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Unsteady natural gas flow within pipeline network, an analytical approach

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ABSTRACT

The natural gas pipeline network (including distribution network) may be subjected to some extreme conditions such as pipeline rapture, sudden changed demand and etc. The behavior of natural gas pipeline network should be properly identified to prevent network failure and have continued pipeline operation under these extreme conditions. These conditions usually cause unsteady behavior of the pipeline network. Consequently, it is necessary to develop an unsteady state mathematical method to study natural gas pipeline network under unsteady conditions. To achieve this goal, an analytical approach has been developed to analyzed natural gas pipeline network. The governing equations derived for one-dimensional isothermal compressible viscous flow with Kirchhoff's laws have been employed to develop a method for studying the pipeline network under unsteady conditions. The proposed method has been compared with previous studies for validation purposes. The validation shows the proposed method has an average absolute present derivation (AAPD) less than 0.7%. Finally the effect of a few parameters including: friction factor, natural gas composition and decreasing and increasing demand have been studied. Results show that Weymouth and AGA equation predict highest and lowest pressure drop at the network nodes respectively. Also by increasing natural gas molar mass, the pressure at nodes will be decreased. It could be also concluded that demand rise causes pressure drop to decrease and demand fall causes pressure drop to increase.

1. Introduction

Natural gas transmission and distribution pipeline network are utilized to deliver natural gas from the production points to the consumers. Unlike the other networks (electric and water distribution networks), the interruption in natural gas delivery system may have devastating effects. To prevent the interruption, the behavior of the natural gas under various conditions should be thoroughly known. To know the natural gas network behavior, it is necessary to simulate and analyze the pipeline network. Many researchers have modeled the pipeline and pipeline networks under steady state and unsteady state conditions. Due to existing of measuring devices, valves and possible accidents in pipelines and networks, the unsteady models is more close to reality than steady model. The main priority in the pipeline network modeling is to calculate volumetric flow rate in pipeline and pressure at the network nodes (or pressure drop in pipeline). Consequently, researchers are always looking for a way to calculate the volumetric flow rate and pressure, and the relationship between these two parameters.

Computing the volumetric flow rate in the steady state condition for a pipeline has been given in various publications. Weymouth (Weymouth, 1912) was the first who presented an equation to calculate volumetric flow rate and pressure drop in horizontal and inclined pipelines. Most recently, Tian and Adewumi (Tian and Adewumi, 1994) reported an analytical steady state flow equation without neglecting the kinetic energy term in the momentum equation. Zhou and Adewumi (Zhou and Adewumi, 1998) also presented an analytical flow equation for steady state flow through natural gas pipelines without neglecting any terms in the momentum equation. The well-known equations for flow equations in the steady state condition are included: Weymouth, Panhandle A & B, American Gas Association (AGA) and Colebrook-White. These equations are very practical and have been used widely in the natural gas industry. Stoner (Stoner, 1969) also presented new solutions for natural gas pipeline equations. In the Stoner study, the

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mathematical model of a system consists of node continuity equations which are solved by the dimensional Newton-Raphson Method. In one new research, Farzaneh-Gord et al. (Farzaneh-Gord et al., 2013) studied the effects of natural gas hydrate on the underground transmission pipeline in the steady state condition. In their study, the effect of natural gas composition has been also studied.

For unsteady (transient) flow in natural gas pipelines many researchers have been reported algorithms and numerical methods to solve governing equations such as: finite difference (implicit and explicit), method of characteristics and similar methods (Wylie and Streeter, 1978; Luongo, 1986; Yow, 1971; Osiadacz, 1983; Ibraheem and Adewumi, 1996; Greyvenstein, 2002). Though, all of these reported methods are time-consuming especially for analyzing pipeline network. The mentioned method (such as CFD methods) spent a lot of time to solve the governing equation and relatively has a high CPU time to analyze a single gas pipeline (Wang et al., 2015). Consequently, for a complex gas pipeline network, it can be very time consuming. Therefore, some of the researchers applied other methods to provide simpler solutions. Luongo (Luongo, 1986) and Wylie et al. (Wylie et al., 1971) linearized partial differential set of equations by neglecting the inertia term in momentum equation. Yow (Yow, 1971) formulated the transient flow problem including the inertial multiplier by the characteristics method. Osiadacz (Osiadacz, 1987) simulated transient gas flow with isothermal assumption without neglecting any terms in momentum equation for gas networks. Kiuchi (Kiuchi, 1994) analyzed unsteady gas network with isothermal conditions by applying implicit finite difference method. In Kiuchi study, the inertia term of momentum equation was neglected (Kiuchi, 1994). Dukhovnaya and Adewumi (Dukhovnaya and Adewumi, 2000) and Zhou and Adewumi (Zhou and Adewumi, 1996) simulated non-isothermal transient flow of gas in pipelines. These researches were applied total variation diminishing (TVD) scheme to formulated system of governing equations. Osiadacz and Chaczykowski (Osiadacz and Chaczykowski, 2001) compared isothermal and non-isothermal pipeline gas flow models in the unsteady conditions. Tentis et al. (Tentis et al., 2003) have used an adaptive method of lines to simulate the transient gas flow in pipelines. Gato and Henriques (Gato and Henriques, 2005) simulated unsteady, one-dimensional compressible flow in natural gas pipeline using the Runge–Kutta discontinuous Galerkin method, with third order approximation in space and time. Chaczykowski (Chaczykowski, 2009, 2010) studied sensitivity of model to the selection of the equation of state and also investigated the effect of thermal model for analyzing unsteady gas pipelines. Adeosun et al. (Adeosun et al., 2009) developed Weymouth equations for unsteady gas volumetric flow rate in horizontal and inclined pipelines. Also Olatunde et al. (Olatunde et al., 2012) presented direct calculations method of Weymouth equations for unsteady gas volumetric flow rate with different friction factors in horizontal and inclined pipelines. Ebrahimzadeh et al. (Ebrahimzadeh et al., 2012) simulated transient gas flow using the orthogonal collocation method. Their method were simulated successfully the isothermal and non-isothermal unsteady flow in gas transmission systems. Helgaker et al. (Helgaker et al., 2014) simulated one-dimensional compressible flow of natural gas pipelines using GERG 2004 equation of state. Wang et al. (Wang et al., 2015) compared four forms of hydraulic equation of a natural gas pipeline based on linearized solution from the point of view the accuracy and efficiency.

Steady state analysis of volumetric flow rate and pressure drop in pipeline network has been studied extensively in literature. Ferguson (Ferguson, 2002) and Ohirhian (Ohirhian, 2002) reported solution techniques for steady state pipeline network analysis. Their studies presented a new method according to the hydraulic equation of pipeline also nodes and close loops of pipeline network. Rios-Mercado et al. (Ríos-Mercado et al., 2002) presented a reduction technique for solving natural gas transmission network optimization problems. Their results are valid for steady-state compressible flow through a network pipeline. Abd Majid (Woldeyohannes and Majid, 2011) developed a simulation model for the steady state analysis of transmission pipeline network system (TPNS) with detailed characteristics of compressor stations. Brikic (Brikić, 2011) solved a looped gas pipeline network according to principles of Hardy Cross method for determination of appropriate friction factor and selection of a representative equation for natural gas flow. El-Shiekh (El-Shiekh, 2013) presented a mathematical formula to design natural gas transmission pipelines in the steady state condition. Natural gas network have been optimized to select the optimum diameter, number of compressor stations, length between each two compressor stations, suction and discharge pressures at compressor stations.

All of the above researches focus on simulations of gas pipeline network (transmission and distribution) in the steady state conditions. There have been limited researches in the field of unsteady (transient) gas pipeline network modeling. Tao and Ti (Tao and Ti, 1998) and Ke and Ti (Ke and Ti, 1999) presented an electrical analogy method to analyze isothermal unsteady gas flow in the natural gas pipeline network. Reddy et al. (Reddy et al., 2006, 2006) simulated transient flow in natural gas pipeline networks using a transfer function model. Accuracy and computational efficiency of their method are evaluated by comparing results with those obtained using a fully nonlinear second order accurate finite difference method. Gonzales et al. (Gonzales et al., 2009) modeled and simulated the gas distribution pipeline network. Their study was presented two numerical schemes and a MATLAB-Simulink library to solve the system proposed numerical schemes. In the recent study, researchers applied new methods that have simpler and shorter time to simulate the gas pipeline network. Behbahani-Nejad and Bagher (Behbahani-Nejad and Bagheri, 2010) presented an efficient method for transient flow simulation of gas pipelines networks. Their method was based on the transfer function models and MATLAB-Simulink. Results showed that the proposed simulation method has a sufficient accuracy and it is computationally more efficient than the other methods. Behbahani-Nejad and Shekari (Behbahani-Nejad and Shekari, 2010) proposed a reduced order modeling approach for natural gas transient flow in pipelines. They derived the linearized form of the Euler equations and obtained the corresponding Eigen system. Then, they used a few dominant flow Eigen modes to construct an efficient reduced-order. Alamian et al. (Alamian et al., 2012) proposed a method based on the state space model to simulated transient flow for gas pipelines and networks. Their results compared with those of the conventional finite difference schemes such as total variation diminishing algorithms, method of lines, and other finite difference implicit and explicit schemes. Ahmadian Behrooz and Bozorgmehry (Ahmadian Behrooz and Bozorgmehry Boozarjomehry, 2015) developed a robust general simulation framework for natural gas transmission networks. The non-isothermal models of natural gas in pipelines and governing equation for gas transmission network have been solved efficiently using the orthogonal collocation method.

As discussed, the natural gas pipeline network consists of so many pipes and nodes which make it difficult to be simulated by a computational fluid dynamics method. Also, due to possible creation of some extreme boundary conditions within the pipeline network, it is necessary to model the network under unsteady state conditions. For this purpose, in the current study, a new equation for analyzing flow of natural gas within a pipeline has been developed. The equation is derived from fundamental governing equations of flow through a pipeline without neglecting any term in the momentum equation. By applying this equation and Kirchhoff laws for pipeline network, a new method for modeling natural gas pipeline network under unsteady condition has been presented. A well-known natural gas pipeline network has been analyzed for validation purpose. Finally, the effect of the important parameter such as: friction factor, natural gas composition and increasing and decreasing demand have been studied on the studied network.

2. The problem description

The transmission and distribution network are essential components to transfer natural gas from well production point to consumption areas. The transmission network is a natural gas pipeline at high-pressure performance. The transmission network composed from complex pipeline network and massive compressor station. The main objective of this network is to transport natural gas from primary source to areas with high-pressure demand. The main pipelines along the transmission network included: the gathering system, the interstate pipeline system and the distribution system. The gathering pipeline system transport crude natural gas from well point to the natural gas treatment refinery. The gathering system comprised of the small diameter natural gas pipelines in low-pressure condition. The interstate system transport natural gas from refinery to area with great stipulations such as: large and populated urban areas. The distribution system is a pipeline network that is a final stage to transport natural gas to consumer. Because of the connections node and measurement devices, natural gas flow within distribution network always in unsteady conditions.

The distribution pipeline network consists of two main items: node and node connecting. Nodes are the point where a pipeline ends or where two or more node connecting connects or where there is an injection or delivery of natural gas. The node connection comprised of pipeline, compressor station, valves, regulators and underground natural gas storage. Fig. 1 show a natural gas distribution pipeline system. The distribution network components include: pipeline, node, close loops and demand. In the current study, author focused on analyzing a network such as shown in Fig. 3 under unsteady state conditions. Pipeline network analyzed with solution of governing equation by employing an analytical approach.

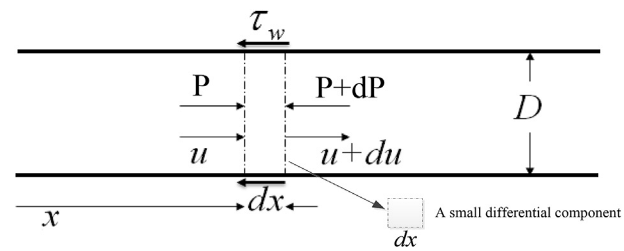

Fig. 2. A differential control volume in natural gas pipeline.

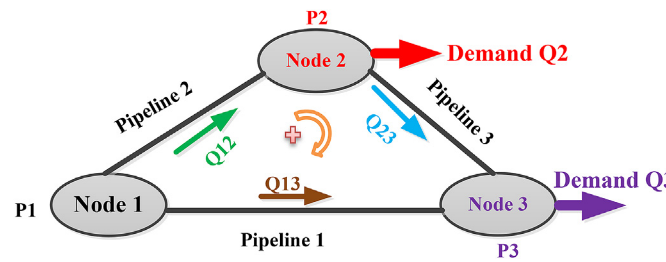

Fig. 3. A typical gas pipeline network under investigation.

3. Mathematical modeling

For analyzing a pipeline network, firstly, a pipeline as an important component of network has been analyzed. Fig. 2 shows a differential control volume of natural gas pipeline for obtaining the governing equations. Rectangular dash dot line in Fig. 2 is represents of a small differential component.

The momentum equation for the control volume could be presented as follow (Helgaker et al., 2014):

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial P}{\partial x} = -\frac{f \rho u^2}{2D} \tag{1}$$

In Equation (1), ρ is density, u is velocity, P is pressure, f is Moody friction factor and D is pipeline diameter. Derivative respect to position x and time t and introducing field units, Equation (1) becomes (Adeosun et al., 2009):

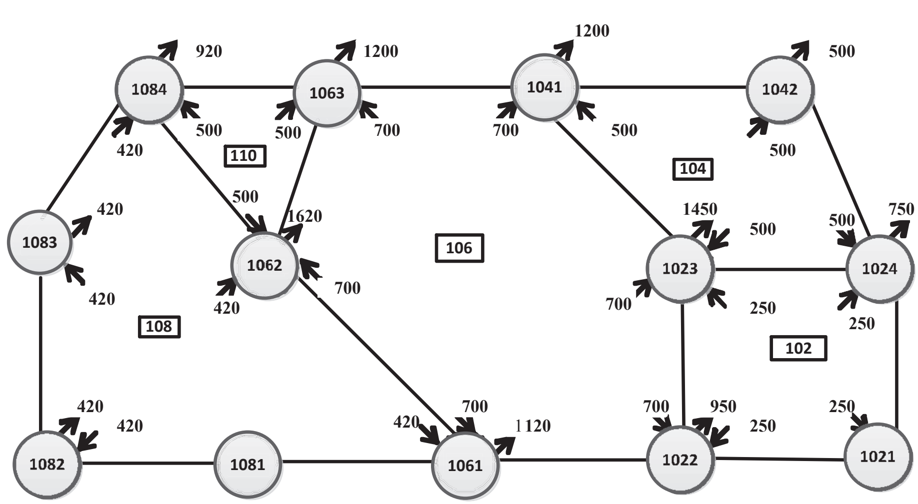

Fig. 1. A typical natural gas distribution network.

$$\frac{\rho}{g_c}\frac{du}{dt}+\frac{\rho u}{g_c}\frac{du}{dx}+\frac{dP}{dx}=-\frac{f\rho u^2}{2g_cD} \tag{2}$$

In Equation (2), g_c is gravitational conversion factor. g_c is equal to one in SI unit as well as in this study. The 1D form of the energy equation for gas flow could be written as:

$$\frac{dP}{dx}=-\frac{\rho}{g_c}\frac{du}{dt}-\frac{\rho u}{g_c}\frac{du}{dx}-\frac{f\rho u^2}{2g_cD} \tag{3}$$

In the right hand of side Equation (3), the first term is pressure gradient due to accumulation, the second term is pressure gradient due to kinetic energy and the third term is pressure gradient due to frictional losses. Applying Kolomogorov and Fomin (Kolomogorov and Fomin, 1957) transformation method (this transformation also have been as mentioned in (Adeosun et al., 2009)), with this transformation the velocity terms in the right hand of side equation (3) have been same order and equation (3) becomes (Adeosun et al., 2009):

$$\frac{dP}{dx}=-\frac{\rho u}{g_c}\frac{du}{dt}-\frac{\rho u}{g_c}\frac{du}{dx}-\frac{f\rho u^2}{2g_cD} \tag{4}$$

Equation (4) can be rewritten in following form (Adeosun et al., 2009):

$$\frac{1}{\rho}\frac{dP}{dx}=-\frac{u}{g_c}\frac{du}{dt}-\frac{u}{g_c}\frac{du}{dx}-\frac{fu^2}{2g_cD} \tag{5}$$

For real gases, ρ can be written in following form:

$$\rho=\frac{Mw_aP\gamma_g}{ZRT} \tag{6}$$

In Equation (6), Mw_a is molar mass of air, Z is compression factor of natural gas, γ_g is gas specific gravity, T is temperature and R is gas constant. Velocity according to volumetric mass flow rate could be defined as follow:

$$u=(Q)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{Z}{1}\right)\left(\frac{4}{\pi}\right)\left(\frac{1}{D^2}\right) \tag{7}$$

In Equation (7), Q is volumetric gas flow rate and T_b, P_b are temperature and pressure in the base condition respectively. Substituting Equations (6) and (7) into Equation (5), we have:

$$\left(\frac{RZT}{Mw_a\gamma_g}\right)\frac{dP}{P}=-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bPD^2}\right)^2\frac{dx}{dt}-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bPD^2}\right)^2-\frac{f}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bPD^2}\right)^2\frac{dx}{D} \tag{8}$$

By arranging Equation (8), following equation could be driven:

$$\left(\frac{RZT}{Mw_a\gamma_g}\right)PdPdt=-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2dx-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2dt-\frac{f}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\frac{dxdt}{D} \tag{9}$$

Integrate respect to time t, equation (9) could be converted to a first order ordinary differential equation, then by solving this equation respect to pressure P and position $x$equation (10) could be obtained:

$$\int_t\int_P\left(\frac{RZT}{Mw_a\gamma_g}\right)PdPdt=-\int_t\int_x\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2dx-\int_t\times\int_x\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2dt-\int_t\times\int_x\frac{f}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\frac{dxdt}{D} \tag{10}$$

The integration processes of equation (10) are as follows:
In first step, integrate respect to time t:

$$\int_P\left(\frac{RZT}{Mw_a\gamma_g}\right)PdP\int_0^t dt=-\int_x\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2dx-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\int_0^t dt-\int_x\frac{f}{2g_cD}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2dx\int_0^t dt \tag{11}$$

And in second step, integrate respect to pressure P and position x:

$$\left(\frac{RZT}{Mw_a\gamma_g}\right)\int_{P_1}^{P_2}PdP\int_0^t dt=-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\int_0^L dx-\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\int_0^t dt-\frac{f}{2g_cD}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\int_0^L dx\int_0^t dt \tag{12}$$

By integrating equation (12), the following equation could be obtained:

$$\left(\frac{RZT}{Mw_a\gamma_g}\right)\left(\frac{P_1^2-P_2^2}{2}\right)\Delta t=\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2L+\frac{1}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\Delta t+\frac{f}{2g_c}\left(\frac{4QTP_bZ}{\pi T_bD^2}\right)^2\frac{L\Delta t}{D} \tag{13}$$

Substituting g_c = 1 kg.m/N.s^2, R = 8.314 kJ/kmol.K, Mw_a = 29 kg/kmol in Equation (13) and simplifying terms, the unsteady-state form of volumetric flow rate could be derived:

$$Q=\sqrt{\frac{8.314}{29\times10^3\times\left(\frac{4\times10^3}{\pi}\right)^2}}\left(\frac{T_b}{P_b}\right)\sqrt{\frac{\left(P_1^2-P_2^2\right)}{ZT\gamma_g}D^5\left[\frac{\Delta t}{fL\Delta t+\frac{D\Delta t}{10^6}+\frac{DL}{10^3}}\right]} \tag{14}$$

The units of parameters in Equation (14) are as follows: Q is (m^3/s), D is (mm), L is (km), T is (K) and P is (kPa). Finally, Equation (15) could be developed to compute the natural gas volumetric flow rate within a pipeline under unsteady-state condition as:

$$Q = 1.329 \times 10^{-8}\left(\frac{T_b}{P_b}\right)\sqrt{\frac{\left(P_1^2 - P_2^2\right)}{ZT\gamma_g}D^5\left[\frac{\Delta t}{fL\Delta t + \frac{D\Delta t}{10^6} + \frac{DL}{10^3}}\right]} \quad (15)$$

In Equation (15), P_1 is upstream pressure, P_2 is downstream pressure, Z is compression factor of natural gas at average flowing temperature and average pressure (here calculated from AGA8 equation of state), T is average flowing temperature, L is pipe segment length andΔt is time change. Equation (15) gives unsteady volumetric flow rate for pressure change in natural gas pipeline. Equation (15) could be also written as follows:

$$P_1^2 - P_2^2 = K(t)\,Q^2 \quad (16)$$

In Equation (16), $K(t)$ is a function of pipeline properties and time that defined as following equation:

$$K(t) = \left(\frac{P_b}{1.329 \times 10^{-8}T_b}\right)^2\left[\left(\frac{ZT\gamma_g}{D^5}\right) \times \left(\frac{fL\Delta t + \frac{D\Delta t}{10^6} + \frac{DL}{10^3}}{\Delta t}\right)\right] \quad (17)$$

By applying Equations (16) and (17) for a pipeline, a model could be developed for the gas pipeline network system using the Kirchhoff's laws. Kirchhoff's laws was originally introduced for the flow of electricity in electrical networks.

Kirchhoff's first law describe that the algebraic sum of natural gas flow at each node is equal to zero. In the other words, the load at each node is equal to the sum of flow that entering in and leaving out of the node. This law could be written as following equation:

$$\sum_{i=1}^{n} Q_i = 0 \quad (18)$$

Where in Equation (18), n is number of node connecting that meets at the node. Q is flow rate which is positive for gas flow entering the node and is negative for gas flow leaving out from the node.

Kirchhoff's second law discussed that the algebraic sum of the pressure drop around the closed loop is zero (according to arbitrary direction). A closed loop starts on a node and ends at the same node. Therefore, there could been no pressure around the loop. if n is the number of node connecting in the closed loop, then for a pipeline network Kirchhoff's second law could be written as the following equation:

$$\sum_{i=1}^{n}\left(P_1^2 - P_2^2\right)_i = 0 \quad (19)$$

In a typical problem, the flow rate and pressure in one node is known (source node), and it is required to find the flow rates and pressures at all the other nodes. For a looped system, the direction element for flow is important.

Briefly, for analyzing a pipeline network and to calculate flow rate in pipes and pressure at nodes, the following steps should be performed:

1) Input temperature, period of time and other pipeline properties.
2) Guess flow rate in pipelines of network. The initial guesses for unsteady state condition are actually the solution of the governing equations under steady condition.
3) Applying Kirchhoff's first and second laws for each node of pipeline network and each closed loop of pipeline network.
4) Calculated volumetric flow rate in pipes using equation (18) and pressure at nodes using equation (19) in the pipeline network.
5) Compare calculated values with previous ones. If difference is less than 10^{-6} the calculation is ended, otherwise update initial guess with the new flow rate calculations and go to step 2 and repeat steps.

3.1. Real gas effect

The effect of real gas in simulations has been investigated by using AGA8 equation of state. The AGA8 EOS was presented by American Gas Association (AGA8-DC92 EoS, 1992). AGA8 EOS predicted compression factor and density for natural gas with high accuracy that used for custody transfer. In this EOS, natural gas considered a mixture of 21 gases so the interaction between the gases in the mixture considered too. This EOS is wide application range and for temperature between 143.15 K and 676.15 K, and pressure up to 280 MPa has high precision for engineering applications. Farzaneh-Gord and Rahbari (Farzaneh-Gord and Rahbari, 2012, 2011) developed this EOS and by using of this EOS calculated the important thermodynamics properties of natural gas mixture such as: enthalpy, entropy, etc., also developing some correlation to study thermodynamics properties of natural gas mixture as well as effect of natural gas compositions on different processes of natural gas industry.

The compression factor in AGA8 EOS defined by the following equation (AGA8-DC92 EoS, 1992):

$$Z = 1 + B\rho_m - \rho_r\sum_{n=13}^{18} C_n^* + \sum_{n=13}^{58} C_n^* D_n^* \quad (20)$$

In Equation (20), ρ_m is molar density and ρ_r is reduced density. Relation between reduced and molar density defined as follows:

$$\rho_r = K^3\rho_m \quad (21)$$

In Equation (21), K is mixture size parameter and defined by the following equation (AGA8-DC92 EoS, 1992):

$$K^5 = \left(\sum_{i=1}^{N} x_i K_i^{\frac{5}{2}}\right)^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i x_j\left(K_{ij}^5 - 1\right)(K_iK_j)^{\frac{5}{2}} \quad (22)$$

In Equation (22), x_i is the mole fraction of component$i$ in the mixture,x_j is mole fraction of component j in mixture,K_i is size parameter of component$i$,K_j is size parameter of component j, K_{ij} is the binary interaction parameter for size and N is the number of compositions in the natural gas mixture.

Second virial coefficient B in Equation (16) defined by the following equation (AGA8-DC92 EoS, 1992):

$$B = \sum_{n=1}^{18} a_n T^{-u_n}\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j B_{nij}^* E_{ij}^{u_n}(K_iK_j)^{\frac{3}{2}} \quad (23)$$

B_{nij}^* and E_{ij} are defined by Equations (24) And (25) respectively (AGA8-DC92 EoS, 1992):

$$B^*_{nij} = (G_{ij} + 1 - g_n)^{g_n} (Q_iQ_j + 1 - q_n)^{q_n} \left(F_i^{1/2}F_j^{1/2} + 1 - f_n\right)^{f_n} (S_iS_j + 1 - s_n)^{s_n} (W_iW_j + 1 - w_n)^{w_n} \quad (24)$$

Table 1
Pipeline specifications data for the studied network.

Gas pipe ID	From node	To node	Diameter (m)	Length (km)
1	1	3	0.6	80
2	1	2	0.6	90
3	2	3	0.6	100

$$E_{ij} = E^*_{ij}(E_iE_j)^{1/2} \quad (25)$$

G_{ij} that used in Equation (24) defined by the following equation (AGA8-DC92 EoS, 1992):

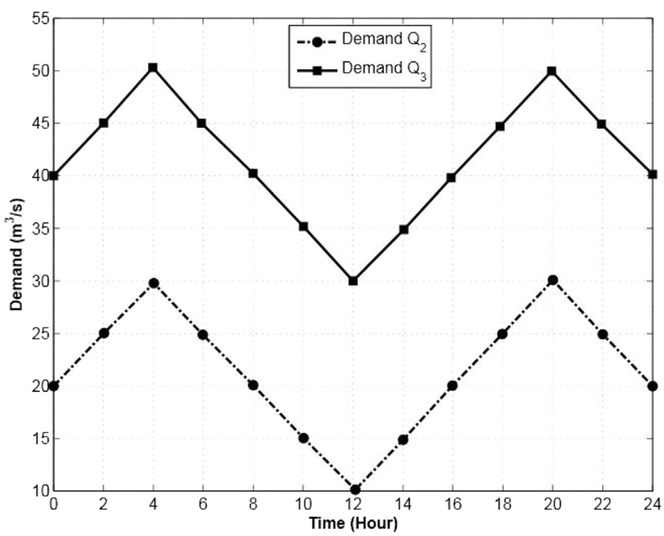

Fig. 4. Demands versus time for nodes 2 and 3 of the studied pipeline network.

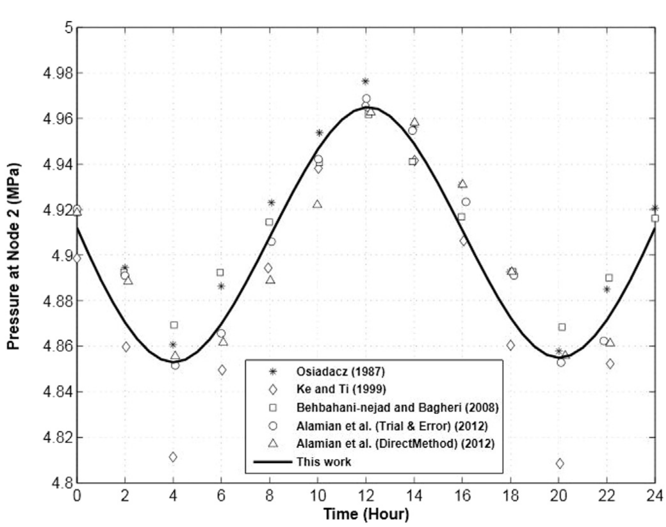

Fig. 5. Pressure at nodes 2 versus time with this work in compared with previous studies.

$$G_{ij} = \frac{G^*_{ij}(G_i + G_j)}{2} \quad (26)$$

In equations (23)–(26), T is Temperature, N is number of component in gas mixture, a_n,f_n,g_n,q_n, s_n,u_n, w_n are the equation of state parameters, E_i,F_i,G_i,K_i,Q_i,S_i,W_i are the corresponding characterization parameters and E^*_{ij}, G^*_{ij} are corresponding binary interaction parameters.

Temperature dependent coefficients $C^*_n; n = 1, \ldots, 58$ in equation (20) defined as follow (AGA8-DC92 EoS, 1992):

$$C^*_n = a_n(G + 1 - g_n)^{g_n}\left(Q^2 + 1 - q_n\right)^{q_n}(F + 1 - f_n)^{f_n} U_n^{u_n} T^{-u_n} \quad (27)$$

G,F,Q,U in equation (27) are the mixture parameters and defined by the equations (28) and (31) (AGA8-DC92 EoS, 1992):

$$U^5 = \left(\sum_{i=1}^{N} x_i E_i^{\frac{5}{2}}\right)^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_ix_j\left(U^5_{ij} - 1\right)(E_iE_j)^{\frac{5}{2}} \quad (28)$$

$$G = \sum_{i=1}^{N} x_iG_i + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_ix_j\left(G^*_{ij} - 1\right)(G_i + G_j) \quad (29)$$

$$Q = \sum_{i=1}^{N} x_iQ_i \quad (30)$$

$$F = \sum_{i=1}^{N} x_i^2F_i \quad (31)$$

In equation (28), U_{ij} is the binary interaction parameter for mixture energy and in equation (22) D^*_n is defined by the following equation:

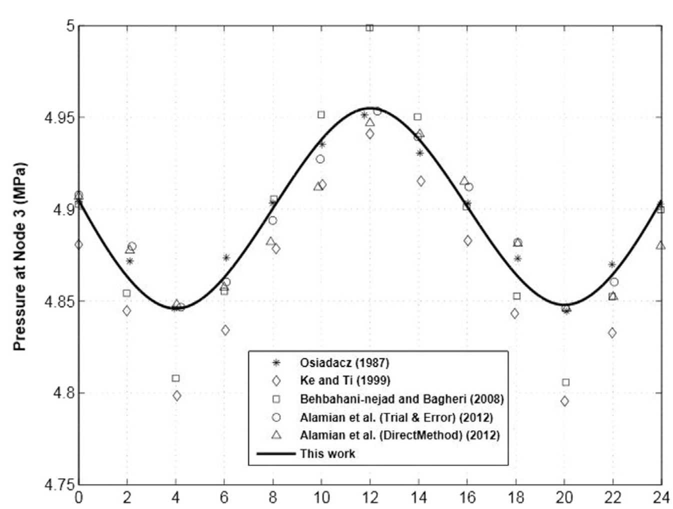

Fig. 6. Pressure at nodes 3 versus time with this work in compared with previous studies.

Table 2
Results of this work compared to previous studies.

	Osiadacz (1987)	Ke and Ti (1999)	Behbahani-nejad and Bagheri (2008)	Alamian et al. (Trial & error) (2012)	Alamian et al. (DirectMethod) (2012
Pressure Node	AAPD (%)				
P2	0.199978885	0.405075914	0.142750947	0.171944766	0.247371789
P3	0.074859113	0.633221314	0.440594046	0.150004766	0.240840616

Table 3
Various type of friction factor.

Type of equation	Friction factor
Weymouth	$f = \frac{0.032}{D^{1/3}}$
Panhandle A	$f = \frac{0.085}{\mathrm{Re}^{0.147}}$
Panhandle B	$f = \frac{0.015}{\mathrm{Re}^{0.0392}}$
AGA	$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.825}{\mathrm{Re}\sqrt{f}}\right)$
Colebrook–White	$\frac{1}{\sqrt{f}} = 1.74 - 2\log_{10}\left(\frac{2\varepsilon}{D} + \frac{18.7}{\mathrm{Re}\sqrt{f}}\right)$

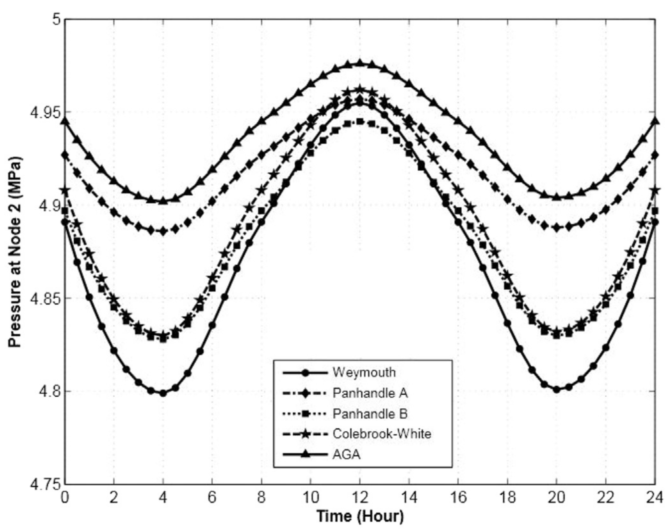

Fig. 7. Pressure at nodes 2 versus time for various friction factors.

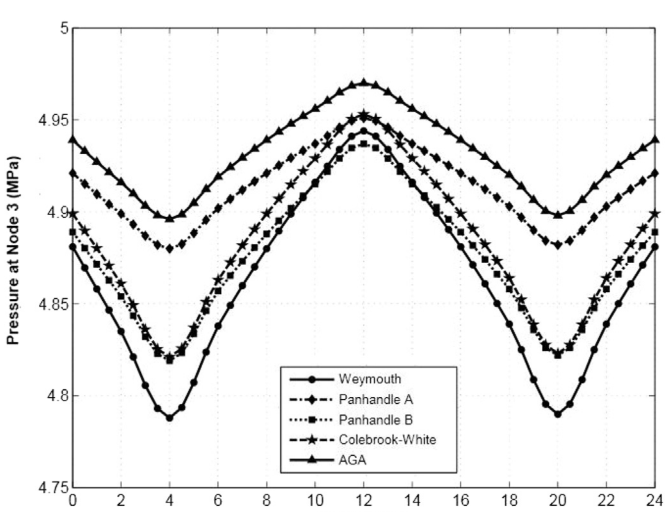

Fig. 8. Pressure at nodes 3 versus time for various friction factors.

$$D_n^* = \left(b_n - c_n k_n \rho_r^{k_n}\right)\rho_r^{b_n}\exp\left(-c_n \rho_r^{k_n}\right) \quad (32)$$

Coefficients of all parameter in equations are presented in reference (AGA8-DC92 EoS, 1992). Given the temperature, pressure and mole fraction of natural gas composition, by using AGA8 EOS compression factor and density of natural gas can be calculated.

4. Results and discussion

To show the method ability for studying a pipeline network under unsteady conditions, a typical network has been selected as a case study. Fig. 3 shows a typical natural gas pipeline network under investigation in the current study. The case study has been selected due to previous investigation for validation.

In this section, firstly the set of equations has been developed for the case study and then the proposed method has been validated against the previous studies. Finally, the effects of various parameters on the behavior of pipeline network under unsteady conditions are investigated.

By applying Kirchhoff's laws for the studied network shown in Fig. 3, set of equations of (29) could be obtained as follow:

$$\begin{cases} Q_{12}(t) = Q_{23}(t) + Q_2(t) \\ Q_{23}(t) + Q_{13}(t) = Q_3(t) \\ \left(P_1^2 - P_2^2\right) + \left(P_2^2 - P_3^2\right) - \left(P_1^2 - P_3^2\right) = 0 \end{cases} \quad (33)$$

In equation (33), two equations are Kirchhoff's first law and one equation is Kirchhoff's second law. The positive arbitrary direction for close loop of this pipeline network has been selected in the clockwise. Applying equation (16), equation (33) converts to following equation:

$$\begin{cases} Q_{12}(t) = Q_{23}(t) + Q_2(t) \\ Q_{23}(t) + Q_{13}(t) = Q_3(t) \\ K_2(t)Q_{12}^2 + K_3(t)Q_{23}^2 - K_1(t)Q_{13}^2 = 0 \end{cases} \quad (34)$$

For computing flow rate in pipes, the problem requires an iterative scheme. As the governing equations are not linear, the iterative method (trial and error) is used to obtain the solution. Finally pressure in nodes could be calculated with following equation:

$$\begin{cases} P_2^2 = P_1^2 - K_2(t)Q_{12}^2 \\ P_3^2 = P_1^2 - K_3(t)Q_{13}^2 \end{cases} \quad (35)$$

4.1. Validation

A common pipeline network, which analyzed and discussed by many researchers (Tao and Ti, 1998; Ke and Ti, 1999; Gonzales et al., 2009; Behbahani-Nejad and Bagheri, 2010; Behbahani-Nejad and Shekari, 2010; Alamian et al., 2012), has been investigated and simulated to validate the present approach. This pipeline network shows in Fig. 3. The specifications of pipelines such as: diameter, length and node connection in the studied network, are presented in Table 1. The natural gas demand according to time at the nodes 2

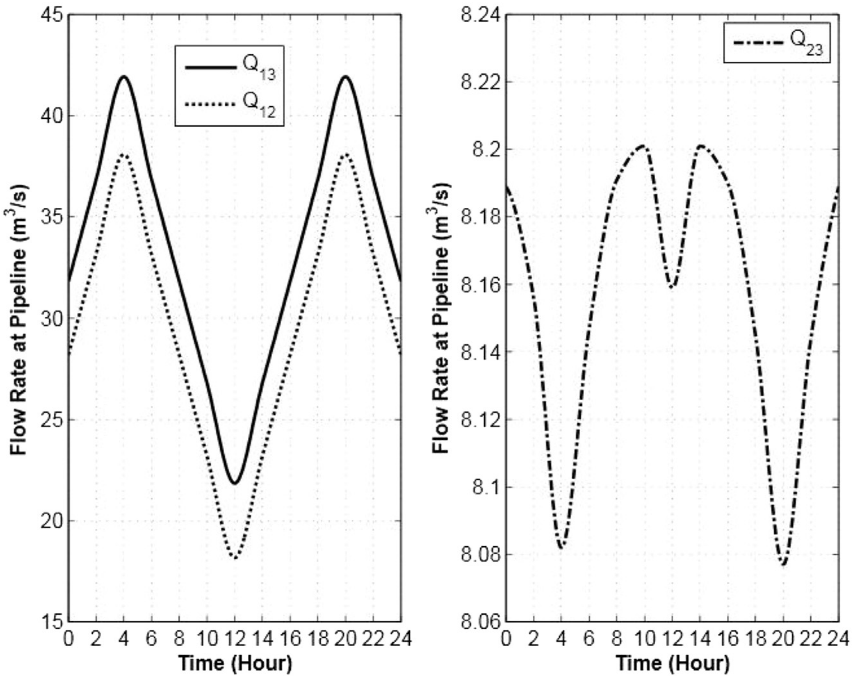

Fig. 9. Flow rate in pipelines of network versus time.

Table 4
Mole fraction of natural gas from different Iran's fields (National Iran Gas Company officialebsite).

Component	Mole fraction (%)		
	Khangiran	Kangan	Pars
CH_4	98.6	90.04	87
C_2H_6	0.59	3.69	5.4
C_3H_8	0.09	0.93	1.7
i-C_4H_{10}	0.02	0.2	0.3
n-C_4H_{10}	0.04	0.29	0.45
i-C_5H_{12}	0.02	0.14	0.13
n-C_5H_{12}	0.02	0.08	0.11
n-C_6H_{14}	0.07	0.14	0.07
C_7^+	0	0.01	0.03
N_2	0.56	4.48	3.1
CO_2	0	0	1.85
Molar Mass (kg/kmol)	**16.31**	**17.79**	**18.68**

and 3 are shown in Fig. 4. Node 1 in the network has been considered as suppliers therefore the pressure at this node assumed constant at 5 MPa. The natural gas specific gravity is considered almost 0.6, and the usable temperature is considered constant and equal to 278 K, and the friction factor used in the pipeline network is considered constant and equal to 0.003.

Figs. 5 and 6 compare pressure at nodes 2 and 3 versus time obtained from the proposed method and previous studies. According to Figs. 2 and 3, there is a good agreement between the current results and previous studies.

Table 2 compare AAPD obtained from the current study and previous works. According to Table 2, results of this study are in good agreement with previous works in which AAPD is less than 0.7%. The agreement shows that the proposed method could be used as a precise tool for analyzing the pipeline networks.

4.2. Effect of friction factor

Table 3 shows various type of friction factor that studied in this work. These equations have been used widely in natural gas

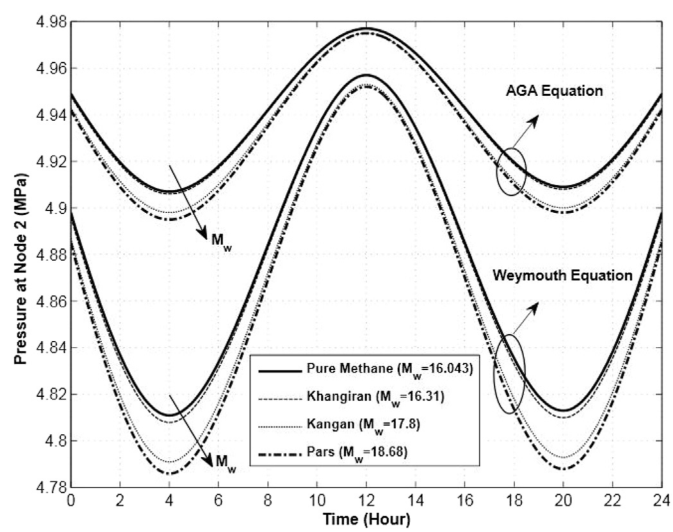

Fig. 10. Pressure at nodes 2 versus time for various natural gas and Weymouth and AGA equations.

industry to calculate friction factor. In Weymouth equation, friction factor is a function of inside diameter of pipeline, in Panhandle A and B equation, it is a function of Reynolds number, in AGA equation, it is a non-linear equation that function of Reynolds number and in Colebrook-White equation, it is a function of relative roughness and Reynolds number. In the previous studies, friction factor has been assumed constant. In this work, the effect of various definition of friction factor has been studied. Figs. 7 and 8 show pressure at nodes 2 and 3 versus time for various friction factors.

According to Figs. 7 and 8, Weymouth equation shows highest pressure drop in pipeline; therefore pressure at nodes has lowest in this case. Also, AGA equation shows lowest pressure drop and therefore pressure at node is highest when this equation utilized. Results obtained from Weymouth equation have best agreement

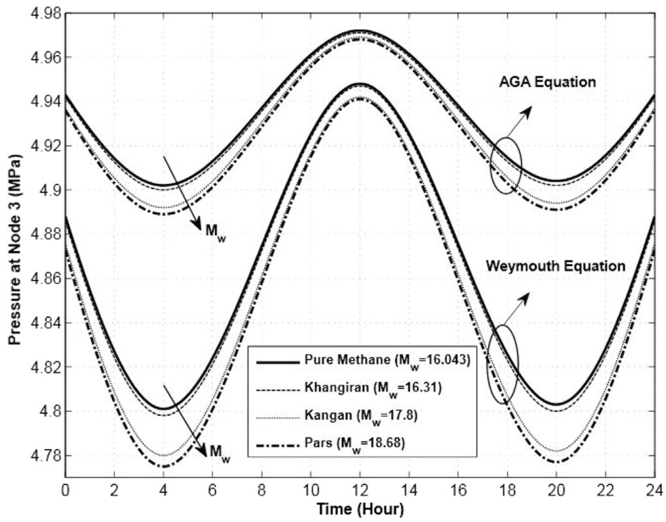

Fig. 11. Pressure at nodes 3 versus time for various natural gas and Weymouth and AGA equations.

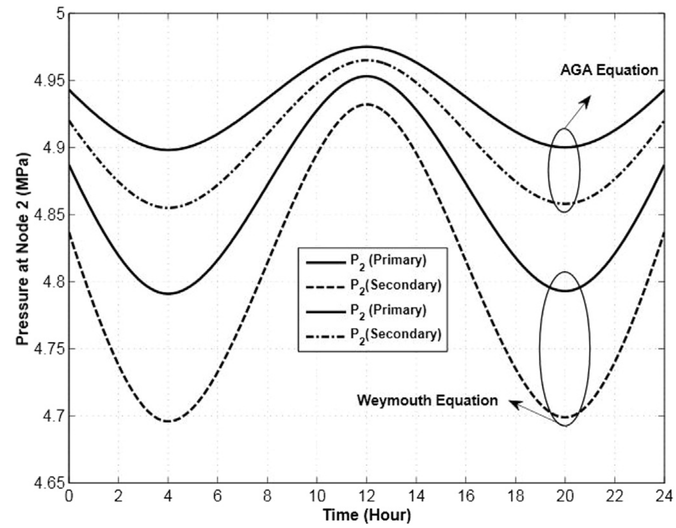

Fig. 13. Primary and secondary pressure at nodes 2 versus time for Weymouth and AGA equations.

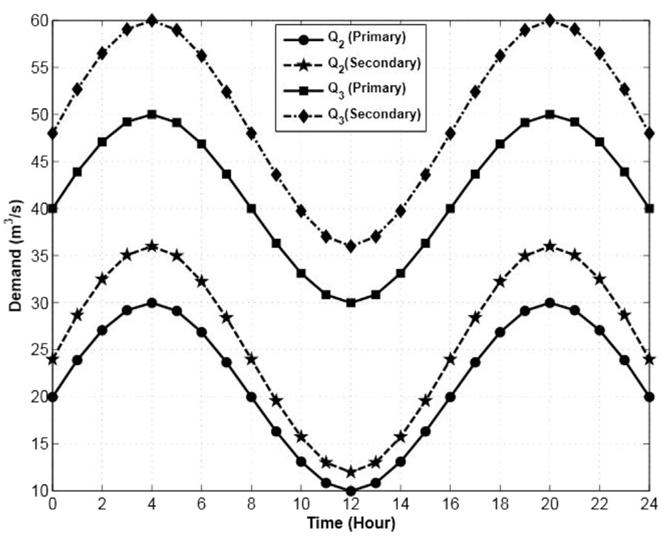

Fig. 12. Increasing pattern of demand at nodes 2and 3 versus time.

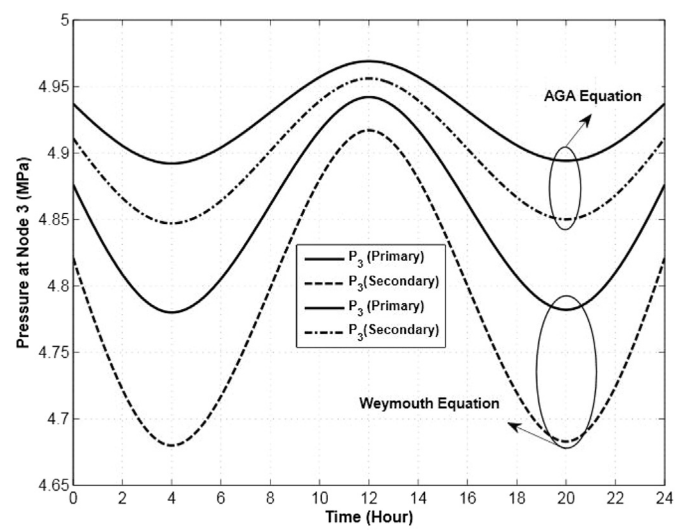

Fig. 14. Primary and secondary pressure at nodes 3 versus time for Weymouth and AGA equations.

with previous studies.

Fig. 9 shows volumetric flow rate in pipes versus time. By applying various friction factors, flow rate in pipes are not shown much difference. Trend of flow rate Q_{12} and Q_{13} has been same with demand at node 2 and 3 but flow rate Q_{23} has the higher different trend.

4.3. Effect of natural gas composition

Natural gas is a mixture of various gases such as: CO_2, N_2 and hydrocarbons such as: CH_4, C_2H_6, C_3H_8, etc. Therefore natural gas properties such as: compression factor and density are depending on the properties of their compositions. Several natural gases, with different compositions, are selected to study the effect of natural gas composition. Table 4 shows mole fraction of these natural gases that have been exploited from different Iran's fields (National Iran Gas Company officialebsite). These natural gases have been picked out from Iran's fields because of the highest difference in their compositions between Iranian pipeline natural gases.

In this section, the effect of natural gas composition has been studied. Figs. 10 and 11 show pressures at nodes 2 and 3 versus time for various natural gas compositions as well for Weymouth and AGA friction factor equations. According to Figs. 10 and 11, by increasing molar mass of natural gas, the pressure at nodes will be decreased. This is due to increase in natural gas specific gravity which caused more pressure drop in pipes and consequently lower pressure at nodes.

4.4. Effect of increasing demand

In this section, the effects of increasing demand have been studied. Fig. 12 shows increasing pattern of demand at nodes 2 and 3 versus time also primary demand of nodes. Figs. 13 and 14 show primary and secondary pressure at nodes 2 and 3 versus time for Weymouth and AGA friction factor equations. According to Figs. 13 and 14, by increasing demand at nodes 2 and 3, pressure in these nodes has been decreased. Increasing demand at nodes causes higher flow rate in pipelines which resulted to pressure decrease at nodes.

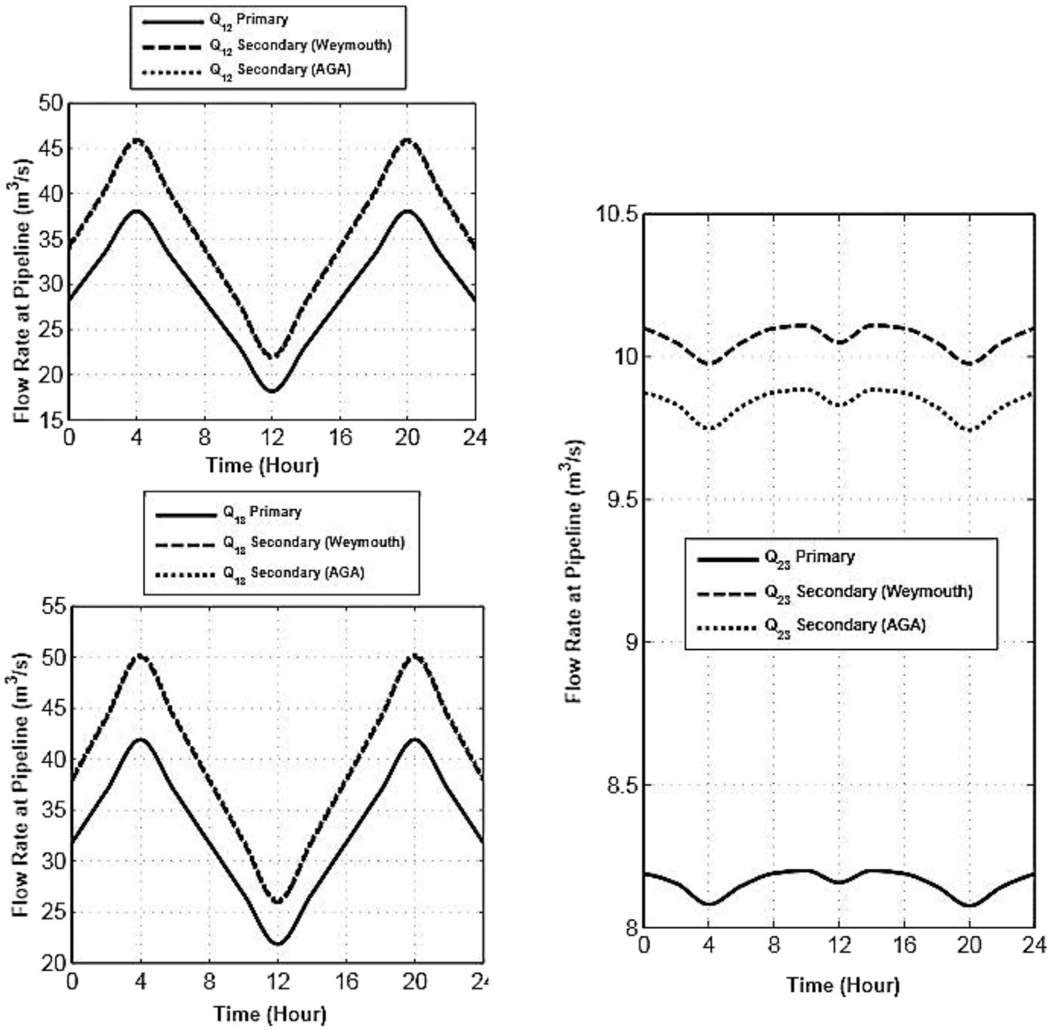

Fig. 15. Primary and secondary flow rate in pipelines versus time.

Fig. 15 shows volumetric flow rate in pipelines in two type of primary and secondary of demand at nodes. According to Fig. 15, for all Q_{12}, Q_{13} and Q_{23} secondary flow rate is higher than primary flow rate. The reason is that, according to Figs. 13 and 14 pressure at nodes in secondary case is less than primary case therefore this decline in pressure led to increase in flow rate within pipelines. Also Q_{23} is higher when Weymouth friction factor has been applied.

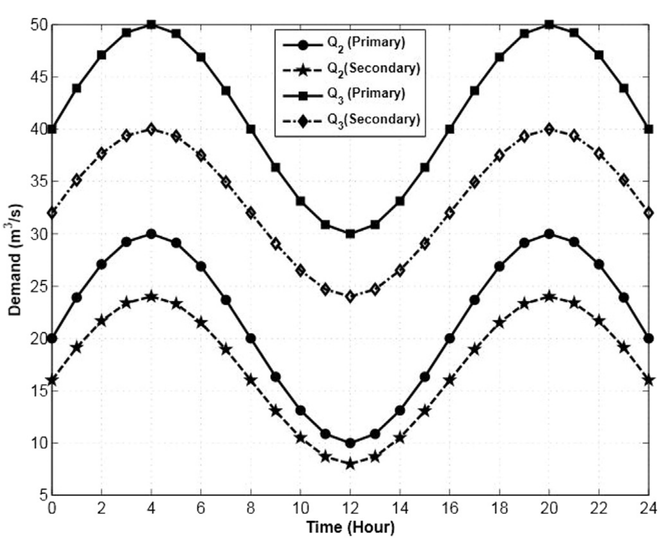

Fig. 16. Decreasing pattern of demand at nodes 2and 3 versus time.

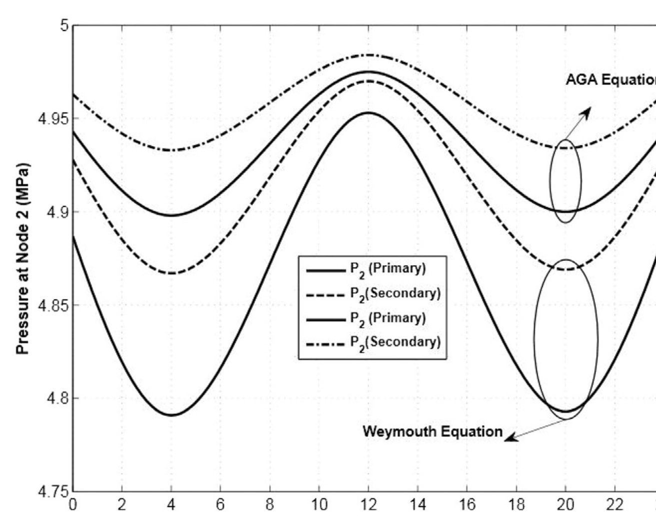

Fig. 17. Primary and secondary pressure at nodes 2 versus time for Weymouth and AGA equations.

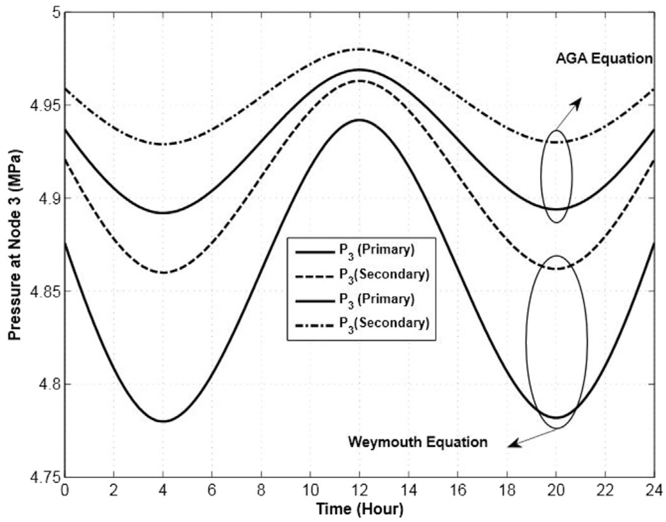

Fig. 18. Primary and secondary pressure at nodes 3 versus time for Weymouth and AGA equations.

4.5. Effect of decreasing demand

In this section, the effect of decreasing demand has been studied. Fig. 16 shows decreasing pattern of demand at nodes 2 and 3 versus time also primary demand of nodes. Figs. 17 and 18 show primary and secondary pressure at nodes 2 and 3 versus time for Weymouth and AGA friction factor equations. According to Figs. 17 and 18, by decreasing demand at nodes 2 and 3, pressure in these nodes will be increased. Decreasing demand at nodes causes less flow rate in pipelines which resulted to pressure increase at nodes.

Fig. 19 shows volumetric flow rate in pipelines in two type of primary and secondary of demand at nodes. According to Fig. 19, for all Q_{12}, Q_{13} and Q_{23}, secondary flow rate is less than primary flow rate. The reason is that, according to Figs. 17 and 18 pressure at nodes in secondary case is higher than primary case therefore this rising in pressure led to increase in flow rate. Also Q_{23} is higher when Weymouth friction factor has been applied.

5. Conclusion

To transport natural gas from the production sites to the consumers, the pipeline network is used. The pipeline network (with components such as pipes, nodes, valves) may be subjected to unsteady boundary conditions. These conditions usually occur due to customers demand variation (due to ambient temperature change), pipeline rapture and many other extreme conditions. To have safe and continues pipeline network operation, it is necessary to have knowledge of the pipeline network under these conditions. As a result, it is necessary to develop a numerical method to be able

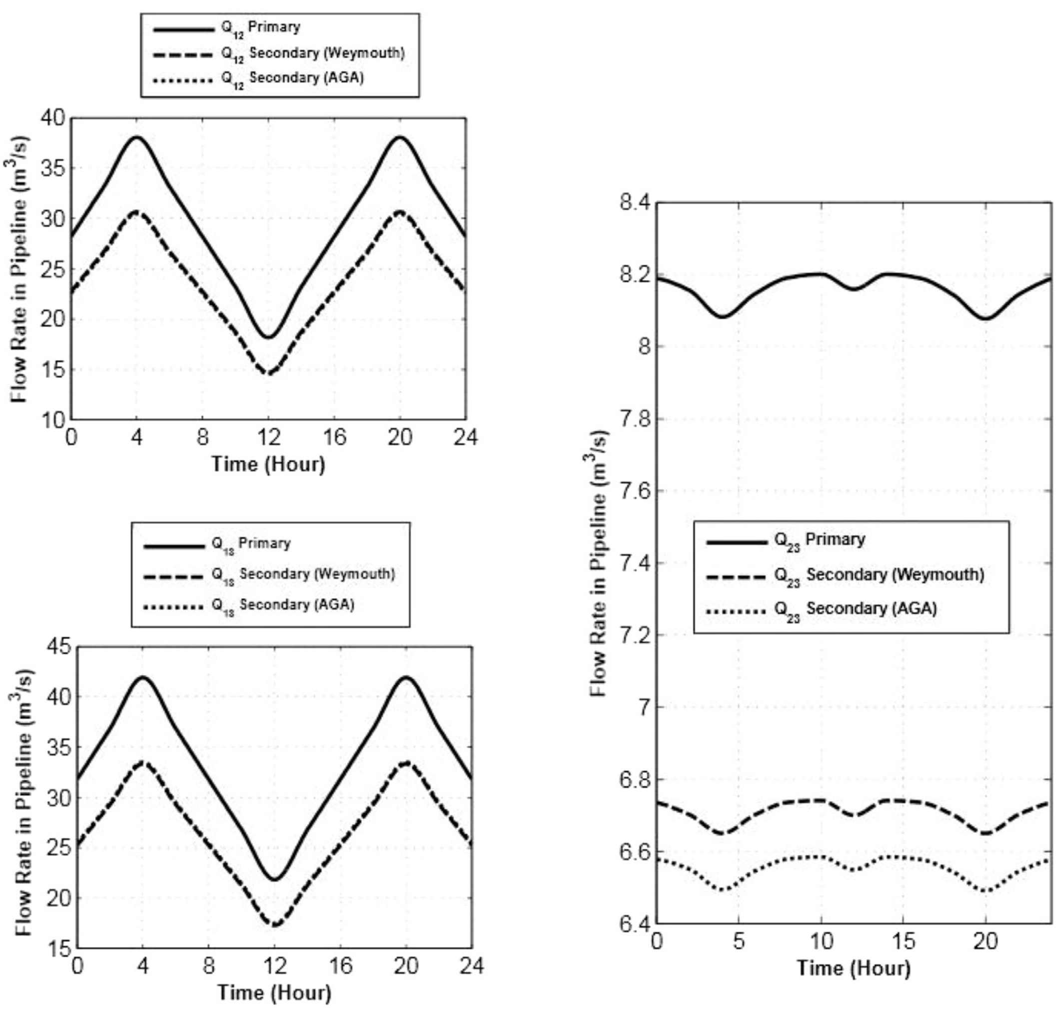

Fig. 19. Primary and secondary flow rate in pipelines versus time.

to simulate the natural gas pipeline network under unsteady condition. In the current study, an analytical approach for simulating a pipeline networks under unsteady conditions has been presented. The model is developed similarly as steady state one. A typical pipeline network has been selected as cases study to validate the proposed method. Validation shows the proposed method have an average absolute present derivation (AAPD) less than 0.7% compared to previous studies. In addition, the effect of important parameter such as: friction factor, natural gas compositions and increasing or decreasing demand have been studied. Results show that unsteady-state form of Weymouth equation has highest pressure drop and closest value to previous studies. Also results show that, natural gas with higher molar mass has less pressure at nodes.

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Glossary

D: pipe inside diameter (mm)
F: friction factor
Δt: Change in time (second)
L: pipe length (km)
K(t): function of pipeline properties
ṁ: Mass flow rate (kg/s)
g_c: Gravitational conversion factor (kg.m/N.s^2)
Mw: Molar mass (kg/kmol)
P: Pressure (MPa)
Q: Volumetric Flow Rate (m^3/s)
R: gas constant (kJ/kmol.K)
Re: Reynolds number
T: Temperature (K or ^{o}C)
U: velocity (m/s)
X: spatial position
Z: Compression factor

Subscript

a: air
b: base condition
g: gas
1: inlet
2: outlet
12: node 1 to node 2
13: node 1 to node 3
23: node 2 to node 3

Greek Letters

ρ: Density (kg/m^3)
γ: Gas specific gravity
ε: Pipeline roughness.

Abbreviations

EOS: Equation of State

AAPD: Average Absolute Present Derivation,

$$\text{AAPD for Pressure} = \frac{\sum_{i=1}^{N}\left|\frac{P_{This\ \ work}-P_{Other\ \ study}}{P_{Other\ \ study}}\right|*100}{Number\ \ of\ \ Data=N}$$

AGA8 EOS Parameters

B: Second Virial coefficient

C_n^**:* Temperature and composition dependent coefficient

K_j*:* size parameter of component *j*

K_{ij}*:* binary interaction parameter

N: number of component in gas mixture

x: mole fraction of component

E_{ij}^**:* Binary energy interaction parameter for second virial coefficient

E_i*:* Characterization energy parameter for *i*-th component (K)

E_{ij}*:* Binary energy parameter for second virial coefficient (K)

F: Mixture high-temperature parameter

F_i*:* high-temperature parameter for *i*-th component

G: Mixture orientation parameter

G_{ij}^**:* Binary interaction parameter for orientation

Q: Quadrupole parameter

U_{ij}*:* Binary interaction parameter for mixture energy

W_i*:* Association parameter for *i*-th component-

ρ_m*:* molar density (kmol/m^3)

ρ_r*:* reduce density (kmol/m^3)