\mu{}^A = \begin{pmatrix}e^a{}_i & 0\\ -e^b{}_i B_{ba} & e_a{}^i\end{pmatrix} \tag{21}$$

where $e_a{}^i$ is the D-dimensional frame, i.e., $e_a{}^i e_b^j\eta_{ij} = G_{ab}$. Using the constraint that fields in the $2D$-dimensional action (20) do not depend on the coordinate $\tilde{x}$, one can reduce it to the following D-dimensional action:

[5]If one uses the generalized metric and dilaton as the dynamical fields which are invariant under the double-Lorentz transformations, then the α'-corrections would appear in the generalized diffoemorphisms [9].

$$\begin{aligned}\mathbf{S}_0^f = -\frac{2}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}(&c_2 G^{ad}G^{bc}\eta_{ji}e_d{}^i\partial_a\partial_c e_b{}^j + 4c_1 G^{ab}\partial_a\Phi\partial_b\Phi + 2c_2 G^{ab}\partial_b\partial_a\Phi + 6c_8 G^{ac}G^{bd}\eta_{ji}\partial_a e_b{}^j\partial_c e_d{}^i\\ &- 4c_1 G^{ac}G^{bd}\eta_{ji}e_b{}^j\partial_a\Phi\partial_c e_d{}^i - 6c_8 G^{ac}G^{bd}\eta_{ji}\partial_b e_a{}^j\partial_c e_d{}^i + 4c_1 G^{ac}G^{bd}\eta_{ji}e_a{}^j\partial_b\Phi\partial_c e_d{}^i - c_2 G^{ad}G^{bc}\eta_{ji}e_d{}^i\partial_c\partial_b e_a{}^j\\ &+ 3c_8 G^{ad}G^{be}G^{cf}\partial_a B_{bc}\partial_d B_{ef} - 6c_8 G^{ad}G^{be}G^{cf}\partial_b B_{ac}\partial_d B_{ef} + 4c_5 G^{ad}G^{be}G^{cf}\eta_{ji}e_f{}^j\partial_a B_{bc}\partial_d e_e{}^i\\ &- 4c_5 G^{ad}G^{be}G^{cf}\eta_{ji}e_f{}^j\partial_b B_{ac}\partial_d e_e{}^i + (-c_2+4c_6)G^{ad}G^{be}G^{cf}\eta_{kj}\eta_{li}e_a{}^k e_c{}^l\partial_b e_f{}^j\partial_d e_e{}^i\\ &- 2c_1 G^{ad}G^{be}G^{cf}\eta_{ki}\eta_{lj}e_a{}^k e_c{}^l\partial_b e_f{}^j\partial_d e_e{}^i + 4c_5 G^{ad}G^{be}G^{cf}\eta_{ji}e_f{}^j\partial_c B_{ab}\partial_d e_e{}^i - 2c_2 G^{ac}G^{bd}\eta_{ji}e_b{}^j\partial_c e_a{}^i\partial_d\Phi\\ &+ (c_2-2c_6)G^{ae}G^{bf}G^{cd}\eta_{kj}\eta_{li}e_b{}^k e_c{}^l\partial_a e_d{}^j\partial_e e_f{}^i + c_1 G^{ae}G^{bf}G^{cd}\eta_{ki}\eta_{lj}e_b{}^k e_c{}^l\partial_a e_d{}^j\partial_e e_f{}^i\\ &- c_2 G^{ad}G^{be}G^{cf}\eta_{kj}\eta_{li}e_b{}^k e_c{}^l\partial_d e_a{}^i\partial_e e_f{}^j + c_2 G^{ad}G^{be}G^{cf}\eta_{ki}\eta_{lj}e_b{}^k e_c{}^l\partial_d e_a{}^i\partial_e e_f{}^j\\ &- 2c_6 G^{ad}G^{be}G^{cf}\eta_{kj}\eta_{li}e_a{}^k e_c{}^l\partial_d e_e{}^i\partial_f e_b{}^j + c_1 G^{ad}G^{be}G^{cf}\eta_{ki}\eta_{lj}e_a{}^k e_c{}^l\partial_d e_e{}^i\partial_f e_b{}^j)\end{aligned} \tag{22}$$

The terms with coefficients c_3, c_4, c_7, which have no $\mathcal{H}^{AB}$, vanish when reducing the couplings (20) to the D-dimensional spacetime. Hence, these terms are zero by the strong constraint. Note that even though the generalized frame contains no derivative on the B-field, the above D-dimensional action contains only terms which have derivative on the B-field. The above D-dimensional action must be equal to (10) plus some D-dimensional total derivative terms. To compare the two actions, one has to rewrite the derivatives of metric in the action (10) in terms of derivatives of the frame $e_a{}^i$. The comparison then fixes both the effective actions (9) and (20) up to an overall factor. The action (9) is fixed as in (13) and the action (20) is fixed as

$$\mathbf{S}_0^f = -\frac{2a_1}{\kappa^2\tilde{V}}\int dx d\tilde{x} e^{-2d}\left(-\mathcal{F}^A\mathcal{F}^B\mathcal{H}_{AB} + 2\partial^A\mathcal{F}^B\mathcal{H}_{AB} + \frac{1}{4}\mathcal{H}^{AB}\mathcal{F}_A{}^{CD}\mathcal{F}_{BCD} - \frac{1}{12}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{ACE}\mathcal{F}_{BDF} + \cdots\right) \tag{23}$$

where dots represent the terms which vanish after using the strong constraint. The above action is the action has been found in [22]. In the next section we consider this approach to find both D-dimensional and $2D$-dimensional effective actions at order α'.

III. EFFECTIVE ACTION AT ORDER α'

The most general four-derivative action which is $O(D,D)$-invariant and is invariant under generalized diffeomorphisms can be constructed by all possible contractions of the following tensors with constant metric η^{AB} and $\mathcal{H}^{AB}$:

$$\begin{aligned}&\mathcal{F}_A\mathcal{F}_B\mathcal{F}_C\mathcal{F}_D + \mathcal{F}_A\mathcal{F}_B\mathcal{F}_C\mathcal{F}_{DEF} + \mathcal{F}_A\mathcal{F}_B\mathcal{F}_{CDE}\mathcal{F}_{FGH} + \mathcal{F}_A\mathcal{F}_{BCD}\mathcal{F}_{EFG}\mathcal{F}_{HIJ} + \mathcal{F}_{ABC}\mathcal{F}_{DEF}\mathcal{F}_{GHI}\mathcal{F}_{JKL}\\ &+ \mathcal{F}_{CDE}\mathcal{F}_{FGH}\partial_A\mathcal{F}_B + \mathcal{F}_{DEF}\partial_A\partial_B\mathcal{F}_C + \partial_A\partial_B\partial_C\mathcal{F}_D + \partial_A\partial_B\partial_C\mathcal{F}_{DEF} + \mathcal{F}_A\mathcal{F}_{DEF}\partial_B\mathcal{F}_C + \mathcal{F}_A\partial_B\partial_C\mathcal{F}_D\\ &+ \mathcal{F}_A\partial_B\partial_C\mathcal{F}_{DEF} + \mathcal{F}_A\mathcal{F}_B\partial_C\mathcal{F}_D + \partial_A\mathcal{F}_B\partial_C\mathcal{F}_D + \mathcal{F}_A\mathcal{F}_B\partial_C\mathcal{F}_{DEF} + \partial_A\mathcal{F}_B\partial_C\mathcal{F}_{DEF} + \mathcal{F}_{ABC}\partial_D\partial_E\mathcal{F}_{FGH}\\ &+ \mathcal{F}_A\mathcal{F}_{BCD}\partial_E\mathcal{F}_{FGH} + \partial_A\mathcal{F}_{BCD}\partial_E\mathcal{F}_{FGH} + \mathcal{F}_{ABC}\mathcal{F}_{DEF}\partial_G\mathcal{F}_{HIJ}\end{aligned} \tag{24}$$

It produces 275 terms, i.e.,

$$\begin{aligned}\mathbf{S}_1^f = -\frac{2}{\kappa^2\tilde{V}}\int dx d\tilde{x} e^{-2d}(&c_1\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{F}_A\mathcal{F}_B\mathcal{F}_C\mathcal{F}_D\\ &+ c_2\mathcal{F}^A\mathcal{F}_A\mathcal{F}^B\mathcal{F}_B + \cdots)\end{aligned} \tag{25}$$

where $c_1, \ldots, c_{275}$ are parameters. The terms which have no tensor $\mathcal{H}^{AB}$, e.g., c_2-term, are zero after using Bianchi identities [28] and the strong constraint. There are 34 such terms. There are also 91 terms with other structures that become zero under these constraints.[6] The remaining 150 terms are in five classes. One class includes terms that cancel each others after using the strong constraint, one

[6]One may use the Bianchi identities found in [28] to demonstrate the vanishing of some of these 125 terms. However, the more convenient way to impose the Bianchi identities and the strong constraint is to reduce the couplings to the D dimensions using the reduction (21) and then use the strong constraint. The above 125 terms vanish when we write the D-dimensional couplings in terms of independent expressions.

class includes terms that are total derivatives, one class includes terms that become zero using appropriate identities, one class includes terms that are reproduced by field redefinitions at order α' of the fields in (23), and the last class includes all other terms in which we are interested. One may examine all terms in details to exclude all terms except the terms in the last class, and then imposes the D-dimensional gauge symmetry. Alternatively, one may impose the D-dimensional gauge symmetry on all 150 terms. In this case, the terms in the first four classes appear in the final action with free parameters which can be chosen arbitrarily.

Using the relation (18), one can write the couplings (25) in terms of dilaton and the generalized frame. Then using the dimensional reduction (21), one can reduce the $2D$-dimensional action (25) to the D-dimensional action, e.g., the reduction of c_1-term is

$$\begin{aligned}\mathbf{S}_1^f = -\frac{2}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}(&16c_1 D\Phi_a D\Phi^a D\Phi_b D\Phi^b + 32c_1 De^{abi} D\Phi_b D\Phi_c D\Phi^c e_{ai} - 32c_1 De^{abi} D\Phi_a D\Phi_c D\Phi^c e_{bi}\\
&+ 16c_1 De^{abi} De^{cdj} D\Phi_b D\Phi_d e_{ai} e_{cj} - 16c_1 De^{abi} De_b{}^{cj} D\Phi_d D\Phi^d e_{ai} e_{cj} + 8c_1 De^{abi} De^c{}_b{}^j D\Phi_d D\Phi^d e_{ai} e_{cj}\\
&- 32c_1 De^{abi} De^{cdj} D\Phi_a D\Phi_d e_{bi} e_{cj} + 8c_1 De_a{}^{cj} De^{abi} D\Phi_d D\Phi^d e_{bi} e_{cj} + 16c_1 De^{abi} De^{cdj} D\Phi_a D\Phi_c e_{bi} e_{dj}\\
&- 16c_1 De^{abi} De_b{}^{cj} De^{dek} D\Phi_e e_{ai} e_{cj} e_{dk} + 8c_1 De^{abi} De^c{}_b{}^j De^{dek} D\Phi_e e_{ai} e_{cj} e_{dk} + 8c_1 De_a{}^{cj} De^{abi} De^{dek} D\Phi_e e_{bi} e_{cj} e_{dk}\\
&+ 16c_1 De^{abi} De_b{}^{cj} De^{dek} D\Phi_d e_{ai} e_{cj} e_{ek} - 8c_1 De^{abi} De^c{}_b{}^j De^{dek} D\Phi_d e_{ai} e_{cj} e_{ek} - 8c_1 De_a{}^{cj} De^{abi} De^{dek} D\Phi_d e_{bi} e_{cj} e_{ek}\\
&+ 4c_1 De^{abi} De_b{}^{cj} De^{dek} De_e{}^{fl} e_{ai} e_{cj} e_{dk} e_{fl} - 4c_1 De^{abi} De_b{}^{cj} De^{dek} De^f{}_e{}^l e_{ai} e_{cj} e_{dk} e_{fl}\\
&+ c_1 De^{abi} De^c{}_b{}^j De^{dek} De^f{}_e{}^l e_{ai} e_{cj} e_{dk} e_{fl} - 4c_1 De_a{}^{cj} De^{abi} De^{dek} De_e{}^{fl} e_{bi} e_{cj} e_{dk} e_{fl}\\
&+ 2c_1 De_a{}^{cj} De^{abi} De^{dek} De^f{}_e{}^l e_{bi} e_{cj} e_{dk} e_{fl} + c_1 De_a{}^{cj} De^{abi} De_d{}^{fl} De^{dek} e_{bi} e_{cj} e_{ek} e_{fl} + \cdots)\end{aligned} \tag{26}$$

where $De_{ab}{}^i \equiv \partial_a e_b{}^i$ and $D\Phi_a \equiv \partial_a \Phi$. The above action is manifestly invariant under T-duality as its parents (25) are invariant under the $O(D,D)$ transformations, however, it is not invariant under the usual D-dimensional gauge transformations for arbitrary parameters. We are going to find these parameters by comparing them with a D-dimensional action at order α' which is not invariant under the T-duality, but is invariant under the conventional gauge transformations, i.e., the standard coordinate transformations, the B-field gauge transformations and the nonstandard Lorentz transformation of the B-field which is required for anomaly cancellations.

The most general D-dimensional action which is invariant under the conventional gauge transformations has four class of terms. One class contains terms that are zero by Bianchi identities, one class contains terms that are total derivative terms, one class contains terms that are reproduced by the field redefinitions at order α' of the fields in (13), and the last class contains all other terms in which we are interested. One may choose the couplings to be [29]

$$\begin{aligned}\mathbf{S}_1^c = \frac{-2}{\kappa^2}\alpha'\int d^{d+1}x e^{-2\Phi}\sqrt{-G}(&b_1 R_{abcd}R^{abcd}\\
&+ b_2 R_{abcd}H^{abe}H^{cd}{}_e + b_3 H_{fgh}H^f{}_a{}^b H^g{}_b{}^c H^h{}_c{}^a\\
&+ b_4 H_f{}^{ab} H_{gab} H^{fch} H^g{}_{ch} + b_5 (H^2)^2\\
&+ b_6 H_{acd} H_b{}^{cd} \partial^a\Phi\partial^b\Phi + b_7 H^2 \partial_a\Phi\partial^a\Phi\\
&+ b_8 (\partial_a\Phi\partial^a\Phi)^2 + d_1 H^{abc}\Omega_{abc})\end{aligned} \tag{27}$$

where $b_1, b_2, \ldots, b_8$ are eight parameters. The field redefinitions freedom allows us to choose the eight arbitrary couplings in (27) in many different schemes. The above is one particular scheme. The last term is zero for the bosonic string theory because the B-field gauge transformation is the standard transformation, i.e., $B_{ab} \to B_{ab} + \partial_{[a}\lambda_{b]}$. This term is nonzero for the heterotic string theory and its coefficient is $d_1 = -a_1/6$ in this case. This term is a result of nonstandard gauge transformation of B-field which is

$$B_{ab} \to B_{ab} + \partial_{[a}\lambda_{b]} + \alpha'\partial_{[a}\Lambda_i{}^j\omega_{b]j}{}^i \tag{28}$$

where $\Lambda_i{}^j$ is the matrix of the Lorentz transformations and $\omega_{bi}{}^j$ is spin connection.[7] The Chern-Simons three-form Ω which is defined as

$$\begin{aligned}\Omega_{abc} &= \omega_{[ai}{}^j\partial_b\omega_{c]j}{}^i + \frac{2}{3}\omega_{[ai}{}^j\omega_{bj}{}^k\omega_{c]k}{}^i;\\
\omega_{ai}{}^j &= \partial_a e_b{}^j e^b{}_i - \Gamma_{ab}{}^c e_c{}^j e^b{}_i\end{aligned} \tag{29}$$

makes $H_{abc} + \alpha'\Omega_{abc}$ to be invariant under the Lorentz transformations, i.e., $H_{abc} + \alpha'\Omega_{abc} \to H_{abc} + \alpha'\Omega_{abc}$. The action (27) is not invariant under T-duality for arbitrary parameters $b_1, b_2, \ldots, b_8$. However, one expects for some specific values for these parameters, the couplings become invariant under T-duality. We are going to find

[7] At order α', there is a scheme in which the appearance of the B-field in the effective action simplifies through extending the spin connection to the spin connection with torsion [30]. Since we are working with the most general covariant action (27), we do not assume such simplification.

these parameters by comparing them with the manifestly T-duality invariant action (26). To compare the two actions, one has to rewrite the couplings in (27) in terms of frame $e_a{}^i$ and B-field. If one compares the above D-dimensional action with the reduced action (26), one would find zero value for all parameters. So the actions are not the same. Let us check their Lagrangians. So we have to include some total derivative terms.

To construct all D-dimensional total derivative terms in terms of frame $e_a{}^i$, B-field and dilaton Φ, we have to construct all contractions of the following tensors with metric G^{ab} to produce the current $\mathbf{I}^a$:

$$\begin{aligned}&\partial_a\partial_b\partial_c B_{de}+\partial_a\partial_b\partial_c e_d{}^e+\partial_a\partial_b\partial_c\Phi+\partial_a\partial_b\Phi\partial_c e_d{}^e+\partial_a\partial_b\Phi\partial_c\Phi+\partial_a\Phi\partial_b\Phi\partial_c\Phi\\&+\partial_a B_{bc}\partial_d\partial_e B_{fg}+\partial_a B_{bc}\partial_d\partial_e e_f{}^g+\partial_a B_{bc}\partial_d\partial_e\Phi+\partial_a\partial_b B_{cd}\partial_e e_f{}^g+\partial_a\partial_b e_c{}^d\partial_e e_f{}^g\\&+\partial_a\partial_b B_{cd}\partial_e\Phi+\partial_a\partial_b e_c{}^d\partial_e\Phi+\partial_a B_{bc}\partial_d\Phi\partial_e\Phi+\partial_a e_b{}^c\partial_d\Phi\partial_e\Phi+\partial_a B_{bc}\partial_d B_{ef}\partial_g B_{hhh}\\&+\partial_a B_{bc}\partial_d B_{ef}\partial_g e_h{}^{hh}+\partial_a B_{bc}\partial_d e_e{}^f\partial_g e_h{}^{hh}+\partial_a e_b{}^c\partial_d e_e{}^f\partial_g e_h{}^{hh}+\partial_a B_{bc}\partial_d B_{ef}\partial_g\Phi\\&+\partial_a B_{bc}\partial_d e_e{}^f\partial_g\Phi+\partial_a e_b{}^c\partial_d e_e{}^f\partial_g\Phi\end{aligned}\tag{30}$$

where $\partial_a e_b{}^c\equiv e^c{}_i\partial_a e_b{}^i$, $\partial_a\partial_d e_b{}^c\equiv e^c{}_i\partial_a\partial_d e_b{}^i$ and $\partial_a\partial_d\partial_f e_b{}^c\equiv e^c{}_i\partial_a\partial_d\partial_f e_b{}^i$. Then the following expression produces all total derivative terms:

$$\mathbf{J}=\frac{-2}{\kappa^2}\alpha'\int dx\partial_a(e^{-2\Phi}\sqrt{-G}\mathbf{I}^a)\tag{31}$$

One may extend the list of currents in (30) by including terms $\partial_a e_b{}^i\partial_c e_{di}$ and $\partial_a\partial_b e_c{}^i\partial_d e_{ei}$ as well, however, they do not produce independent total derivative terms in $\mathbf{J}$. We have examined the equality $\mathbf{S}_1^f=\mathbf{S}_1^c+\mathbf{J}$ and again found zero result for all parameters. This is an indication of the observation made in [21] that fields in the conventional D-dimensional action (27) are not the same as the fields defined in the reduction of the $2D$-dimensional fields. In particular, the fields B, $e_a{}^i$ in (21) are not the same as the dynamical B-field and frame used in (27). So we have to use field redefinitions on the D-dimensional fields in (13), (27) and then compare them with (26).

The variation of action (13), for $a_1=1$, under field redefinition $G_{ab}\to G_{ab}+\delta G_{ab}$, $B_{ab}\to B_{ab}+\delta B_{ab}$ and $\Phi\to\Phi+\delta\Phi$ is

$$\begin{aligned}\delta\mathbf{S}_0^c=-\frac{2}{\kappa^2}\int dx e^{-2\Phi}\sqrt{-G}\Big[&\Big(\frac{1}{2}R+2\nabla_c\nabla^c\Phi\\&-2\nabla_c\Phi\nabla^c\Phi-\frac{1}{24}H^2\Big)(G^{ab}\delta G_{ab}-4\delta\Phi)\\&-\Big(R^{ab}+2\nabla^b\nabla^a\Phi-\frac{1}{4}H^{acd}H^b{}_{cd}\Big)\delta G_{ab}\\&+\frac{1}{2}(\nabla_c H^{cab}-2\nabla_c\Phi H^{cab})\delta B_{ab}\Big]\end{aligned}\tag{32}$$

where we have also ignored some covariant total derivative terms. δG_{ab}, $\delta\Phi$ and δB_{ab} can be constructed by all contractions of the following tensors with metric G^{ab}:

$$\begin{aligned}&\partial_a\partial_b B_{cd}+\partial_a\partial_b e_c{}^d+\partial_a\partial_b\Phi+\partial_a\Phi\partial_b\Phi+\partial_a B_{bc}\partial_d B_{ef}\\&+\partial_a B_{bc}\partial_d e_e{}^f+\partial_a e_b{}^c\partial_d e_e{}^f+\partial_a B_{bc}\partial_d\Phi+\partial_a e_b{}^c\partial_d\Phi\end{aligned}\tag{33}$$

We then examine the following equality:

$$\mathbf{S}_1^f=\mathbf{S}_1^c+\mathbf{J}+\delta\mathbf{S}_0^c\tag{34}$$

It produces many algebraic equations for the parameters with nonzero result for them.

The most important part of the result is that they fix uniquely all eight parameters in the D-dimensional action (27) in terms of b_1, i.e.,

$$\begin{aligned}\mathbf{S}_1^c=\frac{-2}{\kappa^2}\alpha'\int d^{d+1}x e^{-2\Phi}\sqrt{-G}&\\\times\Big[b_1\Big(R_{abcd}R^{abcd}-\frac{1}{2}R_{abcd}H^{abe}H^{cd}{}_e&\\+\frac{1}{24}H_{fhg}H^f{}_a{}^bH^h{}_b{}^cH^g{}_c{}^a-\frac{1}{8}H_f{}^{ab}H_{hab}H^{fcg}H^h{}_{cg}\Big)&\\+d_1H^{abc}\Omega_{abc}\Big]&\end{aligned}\tag{35}$$

In the scheme (27), the effective action (35) then has no derivative of dilaton. Up to the overall factor b_1, the above couplings are the standard effective action of the bosonic and heterotic string theories which has been found in [29] by the S-matrix calculations. This action now is invariant under T-duality. Since we have used field redefinitions to relate the manifestly T-duality invariant action (26) to the manifestly gauge invariant action (27), the T-duality transformations rules in above action would be the Buscher rules plus their α'-corrections. They have be found directly by using the T-duality approach [15,16].

The parameters in the $2D$-dimensional action (25), however, are not fixed uniquely in terms of b_1, d_1. There remain many parameters in (25) to be arbitrary. They reflect the four classes of terms in (25) that we did not removed them from the list of independent couplings in (25). Choosing different values for the arbitrary parameters correspond to different scheme in which the $2D$-dimensional action can be written. One particular choice for the parameters should reproduce the $2D$-dimensional couplings found in [11] which include derivatives of the generalized dilaton and frame. However, as in the D-dimensional action (35), we choose a scheme in which the $2D$-dimensional dilaton appears only as an overall factor. Since the derivative of the dilaton appears in the flux $\mathcal{F}_A$, we choose the arbitrary parameters to have no terms in (25) which has $\mathcal{F}_A$ or its derivatives. Using this constraint, there are still some residual parameters in this scheme. We further constrain the scheme to have no terms with structure $\partial_A\partial_B\partial_C\mathcal{F}_{DEF}$, $\partial_A\partial_B\mathcal{F}_{CDE}$, nor $\partial_A\mathcal{F}_{BCD}\partial_E\mathcal{F}_{EFH}$. Then the effective action in this scheme for the bosonic string theory becomes

$$\begin{aligned}\mathbf{S}_1^{fe} = \frac{-2b_1}{\kappa^2\tilde{V}}\alpha'\int dxd\tilde{x}e^{-2d}\Big(&-\frac{1}{8}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GH}\mathcal{H}^{IJ}\mathcal{H}^{KL}\mathcal{F}_{ACE}\mathcal{F}_{BDG}\mathcal{F}_{FIK}\mathcal{F}_{HJL}+\frac{1}{4}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{F}_A{}^{EF}\mathcal{F}_B{}^{GH}\mathcal{F}_{CE[F}\mathcal{F}_{G]DH}\\&+\frac{1}{4}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GH}\mathcal{F}_{ACE}\mathcal{F}_{BDG}\mathcal{F}_F{}^{IJ}\mathcal{F}_{HIJ}+\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{F}_{AC}{}^{E}\mathcal{F}_B{}^{FG}\mathcal{F}_{DF}{}^{H}\mathcal{F}_{EGH}\\&+\frac{1}{2}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{F}_{AC}{}^{E}\mathcal{F}_{BE}{}^{F}\mathcal{F}_D{}^{GH}\mathcal{F}_{FGH}+\frac{3}{8}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GH}\mathcal{F}_{AC}{}^{I}\mathcal{F}_{BE}{}^{J}\mathcal{F}_{DGJ}\mathcal{F}_{FHI}\\&-\frac{1}{2}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GH}\mathcal{F}_{ACE}\mathcal{F}_{BG}{}^{I}\mathcal{F}_{DH}{}^{J}\mathcal{F}_{FIJ}+\frac{1}{24}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GH}\mathcal{H}^{IJ}\mathcal{H}^{KL}\mathcal{F}_{ACE}\mathcal{F}_{BGI}\mathcal{F}_{DHK}\mathcal{F}_{FJL}\\&-2\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{F}_{AC}{}^{E}\mathcal{F}_B{}^{FG}\partial_{[F}\mathcal{F}_{E]DG}+\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GH}\mathcal{F}_{ACE}\mathcal{F}_{BG}{}^{I}\partial_{(H}\mathcal{F}_{I)DF}+\cdots\Big)\end{aligned}\tag{36}$$

where dots represent terms that are zero under the strong constraint. Our notation for antisymmetrization is $L_{[A}P_{B]} = (L_AP_B - L_BP_A)/2$, similarly for symmetrization. This action now must be invariant under double-Lorentz transformations. Since we have used field redefinitions to relate the manifestly T-duality invariant action (26) to the manifestly Lorentz invariant action (27), the double-Lorentz transformations in above action would be the standard Lorentz transformations plus their α'-corrections. The above action is even under $B \to -B$. It should be the same as the even part of the action has been found in [11] up to the terms that are zero under the strong constraint and up to $2D$-dimensional field redefinitions.

The $2D$-dimensional effective action of the heterotic string theory at order α' contains the above action plus the following terms which are odd under $B \to -B$:

$$\begin{aligned}\mathbf{S}_1^{fo} = \frac{-2d_1}{\kappa^2\tilde{V}}\alpha'\int dxd\tilde{x}e^{-2d}\Big(&\frac{1}{8}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{AC}{}^{G}\mathcal{F}_{BD}{}^{I}\mathcal{F}_{EG}{}^{H}\mathcal{F}_{FIH}-\frac{1}{8}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GI}\mathcal{H}^{HJ}\mathcal{F}_{ACE}\mathcal{F}_{BGH}\mathcal{F}_{DI}{}^{K}\mathcal{F}_{FJK}\\&-\frac{1}{12}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{AC}{}^{G}\mathcal{F}_{BE}{}^{I}\mathcal{F}_{DF}{}^{H}\mathcal{F}_{GIH}+\frac{1}{24}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{ACE}\mathcal{F}_{BD}{}^{G}\mathcal{F}_F{}^{IH}\mathcal{F}_{GIH}\\&+\frac{1}{4}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{AC}{}^{G}\mathcal{F}_{EG}{}^{H}\partial_B\mathcal{F}_{DFH}-\frac{1}{3}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{ACE}\mathcal{F}_B{}^{GH}\partial_{[D}\mathcal{F}_{G]FH}\\&-\frac{1}{4}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{H}^{GI}\mathcal{H}^{HJ}\mathcal{F}_{ACE}\mathcal{F}_{BGH}\partial_D\mathcal{F}_{FIJ}-\frac{1}{8}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{AC}{}^{G}\mathcal{F}_{EG}{}^{H}\partial_F\mathcal{F}_{BDH}\\&-\frac{3}{8}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\mathcal{F}_{AC}{}^{G}\mathcal{F}_{EG}{}^{H}\partial_H\mathcal{F}_{BDF}+\cdots\Big)\end{aligned}\tag{37}$$

where dots represent terms that are zero under the strong constraint. While the action (36) has even number of $\mathcal{H}^{AB}$, the above action has odd number of $\mathcal{H}^{AB}$. The above action should be the same as the odd part of the action has been found in [11] up to the terms that are zero under the strong constraint and up to $2D$-dimensional field redefinitions.

The algebraic equations fix also the non-covariant field redefinitions and total derivative terms required to relate the two D-dimensional actions. The total derivative terms in the bosonic theory have structures $\partial\partial e\partial\partial e$, $\partial\partial e\partial e\partial e$, $\partial\partial B\partial B\partial e$, $\partial B\partial B\partial\partial e$, $\partial B\partial B\partial e\partial e$, $\partial\partial\Phi\partial e\partial e$, $\partial\partial\Phi\partial\Phi\partial e$, $\partial\Phi\partial e\partial\partial e$, $\partial\Phi\partial\Phi\partial e\partial e$, $\partial\Phi\partial e\partial e\partial e$ and

$\partial\Phi\partial e\partial B\partial B$. They indicate that the current $\mathbf{I}^a$ contain terms with structure $\partial\partial e\partial e$, $\partial e\partial e\partial e$, $\partial\Phi\partial e\partial e$ and $\partial e\partial B\partial B$. The reason that there is no current with structure $\partial\partial\Phi\partial e$, $\partial\partial\Phi\partial\Phi$, $\partial\partial B\partial B$ is that they produce terms in $\mathbf{J}$ with structures $\partial\partial\Phi\partial\partial\Phi$, $\partial\partial\Phi\partial\partial e$, $\partial\partial B\partial\partial B$ which are not in the D-dimensional action (27). There are too many total derivative terms to be able to write them explicitly here. The field redefinitions for the bosonic theory however are

$$\begin{aligned}
\delta G^B_{ab} &= b_1(2De_a{}^{ci}De_{bci} - 4De_{(b}{}^{ci}De_{ca)i} + 2De_{cbi}De^c{}_a{}^i + 8De_{(b}{}^{ci}D\Phi_c e_{a)i} - 8De^c{}_{(b}{}^i D\Phi_c e_{a)i} - 4DDe_{(b}{}^c{}_{ci}e_{a)}{}^i \\
&\quad + 4DDe^c{}_{c(bi}e_{a)}{}^i - 4De_{cd}{}^j De^{cdi}e_{ai}e_{bj} + 4De^{cdi}De_{dc}{}^j e_{ai}e_{bj} + 4De_{(b}{}^{ci}De^d{}_d{}^j e_{a)i}e_{cj} - 4De^c{}_{(a}{}^i De^d{}_d{}^j e_{b)i}e_{cj} \\
&\quad + 2De_{(b}{}^{ci}De_c{}^{dj}e_{a)j}e_{di} - 6De_c{}^{dj}De^c{}_{(b}{}^i e_{a)j}e_{di} - 2De_{(a}{}^{ci}De^d{}_c{}^j e_{b)j}e_{di} + 6De^c{}_{(a}{}^i De^d{}_c{}^j e_{b)j}e_{di} \\
&\quad + 2De_a{}^{ci}De_b{}^{dj}e_{cj}e_{di} - 4De_{(a}{}^{ci}De^d{}_{b)}{}^j e_{cj}e_{di} + 2De^c{}_a{}^i De^d{}_b{}^j e_{cj}e_{di} - 4De_{(a}{}^{ci}De_c{}^{dj}e_{b)i}e_{dj} + 4De_c{}^{dj}De^c{}_{(a}{}^i e_{b)i}e_{dj} \\
&\quad + 4De_{(a}{}^{ci}De^d{}_c{}^j e_{b)i}e_{dj} - 4De^c{}_{(a}{}^i De^d{}_c{}^j e_{b)i}e_{dj}) \\
\delta\Phi^B &= \frac{1}{4}G^{ab}\delta G^B_{ab} \\
\delta B^B_{ab} &= b_1 H^{cd}{}_{[a}(\omega_{b]ij}e_c{}^i e_d{}^j + e_{b]i}De_{cd}{}^i)
\end{aligned} \tag{38}$$

where $DDe_{abc}{}^i \equiv \partial_a\partial_b e_c{}^i$, $De_{ab}{}^i \equiv \partial_a e_b{}^i$ and $D\Phi_a \equiv \partial_a\Phi$. The curved indices are raised with metric G^{ab} and flat indices are lowered by η_{ij}. The above α'-corrections must be added to the metric, dilaton and the B-field in the reductions of $2D$-dilaton, i.e., $e^{-2d} = e^{-2\Phi}\sqrt{-G}$ and the generalized metric (7) in order to make them identical to the corresponding covariant fields in the D-dimensional action (27). The resulting new $2D$-dilaton $\tilde{d}$ and generalized metric $\tilde{\mathcal{H}}$, then should transform covariantly as (16). The above field redefinition for dilaton, however, is such that $\delta d = \delta\Phi - \frac{1}{4}G^{ab}\delta G_{ab} = 0$. So the generalized dilaton remains invariant under the field redefinitions at order α'. Using the α'-corrections to the generalized metric, one may find the α'-corrections to the generalized diffeomorphisms for the fields that are transformed by the Buscher rules under T-duality. Alternatively, one may add the above α'-corrections, with a minus sign, to the D-dimensional covariant fields G_{ab}, Φ, B_{ab} in order to make them identical to the corresponding $2D$-dimensional fields $e^{-2d} = e^{-2\Phi}\sqrt{-G}$ and (7) which are transformed by Buscher rules under T-duality [7]. In this way one may find the α'-corrections to the Buscher rules for the D-dimensional covariant fields.

The total derivative terms in the heterotic theory have the same structures as in the bosonic theory as well as the structures $\partial\partial e\partial\partial B$, $\partial\partial e\partial e\partial B$, $\partial e\partial e\partial\partial B$, $\partial e\partial e\partial\Phi\partial B$, $\partial e\partial\Phi\partial\partial B$, $\partial\partial e\partial\Phi\partial B$, $\partial\Phi\partial\Phi\partial e\partial B$ and $\partial e\partial\partial\Phi\partial B$. They indicate that the current $\mathbf{I}^a$ contain terms with structure $\partial\partial e\partial B$, $\partial e\partial e\partial B$, and $\partial e\partial\Phi\partial B$. The field redefinitions for the heterotic theory are

$$\begin{aligned}
\delta G^H_{ab} &= \delta G^B_{ab} + d_1 H^{cd}{}_{(a}e_{b)i}De_{cd}{}^i \qquad \delta\Phi^H = \frac{1}{4}G^{ab}\delta G^H_{ab} \\
\delta B^H_{ab} &= \delta B^B_{ab} + d_1(De_{[ab]}{}^i De^c{}_{ci} + DDe_{[a}{}^c{}_{ci}e_{b]}{}^i - 4De_{[ab]}{}^i D\Phi^c e_{ci} + DDe_{[acb]i}e^{ci} \\
&\quad + De_c{}^{dj}De^c{}_{[b}{}^i e_{a]j}e_{di} - 2De_{[a}{}^{ci}De^d{}_c{}^j e_{b]j}e_{di} + De^c{}_{[a}{}^i De^d{}_c{}^j e_{b]j}e_{di} \\
&\quad + 2De_{[ba]}{}^i De^{cdj}e_{cj}e_{di} - De_{[a}{}^{ci}De^d{}_{b]}{}^j e_{cj}e_{di} + 2De_{[ab]}{}^i De^{cdj}e_{ci}e_{dj})
\end{aligned} \tag{39}$$

Here also the field redefinition for the generalized dilaton δd is zero. While the field redefinition terms in the bosonic theory do not change the symmetry under $B \to -B$, the field redefinition terms corresponding to the Chern-Simons term in the heterotic theory change the symmetry of fields under $B \to -B$. This is as expected because the field redefinitions should produce odd terms under $B \to -B$ from the even terms in (32). It is interesting to note that in both heterotic and bosonic theories the total derivative term are zero when the D-dimensional frame $e_a{}^i$ is constant. Moreover, the T-duality covariant fields and the gauge covariant fields are identical when $e_a{}^i$ is constant, i.e., $\delta G = \delta\Phi = \delta B = 0$.

We have found the T-duality invariant $2D$-dimensional actions (36) and (37) by constraining them to be invariant under the generalized diffeomorphism and by constraining them to be invariant under the D-dimensional gauge transformations after using appropriate field redefinitions (38) and (39) on the leading order action (14). These actions must be invariant under the double-Lorentz

transformations as well. However, the transformations are the standard double-Lorentz transformations pulse their corresponding α'-corrections [10,11]. Since we have used the $2D$-dimensional field redefinitions freedom to write the α'-order actions (36) and (37) in a scheme in which the dilaton appears only as an overall factor, the field redefinitions (38) and (39) and the corresponding α'-corrections to the double-Lorentz transformations are then fixed uniquely. If we have chosen a different scheme for the $2D$-dimensional actions, then the field redefinitions (38) and (39) would be different. The form of the corrections to the double-Lorentz transformations might be different from those have been found in [10,11], because the $2D$-dimensional actions in [10,11] is written in a different scheme in which the derivative of dilaton also appears in the actions. Unlike our approach that the $2D$-dimensional action is fixed first, in [10,11] the authors first fix the α'-corrections to the double-Lorentz transformations and then find the corresponding $2D$-dimensional actions. It would be interesting to find the α'-corrections to the double-Lorentz transformations corresponding to the actions (36) and (37) and then compare them with the transformations found in [10,11] under the appropriate field redefinitions that change the actions (36) and (37) to the corresponding actions in [10,11].

Using the generalized frame and dilaton as the dynamical fields, we have found the $2D$-dimensional actions (36) and (37). One may wish to use the generalized metric and dilaton as the dynamical fields to find the $2D$-dimensional action at order α', i.e., extension of the action (14) to the order α'. We have done this calculation. We have found that for the case that B-field is zero there are nonzero D-dimensional and $2D$-dimensional actions, however, in the presence of the B-field one would find no effective action at order α'. In fact, when we write the couplings (27) in terms of B-field and metric, for instance, the second coupling in (27) produces, among other things, the following terms:

$$\begin{aligned} &-2G^{fg}\partial_a G_{dg}\partial^a B^{bc}\partial_b B^{de}\partial_c G_{ef} \\ &+G^{fg}\partial_a B^{de}\partial^a B^{bc}\partial_b G_{dg}\partial_c G_{ef} \end{aligned} \tag{40}$$

None of them is reproduced by any $2D$-couplings, any noncovariant field redefinition or any total derivative term at order α'. This confirms the observation that there is no manifestly background independent and duality invariant formulation of the bosonic theory at order α' in terms of the generalized metric, however, there is such formulation in terms of generalized frame [31]. So one expects the convenient framework for studying the higher derivative couplings in the D-dimensional string theory and in DFT is the framelike formulation of DFT.

The gravity and dilaton couplings in the effective actions of string theories at orders α'^2 and α'^3 are known in the literature. Using the T-duality approach, it has been shown in [19,20] that they are invariant under the T-duality transformations which are the Buscher rules and their α'-corrections. However, the B-field couplings at these orders are not known in the literature. It would be interesting to use the method in this paper to find these couplings as well as their corresponding $2D$-dimensional actions. The B-field couplings at order α'^2 for Hohm-Siegel-Zwiebach double field theory [32] have been found in [33].

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