Design of Fractional Order Sliding Mode Controller for a class of nonlinear systems

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Abstract
In this article, a novel nonlinear sliding mode controller is proposed to control a class of nonlinear systems. The proposed control scheme is based on conformable fractional order operators. The stability analysis is performed using Lyapunov direct method. Simulation results show high convergence speed, chattering reduction and small control effort.
Keywords: Fractional order calculus, Chaotic system, Conformable fractional order derivative, Sliding mode control.

1 Introduction

Chaos control has been attracting much attention of researchers in the control field. So far, various methods have been presented for the control of chaos [1-2]. Sliding mode control (SMC) is an effective strategy to suppress chaotic behavior for uncertain systems with disturbance. On the other hand, using fractional order (FO) calculus in the design of controller can improve the control performance. However, FO operators have complex calculation. Recently, conformable fractional order (CFO) derivative is defined [3]. It has simple calculation. So, in this paper a conformable sliding mode controller is proposed to control uncertain chaotic systems with disturbances. In the simulation results, Duffing system is selected. It is a nonlinear vibration system which is the basis of many complex dynamics. So far several methods are proposed to control Duffing system [4-5]. The numerical simulations show efficiency of the method in the chaos control.

2 Mathematical Preliminaries

In this section, some necessary definitions are presented.

Definition 1 [3]. For $f : [0,\infty) \rightarrow R$, the ‘‘CFO derivative’’ is defined as

$$T^{\alpha}(f)(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon} \tag{1}$$

Definition 2 [3]. The CFO integral is defined as:

$$T_{\alpha}^{-a} f(t)=\int_{a}^{t}\frac{f(t)}{t^{1-\alpha}}dt \tag{2}$$

Remark 1. In this work, the notation T^{β} indicates the conformable derivative.

3 Designing Fractional-Order Sliding Mode Controller

Consider the following class of the dynamical systems in the presence of uncertainty and disturbances as follows:

$$\begin{aligned}\dot{x}_1(t)&=x_2(t)\\ \dot{x}_2(t)&=f(t)+\Delta f(X)+d(t)+u(t)\end{aligned} \tag{3}$$

Where $|d(t)|\le D$ and $|\Delta f(X)|\le\Delta$ are external disturbances and system uncertainties respectively.

The novel proposed FO sliding-mode surface is proposed as follows:

$$s(t)=m\,x_2(t)+a_1\int[\tanh(x_1(t))+a_2\tanh(x_2(t))+b_1\tanh(c_1T^{-\alpha}.x_1(t))+b_2\tanh(c_2.T^{-\alpha}x_2(t))-zx_1] \tag{4}$$

Where $\alpha\in(0,1)$ and $q\in(0,1)$.
In order to obtain the equivalent control law, we have:

$$\begin{aligned}\dot{s}(t)=m\,\dot{x}_2(t)+a_1\tanh(x_1(t))+a_2\tanh(x_2(t))+b_1\tanh(c_1.T^{-\alpha}x_1(t))+b_2\tanh(c_2T^{-\alpha}.x_2(t))\\ -zx_1=0\end{aligned}. \tag{5}$$

Substituting $\dot{x}_2$ from Eq. (3) in the above equation leads to:

$$\begin{aligned}\dot{s}(t)=m(f(t)+u(t))+a_1\tanh(x_1(t))+a_2\tanh(x_2(t))+b_1\tanh(c_1.T^{-\alpha}x_1(t))\\ +b_2\tanh(c_2T^{-\alpha}.x_2(t))-zx_1)=0\end{aligned} \tag{6}$$

The equivalent control is as follows:

$$\begin{aligned}u(t)=-\frac{1}{m}(mf(t)+a_1\tanh(x_1)+a_2\tanh(x_2)+b_1\tanh(c_1.T^{-\alpha}x_1(t))+b_2\tanh(c_2.T^{-\alpha}x_2(t))\\ -zx_1)\end{aligned} \tag{7}$$

The switching law is introduced as:

$$u_r=-K\tanh(s(t)/h) \tag{8}$$

Where, h is a positive constant. Finally the control law is as:

$$u(t) = u_{eq}(t) + u_r(t) \tag{9}$$

4 Simulation Results

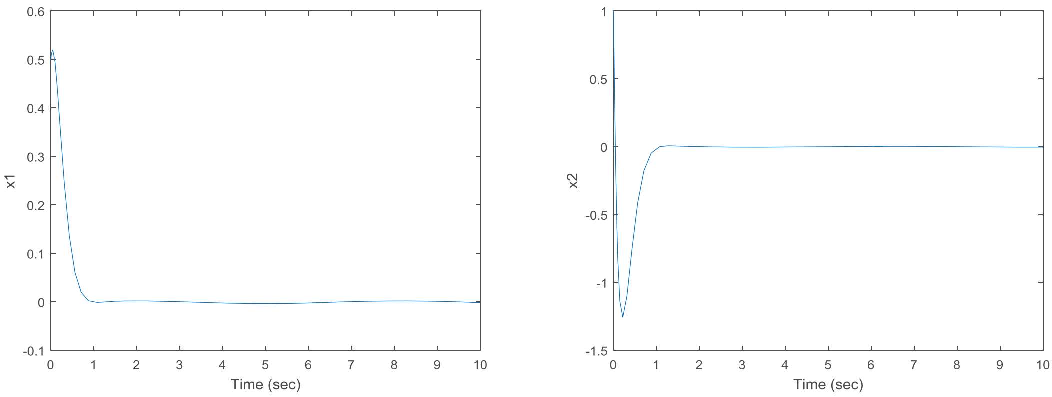
In this section, numerical simulations confirm the performance of the designed scheme.

Consider the following Duffing forced-oscillation system [5].

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.3x_2 - x_1^3 + \Delta f(X) + d(t) + u(t) \end{aligned} \tag{10}$$

Where $\Delta f = 0.04\sin(0.1t)$, the external disturbance is assumed as $d(t) = 0.07 + 0.05\sin(t)$. Parameters of the controller are as follows:
$m = 0.01,\quad a_1 = 0.35,\quad a_2 = 0.1,\quad z = 0.1,\quad b_1 = 1, b_2 = 1\quad, c_1 = 50,\quad c_2 = 50,\quad \alpha = 0.97,\quad q = 0.75$

Fig. 1 shows the state trajectories of Duffing forced-oscillation system. Also, the time response of the controller is demonstrated in fig. 2. The simulation results demonstrate a high speed of trajectory states convergence.



a) State trajectory of x_1 b) State trajectory of x_2

Fig 3. State trajectories

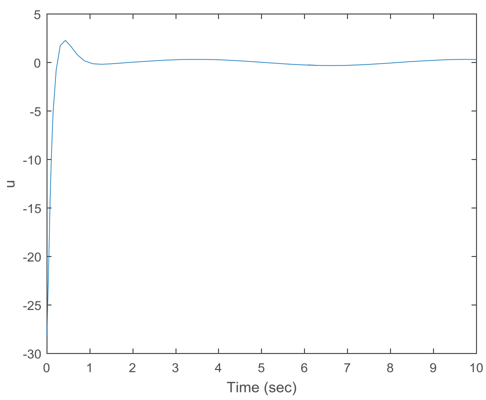


Fig.4. Control effort

5 Conclusion

In this paper, an application of CFO derivative as a simple FO derivative in the design of a FO controller is introduced. A novel nonlinear FO SMC based on CFO derivative to suppress the chaotic behavior of a class of integer-order nonlinear uncertain chaotic system in the presence of disturbance is proposed. The finite-time stability of the proposed controller is proved using the classic Lyapunov theory. To verify the effectiveness of the proposed control scheme, it is applied to Duffing forced-oscillation system with uncertainties and external disturbances. Small control effort, high convergence speed and chattering reduction are the advantages of the proposed controller.

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