

# Impact of mean velocity accuracy on the estimation of rating curves in compound channels

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## ABSTRACT

The stage-discharge relationship in open channels plays a significant role in analyzing and designing the channels and hydraulic structures. Recently a new stage-discharge relationship is introduced (Maghrebi et al., 2017). The proposed relationship can be used in arbitrary shaped channels including compound channels, where the traditional formulations are not able to accurately estimate the discharge. In this study, the stage-discharge curves in 6 laboratory prismatic compound cross sections are calculated based on the proposed stage-discharge relationship. The mean velocity parameter appearing in this relationship can be substituted with any equivalent value. As an alternative, the mean extracted velocity using the SKM is used in the proposed relationship in order to produce the stage-discharge curves for the above mentioned cross sections. Then, the obtained results are compared with the ones of the proposed relationship. The accuracy of the two models is tested successfully against available experimental results, which are taken from the Flood Channel Facility (FCF) laboratory. Finally, the normalized root mean square error (NRMSE) and the mean absolute percentage error (MAPE) calculated based on the SKM method produced by the CES are within 0.61% and 10%, respectively. However, the corresponding calculated values based on application of mean velocity  $U$  extracted from the depth-averaged velocity  $U_d$  of the SKM do not exceed 0.23% and 4%, respectively, which shows a better agreement.

## Nomenclature

$A$	Cross-sectional area of flow
$a, b, c, d$	Constants
$A_e$	Area at estimated water level
$A_r$	Area at referenced water level
$c_1$	A factor that depends on shear velocity, roughness on the wall and turbulent intensity
$ds$	A finite element of boundary
$du$	Differential velocity deviation between an element of the boundary and an arbitrary point in the flow field
$fO$	A function of
$H$	Water depth along y-axis at a cross section
$MAPE$	Mean absolute percentage error
$N$	Number of observed or estimated data
$NRMSE$	Normalized root mean square error
$n$	Manning roughness coefficient
$P$	Total wetted perimeter $P = P_w + T$

$P_e$	Total wetted perimeter at estimated water level
$P_r$	Total wetted perimeter at referenced water level
$P_w$	Wetted perimeter
$Q_e$	Estimated discharge
$Q_r$	Referenced discharge
$\mathbf{r}$	Position vector of arbitrary point in field
$S_0$	Slope of the channel bottom
$SKM$	Shiono-Knight method
$SPM$	Single point method
$T$	Width of channel section at the free surface
$U$	Cross sectional mean flow velocity in the streamwise direction
$u$	Streamwise velocity at a point in the channel section
$U_d$	depth-averaged velocity
$U_e$	Mean velocity at estimated water level
$U_r$	Mean velocity at reference water level
$V$	Average velocity of flow
$y$	Normal distance from boundary
$z$	Distance measured in lateral direction

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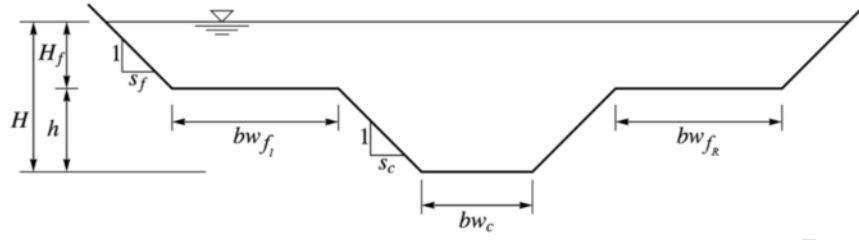


Fig. 1. General schematic cross section of a compound channel.

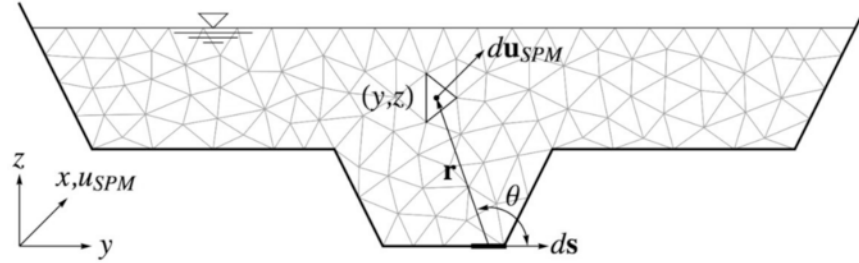


Fig. 2. The flow cross section with triangle mesh along with other existing variables in SPM method.

**Table 1**  
Geometrical and hydraulic specifications of the compound channel cross sections.

Test	$s_0 (\times 10^{-3})$	$n_c$	$n_f/n_c$	$s_c$	$s_f$	$bw_c(m)$	$bw_{f1}(m)$	$bw_{f2}(m)$	$h(m)$	$H_f(m)$
FCF-Series01	1.027	0.01	1	1	0	1.5	4.1	4.1	0.15	0.15
FCF-Series02	1.027	0.01	1	1	1	1.5	2.25	2.25	0.15	0.15
FCF-Series03	1.027	0.01	1	1	1	1.5	0.75	0.75	0.15	0.15
FCF-Series06	1.027	0.01	1	1	1	1.5	2.25	0	0.15	0.15
FCF-Series08	1.027	0.01	1	0	1	1.5	2.25	2.25	0.15	0.15
FCF-Series10	1.027	0.01	1	2	1	1.5	2.25	2.25	0.15	0.15

$\theta$  Angle between the positional vector and the boundary element

## 1. Introduction

Stage-discharge relationships in open channels are important for a variety of applications such as water resources planning, design of hydraulic structures and hydraulic and hydrologic modeling. Compound open channels both in its artificial and natural types occur frequently in hydrological modeling. When over bank flow occurs, using the Manning equation for discharge computation in compound channels and considering the whole cross section as one unit, this classical formula either overestimate or underestimate the discharge [2]. A compound open-channel is a channel consisting of a main channel flanked by one or two-side floodplains (Fig. 1). In dry seasons or in low flows, normally the main channel conveys these flows. Floodplains are used mainly to pass the major flows during the floods [6].

The flow behavior in compound channels when the level of water surpasses the floodplain level, is very complicated and such complexity is of direct impact on stage-discharge relationship. The momentum transfer of the flow in line with the main flow along the floodplains results in more complexity regarding the flow behavior; thus, discharge estimation in 1D models involve numerous uncertainties. Stage-discharge curves are among the most valuable results in the field of discharge calculation. Reading the stage of an open channel is much simpler than calculating its discharge at some levels; consequently, offering a stage-discharge relationship can reduce the work load required for discharge estimation in various levels to a considerable extent.

Birkhead and James [3] presented a discharge calculation method based on flow resistance relationships which is only efficient for predicting the discharge in the main channel of a compound cross section. An analytical solution to the Navier–Stokes equation was proposed by Shiono and Knight [13] in order to describe the lateral variations of the depth-averaged streamwise velocity,  $U_d$ . This method is often presented as an abbreviation, i.e. *SKM*.

Maghrebi [9] presented a method through which he was able to calculate the discharge in any type of channels including compound channels using only one measurement point; employing this approach in the River Severn, UK, demonstrated the proper accuracy of the method. The method is known as a Single Point Method (*SPM*). The results of the proposed method were in good accuracy when compared with the observed stage-discharge data [10].

Through the concept of energy and 2D models, Yang et al. [14] calculated the discharge in compound channels and compared them to similar models. They also concluded that the obtained discharge values using the DCM (Divided Channel Method) are to some extent high, because this method does not support momentum distribution in compound channels along with the fact that energy loss is also not considered. One of the merits of the proposed method is that it does not require model calibration. A number of other methods are also presented such as, SCM (Single Channel Method), COH (Coherence), and WDCM (Weighted Divided Channel Method) which are focused on compound open channels. The flow cross section in SCM is considered as a single cross section whereas with certain assumptions, the DCM divides the cross section to a number of appropriate sub-cross sections and perform calculations on each sub-cross section separately. Sahu et al. [12] have shown that the SCM and DCM methods do not yield proper and reliable discharge estimations in compound channels. In the COH method

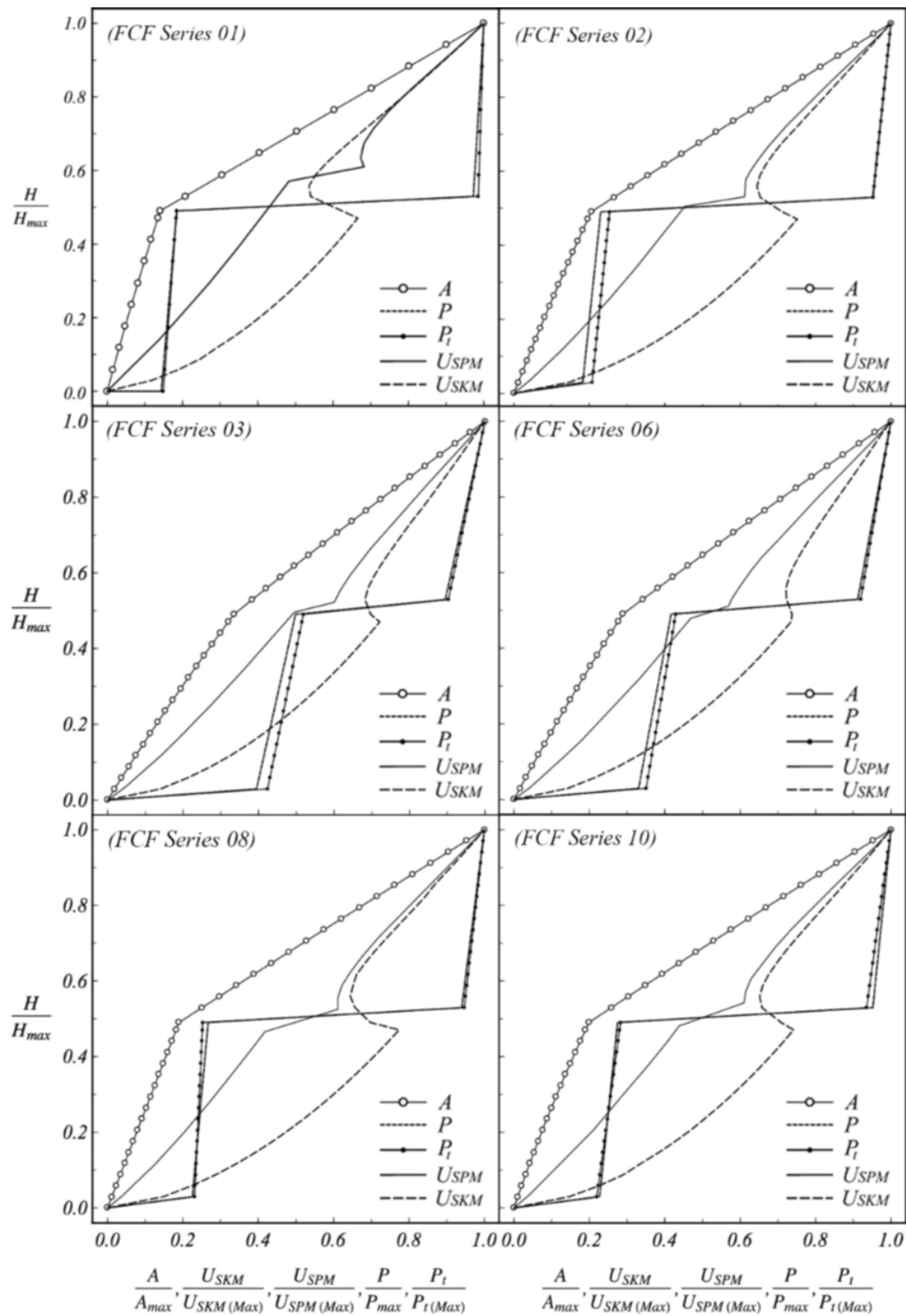


Fig. 3. Relative values of  $A$ ,  $P$ ,  $U_{SPM}$ , and  $U_{SKM}$ , relative to maximum values.

introduced by Ackers [1], a ratio called “Coherence” is used which can be obtained from the calculated discharge ratio in SCM and DCM methods and would justify and modify the internal effect of subsections. Lambert and Myers [8] proposed a weight-based method according to

the DCM called WDCM in which the weight correction factor was introduced. Bousmar and Zech [4] presented a method called the Exchange Discharge Model (EDM) which is based on the other two corrective parameters obtained by a set of relationships.

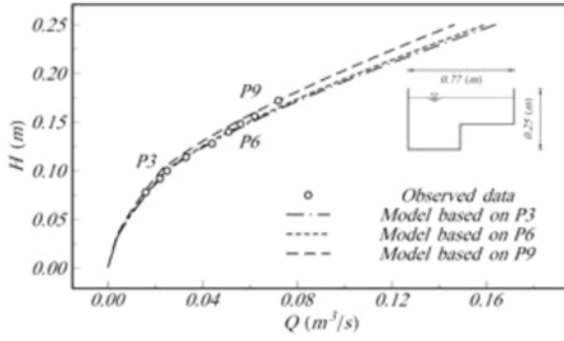


Fig. 4. Stage-discharge curve for a model channel with observed data in main channel.

The purpose of this study is to estimate the stage-discharge curves in six laboratory compound cross sections (FCF Series). In a relationship previously presented by Maghrebi et al. [11], the velocity parameter with a power of unity has played a key role in stage-discharge prediction. In this study, it is attempted to replace the velocity parameter in the proposed stage-discharge relationship with the mean velocity obtained from the Shiono-Knight method and then the results of the existing methods are compared to laboratory obtained values as well as the rating curves directly obtained by the SKM.

## 2. The proposed stage-discharge relationship

Maghrebi et al. [11] stated that the discharge at any level of the channel is a function of the following parameters:

$$Q \propto A^{a_1} P^{a_2} P_t^{a_3} U^{a_4} n^{a_5} S_0^{a_6} \quad (1)$$

where  $Q$  is the discharge,  $A$  is the cross section area,  $P$  is the wet perimeter of the flow section,  $P_t$  is the sum of  $P$  and the width of water surface ( $P_t = P + T$ ),  $U$  is the mean flow velocity,  $n$  is the Manning roughness and  $S_0$  is the gradient of channel floor. It must be noted that  $U$ , i.e. the mean velocity which is placed in Eq. (1) using the SPM method, can be replaced with the velocity obtained from other methods such as SKM. Then, the obtained results are compared with the ones of the proposed relationship. The accuracy of the two models is tested against available experimental results, which are taken from the Flood Channel Facility (FCF) laboratory.

### 2.1. The SPM method

Maghrebi [9] presented a model that is capable of calculating the dimensionless velocity contours in the cross sections of open channels and natural rivers with irregular roughness distributions. This method

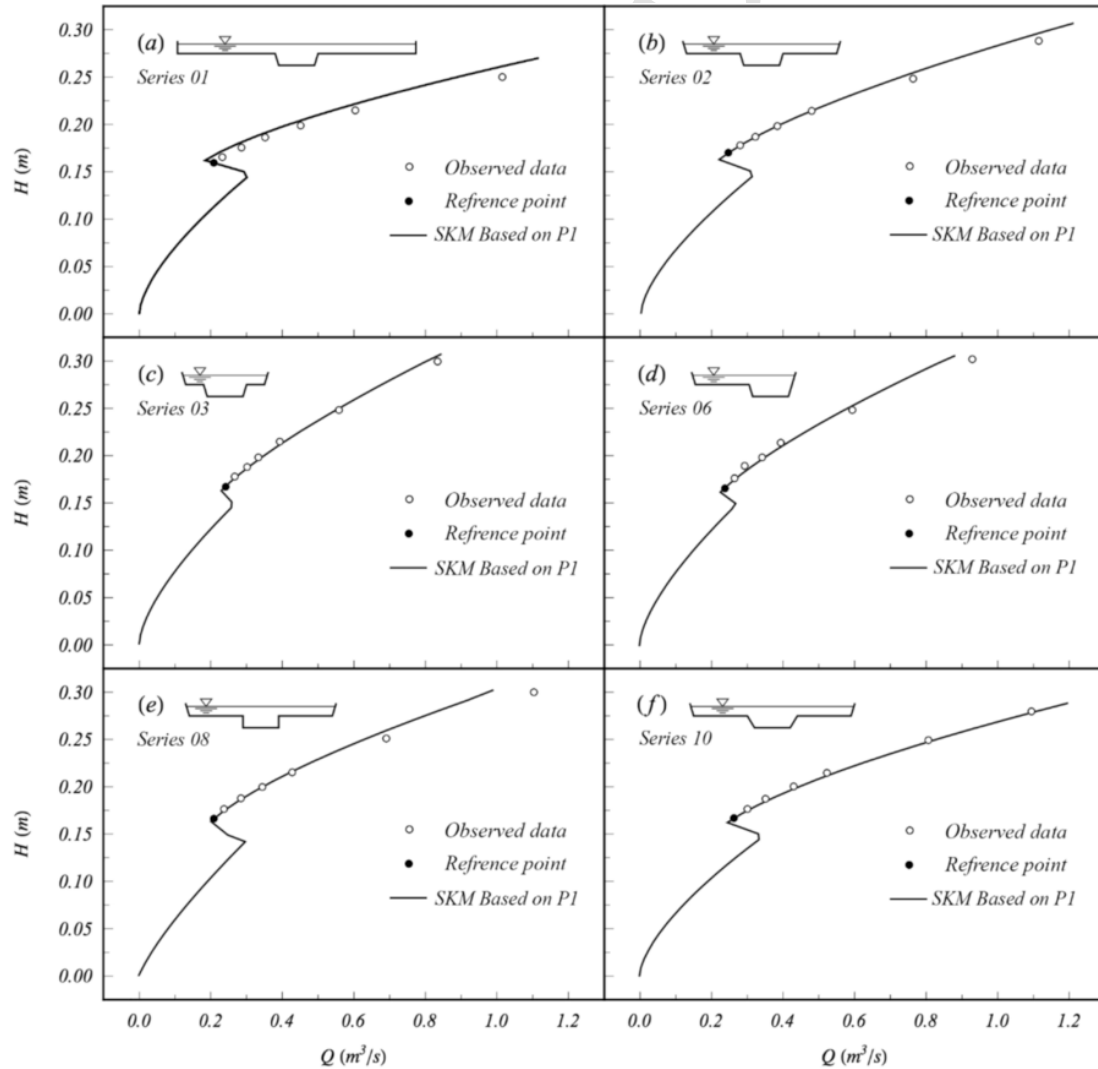


Fig. 5. The resulting stage-discharge from speed values of Shiono-Knight method in the recommended stage-discharge relationship using P1 referenced point.

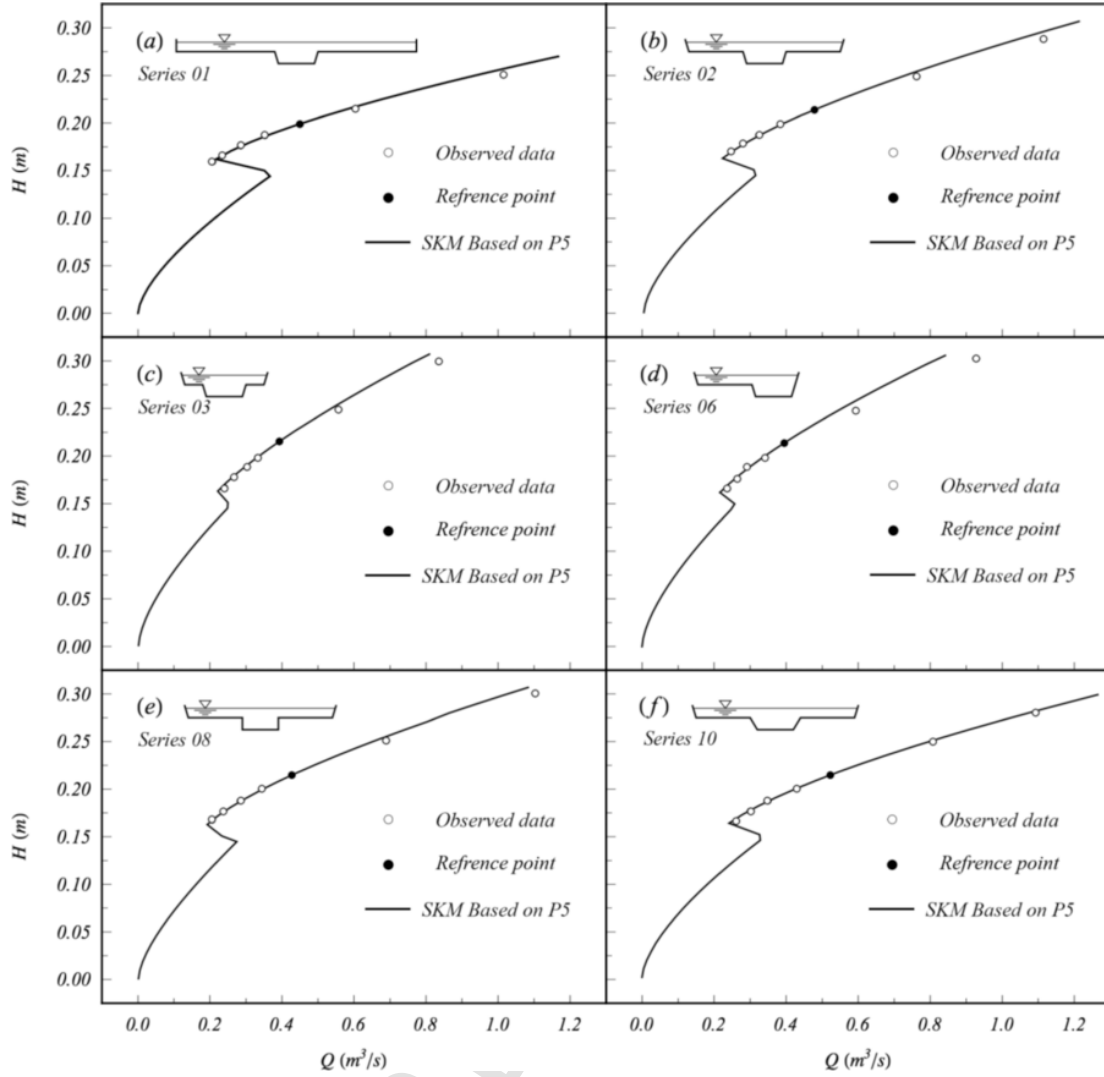


Fig. 6. The resulting stage-discharge from speed values of Shiono-Knight method in the recommended stage-discharge relationship using P5 referenced point.

is based on the idea that each element of the wetted perimeter affects the velocity of an arbitrary point on the flow cross section. Then, through integration, the impact of all rigid boundaries on the considered point in flow cross section can be calculated. Fig. 2 illustrates a flow cross section covered by triangle meshes and the center of each is where the boundary impacts are applied. As can be seen in Fig. 2, the wetted perimeter of the channel is divided into a set of limited elements called  $ds$ . The impact of  $ds$  on arbitrary streamwise velocity in the space with the coordinates of  $(y, z)$  is shown with  $du_{SPM}$  which is calculated using the following vector equation:

$$du_{SPM} = f(r) \times c_1 ds \quad (2)$$

Consequently, the overall boundary impact on the velocity of each element can be integrated as:

$$u_{SPM}(y, z) = \int_{boundary} c_1 f(r) \sin \theta ds \quad (3)$$

where  $c_1$  is a constant which depends on the boundary shear stress, turbulent intensity and relative roughness,  $\theta$  is the angle between the position vector  $r$  and the boundary element  $ds$ , and  $f(r)$  is the dominant velocity function. Chen [5] derived the power velocity distribution from the partial differential equation that represents the steady uni-

form turbulent flow in a pipe or a channel, as follows:

$$\frac{u}{u_*} = c \left( \frac{y}{k_s} \right)^{1/m} \quad (4)$$

where  $u$  is the streamwise time-averaged local velocity in distance  $y$  from the wall,  $u_*$  is the boundary shear velocity ( $u_* = \sqrt{\tau_0/\rho}$ ) in which  $\tau_0$  is the boundary shear stress and  $\rho$  is mass density of liquid. Furthermore,  $k_s$  is the Nikuradse sand equivalent roughness,  $c$  is a coefficient dependent on the Reynolds number and/or roughness, and power  $m$  varies usually between 4 and 12, depending on the extent of turbulence [15]. Eq. (4) can be rewritten as the following:

$$f(r) = u_* \left( c_2 r^{\frac{1}{m}} \right) \quad (5)$$

where  $c_2$  is related to the roughness and nature of flow. Replacing  $f(r)$  from Eq. (5) into Eq. (3), the local point velocity at an arbitrary position in the channel section with coordinates of  $(y, z)$  in Fig. 2,  $u_{SPM}(y, z)$  is obtained as:

$$u_{SPM}(y, z) = \int_{boundary} c_1 c_2 u_* \left( r^{1/m} \right) \sin \theta ds \quad (6)$$

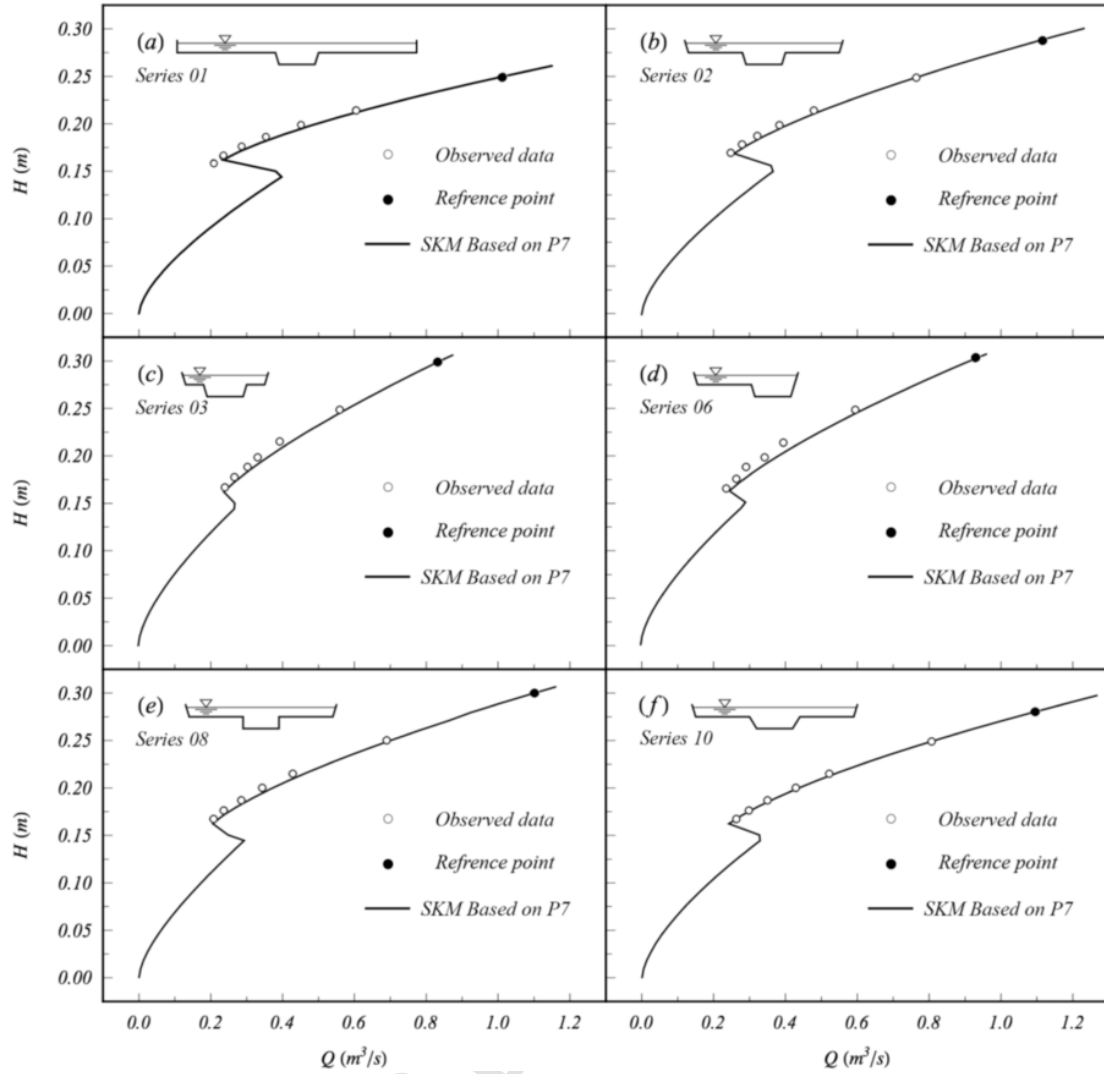


Fig. 7. The resulting stage-discharge from speed values of Shiono-Knight method in the recommended stage-discharge relationship using P7 referenced point.

By considering the value of  $c_1 c_2 u_s$  equal to  $c_3$  and  $m = 7$ , Eq. (6) can be written as:

$$u_{SPM}(y, z) = \int_{boundary} c_3 \left( r^{\frac{1}{7}} \right) \sin \theta ds \quad (7)$$

Finally, the mean value of  $u_{SPM}$  over the whole cross sectional area which is shown by  $U_{SPM}$  can be obtained as follows:

$$U_{SPM} = \frac{\int_A u_{SPM}(y, z) dA}{A} \quad (8)$$

where  $dA$  is the area of each triangular mesh and  $A$  is the total area of the whole flow section. It should be noted that the value of  $U_{SPM}$  is not necessarily the mean velocity of the cross section. It can be assumed that a multiplier like  $\eta$  exists that can convert  $U_{SPM}$  into the real mean velocity  $V$ , and such a factor can be obtained through laboratory or field works. In other words, we have:

$$V = \eta U_{SPM} = \frac{\int_A \eta u_{SPM}(y, z) dA}{A} \quad (9)$$

In Eq. (9) it is assumed that  $\eta$  takes a fixed value for the whole cross section.

## 2.2. Depth-integrated formulation

Shiono and Knight [13] presented a relationship to estimate the depth-averaged velocity  $U_d$  and shear stress  $\tau_0$  on the bed of prismatic channels. To extract such an equation, the channel cross-section is usually divided into a number of discrete panels or regions. Then, an analytical approach based on depth-averaged and integration from the momentum equation (Navier-Stokes equations) along the flow depth while assuming a uniform, steady and incompressible flow, is performed. Expanding the momentum equation within the desired control volume which is a vertical slice of the flow cross section extends from the bed to the water surface, they finally reached the following equation:

$$\rho g S_0 H - \left( 1 + \frac{1}{s^2} \right)^{1/2} \frac{\rho f}{8} U_d^2 + \frac{\partial}{\partial z} \left\{ \rho \lambda H^2 \left( \frac{f}{8} \right)^{1/2} U_d \frac{\partial U_d}{\partial z} \right\} = \frac{\partial H(\rho u w)_d}{\partial z} \quad (10)$$

where  $\rho$  is the fluid density,  $g$  is the gravitational acceleration,  $S_0$  is the floor gradient of the channel which is assumed as equal to the energy line gradient in a uniform and steady flow,  $H$  is the water depth,  $s$  is the side slope,  $f$  is the local friction factor ( $f = 8\tau_b/\rho U_d^2$ ),  $\lambda$  is dimen-

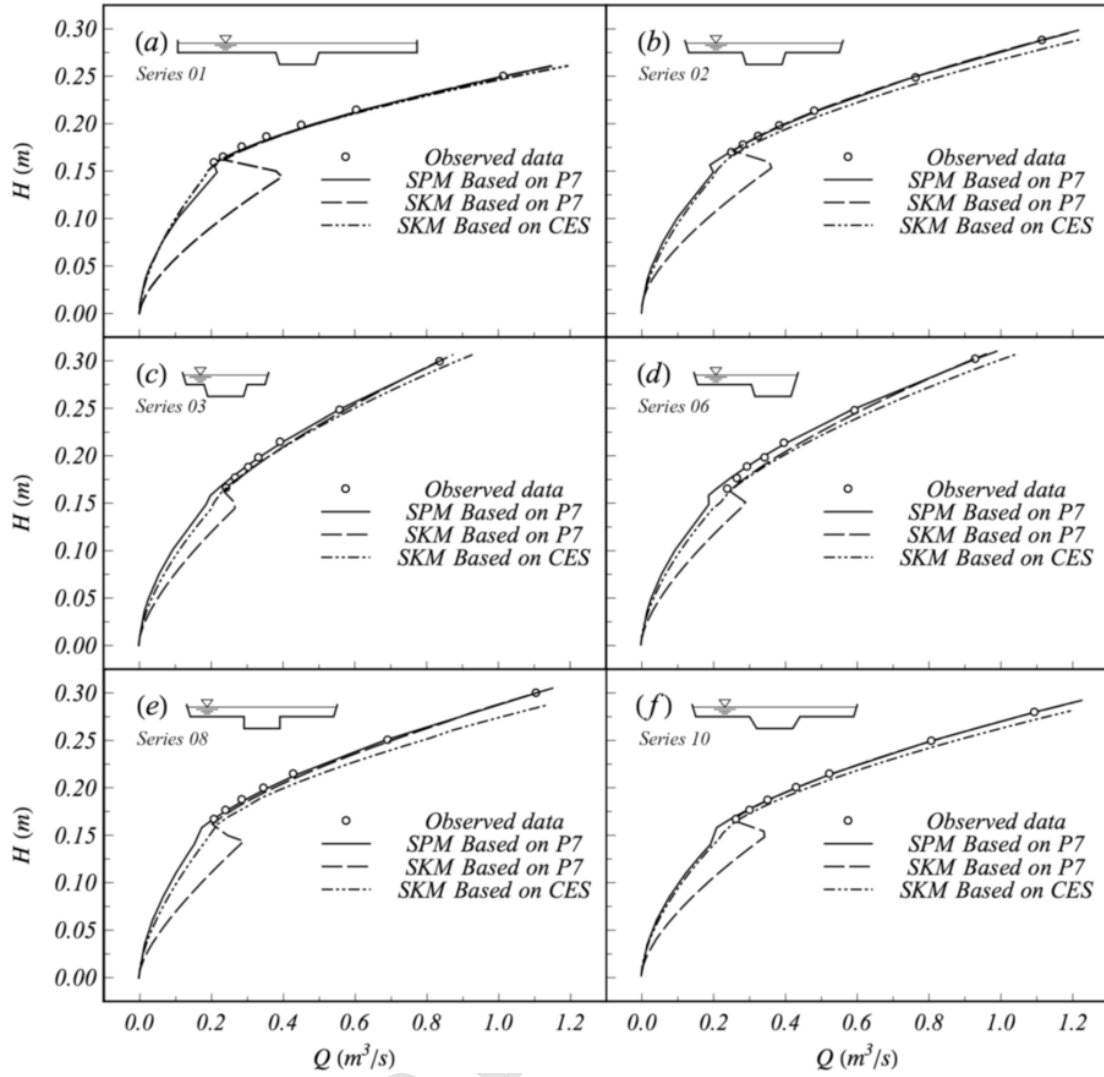


Fig. 8. Comparing stage-discharge curves obtained from various methods.

sionless eddy viscosity, and  $u$ ,  $v$ , and  $w$  are the velocity components along  $x$ ,  $y$ , and  $z$ , respectively. Solving this equation gives:

$$U_d = \sqrt{C_1 e^{r_1 z} + C_2 e^{r_2 z} + A_0} \quad (11)$$

where,

$$r_1 = \sqrt{\frac{2}{\lambda H_0^2} \left( \frac{f}{8} \right)^{1/2}} \quad (12)$$

$$r_2 = -\sqrt{\frac{2}{\lambda H_0^2} \left( \frac{f}{8} \right)^{1/2}} \quad (13)$$

$$A_0 = \frac{\rho g S_0 H_0 - \Gamma}{\rho f_8} \quad (14)$$

If the effects of secondary flow in Eq. (14) is disregarded, then  $\Gamma = 0$ . All of the parameters of Eq. (1) except  $U$  and  $Q$  can be calculated using geometrical specifications of the cross section at an arbitrary water level. According to Eq. (1), the most general form of the

stage-discharge relationship is as follow:

$$\frac{Q_e}{Q_r} = \left( \frac{A_e}{A_r} \right)^{a_1} \left( \frac{P_e}{P_r} \right)^{a_2} \left( \frac{(P_t)_e}{(P_t)_r} \right)^{a_3} \left( \frac{U_e}{U_r} \right)^{a_4} \left( \frac{n_e}{n_r} \right)^{a_5} \quad (15)$$

where the subscripts  $r$  and  $e$  refer to the referenced and estimated values, respectively. The continuity equation i.e.  $Q=AU$  is one of the most fundamental equations. In this equation, velocity has a power of 1. Moreover, since  $U$  is the velocity, we do not expect any other values for  $a_4$  except 1. In addition, we have  $a_5 = -1$  due to the inverse relationship between discharge and roughness as the Manning formulae. Therefore,  $a_1$ ,  $a_2$  and  $a_3$  are remained to be evaluated. The multivariate Newton's method is applied to the difference between the observed and estimated data to optimize the exponent values of the governing parameters. The first step is collection of some of the data taken from the observational and theoretical rating curves for different hydraulic cross-sections. The stage-discharge values are calculated for the rectangular and compound sections at different water levels using the Manning formula. Maghrebi et al. [11] presented the most accurate relationship which is associated with the least values of *NRMSE*. This can be achieved by the minimization of the summation of the statistical measures *NRMSEs* for both rectangular and compound channels, which

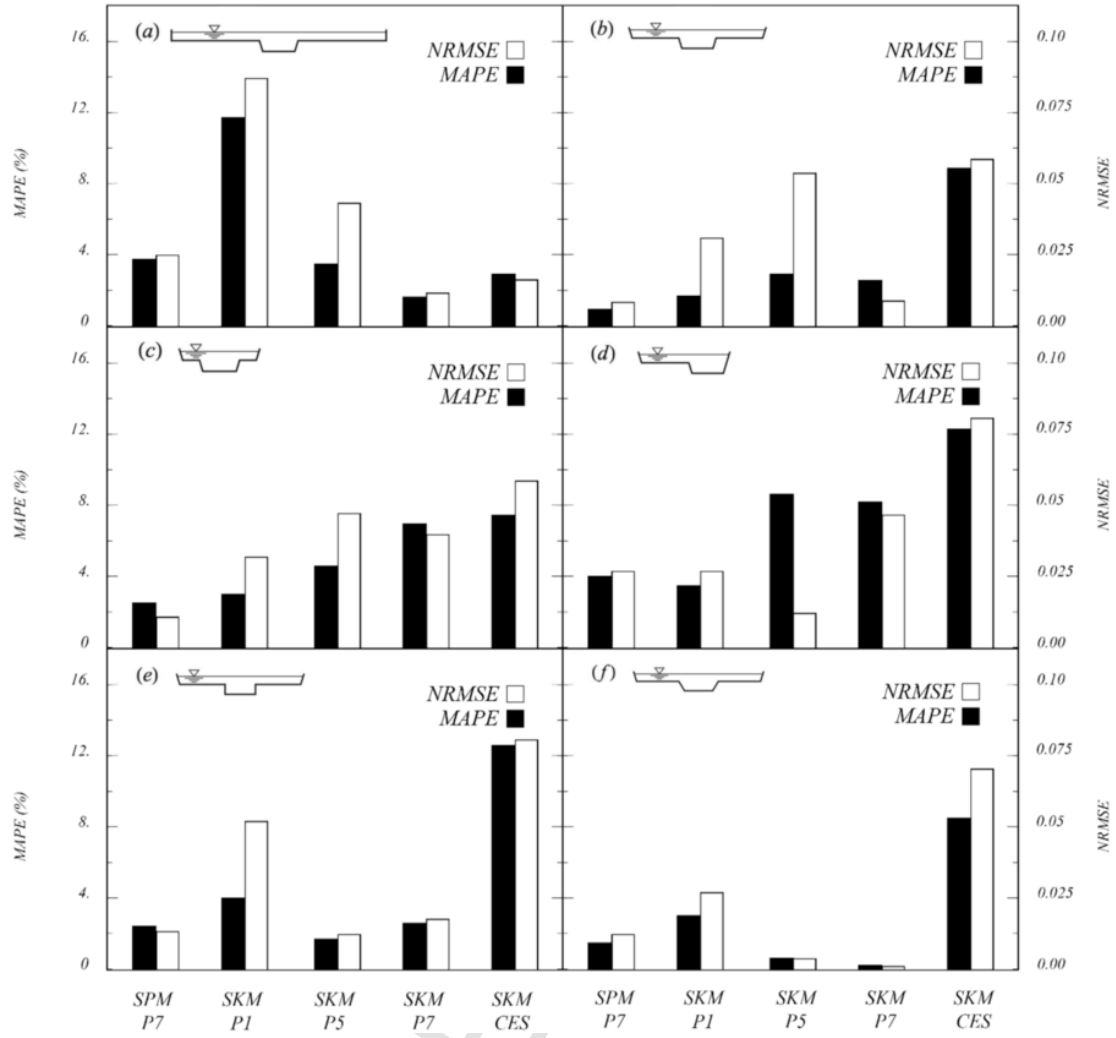


Fig. 9. Error analysis of various methods.

is defined in the following form:

$$\begin{aligned}
 NRMSE &= \frac{\frac{1}{N} \sum_{i=1}^N \sqrt{\left[ \left( Q_r \left( \frac{A_e}{A_r} \right)^a \left( \frac{P_e}{P_r} \right)^b \left( \frac{(P_t)_e}{(P_t)_r} \right)^c \left( \frac{U_e}{U_r} \right)^1 \left( \frac{n_e}{n_r} \right)^{-1} \right)_i - (Q_r)_i \right]^2}}{(Q_r)_{\max} - (Q_r)_{\min}} \\
 &= \frac{RMSE}{(Q_r)_{\max} - (Q_r)_{\min}}
 \end{aligned}$$

After the optimization, they suggested their final relationship as following:

$$Q_e = Q_r \left( \frac{A_e}{A_r} \right)^{0.972} \left( \frac{P_e}{P_r} \right)^{-1.268} \left( \frac{(P_t)_e}{(P_t)_r} \right)^{0.832} \left( \frac{U_e}{U_r} \right)^1 \left( \frac{n_e}{n_r} \right)^{-1} \quad (17)$$

where the subscripts  $r$  and  $e$  refer to the referenced and estimated values, respectively. In the present study all of the selected compound channels have uniform roughness both in the main channel and floodplain. So, the last item on the right hand side of Eq. (17) is equal to 1. Eq. (17) is considered as a universal relationship which can be applied to any natural or artificial compound channels with uniform roughness distribution along the wetted perimeter. It should be mentioned that the parameter  $U$  in Eq. (17), plays the role of mean cross sectional ve-

locity. This parameter not only can be obtained from the SPM, but also from any other techniques such the SKM.

Actually, in order to estimate the discharge by the use of Eq. (17), all of the parameters including  $A$  the cross section area,  $P$  the wetted perimeter of the flow section,  $P_t$  the sum of  $P$  and the width of water surface ( $P_t = P + T$ ),  $U$  the mean flow velocity and  $n$  the Manning roughness are needed to be calculated at all water levels in the range of the required rating curve. In other words, for an arbitrary open channel we have:

$$\begin{aligned}
 A &= A(H), P \\
 &= P(H), P_t \\
 &= P_t(H), n \\
 &= n(H), U \\
 &= U(H)
 \end{aligned}$$

All of the geometric parameters of the channel sections will take fixed values at a certain water level. However, the last item i.e. mean cross sectional velocity  $U$  can be calculated by a number of methods namely SKM, SPM and ... and usually they are different from each other especially when we are engaged with compound channels. The accuracy of the estimated mean cross sectional velocity affects the accuracy of the estimated discharge and the whole rating curve. Since the discharge at a known reference level is known, then the whole right hand side of Eq. (17) will be ready. Consequently, the discharge at the



required level can be estimated easily. In summary the following steps should be taken in order to obtain the stage-discharge curves:

1. The observed discharge is known at a referenced water level  $H_r$ .
2. The mean cross sectional velocity  $U$  can be measured or estimated at the reference level and it can be estimated at other levels.
3. The geometric parameters such as  $A$ ,  $P$ ,  $P_t$  and also  $n$  are known both for the reference and the estimated levels.
4. Replacing the estimated values in Eq. (17), one can easily calculate the estimated discharge.
5. By adjoining the calculated values of  $(Q_e, H_e)$  at different levels, the stage-discharge curve will be obtained.

### 3. Applying the stage-discharge relationship

In this study, as an application, six compound channels selected from the experimental works of Knight [7] on *FCF* models, are implemented. The details are given in Table 1. The subscripts  $f$  and  $c$  refer to the floodplain and main channel, respectively;  $s$  is the side slope,  $bw$  is the base width,  $H$  is the total water depth,  $h$  is the water depth of the main channel and  $H_f$  is the depth of the floodplain. These specifications are shown in Fig. 1. Note that the water level of all the observational or referenced sections are above the floodplain.

*CES* is a software which is developed to calculate the depth-averaged velocity  $U_d$  at any lateral position of the flow cross section based on the *SKM*. Then the mean cross sectional velocity which is required for the proposed method can be calculated. The calculated mean velocity values in a number of water levels are implemented in Eq. (17) and the produced stage-discharge curves through *SPM* and *SKM* velocity values will be compared with the observational values of the *FCF* models.

In the next step, the mean velocity is calculated using the *SKM* and other required hydraulic specifications as appear in Eq. (17) at 35 selected water levels throughout the channel cross sections; the specifications of sections are applied to *CES* software and the mean velocity values are extracted. The results are presented in Fig. 3. In this figure, it can be seen that the presented ratios on horizontal axes are the required parameters for Eq. (17). By selecting any arbitrary level from vertical axes in Fig. 3, the required ratios for calculating discharge can be obtained from Eq. (17) and as a result, the corresponding discharge can be estimated easily.

As can be seen, the velocity values extracted using the *SPM* are compared to the values extracted from the *SKM*. The differences between the velocity values of the *SPM* and *SKM* are subtle at all water levels in the floodplains especially in *FCF*-series S2, S8, and S10. As seen in Fig. 3, the velocity values of the *SKM* is larger than the *SPM*. Therefore, it can be concluded that the discharge values of the *SKM* to be larger than those of the *SPM*.

To examine the model behavior in prediction of the discharge when the stage level is below the floodplain bed level, a compound channel with one sided floodplain is used. The designed channel is an asymmetric rectangular compound channel with a width of 0.77 m, height of 0.25 m. The floodplain level is 0.104 m above the main channel bed level with a longitudinal bottom bed slope of 0.005. The roughness distribution along the wetted perimeter is uniform. Discharges are measured at nine water levels by electromagnetic flow meter with accuracy of  $\pm 0.2\%$  [16]. Four water levels are located below the flood plain bed level including P3. The rating curves for three water levels as well as the experimental observations are plotted in Fig. 4. The predicted stage-discharge curves for P3 which is related to lower water level as well as two other stages corresponding to P6 and P9 are plotted in Fig. 4. As can be seen, they are very close to each other. More specifically speaking, the two curves of P3 and P6 are much closer to each other.

The observed data are mainly coincided with the rating curves based on P3 and P6.

Now, the velocity values of the *SKM* and geometrical specifications of cross sections are replaced in Eq. (17). The resulting stage-discharge curves based on three referenced levels of P1, P5 and P7 are shown in Figs. 5–7, respectively.

Generally, the predicted rating curves are close to the observed data around the reference section. However, as going away from the reference section, the difference between the observed and predicted data will be increased. When P5 and P7 are selected as the reference levels, the resulting curves will be in better agreement with the observed data. Figs. 5(f)–7(f) present the best agreement between the stage-discharge curves and laboratory observations, however, the most considerable disagreement can be observed in Figs. 5(a)–7(a).

In Fig. 8(a–f), the rating curves calculated based on the *SPM* on P7, *SKM* on P7, and *SKM* based on *CES* for *FCF* series 01, 02, 03, 06, 08 and 10 are plotted, respectively. The rating curves are scattered from each other in low water level below the floodplain stage. However, when the water level rises to the over the floodplain level, a good agreement between the rating curves calculated based on different methods can be observed.

#### 3.1. Error analysis

In order to have more clarification on the performance of the *SKM* and *SPM* models, some of the statistical measures including the mean absolute percentage error (*MAPE*) and the normalized root mean square error (*NRMSE*) are calculated based on the estimated discharge  $Q_e$  and the observed data  $Q_o$  as follows:

$$MAPE(\%) = \frac{100}{N} \sum_{i=1}^N \left| \frac{(Q_o)_i - (Q_e)_i}{(Q_o)_i} \right| \quad (18)$$

$$NRMSE = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N [(Q_o)_i - (Q_e)_i]^2}}{(Q_o)_{\max} - (Q_o)_{\min}} \quad (19)$$

The results of error analysis are presented in Fig. 9. It is obvious that the values of both *MAPE* and *NRMSE* in the upper referenced levels such as P5 and P7 are much lower than that of P1 which is located at the lower part of the flow section above the floodplain stage. The results in Fig. 9 can be considered from three different points of view. First, it can be observed that the results of *SKM* for Fig. 9(a), (b), (e) and (f) for the upper reference sections are associated with lower errors. Second, the results of *SPM* method at P7 can be compared with the corresponding level of the *SKM* method. Implementation of the *SKM* mean velocity in Eq. (17), except those presented in Fig. 9(a), (e) and (f) which are almost in the same degree of error. Third, the calculated stage-discharge curves based on the *SKM* in all cases are associated with larger errors when compared with the results obtained by implementation of the *SPM* method. Based on the whole water levels, the maximum values of *MAPE* and *NRMSE* do not exceed 12% and 0.85, respectively.

### 4. Conclusion

Estimation of stage-discharge curves in compound channels is among the most important information for water resources management and design of hydraulic structures. A number of methods have been proposed for calculating the discharge, which can be considered as a base for the establishment of the stage-discharge curves. Applying the conventional means of discharge estimation in compound channels usually require section division into a number of subsections and in a number of other cases, performing calibration is one of the necessities

of employing them. The velocity parameter appearing in the proposed relationship (Eq. (17)) is actually the mean cross sectional velocity. Since Eq. (17) is presented in the form of a ratio, therefore the exact value of the mean velocity is not required and having a rough estimation of the mean velocity will be adequate. In this study, the calculated mean velocity values using the *SKM* and *SPM* are replaced in this relationship. The results of stage-discharge curves obtained by these models are compared against the experimental data. It is found when the referenced section is adopted from higher water levels such as *P7* in the current research, the proposed method shows a higher accuracy for estimating the rating curve. The statistical measures of MAPE and NRMSE, calculated based on the estimated results by direct application of the *SKM* produced by the *CES*, application of mean velocity  $U$  extracted from the depth-averaged velocity  $U_d$  of the *SKM* and the proposed model in comparison to the observed data taken from *FCF* laboratory for six compound cross sections, are within (10%, 0.61), (4.0%, 0.23) and (3.1%, 0.023), respectively. This shows that application of the extracted mean velocity by the *SKM* in the proposed relationship in comparison to the rating curves directly obtained by the *CES* leads to much better agreement with the observed data.

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