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Research Paper

Tube-in-tube helical heat exchangers performance optimization by entropy generation minimization approach

M. Farzaneh-Gord [a], H. Ameri [a], A. Arabkoohsar [b,*]

[a] Department of Mechanical Engineering, Shahrood University of Technology, Shahrood, Iran
[b] Department of Mechanical Engineering, Azadshahr Branch, Islamic Azad University, Azadshahr, Iran

HIGHLIGHTS

- Helical heat exchanger performance is optimized in terms of entropy generation rate.
- Laminar and turbulent flows in the inner and outer tubes of the device were studied.
- Optimum diameter ratio for the inner tube and the annulus were determined.
- Optimal flow characteristics for this type of heat exchanger were found.

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ABSTRACT

There are several different types of heat exchangers that are used for various applications. Helically coiled heat exchanger is one of widely used types of heat exchangers in industrial applications as it offers certain advantages, such as smaller size, higher heat transfer rate, efficient performance in high pressure and temperature differentials and lower cost. This study aims at finding the optimal geometry and operational conditions of helically coiled heat exchangers for both laminar and turbulent flows based on the second law of thermodynamics. In order to fulfill the main goals of this work, first, a dimensionless function for entropy generation number comprising four dimensionless variables, i.e. Prandtl number (*Pr*), Dean number (*De*), the ratio of helical pipe diameter to the tube diameter (δ) and the duty parameter of heat exchanger, is derived. Next, the entropy generation number is minimized to develop analytical expressions for the optimal values of δ, *De* number (for laminar flow) and Reynolds number (*Re*, for turbulent flow) of the heat exchangers.

1. Introduction

In spite of relatively complicated design and manufacturing process, helically coiled heat exchangers are widely used in industrial applications because they offer some advantages that make them the most appropriate heat exchanger type in many specific cases. In fact, this heat exchanger type is more suitable for the applications with limited space, lower flow rates, high pressure drop of one or both of the fluids along the heat exchanger and the applications with multiphase fluids [1–6]. In a tube-in-tube helical heat exchanger, the fluids are subjected to a centrifugal force as they flow through spiral shape tubes. This centrifugal force creates counter-rotating vortices, so-called secondary flows, in the fluids and in this way, enhances significantly the rate of heat transfer between the fluids [7].

Although the accidental motion of fluids and the behavior of secondary flows through the curvatures make it difficult to simulate the heat transfer procedure through a helical tube-in-tube [8], several studies have been conducted on these heat exchangers so far. For example, Kumar et al. [9] investigated hydrodynamic and heat transfer characteristics of this type of heat exchanger experimentally. Ghorbani et al. [10] carried out an experimental study on a vertical helically coiled heat exchanger and found the coil surface area a very effective geometrical parameter on overall heat transfer coefficient while the effect of the tubes diameters was negligible. Jayakumar et al. [11] studied the performance of fluid to fluid helical coil heat exchangers under different boundary conditions, such as constant heat flux, constant wall temperature and constant heat transfer coefficient. In another work, Jayakumar et al. [12] studied numerically the variation of Nusselt number (Nu) along the tube length and compared their results with their results derived from experimental tests. Rahul et al. [13] developed a novel correlation for calculating the overall heat transfer

* Corresponding author.
E-mail address: mani.koohsar@yahoo.com (A. Arabkoohsar).

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Nomenclature

A	cross-sectional area of the tube (m^2)
B	heat exchanger duty parameter
c_p	specific heat of tube-fluid (J/kg K)
d	diameter of tube (m)
D	coil diameter (m)
D_i	diameter of inner tube (m)
D_o	diameter of annulus (m)
De	Dean number
f	friction factor
h	heat transfer coefficient (W/m^2 K)
k	thermal conductivity of the tube-fluid (W/m K)
$\dot{m}$	mass flow rate (kg/s)
N_s	scaled dimensionless entropy generation rate
Nu	Nusselt number
q'	heat transfer rate per unit tube length (W/m)
Q	dimensionless heat flux
Re	Reynolds number
$\dot{S}_{gen}$	entropy generation rate (W/K)
St	Stanton number
T	temperature (K)
Greek symbols	
δ	curvature ratio
ρ	density (kg/m^3)
μ	viscosity coefficient (N s/m^2)

coefficient of helical tube-in-tube heat exchangers. Huminic and Huminic [14] assessed the effect of using nano-fluids on the performance of double tube helical heat exchangers, finding 14–19% enhancement in the overall heat transfer coefficient. Most recently, San et al. [15] analyzed the heat transfer performance of a helical heat exchanger comprising a tube with rectangular cross section and two cover plates for water as the helical fluid and air as radial flow material. Several more studies in this area could be addressed in the literature [16,17,4].

On the other hand, entropy generation minimization is a reliable technique for optimizing the performance of different systems. In fact, minimizing entropy generation in a device can optimize the performance of the given system based on the second law of thermodynamics [18]. Comprehensive information and practical examples about the application of this technique for heat exchangers could be found in several books and research articles [19–21]. Several more studies have also been conducted in this area by other researchers that could be addressed in the literature. For example, Sahin [22,23] accomplished thorough second law analysis studies for various duct geometries with constant wall heat flux and laminar flows. Ko and Ting [24,25] and Ko [26] performed detailed irreversibility analysis studies for helical coil tubes. In the next step, they determined the optimal configuration of curved rectangular cross sectional ducts with internal laminar flow, forced convection and constant wall heat flux based on minimum entropy generation principle [27–29]. Shokouhmand and Salimpour [30,31] investigated the rate of entropy generation in a helical tube with fully developed laminar flow inside, uniform constant wall temperature and forced convection. In addition to the aforementioned studies, several more works may be found in the literature focusing on entropy generation minimization in coiled tubes, however, little attention has been paid to the entropy generation of both turbulent and laminar flows in tube-in-tube helical heat exchangers.

Therefore, the main goal of this work is minimizing the rate of entropy generation in tube-in-tube helical heat exchangers for both turbulent and laminar flows by finding the optimal geometry and operational conditions of this type of heat exchanger. Overall, four parameters, namely, the ratio of the helical pipe diameter to the diameter of both of the inner and outer tubes (δ_1 and δ_2), *Re* number (for turbulent flow) and *De* number (for laminar flow), are going to be optimized for obtaining the lowest rate of entropy generation in this work. It is worth mentioning that fixed geometry is considered for the heat exchanger for finding optimal operational conditions, and by the same token, fixed operational conditions are considered for the heat exchanger to optimize its geometry.

2. Problem description and formulation

In this section, detailed mathematical modeling of tube-in-tube helical heat exchangers based on the second law of thermodynamics is presented. Fig. 1a and b illustrates the schematic diagram and the geometry of a tube-in-tube helical heat exchanger, respectively. According to Fig. 1a, in this type of heat exchanger, the fluid passing through the internal helical tube (called tube) exchanges thermal energy with a secondary fluid that passes through the space between the inner and outer tubes walls (called annulus). Fig. 1b gives information about the details of the heat exchanger geometry.

Hereafter, the formulation related to the rate of entropy generation and other relevant equations for two different cases, i.e. laminar and turbulent flows within the tube, are presented. It is worth mentioning that, in this study, first, the flow through the tube and then, the flow through the annulus are optimized in terms of entropy generation. Generally, for each of the fluids, the rate of entropy generation (per unit length of the tube) in the heat transfer process could be given by [20–22]:

$$\dot{S}_{gen} = \frac{q'^2 d}{4T^2 \dot{m} c_p St} + \frac{2\dot{m}^3 f}{\rho^2 T d A^2} \quad (1)$$

In this equation, the first term on the right side of the equation is in connection with heat transfer and the second part is about the friction between the fluid and the tube wall. The parameters T, c_p and ρ are the fluid mean temperature through the tube, its specific heat capacity at constant pressure and its density, respectively. Also, the parameter *St* is Stanton number which is defined as [20]:

$$St = \frac{hA}{\dot{m} c_p} \quad (2)$$

where h, A and $\dot{m}$ refer to convective heat transfer coefficient, the cross sectional area of tube and the fluid mass flow rate, respectively. Note that this equation is extracted for a straight pipe and can be securely used for a helical twist tube provided that $\dot{S}_{gen}$ is computed based on unit center-line length of the pipe. Obviously, higher value of *St* decreases the rate of entropy generation as the contribution of heat transfer term in entropy generation falls. On the other hand, higher values of friction factor (f) increase the entropy generation rate as a result of viscous effects.

According to Bejan [20], Eq. (1) could also be written as:

$$\dot{S}_{gen} = \frac{q'^2}{\pi k T^2 Nu} + \frac{32\dot{m}^3 f}{\pi^2 \rho^2 T d^5} \quad (3)$$

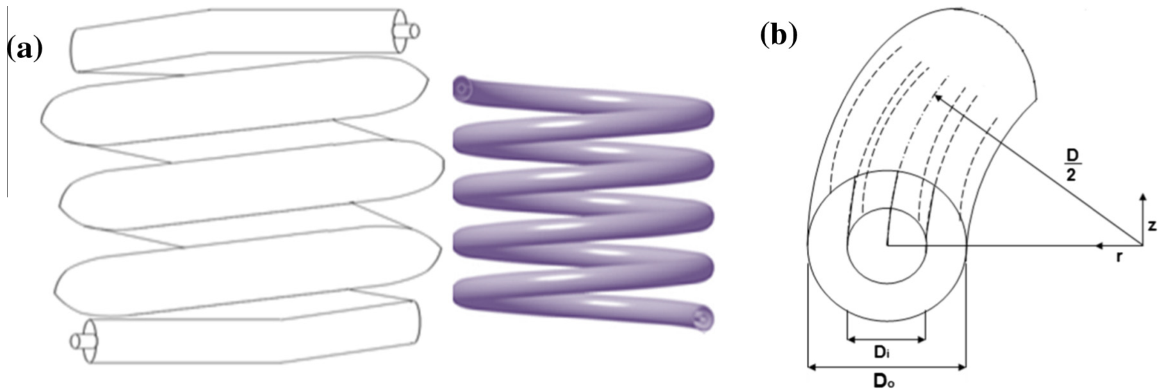

Fig. 1. The schematic diagram (a) and the details of the geometry (b) of tube-in-tube helical heat exchanger.

where *Nu* and k represent Nusselt number and thermal conductivity of the fluid, respectively. In order to generate a dimensionless function for entropy generation rate of the system, the following dimensionless parameters must be determined:

$$Q = \frac{q'}{kT} \quad Re = \frac{4\dot{m}}{\pi\mu d} \quad B^2 = \frac{q'^2\rho^2\dot{m}^2}{kT\mu^5} \tag{4}$$

Putting these dimensionless functions in Eq. (3), one has:

$$N_s = \frac{1}{\pi Nu} + \frac{\pi^3 f Re^5}{32B^2} \tag{5}$$

where N_s is scaled dimensionless entropy generation rate function which could also be re-written as:

$$N_s = \dot{S}_{gen}/kQ^2 \tag{6}$$

The flow through the tubes may be either laminar or turbulent. As the formulations of calculating *Nu* number and friction factor for laminar and turbulent flows are different, the formulation corresponding with each of these cases is presented in a specific subsection hereunder. It is re-mentioned that the ratio of the helical pipe diameter to the diameter of the inner tube (δ_1) and the ratio of the helical pipe diameter to outer tube diameter (δ_2) are the parameters that are going to be optimized for both laminar and turbulent cases, *De* number is optimized for laminar flow and *Re* numbers is the desired parameter for being optimized for turbulent flow.

2.1. Laminar flow

Janssen and Hoogendoorn [32] developed a correlation for calculating the *Nu* number of an internal laminar flow through a tube (or between concentricity tubes) as follow:

$$Nu = 0.7Re^{0.43}Pr^{\frac{1}{6}}\left(\frac{1}{\delta}\right)^{0.07} \tag{7}$$

where δ is the ratio of helical pipe diameter to the tube diameter. Besides, Ito [33] indicated that the friction factor of internal tube with a laminar flow could be given by:

$$f = 0.37\left(\frac{64}{Re}\right)De^{0.36} \tag{8}$$

Having these two equations and dimensionless equations presented in the previous section, one could formulate the tube-in-tube helical heat exchanger for laminar flow through the inner tube and the annulus to find the optimal characteristics and operational conditions.

(a) For the inner tube, combining Eqs. (4)–(8), one could rewrite the dimensionless entropy generation rate term as:

$$N_s = \frac{0.455}{De^{0.43}Pr^{\frac{1}{6}}\delta_1^{0.145}} + 22.9\frac{\delta_1^2 De^{4.36}}{B^2} \tag{9}$$

where δ_1 is the ratio of helical pipe diameter to the inner tube diameter. From engineering mathematics, by using the chain rule of differentiation and considering the derivation of the above formula relative to δ_1 as the heat exchanger design parameter equal to zero $\left(\frac{\partial N_s}{\partial \delta_1} = 0\right)$, one could find the values of δ_1 at which the extremum (either maximum or minimum) entropy generation rates happen. Then, one could find the geometry corresponding with the minimum rate of entropy generation ($\delta_{1,opt}$). In this case, this function is found as below:

$$\delta_{1,opt} = \frac{B^{0.93}}{21.12Pr^{0.078}De^{2.23}} \tag{10}$$

The optimal *De* number (De_{opt}) is then found as:

$$De_{opt} = \frac{B^{0.418}}{4.007Pr^{0.035}\delta_1^{0.45}} \tag{11}$$

(b) For the annulus with a laminar flow, the *Nu* number may be calculated by Eq. (7) provided that the modified *De* number is utilized. The modified *De* number for the annulus is defined as:

$$De = \sqrt{2}Re\left(\frac{D_o - D_i}{D}\right)^{0.5} \tag{12}$$

Simply, this correlation could be re-written as follow:

$$De = \sqrt{2}Re\left(\frac{\delta_1 - \delta_2}{\delta_1 \cdot \delta_2}\right)^{0.5} \tag{13}$$

where δ_2 is the ratio of helical pipe diameter to the outer tube diameter. It is worth mentioning here that the hydraulic radius of the annulus must be used for calculating the *Re* number in this equation [31]. Taking Eqs. (4)–(8) and (13) into account, the expression of the rate of entropy generation for annulus with laminar flow may be written as:

$$N_s = \frac{0.392(\delta_1 - \delta_2)^{0.14}}{De^{0.43}Pr^{\frac{1}{6}}(\delta_1 \cdot \delta_2)^{0.14}} + 91.6\frac{(\delta_1 \cdot \delta_2)^2 De^{4.36}}{B^2(\delta_1 - \delta_2)^2} \tag{14}$$

By the same approach as that used for the inner tube, the optimal value of δ_2 that minimizes the rate of entropy generation (N_s) is obtained as:

$$\delta_{2,opt} = \frac{57.7B\delta_1^2 Pr^{\frac{1}{12}}De^{0.24} - \delta_1 B^2}{3323Pr^{\frac{1}{6}}De^{4.79}\delta_1 - B^2} \tag{15}$$

And, for the optimal *De* number, one has:

$$De_{opt} = \frac{B^{0.48}(\delta_1 - \delta_2)^{0.45}}{5.0642 Pr^{0.35}(\delta_1 \cdot \delta_2)^{0.039}} \quad (16)$$

2.2. Turbulent flow

In this section, the entropy generation analysis associated with internal turbulent flow in helical tube-in-tube heat exchanger is presented. In contrast with laminar flow in which *De* number was used, in this case, *Re* number is used for predicting the friction factor as *De* number may not correlate the flow measurements data as much as *Re* number. The corresponding correlations for calculating *Nu* number [32] and friction factor [33] of a turbulent flow through helically coiled heat exchangers are respectively as below:

$$Nu = 0.00619 Re^{0.92} Pr^{0.4}\left(1 + \frac{3.46}{\delta}\right) \quad (17)$$

$$f = \frac{64}{Re^{1.95}\delta^{0.1}} \quad (18)$$

Having these equations, just the same as previous section, one could present dimensionless entropy generation rate correlations for the inner tube and the annulus in case of having a turbulent flow.

(a) For the inner tube, substituting the values of f and *Nu* number from Eqs. (17) and (18) in Eq. (4), one obtains:

$$N_s = \frac{51.423}{Re^{0.92} \cdot Pr^{0.4}\left(1 + \frac{3.46}{\delta_1}\right)} + 15.503\frac{Re^{4.05}}{B^2 \cdot \delta_1^{0.1}} \quad (19)$$

Differentiating N_s with respect to δ_1 and *Re* number and equating the results to zero, one may determine the optimal δ_1 and *Re* number values leading to the minimum irreversibility in the heat exchanger. This operation results to the following equations:

$$\frac{\delta_{1,opt}^{1.1}}{(\delta_{1,opt} + 3.46)^2} = \frac{Re^{4.97} Pr^{0.4}}{28.61 B^2} \quad (20)$$

$$Re_{opt} = \frac{1.306\delta_1^{0.23} B^{0.402}}{Pr^{0.503}} \quad (21)$$

(b) For the annulus, combining Eqs. (14), (17) and (18) and substituting the values of *Nu* number and friction factor into Eq. (5), one obtains the following equation for calculating the dimensionless entropy generation rate in turbulent flow:

$$N_s = \frac{51.55}{Re^{0.92} Pr^{0.4}\left[1 + 3.45\left(\frac{\delta_1 - \delta_2}{\delta_1 \cdot \delta_2}\right)\right]} + 62.01\frac{(\delta_1 - \delta_2)^{0.1} Re^{4.05}}{B^2(\delta_1 \cdot \delta_2)^{0.1}} \quad (22)$$

Following the same mathematical procedure as that for previous cases, the optimum Re_{opt} that minimizes the N_s value is found as below:

$$Re_{opt} = \frac{1.3065 B^{0.4}(\delta_1 \cdot \delta_2)^{0.04}}{Pr^{0.5}} \quad (23)$$

In the end, it should be mentioned that as the correlations used for *Nu* number, friction coefficient, etc., are experimentally developed, there are some restrictions for the values of δ, *Re* number and *De* number in our simulations. According to [29], *De* number is restricted in the range of 100–8300 and according to [34], *Re* number, *Pr* number and δ are respectively restricted to the ranges of 5000–100,000, 0.7–5 and 10–37.

3. Results and discussion

In this section, the results of numerical simulation accomplished for entropy generation minimization of helical tube-in-tube heat exchangers with laminar and turbulent flows are presented.

Fig. 2 illustrates the effect of changing the ratio of inner tube and annulus diameters for laminar flows, in specific *De* numbers, on the rate of entropy generation. Note that Eqs. (10) and (15) are in connection with this figure and the constant values of $De = 500$, $B = 10^9$ and $Pr = 2.5$, adapted from Ref. [35], have been considered for depicting this figure. According to the figure, the lowest rate of entropy generation for the tube occurs in $\delta_1 = 10$. As can be seen, the value of N_s associated with this optimal point is almost 0.0204 and this value increases as δ_1 scats from 10 for either higher or lower values. For the annulus, on the other hand, there is no specific δ_2 associated with the minimum value of N_s. In other words, N_s remains at its optimal value of 0.02 when δ_2 value is in range of 3.5–5.

Similarly, for the same values of B and *Pr* number and employing Eqs. (19) and (22), Fig. 3 illustrates the effect of changing the values of curvature ratios (δ_1 and δ_2) on the rate of entropy generation in the inner tube and the annulus of the helical heat exchanger in case of having turbulent flows, respectively. According to the given information in previous section, in contrast with the laminar flow analysis in which *De* number was considered in flow modeling, *Re* number is applied here.

Obviously from the figure, in contrast with the inner tube for which there is a specific curvature ratio corresponding with the minimum rate of entropy generation ($\delta_1 \approx 16.5$), for the annulus, after sharp collapse in entropy generation rate by increasing the value of δ_2 from 0 to 1.5, entropy generation rate decreases with an almost constant mild pace as the value of δ_2 increases further. It is noteworthy that the optimal N_s value for the inner tube is 0.0177 while, for the annulus, the value of N_s decreases from 0.06 down to 0.057 for $1.5 < \delta_2 < 10$. This implies to this fact that although increasing the annulus curvature ratio could enhance the entropy generation performance of the device in a turbulent flow, its effect is negligible and $\delta_2 \approx 2$ may be a very good choice.

Hereafter, the results are divided into two categories, i.e. those related to the inner tube and the others in connection with the annulus of the helical tube-in-tube heat exchanger. In this context, Figs. 4–6 are presented for the inner tube analysis and the last three figures are related to the annulus simulation.

Fig. 4, which is prepared by applying Eq. (11), shows the effect of *De* number value change on the rate of entropy generation in the inner tube in case of having a laminar flow. This figure is provided based on specific constant B and *Pr* number values ($B = 10^9$ and $Pr = 2.5$) and includes different graphs corresponding with different values of δ_1 each.

According to the figure, the same trends with trifle different paces are observed for N_s value changes versus *De* number variations in various δ_1 values. The lowest rate of entropy generation is expected to increase trivially as the δ_1 value comes up. Also, the *De* number associated with the lowest value of N_s decreases as δ_1 picks up. Specifically, for $\delta_1 = 10$ which has already been found as the optimal value for the inner tube of laminar flow, the lowest rate of entropy generation occurs in approximately $De = 500$.

Similarly, Fig. 5 shows the effect of *Re* number changes on the rate of entropy generation in the inner tube for a turbulent flow. In fact, this figure, which is based on Eq. (19), indicates the optimal *Re* number values for the flow within the inner tube in different curvature ratios of the tube and constant B and *Pr* number values as those used for the previous figures, i.e. $B = 10^9$ and $Pr = 2.5$.

As seen, similar to the laminar flow with unique optimal *De* numbers for different cases, there is certainly an optimal *Re*

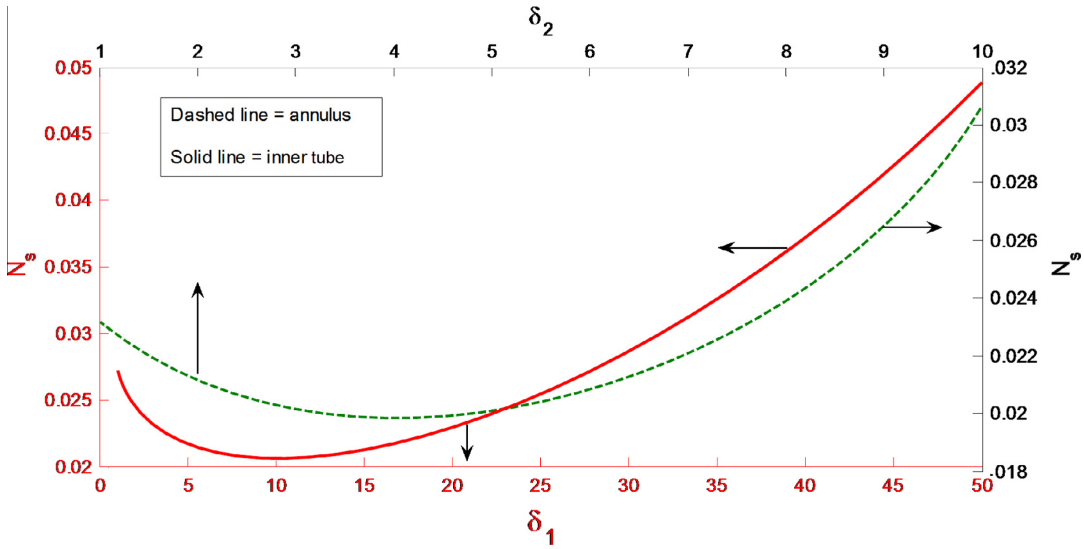

Fig. 2. Variation of N_s value with curvature ratios changes in laminar flow for both the inner tube and the annulus in specific *De* number.

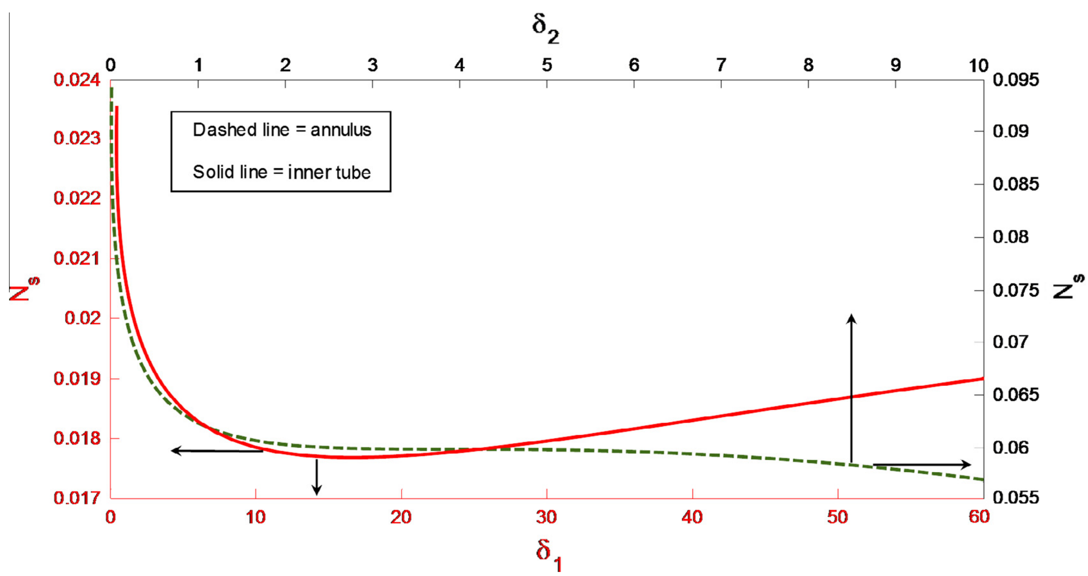

Fig. 3. Variation of N_s value with curvature ratio changes in turbulent flow for both the inner tube and the annulus in specific *Re* number.

number for a turbulent flow as well. According to the figure, the lowest N_s value differs trivially for different curvature ratios so that the N_s values increases from 0.05 to only 0.1 for a wide range of curvature ratio from the low value of 5 up to a very large value of 30. On the other hand, the corresponding *Re* number with the minimum entropy generation rate varies considerably so that the Re_{opt} for $\delta_1 = 5$ is almost 7000 while $Re_{opt} \approx 8500$ for the curvature ratio equal to 30. The other spotlight in the above figure is that the graph associated with $\delta_1 = 5$ makes significant differences relative to the next case ($\delta_1 = 10$) in terms of optimum *Re* number and Ns values while the other cases show very close values to each other for both of these parameters.

Finally, defining the ratio of actual *Re* number and *De* number as well as the dimensionless entropy generation number to their corresponding optimal values as relative *Re*, relative *De* and relative N_s, respectively, one could illustrate the variation of relative N_s versus the variation of relative *De* number (for laminar flow) and relative *Re* number (for turbulent flow) of the inner tube in the heat exchanger (Fig. 6). According to the formulation presented, Eqs. (9), (11), (19) and (21) are in connection with this figure.

Clearly, the best point for each case, in which $N_s = N_{s,opt}$ (i.e. relative $N_s = 1$), is where the values of relative *Re* and relative *De* are equal to 1 ($Re = Re_{opt}$ and $De = De_{opt}$). As the values of these parameters scat from 1, the value of relative N_s increases and this means that the rate of entropy generation and irreversibility in the systems increase. This figure could reveal the pace of entropy generation increase as *De* and *Re* numbers deviate from their optimal values in laminar and turbulent flow of the inner tube. According to the figure, entropy generation and irreversibility pick up at much faster rates as *Re* number scat from its optimum value and this rate is milder for *De* number deviations in a laminar flow. This, in fact, implies the important effects of *Re* number (or fluid velocity) on thermodynamic performance of tube-in-tube helical heat exchangers. The other noteworthy point about this figure is the difference between the slopes of the graphs on the right and left sides of the optimal point. Note that the figures axes are logarithmic and

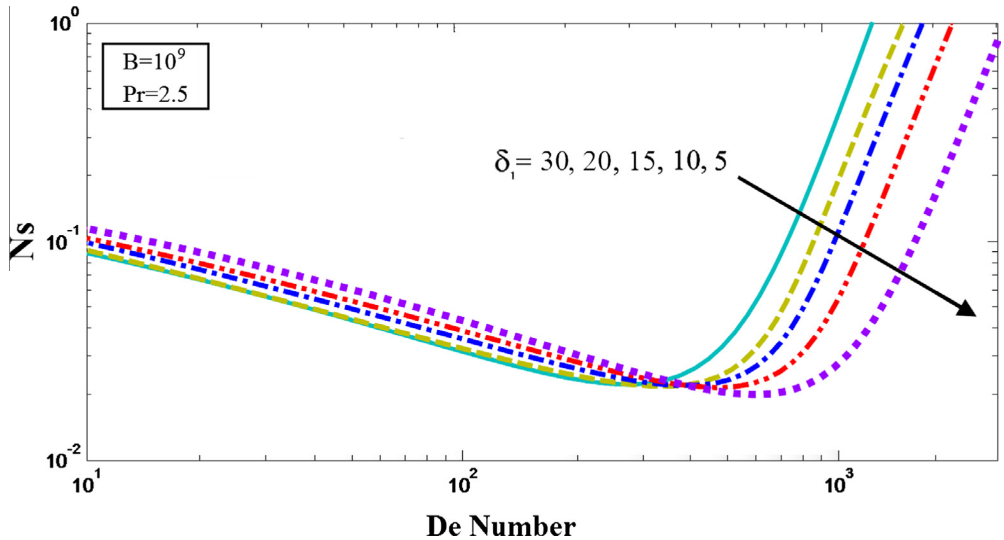

Fig. 4. *De* number variation effect on entropy generation rate for the inner tube in laminar flow and different δ_1 values.

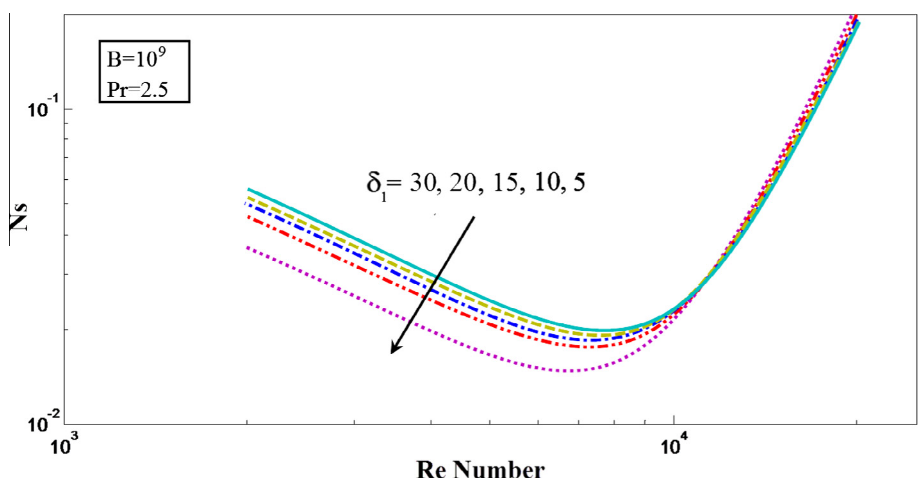

Fig. 5. Influence of *Re* number change on entropy generation rate for the inner tube in turbulent flow.

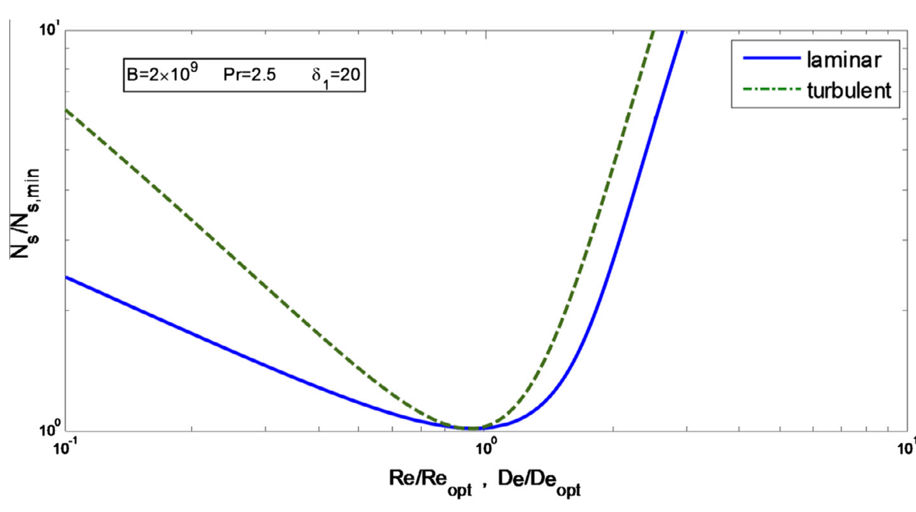

Fig. 6. Relative N_s value changes versus relative *Re* or relative *De* for the in inner tube.

may be misleading about the rate of entropy generation increase in higher or lower values of *Re* and *De* numbers than the optimal values. Paying enough attention to the figure, one could find out that, for both laminar and turbulent flows, decreasing *Re* number and *De* number values increase the rates of entropy generation at higher speeds in comparison with the cases that the values of *Re* and *De* numbers exceed the optimal values.

It should be mentioned here that there is only one similar work in the literature that studies the optimal *De* and *Re* numbers of helically coiled heat exchangers for respectively laminar and turbulent flows, though it only considers the inner tube and it has the restriction of constant wall heat flux condition [36]. In the study, for the constant values of $Pr = 2.5$ and $B = 10^9$ and various curvature ratios from 5 to 50 decreasing *De* numbers in the range 500–1500 (for laminar flow) and descending *Re* numbers of 4500–500 (for turbulent flow) are obtained. Comparing the results with the last four figures given above, one could observe exactly the same trends for variation of optimal *Re* and *De* numbers in various curvature ratios, though the obtained values are fairly different which is due to the constant wall heat flux condition.

Hereafter, the results associated with the simulation carried out on the annulus of the helical tube-in-tube heat exchanger are presented. In this context, Fig. 7 shows the effect of *De* number changes on the rate of entropy generation for a laminar flow in the specific values of $B = 2 \times 10^9$, $Pr = 2.5$ and $\delta_1 = 20$. This figure investigates this matter for various annulus curvature ratios (five different cases from very low to very high values) and it is prepared employing Eq. (14).

As can be seen, lower curvature ratios for the annulus result to lower rates of entropy generation in the system. In this case, the $N_{s,opt}$ varies from 0.018 up to 0.025 for different annulus curvature ratios. On the other hand, the *De* number corresponding with the optimum N_s value decreases significantly so that for $\delta_2 = 1$, the optimal *De* is equal to almost 1040 while a low *De* number value of approximately 170 is observed for $\delta_2 = 9$. Evidently, in this case, i.e. the annulus with laminar flow, both of the entropy generation rate and optimal *De* number value change at uniform paces as the curvature ratio varies.

Fig. 8, based on Eq. (22), aims at investigating the effect of *Re* number changes on the rate of entropy generation for a turbulent flow through the annulus in the specific values of $B = 2 \times 10^9$, $Pr = 2.5$ and $\delta_1 = 20$. According to this figure, $\delta_2 = 1$ results to the lowest value of N_s where the Re_{opt} number is almost equal to 4500. For higher curvature ratios (from 3 up to 9), the values of Re_{opt} and $N_{s,opt}$ increase as δ_2 climbs, though the speed of growth of both of these parameters is much lower than that observed for $1 \leqslant \delta_2 \leqslant 3$.

In the end, by the same approach and objective as that used for plotting Fig. 6 and reconsidering the definitions relative *Re*, relative

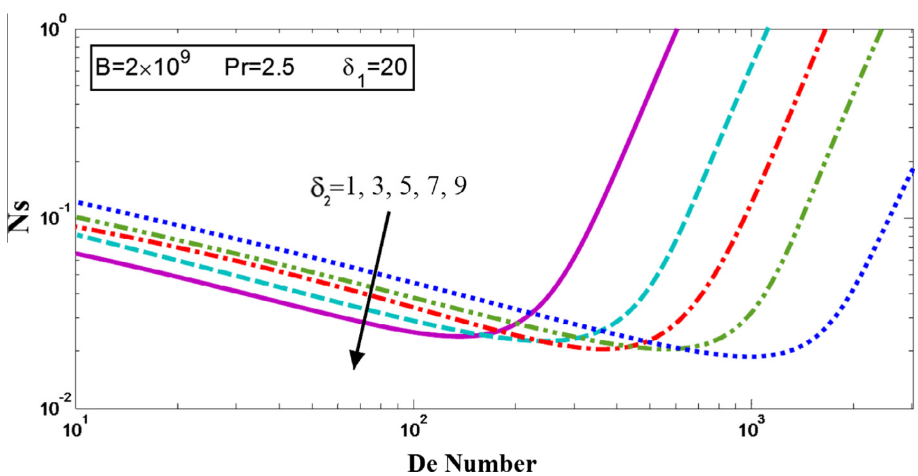

Fig. 7. Influence of *De* number change on entropy generation number for a laminar flow through the annulus.

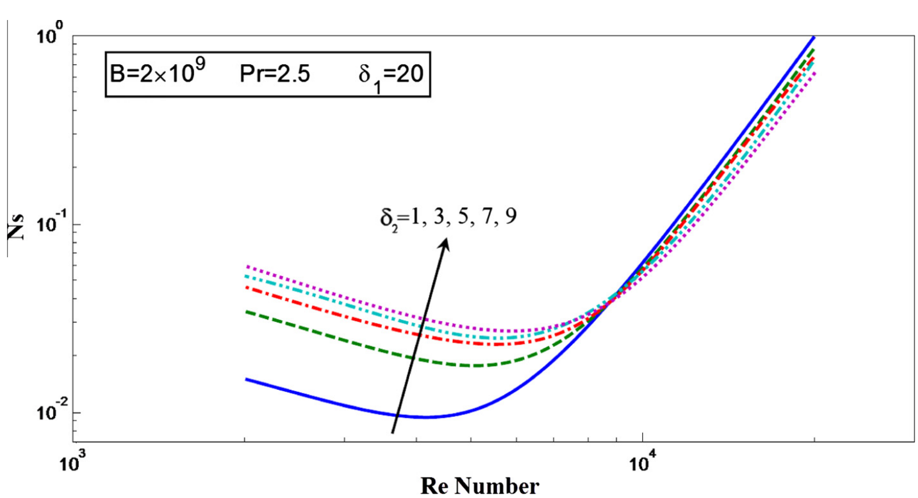

Fig. 8. Entropy generation rate change versus *Re* number alteration for a turbulent flow through the annulus.

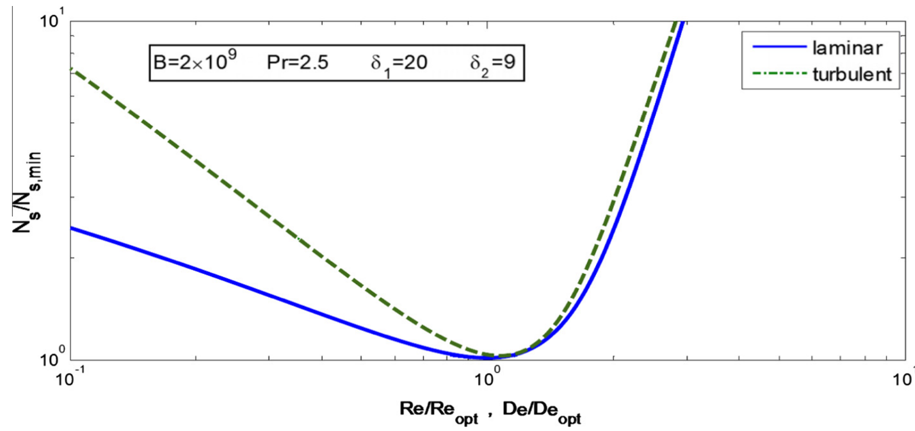

Fig. 9. Relative entropy generation rate changes versus relative *Re* and relative *De* numbers for the annulus with laminar and turbulent flows.

De and relative N_s, one could assess the effect of deviating from the optimal *De* and *Re* values on the rate of entropy generation for different cases. Fig. 9, using Eqs. (14), (16), (22) and (23), studies this issue for specific constant B and *Pr* number, δ_1 and δ_2 values ($B = 2 \times 10^9$, $Pr = 2.5$, $\delta_1 = 20$ and $\delta_2 = 9$) for the annulus of the helical coiled heat exchanger for both laminar and turbulent flows. As can be seen, for the annulus also, the effect of deviation from the optimal *Re* value (in a turbulent flow) on the rate of entropy generation is much more than the effect of deviating from the *De* number on the entropy generation rate in a laminar flow. Of course, according to the figure, this effect is considerably less in relative *Re* number and relative *De* number values greater than 1. The value of relative N_s increases by 250% when relative *De* number is 0.1 while an increase of almost 720% is observed for this parameter in a turbulent flow for a relative *Re* number equal to 0.1. The relative entropy generation value gets tenfold for both laminar and turbulent flows in relative *De* and *Re* number values equal to 3.

4. Conclusion

This work aimed at optimizing tube-in-tube helical heat exchangers geometry and flow characteristics with either laminar flow or turbulent flow based on entropy generation minimization approach. For this objective, a helical tube-in-tube heat exchanger was theoretically investigated, it was mathematically modelled and dimensionless expressions for various parameters of the problem were developed. Defining the dimensionless expression of curvature ratio for both of the inner tube (δ_1) and the annulus (δ_2), it was found out that the geometry of the heat exchanger affects its performance significantly and one could find optimal curvature ratios for both of the inner tube and the annulus to minimize the demotion of thermal energy and viscous dispersion of mechanical energy in the heat exchanger. Then, the rate of entropy generation through the annulus and the inner tube with either laminar flow or turbulent flow was calculated. Finally, taking the results of the numerical simulation implemented into account, the optimal geometry (δ_1 and δ_2) and flow characteristics (*Re* number, *De* number) that optimize the performance of this type of heat exchanger based on the lowest rate of entropy generation were determined. Combining economic analysis with an entropy generation minimization analysis seems to be an interesting investigation for future works in this context to promote application of double-pipe helical heat exchangers.

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