International Journal of Theoretical Physics
https://doi.org/10.1007/s10773-018-3920-2

Next-to-Next-to-Leading Order Calculation of Valence Quark Polarization in the Valon Model

F. Taghavi-Shahri[1] · Z. Alizadeh Yazdi[1] · S. Atashbar Tehrani[2]

Received: 25 June 2018 / Accepted: 5 October 2018 / Published online: 15 October 2018

Abstract
We used the "valon model" to study the valence quarks polarization inside nucleon at Next-to-Next-to-leading order (NNLO) approximation. The results show a good agreement with available experimental data and also with those from global analysis presented in Taghavi-Shahri et al. (Phy. Rev. D **93**, 114024, 2016).

Keywords Valon model · PPDFs · Valence quarks polarization · NNLO approximation

1 Introduction

Understanding the internal structure of matter has been one of the main goals in high energy physics. Electron - proton deep inelastic scattering shows that the proton consists of partons. Each parton carries a specific fraction of the nucleon's momentum and spin . Now we know that only 40 percent of the proton's spin comes from its constituent quarks, and the rest of the spin originating from gluons and orbital angular momentum of quarks and gluons. The main question is that "how the spin of the nucleon is shared between its constituent quarks and gluons?". To answer this question we have to study the "polarized parton distribution functions" or PPDFs.

In this paper we intend to investigate the polarized parton distribution functions. The Polarized parton distribution function is defined as the probability density for finding a parton with a longitudinal momentum fraction x of its parent nucleon's momentum and spin align/ anti-align to the nucleon's spin. It measures the net helicity of partons in a longitudinally polarized nucleon.

✉ F. Taghavi-Shahri
taghavishahri@um.ac.ir

Z. Alizadeh Yazdi
zahralizadeh.y@gmail.com

S. Atashbar Tehrani
atashbart@gmail.com

1 Department of Physics, Ferdowsi University of Mashhad, P.O. Box 1436, Mashhad, Iran

2 Independent Researcher, P.O. Box 1149-834413 Tehran, Iran

These PPDFs set can be calculated from the DGLAP evolution equations at various perturbation approximation. The polarized parton distribution functions have been calculated by different global analyses groups using experimental data [1–13] and they have been computed with various phenomenological models [14–16]. Recently, the polarized splitting functions was proposed at NNLO approximation in Mellin space in [17]. Therefore, we can investigate the PPDFs at this approximation.

In this paper we want to calculate the polarized valence quark distributions and the non-singlet spin structure function using the valon model. The valon model is a phenomenological model originally was introduced by R.C. Hwa [18] in the early 1980s. It was revitalized later by Hwa [19] and others [20–22] and developed to the polarized cases at NLO approximation [23–26]. In this model, a hadron is a bound state of two or three "valons". Each valon includes a valence quark with its associated sea quarks and gluons. The quantum number of the valon is the same as the quantum number of its valence quark. These valons carry all the hadron's momentum and spin.

The probability of finding a valon with momentum fraction y of the hadron's momentum defines the valon distribution function. These distributions are Q^2 independent and universal. They can be considered as the wave- function squared of the constituent quarks in the hadron.

At low $Q^2 (Q^2 < 0.3GeV^2)$, the structure of valons can not be determined. It means at this range of energy, the hadron is only a bound state of three valence quarks that carry all the hadron's momentum and spin. At high Q^2, the structure of valon can be resolved. Solutions of the DGLAP evolution equations in each valon with suitable initial input densities in the valon, led us to the Q^2 dependence of the parton distribution functions in the valon and then in the hadron.

The paper is organized as follows. In Section 2, we briefly review the solutions of the DGLAP evolution equations at NNLO approximation in Mellin space. Then in Section 3, we utilize the valon model to calculate the valence quarks polarization and the non- singlet spin structure function. The results of our calculations are presented in Section 4. Finally our conclusions are given in Section 5.

2 The DGLAP Evolution Equations at NNLO Approximation

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [27–30] are essential tool to study the Q^2- and x- dependence of the parton distribution functions and the hadron's structure. The Q^2 variable is the virtuality of the exchanged vector bozon in the deep inelastic scattering process and "x" is the Bjorken variable. The general form of the DGLAP evolution equations is:

$$\frac{\partial \Delta f(x, Q^2)}{\partial ln Q^2} = \Delta f(x, Q^2) \otimes \Delta P_{ij}(x, Q^2) \tag{1}$$

with

$$a(x) \otimes b(x) = \int_x^1 \frac{dy}{y} a(y) b\left(\frac{x}{y}\right) \tag{2}$$

where $\Delta f(x, Q^2)$ is the polarized parton distribution functions and $\Delta P_{ij}(x, Q^2)$ are known as the splitting functions. The spliting functions at different perturbative approximations are expanded as follows:

$$\Delta P_{ij}(x, \alpha_s(Q^2)) = \Delta P_{ij}^{LO}(x) + \frac{\alpha_s(Q^2)}{2\pi} \Delta P_{ij}^{NLO}(x) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 \Delta P_{ij}^{NNLO}(x) + ... \tag{3}$$

We start with the solutions of the NNLO DGLAP evolution equations in the Mellin space [31]. The related splitting functions at NNLO approximation can be found in Ref. [17]. The Mellin moments are defined as:

$$\Delta f(N) = \int_0^1 \Delta f(x) x^{N-1} dx \tag{4}$$

The main advantage of working in moment space is that it reduces the convolution product $\otimes$ (Eq. (2.2)) into an ordinary product.

3 Study the Valence Quark Polarization in the "Valon" Model

We devoted this section to the details of the valence quark polarization in the valon model. In the valon representation of hadrons, the PPDFs are given by:

$$\Delta q_i^P(x, Q^2) = \sum \int_x^1 \frac{dy}{y} \Delta G_{valon}^P(y) \Delta q_i^{valon}\left(\frac{x}{y}, Q^2\right) \tag{5}$$

where the $\Delta q_i^{valon}(x/y, Q^2)$ in (5) is the polarized parton distribution inside a valon. $\Delta G_{valon}^P(y)$ is the helicity distribution of the valon in the parent hadron i.e (probability of finding the polarized valon inside the polarized hadron). $\Delta G_{valon}^p(y)$ is related to the unpolarized valon distribution, $G_j^p(y)$ by:

$$\Delta G_j^p(y) = \Delta F_j(y) G_j^p(y) = N_j y^{\alpha_j}(1-y)^{\beta_j}(1 + a_j y^{0.5} + b_j y + c_j y^{1.5} + d_j y^2) \tag{6}$$

where j index refers to the U and D type valons [18, 22]. The related free parameters are calculated in [23] and they are summarized in Table 1. They are also shown in Fig. 1.

The Mellin moment of the polarized valon distributions are defined as:

$$\Delta G_{U,D}^p(N) = \int_0^1 y^{n-1} \Delta G_{U,D}^p(y) dy \tag{7}$$

$\Delta G_{U,D}^p(N)$ correspond to the valon helicity densities in the Mellin space for the U and D valons. Before we start our discussion, it is suitable to define our coefficients and conventions that we will used them in the rest of the paper [32]. The Running coupling constant is defined at Next-to-Next-to leading order as follows [32]:

$$a_s(Q^2) = \frac{1}{\beta_0 L_\Lambda} - \frac{1}{(\beta_0 L_\Lambda)^2} b_1 \ln L_\Lambda + \frac{1}{(\beta_0 L_\Lambda)^3} \left[b_1^2(\ln^2 L_\Lambda - \ln L_\Lambda - 1) + b_2\right] \tag{8}$$

Table 1 Numerical values for parameters of (6) for polarized valon distributions inside proton [23]

$valon(j)$	N_j	α_j	β_j	a_j	b_j	c_j	d_j
U	3.44	0.33	3.58	−2.47	5.07	−1.859	2.780
D	−0.568	−0.374	4.142	−2.844	11.695	−10.096	14.47

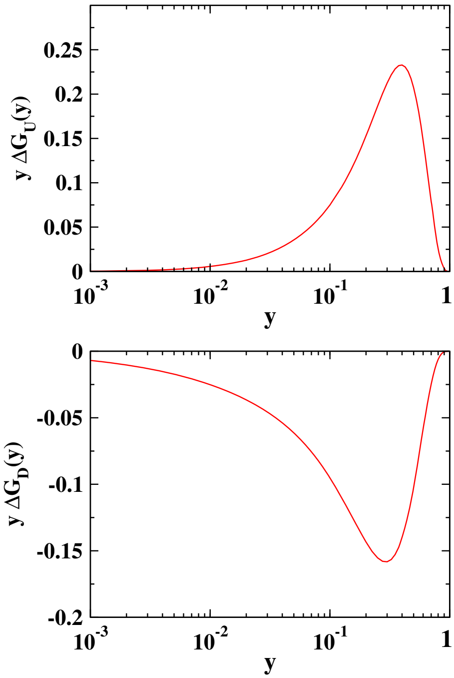


Fig. 1 $y\Delta G_U(y)$ and $y\Delta G_D(y)$ as a function of y

where

$$L_\Lambda = \ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right) \tag{9}$$

The coefficients of the beta function in $\overline{MS}$ scheme (β_0, β_1 and β_2) are [32] :

$$\begin{aligned}
\beta_0 &= 11 - \frac{2}{3}n_f;\\
\beta_1 &= 102 - \frac{38}{3}n_f;\\
\beta_2 &= \frac{2857}{2}n_f - \frac{5033}{18}n_f + \frac{325}{54}n_f^2
\end{aligned} \tag{10}$$

We also have $b_1 = \frac{\beta_1}{\beta_0}$ and $b_2 = \frac{\beta_2}{\beta_0}$. In the valon model, we first calculate the polarized parton distributions inside each valon (with suitable initial inputs). Then we can calculate these distributions in the proton. The solutions of the DGLAP evolution equations at NNLO approximation for the polarized valence quark distributions in the Mellin space are given by:

$$\begin{aligned}
M^v_{NS}(N, Q^2) = \left(\frac{a_s}{a_0}\right)^{-r_0} &(1 - (a_s - a_0)\, r_1^- + \frac{1}{2}(a_s - a_0)^2 (r_1^-)^2\\
&+ \frac{1}{2}(a_0^2 - a_s^2)\, r_2^v)\ \ M^v_{NS}(N, Q_0^2)
\end{aligned} \tag{11}$$

$\Delta P_{NS}^{-(1)}$, $\Delta P_{NS}^{v(2)}$ are the splitting functions and the parameters r_0, r_1^- and r_2^v are as follows [17, 33, 34] ,

$$\begin{aligned} r_0 &= \frac{\Delta P_{qq}^{(0)}(N)}{\beta_0} \\ r_1^- &= \frac{\Delta P_{NS}^{-(1)}(N)}{\beta_0} - b_1 r_0 \\ r_2^v &= \frac{\Delta P_{NS}^{v(2)}(N)}{\beta_0} - b_1 r_0 - b_2 r_1^-; \end{aligned} \tag{12}$$

Finally, the valence quark distributions inside proton are given by:

$$\Delta u_v(N, Q^2) = 2\, M_{NS}^v(N, Q^2) \times \Delta G_U^p(N)$$

$$\Delta d_v(N, Q^2) = M_{NS}^v(N, Q^2) \times \Delta G_D^p(N) \tag{13}$$

The non- singlet spin structure function g_1^{NS} in the moment space at NNLO approximation is as follows:

$$\mathcal{M}[g_1^{NS}, N, Q^2] = \left(\frac{1}{6}\right) \left[(1 + a_s \Delta C_q^1 + a_s^2 \Delta C_{NS}^{(2)}) \left(\Delta u_v(N, Q^2) - \Delta d_v(N, Q^2)\right)\right] \tag{14}$$

Where ΔC_q^1, $\Delta C_{NS}^{(2)}$ are the corresponding coefficient functions [34, 35].

We work in $\overline{MS}$ scheme, with $\Lambda_{QCD} = 0.252 GeV^2$ and $Q_0^2 = 0.283 GeV^2$. Our initial input density at Q_0^2 is the Dirac delta function $\delta(z-1)$. Then, the Mellin transform of this initial input, being equal to one. So we choose $M_{NS}^v(N, Q_0^2) = 1$. This means that when Q^2 approaches to Q_0^2 , the nucleon can be considered as a bound state of three valence quarks which carry all of the nucleon's momentum and spin. Therefore, at this scale of Q_0^2 and when we solve the DGLAP evolution equation in the valon, there is only one valence quark in each valon and this valence quark carry all of the valon's momentum and spin. Therefore, we choose the initial input density in the valon as $\delta(z-1)$. In our calculations, the sea quarks polarization is consistent with zero . This prediction was already suggested in Ref [23] and it was confirm by experiment later [36, 37].

We used (13) to compute the valence quark polarization in moment space; eventually we get help from Jacobi polynomial approach for gaining of the final result at x- space. The Jacobi polynomials expansion method is one of the simplest and fastest algorithm to calculate the structure function from its Mellin moments to x- space. In this method, one can easily develop the polarized structure functions, $xg_1(x, Q^2)$, in terms of the Jacobi polynomials, $\Theta_n^{\alpha,\beta}(x)$, as follows [38, 40, 41]

$$xg_1^{NS}(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} a_n(Q^2)\, \Theta_n^{\alpha,\beta}(x)\,, \tag{15}$$

where n is the order of the expansion terms, $N_{\max}$ is the maximum order of the expansion which normally can be set to 7 and 9. The parameters α and β are a set of free parameters which normally set to 3 and 0.5, respectively. The Q^2–dependence of the structure functions are encoded in the Jacobi polynomials moments, $a_n(Q^2)$. The x-dependence will be presented by the weight function $x^\beta(1-x)^\alpha$ and the Jacobi polynomials $\Theta_n^{\alpha,\beta}(x)$ which can be written as,

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\, x^j \,, \tag{16}$$

where the coefficients $c_j^{(n)}(\alpha,\beta)$ are combinations of Gamma functions in term of n, α and β. The above Jacobi polynomials have to the following orthogonality relation,

$$\int_0^1 dx\, x^{\beta}(1-x)^{\alpha}\, \Theta_k^{\alpha,\beta}(x)\, \Theta_l^{\alpha,\beta}(x) = \delta_{k,l}\,. \tag{17}$$

Therefore, one can obtain the Jacobi moments, $a_n(Q^2)$, by using the above orthogonality relations as,

$$\begin{aligned} a_n(Q^2) &= \int_0^1 dx\, x g_1^{NS}(x,Q^2)\, \Theta_k^{\alpha,\beta}(x) \\ &= \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\, \mathcal{M}[x g_1^{NS}, j+2]\,, \end{aligned} \tag{18}$$

where the Mellin transform $\mathcal{M}[x g_1^{NS}, N]$ introduced as,

$$\mathcal{M}[x g_1^{NS}, \mathrm{N}] \equiv \int_0^1 dx\, x^{\mathrm{N}-2}\, x g_1^{NS}(x,Q^2)\,. \tag{19}$$

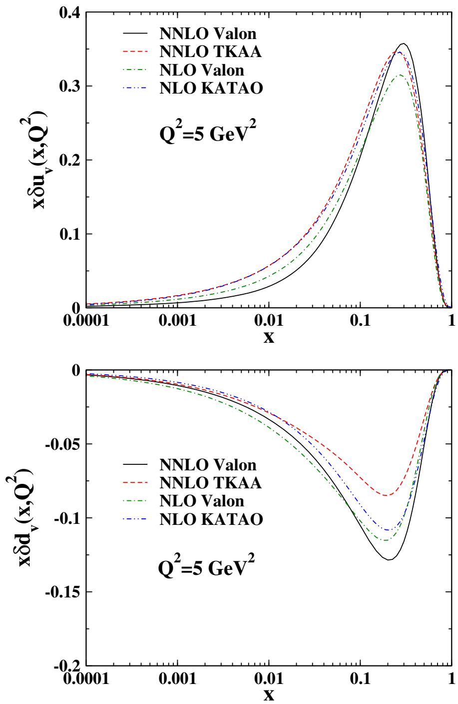


Fig. 2 $x\Delta u_v(x, Q^2)$ and $x\Delta d_v(x, Q^2)$ as a function of x for $Q^2 = 5Gev^2$

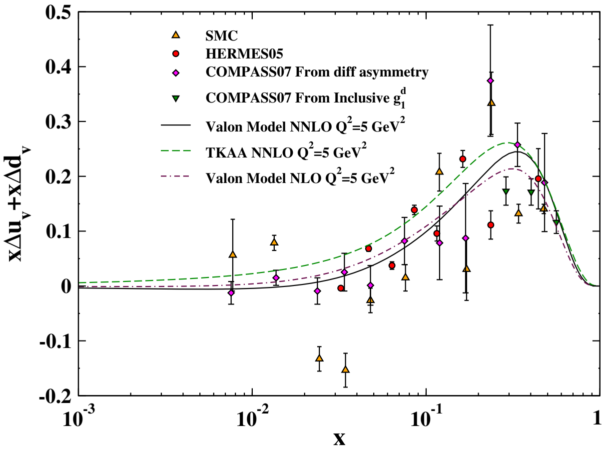


Fig. 3 $x\Delta u_v + x\Delta d_v$ in the valon model and comparison with experimental data [42–45], TKAA global analyses [38] and those obtained at NLO approximation [25]

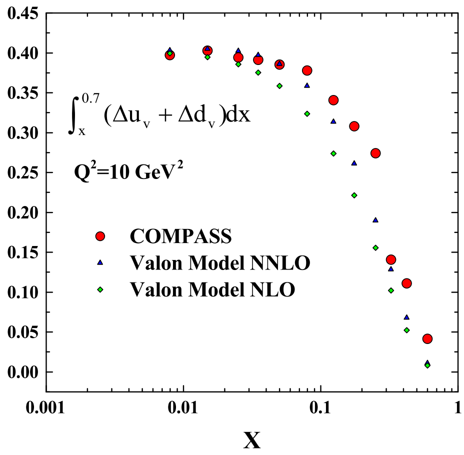
Finally the non- singlet spin structure function $xg_1^{NS}(x, Q^2)$ can be written as follows,

$$xg_1^{NS}(x, Q^2) = x^{\beta}(1-x)^{\alpha} \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \times \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)\, \mathcal{M}[xg_1^{NS}, j+2]\,. \tag{20}$$

4 Results

This section is devoted to our results. We can calculate the polarized valence quark distributions in the Mellin space by using (13); then we use the Jacobi polynomial approach for

Fig. 4 Integral of $\Delta u_v + \Delta d_v$ as function of low x limit of integration and its comparison with experimental data from COMPASS Collaboration [44] and those obtained at NLO approximation [25]



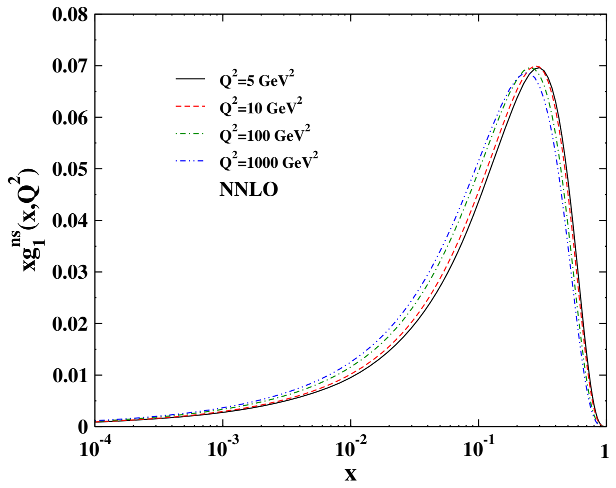


Fig. 5 $xg_1^{NS}(x, Q^2)$ as a function of x for different values of Q^2

gaining of the valence quark polarization and the non-singlet spin structure function as a function of x. In Fig. 2, we show our results for the polarized distribution functions of the valence "u" and "d" quarks, $x\Delta u_v(x, Q^2)$ and $x\Delta d_v(x, Q^2)$ for $Q^2 = 5Gev^2$ in comparision with those obtainin the valon model at NLO approximation and those obtained from KATAO and TKAA global analyses [23, 38, 39].

In Fig. 3, our results for $x\Delta u_v + x\Delta d_v$ are compared with experimental data [42–45] and recent results from global analyses [38]. We compute the integral of $\Delta u_v + \Delta d_v$ over

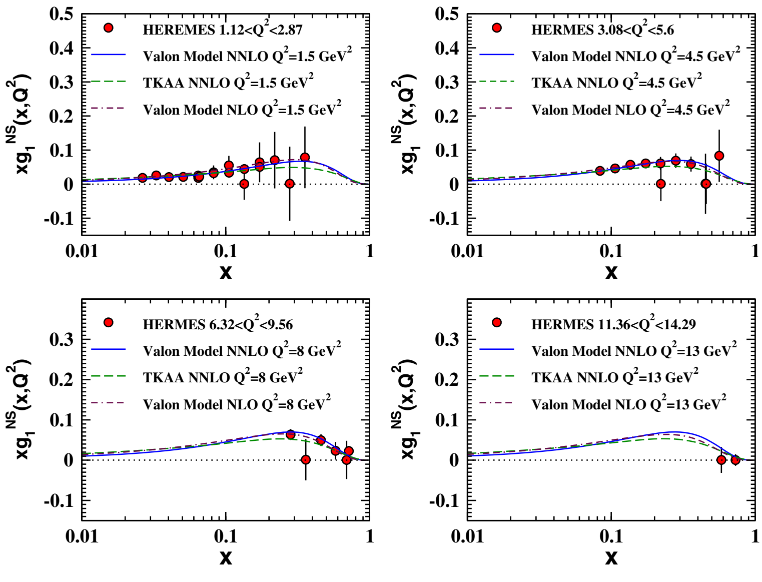


Fig. 6 The results of $xg_1^{NS}(x, Q^2)$ for a different values of Q^2 . We compared our results with HERMES experimental data [36], TKAA global fit [38] and those obtained at NLO approximation [23]

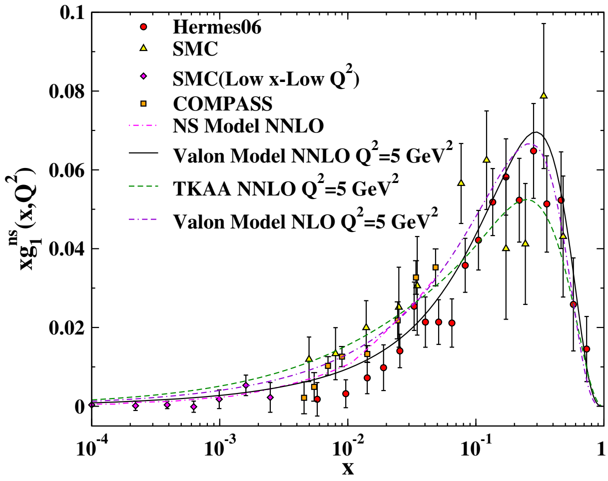


Fig. 7 xg_1^{NS} which is compared with experimental data from SMC, COMPASS and HERMES Collaborations [36, 37, 46–51] NS model [52], TKAA model[38] and NLO results [23] at $Q^2 = 5GeV^2$

the range of $0.006 < x < 0.7$ as a function of low x limit of integration at $Q^2 = 10GeV^2$. Our results are compared with experimental data that released by COMPASS Collaboration [44] in Fig. 4.

The non- singlet spin structure function g_1^{NS} at NNLO approximation are shown in Fig. 5. In Fig. 6 we compared our results for $xg_1^{NS}(x, Q^2)$ with HERMES data [36] and TKAA global analyses [38]. Finally, in Fig. 7 the x dependence of xg_1^{NS} is shown in comparison with experimental data from SMC, COMPASS, HERMES [36, 37, 46–51], NS model [52] and TKAA model[38] at $Q^2 = 5GeV^2$. We also compared our new results at NNLO approximation with those obtained at NLO approximation [23, 25] in Figs. 3, 4, 5, 6 and 7.

5 Summary and Conclusions

We have applied the so- called valon model for calculating the polarized valence quark distributions and the non- singlet spin structure function of the proton at NNLO approximation. Our results have nice agreement with all available experimental data and exist phenomenological models. It also shows that how such a simple model can well rehabilitate the experimental data and therefor provides a good physical picture of the nucleons.

Acknowledgments The authors would like to thank F.Arash for carefully reading the manuscript, fruitful discussion, and critical remarks. This work is supported by Ferdowsi University of Mashhad under Grant No. 2/43305, 1396/02/26.

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