Conformable Fractional Order Sliding Mode Control for a Class of Fractional Order Chaotic Systems

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A B S T R A C T

In this paper, a novel conformable fractional order (FO) sliding mode control technique is studied for a class of FO chaotic systems in the presence of uncertainties and disturbances. First, a novel FO nonlinear surface based on conformable FO calculus is proposed to design the FO sliding mode controller. Then, asymptotic stability of the controller is derived by means of the Lyapunov direct method via conformable FO operators. The stability analysis is performed in the sliding and reaching phase. In addition, the realization of reaching phase is guaranteed in finite time and the reaching time is calculated analytically. The proposed control approach has some superiorities. Reduction of the chattering phenomenon, high robustness against the uncertainty and external disturbance, and fast convergence speed are the main advantages of the proposed control scheme. Moreover, it has simple calculations because of using conformable FO operators in the control design. The numerical simulations verify the efficiency of the proposed controller.

Article Info

I. INTRODUCTION

Chaos phenomenon occurs in different kinds of real systems with prominent attributes, including unpredictable behaviour and dependency on initial values. Therefore, chaos control has become an interesting topic in various fields of science. So far, several control techniques have been reported in the literature in order to control the chaotic systems [1-5].

In recent decades, fractional order (FO) calculus is an interesting and powerful instrument for modelling and controlling of real phenomena. FO Sliding mode control (SMC) is a well-known robust control technique for controlling uncertain systems in the presence of disturbances [6-11]. In [6], a fractional terminal SMC was represented to control a class of nonlinear systems with uncertainty. In [7], a single link flexible manipulator was controlled via a FO SMC. In [8], a FO SMC based on a nonlinear disturbance observer was developed for a class of FO systems in the presence of

mismatched disturbances [9]. In [10], an adaptive SMC was suggested to control FO chaotic systems considering uncertainties and disturbances. In [11], a FO SMC was examined for the output tracking of the desired signal.

However, FO SMC has been successfully applied in a wide range of engineering applications, it suffers from an inevitable problem, namely chattering phenomenon, leading to increasing the control effort and triggering the high-frequency dynamics of the system. Several research works have been dedicated to lessen the effects of the chattering, such as [12,13]. Another important topic in designing the SMC is the convergence speed and reaching phase in the finite time [14,15]. In [14], an adaptive terminal SMC was developed to control a power system. In [15], an adaptive nonlinear SMC scheme was proposed for a class of fourth-order systems.

Up to now, various definitions of FO derivative were presented. Among them, the Riemann–Liouville, Caputo and Grunwald-Letnikov are the most well-known definitions [16,17]. A significant defect of FO operators is the

complexity in calculations. In recent years, a new definition for the FO derivative was introduced, namely conformable FO derivative. The main advantage of this FO definition is simplicity in calculations. In addition, it has some properties of classic operators which other fractional order derivatives do not satisfy them, such as chain rule, product, and quotient [18]. In [19], the superiority of conformable definition was shown via a comparison between the Grünwald-Letnikov and conformable derivatives. In [20-21], some properties based on conformable derivative were studied. In [22], the fractional Newtonian mechanics was addressed using the conformable fractional calculus. The conformable transform method and its applications for conformable fractional differential equations were presented in [23]. In [24], a conformable fractional differential equation was discussed having three-point initial and boundary conditions. In [25], the conformable FO equations were solved using numerical method. Furthermore, chaotic behavior of the conformable FO Lorenz system as an example was examined.

Stability analysis of nonlinear systems is necessary to design a controller. Several works have investigated stability analysis of nonlinear systems by means of fractional calculus [26-30]. In [26], fractional generalization of concept of stability was considered. In [27], a definition for Mittag-Leffler stability and fractional Lyapunov direct method were presented. In [28], stability analysis of FO nonlinear systems was derived using the Lyapunov direct method with Mittag-Leffler stability. In [29], stability of fractional differential systems based on the conformable fractional derivatives was studied. However, there are very few papers considering modelling of the nonlinear systems with conformal FO definition [29,30]. Therefore, application of the conformable FO operators in the design of FO controller is an open area. Accordingly, for the first time, in this paper, a FO sliding mode control is designed for a class of conformable fractional order chaotic system using the conformable fractional derivative and the superiority of the proposed controller is shown. Having these facts in mind, the main contributions of this paper in comparison with previous researches are as follows. A novel FO manifold using conformable FO operators is proposed to control chaotic systems in the presence of uncertainties and disturbances. The conformable FO operator as an interesting definition is applied in designing of the FO sliding mode controller. Based on conformable FO operators, the stability of the controller is derived using the Lyapunov direct method. The main advantage of the proposed control method is fast convergence speed with together less chattering and complexity in calculations.

The paper is structures as follows: Some mathematical preliminaries are presented in Section 2. System description and conformable FO sliding mode controller design methodology are presented in section 3. Section 4 shows the simulation results. Finally, conclusions are given in section 5.

II. MATHEMATICAL PRELIMINARIES

In this section, some definitions and theorems adopted in this paper are given.

Definition 1 [16]. The β th-order fractional integration of function f (t) is given by

$$D_t^{-\beta} f(t) = \frac{1}{\Gamma(\beta)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\beta}} d\tau \tag{1}$$

where Γ (.) is the Gamma function.

Definition 2 [18].   Given a function f: $[0, \infty) \to$ R. Then, the ''conformable derivative'' of f(t) with order β is defined as

$$T^{\beta} f(t) = \lim_{\lambda \to 0} \frac{f(t + \lambda t^{1-\beta}) - f(t)}{\lambda} \tag{2}$$

For all t>0, $\beta \in (0,1)$. If f is differentiable, then

$$T^{\beta}(f) = t^{1-\beta} \frac{df}{dt}(t)$$

Definition 3 [18]. Conformable integral is as

$$T_a^{-\beta} f(t) = I_1^a(t^{\beta-1} f) = \int_a^t \frac{f(x)}{x^{1-\beta}} dx \tag{3}$$

where $\beta \in (0,1)$.

Lemma 1 [20]. Let $g : (a,b) \to R$ be differentiable and $\gamma \in (0,1]$. Then for all $t > a$, we have

$$I_\gamma^a T_\gamma^a(g)(t) = g(t) - g(0) \tag{4}$$

Consider the conformable fractional dynamic system as

$$_{t_0}T_t^\alpha x(t) = f(t, x(t)) \tag{5}$$

where f(.) describes dynamics of system. The above system is conformable stable, if its equilibrium is stable.

Theorem 1 [18]. Consider $f(t)$ be a continuous function such that $_t I_0^\alpha f(t)$ exists. Then,

$$_t T_\alpha(I_0^\alpha f)(t) = f(t), \quad for \ t \geq 0 \tag{6}$$

where $\alpha \in (0,1]$.

Definition 4 [8]. The fractional Laplace transform of order α from t_0 of   f (t) is defined as

$$L_\alpha^{t_0}\{f(t)\}(s) = \int_{t_0}^{\infty} e^{-s\frac{(t-t_0)^\alpha}{\alpha}} f(t)(t-t_0)^{\alpha-1} dt \tag{7}$$

where $\alpha \in (0,1]$ and $f:[t_0,\infty) \to R$.

Lemma 2 [20].   For $f : R^+ \to R$, we have

$$L_\alpha^{t_0}\{f(t)\}(s) = L\{f(t_0 + (\alpha t)^{\frac{1}{\alpha}})\}(s) \tag{8}$$

where $L\{g(t)\}(s) = \int_0^\infty e^{-st} g(t)dt$.

Theorem 2 [20]. Consider $a \in R$ and $f(t)$ be differentiable real valued function. Then,

$$L_\alpha^a \{T_\alpha f(t)\}(s) = sF_\alpha(s) - f(a) \tag{9}$$

In this paper, the notations T^α and $T^{-\alpha}$ denote the conformable FO derivative and integral, respectively.

Theorem 3. Let $V(t, x(t))$ be a Lyapunov function. If $V(t, x(t))$ satisfies the next condition, the equilibrium point x=0 is conformable stable.

$$\mu_1 \|x\|^m \le V(t, x(t)) \le \mu_2 \|x\|^{mn}, \tag{10}$$

$$T_t^\alpha V(t, x(t)) \le -\mu_3 \|x\|^{mn}, \tag{11}$$

where $t \ge 0, \alpha \in (0,1), \mu_1, \mu_2, \mu_3, m$ and n are the arbitrary positive constants.

Proof. According to [30], Eqs. (10) and (11) results

$$T^\alpha V(t, x(t)) \le -\mu_3 \mu_2^{-1} V(t, x(t)). \tag{12}$$

Applying the fractional Laplace transform, the following equation is derived.

$$sV(s) - V(0) \le -\mu_3 \mu_2^{-1} V(s) \tag{13}$$

So , we have

$$V(s) \le \frac{V(0)}{s + \dfrac{\mu_3}{\mu_2}} \tag{14}$$

where $V(0) = V(0, X(0)) \ge 0$.

Applying inverse fractional Laplace transform to (14),

is obtained. Based on (10), (11), $\|x\| \le [\dfrac{V(0)}{\mu_1} e^{(\frac{-\frac{\mu_3}{\mu_2}(t-t_0)^\alpha}{\alpha})}]^{\frac{1}{\alpha}}$

is achieved. If $k = \dfrac{V(0)}{\mu_1} \ge 0$, then we have

$\|x(t)\| \le [ke^{\frac{-\frac{\mu_3}{\mu_2}(t-t_0)^\alpha}{\alpha}}]^{\frac{1}{\alpha}}$, which indicates that the system (5) is conformable stable. This complete the proof.

III. MAIN RESULTS

In this paper, we consider a class of FO chaotic systems as

$$T^\alpha x = f_1(t) + \Delta f(t) + d(t) + u(t) \tag{15}$$

$$T^\alpha y = f_2(t)$$

$$T^\alpha z = f_3(t)$$

where $f_1(.)$, $f_2(.)$ and $f_3(.)$ are the nonlinear functions which are show dynamics of system . Also, $|\Delta f(t)| \le \Delta$ and $|d(t)| \le D$ denote the uncertainty and disturbances, and $u(t)$ is the control signal.

In the following, a novel conformable FO sliding manifold is proposed as

$$s = a_1 x + a_2 y + a_3 z + p_1 T^{-\alpha} y + T^{-\alpha}[b_1 \tanh(c_1 x) + b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) + b_3 \tanh(c_3 z)] \tag{16}$$

where $\alpha, \beta_1, \beta_2, \gamma \in (0,1)$.

Applying conformable FO derivative in both sides of (16) helps to reach the equal control law in sliding mode strategy in the following form.

$$T^\alpha s = a_1 f_1 + a_1 u + a_2 f_2 + a_3 f_3 + p_1 y + b_1 \tanh(c_1 x) + b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) + b_3 \tanh(c_3 z) = 0 \tag{17}$$

The equal control law is expressed as

$$u_{eq}(t) = -\frac{1}{a_1}[a_1 f_1 + a_2 f_2 + a_3 f_3 + p_1 y + b_1 \tanh(c_1 x) + b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) + b_3 \tanh(c_3 z)] \tag{18}$$

and the switching law is as

$$u_{sw} = -k \tanh(n s) \tag{19}$$

So, the proposed controller is obtained as

$$u(t) = -\frac{1}{a_1}[a_1 f_1 + a_2 f_2 + a_3 f_3 + p_1 y + b_1 \tanh(c_1 x) + b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) + b_3 \tanh(c_3 z)] - k \tanh(ns) \tag{20}$$

Theorem 4. The state trajectories of the controlled system using the proposed FO SMC converges to the sliding surface s=0 in the finite time.

Proof: Suppose the following Lyapunov candidate.

$$V(x, y, z) = \frac{1}{2}s^2 \tag{21}$$

Conformable derivative of the candidate function is as

$$\begin{aligned} T^\alpha V(x, y, z) = T^\alpha(s).s &= a_1 f_1 + a_1 \Delta f + a_1 d + a_1 u \\ &+ a_2 f_2 + a_3 f_3 + p_1 y + b_1 \tanh(c_1 x) \\ &+ b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) \\ &+ b_3 \tanh(c_3 z)].s \le -\lambda.|s| \end{aligned} \tag{22}$$

Substituting the control signal into (22) leads to

$$\begin{aligned} T^\alpha V(x, y, z) = T^\alpha(s).s &= a_1 f_1 + a_2 f_2 + a_3 f_3 + a_1 \Delta f + a_1 d \\ &+ p_1 y - a_1 \frac{1}{a_1}[a_1 f_1 + a_2 f_2 + a_3 f_3 + p_1 y \\ &+ b_1 \tanh(c_1 x) + b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) \\ &+ b_3 \tanh(c_3 z)] - a_1 k \tanh(ns)] + b_1 \tanh(c_1 x) \\ &+ b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y) + b_3 \tanh(c_3 z)].s \end{aligned} \tag{23}$$

Based on the upper bound of uncertainties and disturbances, we have

$$T^\alpha V(x,y,z) \le [a_1 f_1 + a_1 \Delta + a_1 D + a_2 f_2 + a_3 f_3 + p_1 y$$
$$- [a_1 f_1 + a_2 f_2 + a_3 f_3 + p_1 y + b_1 \tanh(c_1 x)$$
$$+ b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)) + b_3 \tanh(c_3 z)] \quad (24)$$
$$- k \tanh(ns)] + b_1 \tanh(c_1 x)$$
$$+ b_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y))$$
$$+ b_3 \tanh(c_3 z)].|s| \le -\lambda |s|$$

Then, we conclude

$$T^\alpha(x,y,z) = (T^\alpha s).s = (a_1 D + a_1 \Delta - |k.a_1.\tanh(s)|).|s| \quad (25)$$
$$\le -\lambda.|s|$$

Simplifying the above equation results in

$$T^\alpha V(x,y,z) = (D + \Delta - |k \tanh(s)|) \le -\frac{\lambda}{a_1} \quad (26)$$

From there, we have

$$D + \Delta + \frac{\lambda}{a_1} \le |k \tanh(s)| \le k \quad (27)$$

Therefore, based on Theorem 3, if $D + \Delta + \dfrac{\lambda}{a_1} \le k$, then the

sliding mode surface $s = 0$ will be reached in the finite time. To calculate the time, according to reaching condition, we have $T^\alpha V = sT^\alpha s < -\eta |s|$. Therefore,

$$\frac{s}{|s|} T^\alpha s < -\eta \quad (28)$$

Hence, we get

$$\mathrm{sgn}(s).T^\alpha s < -\eta \quad (29)$$

$$\mathrm{sgn}(s).T^\alpha s = \begin{cases} T^\alpha s < -\eta, & s > 0 \\ -T^\alpha s < -\eta, & s < 0 \end{cases} \quad (30)$$

Taking conformable fractional order integrator of both sides of the above inequality leads to the below inequality:

$$\begin{cases} {}^{t_r}_0 T^{-\alpha} T^\alpha s < -T^{-\alpha}\eta = \int_0^{t_r} -\frac{\eta}{\tau^{1-\alpha}} d\tau, & s > 0 \\ -{}^{t_r}_0 T^{-\alpha} T^\alpha s < -T^{-\alpha}\eta = \int_0^{t_r} -\frac{\eta}{\tau^{1-\alpha}} d\tau, & s < 0 \end{cases} \quad (31)$$

Based on definition 3 and Lemma 1, we obtain

$$\begin{cases} [s(t) - s(0)]_0^{t_r} = 0 - 2s(0) < \int_0^{t_r} \frac{\eta}{\tau^{1-\alpha}} d\tau = \frac{\eta}{\alpha}(t_r)^\alpha, s > 0 \\ -[s(t) - s(0)]_0^{t_r} = 2s(0) - 0 < \int_0^{t_r} \frac{\eta}{\tau^{1-\alpha}} d\tau = \frac{\eta}{\alpha}(t_r)^\alpha, s < 0 \end{cases} \quad (32)$$

Therefore, the reaching time t_r is calculated as

$$\begin{cases} 0 - 2s(0) \le -\frac{\eta}{\alpha} t_r{}^\alpha, & t_r \le (\frac{2\alpha s(0)}{\eta})^{\frac{1}{\alpha}} & s > 0 \\ 2s(0) - 0 < \frac{\eta}{\alpha} t_r{}^\alpha, & t_r < (-\frac{2\alpha s(0)}{\eta})^{\frac{1}{\alpha}} & s < 0 \end{cases} \quad (33)$$

Finally, t_r is $t_r \le \dfrac{|2\alpha s(0)|^{\frac{1}{\alpha}}}{\eta}$.

Theorem 5. The system (15) under the proposed controller (20) is asymptotically stable.

Proof. Consider the candidate Lyapunov function as

$$v(x,y,z) = \frac{1}{2}(a_1 x^2 + \frac{a_2}{Y} Xy^2 + \frac{a_3}{Z} Xz^2) \quad (34)$$

The time derivative of the Lyapunov function considering (15) yields

$$\dot{V}(x,y,z) = a_1 x f_1 + \frac{a_2}{Y}(Xy f_2) + \frac{a_3}{Z}(Xz f_3)$$
$$\le a_1(Xf_1 + Xdf + Xu) + \frac{a_2}{Y}(XYf_2) \quad (35)$$
$$+ \frac{a_3}{Z}(XZf_3) = a_1[Xf_1 + Xdf$$
$$- \frac{1}{a_1}[a_1 Xf_1 + a_2 Xf_2 + a_3 Xf_3 + p_1 Xy$$
$$+ Xb_1 \tanh(c_1 x) + Xb_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y))]]$$
$$+ Xb_3 \tanh(c_3 z)] - Xk \tanh(n.s) + a_2 Xf_2 + a_3 Xf_3$$

So, we have:

$$\dot{v}(x,y,z) = [(a_1 Xdf - p_1 Xy - Xb_1 \tanh(c_1.x)$$
$$- Xb_2 \tanh(c_2 T^{\beta_1} T^{-\beta_2}(y)))] - Xb_3 \tanh(c_3 z)] \quad (36)$$
$$+ Xk \tanh(n.s)$$

From [31], since the chaotic systems are dissipative, so all the states have limited values, that is, $|x| \le X$, $|y| \le Y$, $|z| \le Z$, where X, Y, Z are the positive constants. Thus, we have

$$\dot{V}(x,y,z) \le X [(a_1 \Delta - [p_1 Y + b_1 + b_2 + b_3] + a_1 k] \quad (37)$$

Finally, for $\dfrac{1}{a_1}[(a_1 \Delta - [p_1 Y + b_1 + b_2 + b_3] \le k$, the

controlled system (15) is asymptotically stable.

Remark 1. The main results can be adopted for the integer case. Indeed, the proofs of Theorems 3-5 can be derived again for the integer case considering $\alpha = 1$.

IV. SIMULATION RESULTES

Here, three numerical simulations are illustrated to show the effectiveness of the proposed control scheme, including the fractional and integer order systems adopted from the literature.

Example 1. Consider the conformable FO Lorenz chaotic system as follows [25]:

$$T^{\alpha}x = -a(x-y) + \Delta f + d + u \qquad (38)$$
$$T^{\alpha}y = rx - y - xz$$
$$T^{\alpha}z = -bz + xy$$

where $a = 10$, $r = 28$, $b = \dfrac{8}{3}$ with $\alpha = 0.98$, and $|\Delta f| = |0.1 - 0.1\sin(\pi x)| \le 0.2$, $|d| = |0.1\cos(t)| \le 0.1$. The initial values are $x(0) = 1$, $y(0) = 1$, $z(0) = 1$. The control parameters are designed as follows:

$$a_1 = 1.5, \quad a_2 = 1.2, \quad a_3 = 0.5, \quad p_1 = 1.5, \quad p_2 = 0.7,$$
$$b_1 = 4 \quad b_2 = 8, \quad b_3 = 0.7 \quad c_1 = 30, \quad c_2 = 50, \quad c_3 = 50,$$
$$k = 1.2, \quad \beta_1 = 0.97, \quad \beta_2 = 0.8$$

Figs. 1 and 2 illustrate the results of applying the proposed controller in comparison with the designed controller adopted from [10]. Fig. 1 shows the state trajectories of the controlled system and the ability of the proposed controller in improving the convergence rate in comparing with [10]. Fig. 2 demonstrates the control inputs for both controllers. From Fig. 2, we can conclude that the proposed controller can reduce chattering phenomenon and control effort.
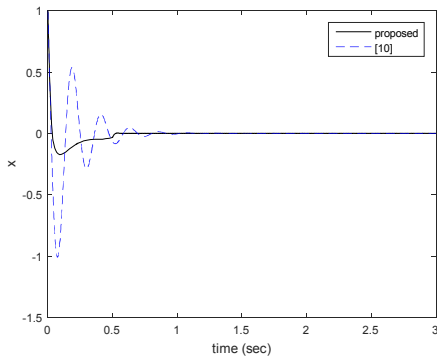
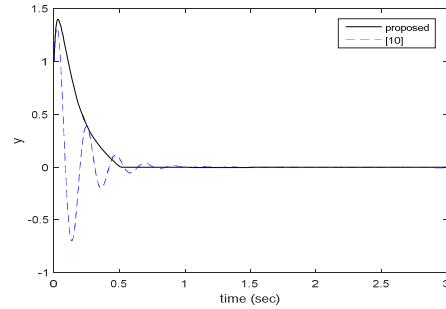

Fig. 1(a).State trajectory of x
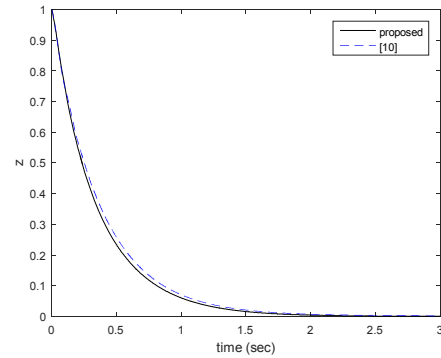

Fig. 1(b).State trajectory of y

Fig. 1 (c) .State trajectory of z

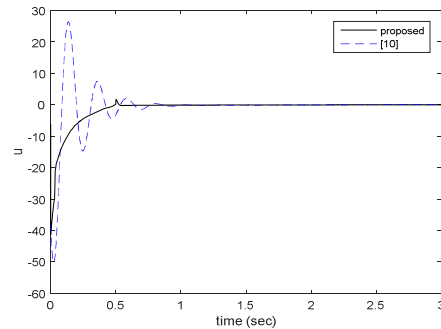Fig. 1. State trajectories for example 1

Fig. 2. Control effort for example 1

For comparison, the designed controller with the following sliding surface is adopted from [11].

$$s(t) = 1.5x(t) + 1.2y(t) + 0.5z(t) \qquad (39)$$

The state trajectories of controlled system with both controllers are illustrated in Fig. 3. The control input is illustrated in Fig. 4. From the results, chattering suppression and fast convergence rate are the superiorities of the proposed controller.
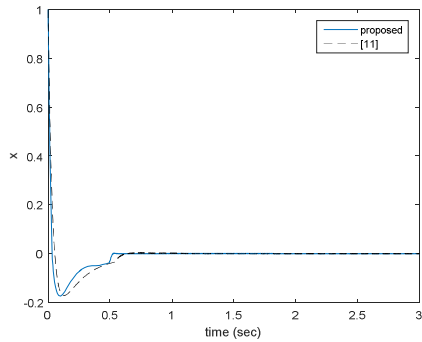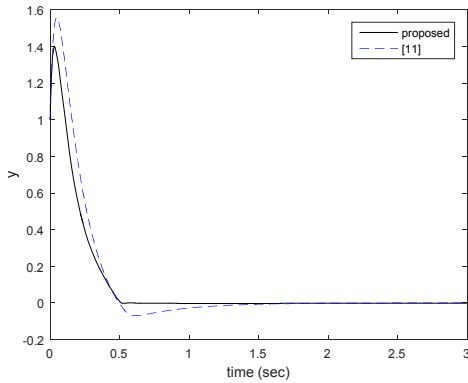
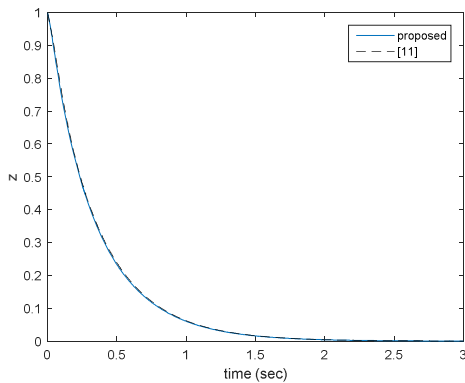Fig. 3(a).State trajectory of x

Fig. 3(a).State trajectory of y

Fig. 3(b).State trajectory of z
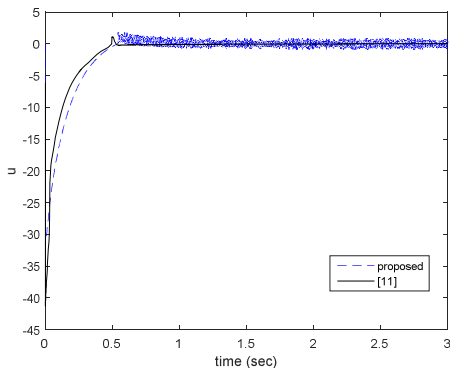Fig. 3. State trajectories for example 1

Fig. 4. Control effort for example 1

Example 2. Consider the following modified conformable FO Lotka-Volterra systems [29]:

$$T^{\alpha_1}x(t) = x(t)(r - ax(t) - by(t))$$
$$T^{\alpha_1}y(t) = y(t)(-d + cx(t)) \qquad (40)$$

where $\alpha_1, \alpha_2 \in (0,1)$.

According to [25], for the below parameters, the system is unstable.

$$r = 1, \quad a = 0, \quad b = 1, \quad c = 4, \, d = 2$$

where $|\Delta f| = |0.1 - 0.1\sin(\pi x)| \le 0.2$,

$|d| = |0.1\cos(t)| \le 0.1$ .Considering initial conditions

$x(0) = 0.2, \quad y(0) = 0.8$, numerical simulations reveal that the proposed controller with the following control parameters can stabilize the system as shown in Figs. 5 and 6.

$$a_1 = 7, \quad a_2 = 1.2, \quad p_1 = 1, \quad b_1 = 0.8075, \quad b_2 = 0.935, \quad c_1 = 40,$$
$$c_2 = 100, \quad k = 1.2, \quad \beta_1 = .97, \quad \beta_2 = 0.8$$

Figs. 5 and 6 represent the results applying the proposed controller in comparison with the designed controller adopted from [11] with the sliding surface as

$$s(t) = 7x(t) + 2.2y(t) + 3.5y(t) \qquad (41)$$

Fig. 5 shows the state trajectories of the both controllers and their control inputs are provided in Fig. 6. From the results, it can be deduced that the proposed controller has outstanding performance in terms of convergence speed.
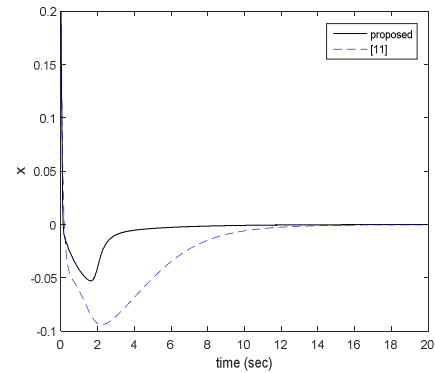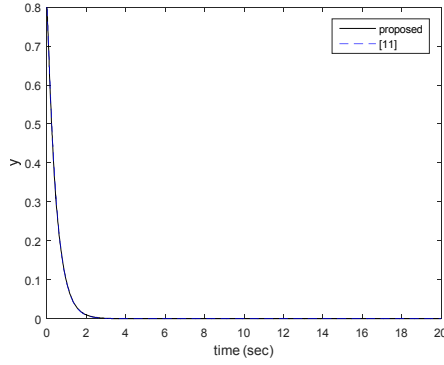

Fig. 5(a).State trajectory of x

Fig. 5 (b). State trajectory of y
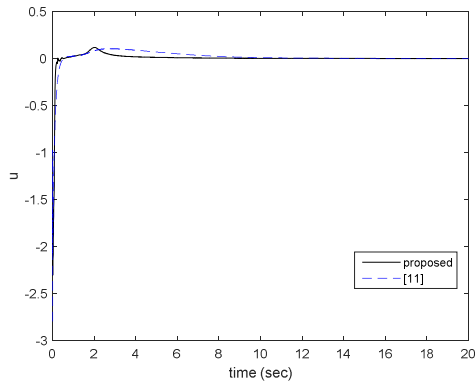
Fig. 5. State trajectories for example 2

Fig. 6. Control effort for example 2

In the follow up, to show the feasibility of the proposed control strategy shown in Remark 1, the following exampled is considered.

Example 3. Let us consider the following controlled Duffing forced-oscillation system [2].

$$\dot{x}(t) = y(t)$$

$$\dot{y}(t) = -0.3y(t) - x^3(t) + \Delta f(X) + d(t) + u(t) \quad (42)$$

where $\left|\Delta f\right| = \left|0.4\sin(0.1t)\right| \le 0.04$, and the external

disturbance is assumed as $\left|d(t)\right| = \left|0.07 + 0.05\sin(t)\right| \le 0.12$. Parameters of the controller are as follows:

$m = 0.01, \quad a_1 = 5.5, \quad a_2 = 1.2, \quad p_1 = 5.5, \quad b_1 = 1.32, \quad b_2 = 0.75$

, $c_1 = 40, \quad c_2 = 50, \quad \alpha = 0.97, \quad q = 0.75$.

Fig. 7 shows the state trajectories of the system. Also, the time response of the controller is demonstrated in Fig. 8. High convergence speed and elimination of chattering are the main advantages of the proposed controller. Simulation results demonstrate a remarkable increasing convergence speed and the chattering reduction in comparison with the presented controller with the following traditional sliding surface [4].
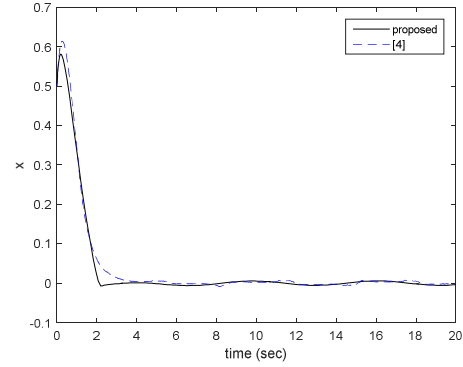
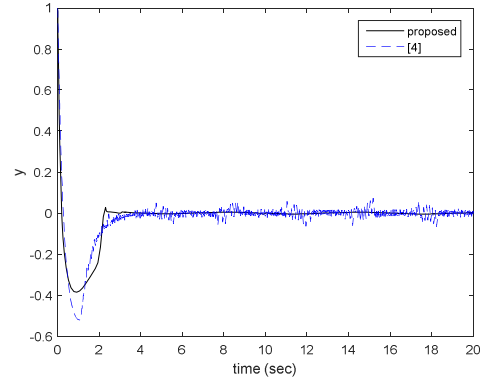$$s(t) = 4x_1(t) + 2.2x_2(t) \quad (43)$$

Fig. 7(a).State trajectory of x

Fig. 7(b).State trajectory of y

Fig. 7. State trajectories (x,y) for example 3
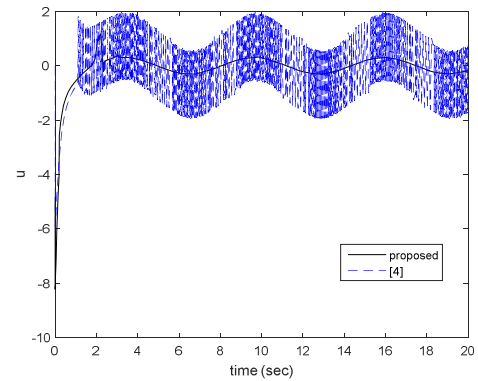

Fig. 8. Control effort for example 3

Example 4. Let us consider the following Gero system [11,32].

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -100\frac{(1 - \cos(x_1))^2}{\sin^3(x_1)} - 0.5x_2 \quad (44)$$

$$- 0.05x_2^3 + \sin(x_1) + 35.5\sin(25t)\sin(x_1)$$

$$+ \Delta f(x,t) + d(x,t) + u(t)$$

where $\Delta f(X,t) + d(t) = 0.25\cos(2\pi t)x_2 + 0.1\sin(2t)$ and

k=0.4 The initial condition of Gyro system is $(x_1(0), x_2(0)) = (1, -1)$.

$m = 0.01, \quad a_1 = 0.7, \quad a_2 = 0.2, \quad p_1 = 35.9, \quad b_1 = 55.38, \quad b_2 = 0.936,$
$c_1 = 11, \quad c_2 = 5, \quad \alpha = 0.97, \quad q = 0.75$.

Fig. 9 shows the state trajectories of the system. Also, the time response of the controller is demonstrated in Fig. 10.
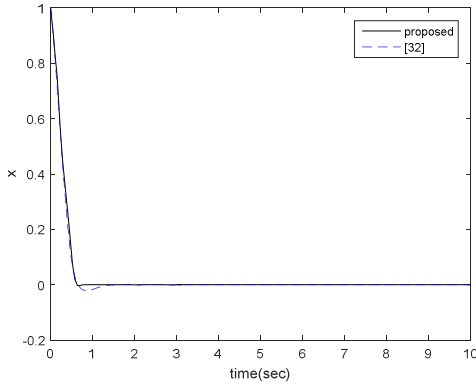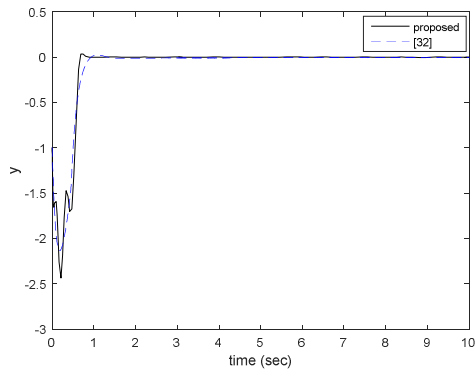

Fig. 9(a).State trajectory of x

Fig. 9(b). State trajectory of y
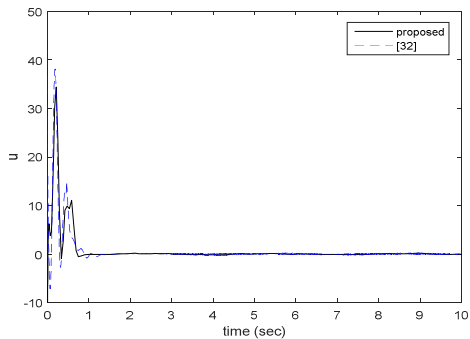Fig. 9. State trajectories (x,y) for example 4

Fig. 10. Control effort for example 4

Simulation results show a remarkable increasing convergence speed in comparison with [32]. Also, the suggested controller can reduce chattering phenomena.

The performance criteria are employed to evaluate the performance of controllers as follows

(a) Integral of the absolute value of the error (IAE)

$$IAE = \int_0^t |e(t)| dt .$$

(b) Integral of the square value (ISV) of the control input

$$ISV = \int_0^t u^2(t) dt$$

The results are shown in table 1.

TABLE I
PERFORMANCE CRITERIA

Systems	Control methods	IAE_x	IAE_y	IAE_z	ISV
CFO Lorem system	The proposed method	0.06395	0.1655	0.3495	68.55
CFO Lorem system	[10]	0.1936	0.1669	0.3734	181.4
CFO Lorem system	[11]	0.06826	0.3707	0.3545	77.29
CFO Lotka-Volterra systems	The proposed method	0.1246	0.38	---	0.1444
CFO Lotka-Volterra systems	[11]	0.4808	0.3816	---	0.2706
Duffing forced-oscillation system	The proposed method	0.718	0.6973	---	9.306
Duffing forced-oscillation system	[4]	0.7704	0.8386	---	48.02
Gero system.	The proposed method	0.2809	1.01	—	140.8
Gero system.	[33]	0.2813	1.08	—	152.8

By evaluating the performance criteria, the proposed control is superior to the other methods.

V. CONCLUSIONS

In this paper, a novel conformable FO nonlinear sliding surface was proposed for a class of FO chaotic systems. The proposed control approach has some superiorities, including low chattering and fast convergence speed. Moreover, it has simple calculations because of using conformable FO operators in the control design. Stability of conformable FO controlled system was guaranteed using the Lyapunov direct method based on conformable FO operators. Simulation results verified the feasibility of the proposed control method.

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